1 Orienting Yourself: The Use of Coordinates

1.1 Introduction

A system of coordinates is a structured framework (like the grid lines on a map or graph paper) that enables us to use numbers to describe the exact physical locations of points or objects.

The idea of 'grid-based thinking' and the geometry required to define the locations of points in space—indeed has deep roots in Bhārat. The first systematic use of grids occurred thousands of years ago — on a massive urban scale—in the Sindhu-Sarasvatī Civilisation, where city streets were constructed with striking precision in North–South and East–West directions at uniform distances of about 10 metres apart. This was a coordinate system in practice: a merchant could find a shop or a warehouse by counting North–South and East–West units of distance from the city centre. Baudhāyana (c. 800 C.E.), as we have seen, later used East–West and North–South lines for his deep geometric constructions, developing the Baudhāyana–Pythagoras Theorem and thus laying the foundation of coordinate geometry.

Putting coordinates on the Earth's surface later became important for navigation. Ujjayinī was described in the ancient world—at least as early as the 4th century BCE in the early Siddhāntas—as the point marking the central longitude meridian from which all other locations were measured. The Greek mathematician Ptolemy (c. 150 BCE), building on earlier works including that of Hipparchus, later described the latitudes and longitudes of thousands of locations, including 'Ozine' (Ujjayinī). Āryabhaṭa (c. 499 CE) replaced the Greek 'chords' with 'sines', making it much easier to calculate the coordinates of a star or a city. He mapped the sky using Celestial Coordinates, measuring coordinate distances from the ecliptic (the path of the sun).

Brahmagupta (c. 628 CE) formalised the notion and use of zero and the negative numbers as algebraic entities; in modern coordinate systems, the 'origin' is zero and the 'negative axes' represent values less than zero. Without Brahmagupta's work, the four-quadrant Cartesian plane, as we will study in this chapter, would be impossible.

Brahmagupta's work was translated into Arabic (as the Sindhind), and the Ujjayinī meridian entered Arabic geography under the name 'Arin,' serving as the zero-longitude reference for early Arabic maps which also then made use of negative numbers. The influential Arab scholar Al-Bīrūnī (c. 1000 CE) travelled to India, studied the Siddhāntas, and used Indian trigonometric methods to calculate the coordinates of various cities across Asia. Al-Bīrūnī also later perfected the 'astrolabe', a handheld device that allowed sailors to find their coordinates by looking at the stars. Ömar Khayyām (c. 1100 CE), who had become an expert in the Indian decimal system and algebraic formalism, was the first mathematician to solve algebraic problems using geometry by interpreting them in terms of coordinates in the plane.

These concepts eventually reached Europe in the 12th century. The final leap occurred when following the related work of Fermat (1636 CE), René Descartes (1637 CE) formalised the fact that any point in a two-dimensional plane could be defined by simply two numbers—representing the point's distances from two perpendicular axes. Points and more complex geometric shapes could then be described precisely using algebra and equations, thus bringing the areas of geometry and algebra even closer together.

In Grades 9 and 10, you will have a chance to study this amazing coordinate system which has such a rich history in human thought and endeavour. You will be able to locate objects with pinpoint accuracy. You will also see how using coordinates enables us to visualise algebraic equations as geometric shapes, and vice versa.

We begin our study of coordinates with a story that will help you understand these new terms better.

1.2 Settling In

It is the beginning of the academic year and Reiaan is both excited and nervous. The family has just moved to a new city. He and his sister, Shalini, will be attending a new school. Today, Shalini will help him settle into the new environment. When someone is not able to see, this can be a very big challenge, but with their mother's transferable job, the siblings have done it often, and it has become easier with each move.

Shalini has just completed Grade 9 and this time, she decided to put to use what she has learnt in Coordinate Geometry in Mathematics to guide Reiaan.

Shalini wanted Reiaan to feel the directions, so she used a rectangular grid on which she had fixed pins and threads. This showed the floor of the room. Points in the sketch were marked using pins. Shalini was using a scale of 1 cm : 1 foot. She used pins to mark out various key points of the room. Points representing the corners of objects were connected with thick wool so that Reiaan could feel their positions with his fingers.

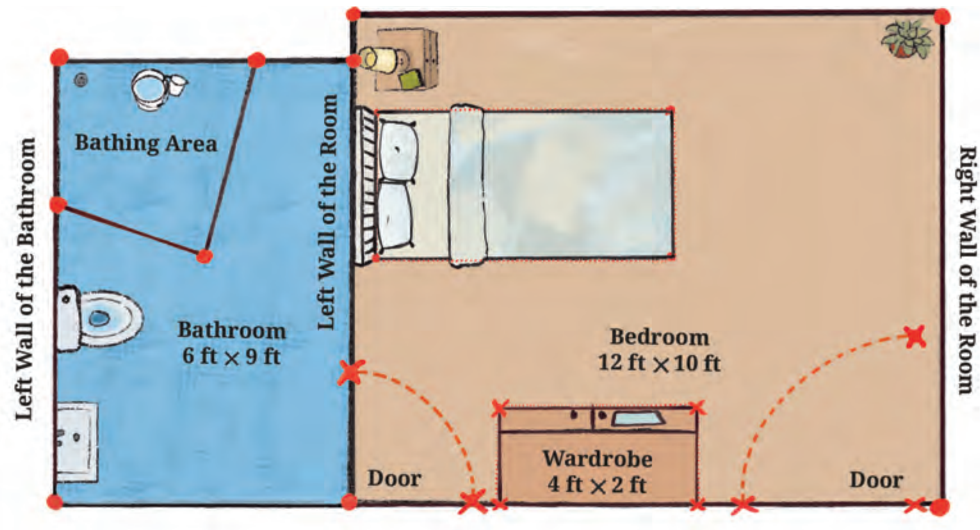

Fig. 1.1: Sketch of Reiaan's room

Let us examine Fig. 1.1 to understand the layout of the room. Notice that this only shows the map of the floor. Do you see why the position of the windows cannot be marked on this map?

1.3 The 2-D Cartesian Coordinate System

In the chapters on integers, rational numbers, and decimals in earlier grades, you studied the number line which is one-dimensional. The two-dimensional coordinate system uses two lines at right angles to each other to mark points in two-dimensional space (short form: 2-D space). For convenience, we consider one of the lines to be horizontal; it is called the x-axis. The other line is vertical; it is called the y-axis. The point of intersection of the x-axis and y-axis is called the origin O; its coordinates are (0, 0). Coordinate axes (this is the plural of 'axis') help us to locate any point in 2-D space using the point's 'coordinates'. Distances from O are marked off in equal units, on both the axes. Distances to the right of O or upwards from O are considered positive, and distances to the left of O or downwards from O are considered negative (Fig. 1.2).

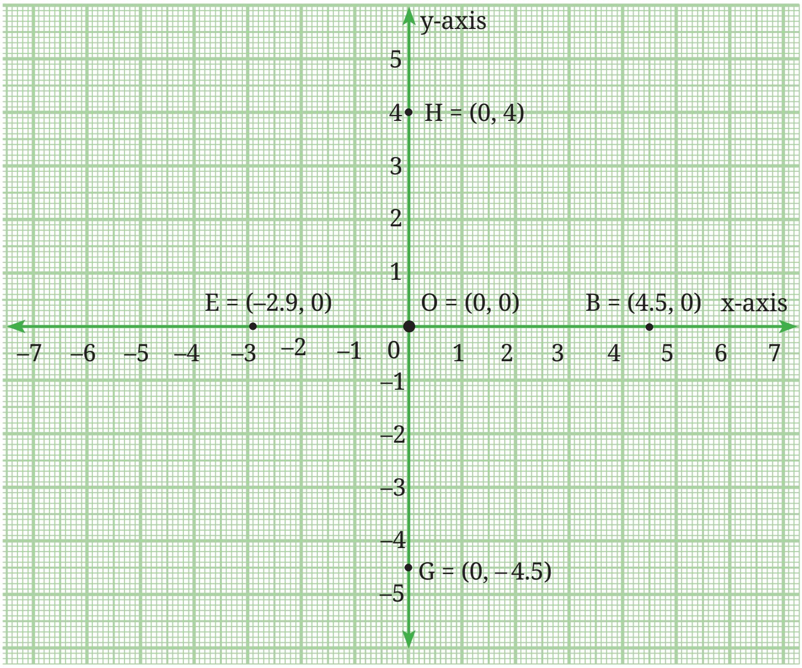

Fig. 1.2: Structure of the coordinate plane

Point B is on the x-axis and 4.5 units to the right of O, its coordinates are (4.5, 0). This fact is written as B = (4.5, 0). Point G is on the y-axis and 4.5 units downward from O, its coordinates are (0, –4.5), that is, G = (0, –4.5). Point H is also on the y-axis but 4 units above O, its coordinates are (0, 4), that is, H = (0, 4).

A point P = (x, 0) lies on the x-axis. If x is positive, then P lies to the right of O. If x is negative, P lies to the left of O. A point P = (0, y) lies on the y-axis. If y is positive, P lies above O; if y is negative, P lies below O.

While writing the coordinates of a point, it is often convenient to drop the '=' sign and write P = (x, y) simply as P (x, y). This is especially true while marking points on a graph.

Exercise Set 1.1

Fig. 1.3 shows Reiaan's room with points OABC marking its corners. The x- and y-axes are marked in the figure. Point O is the origin.

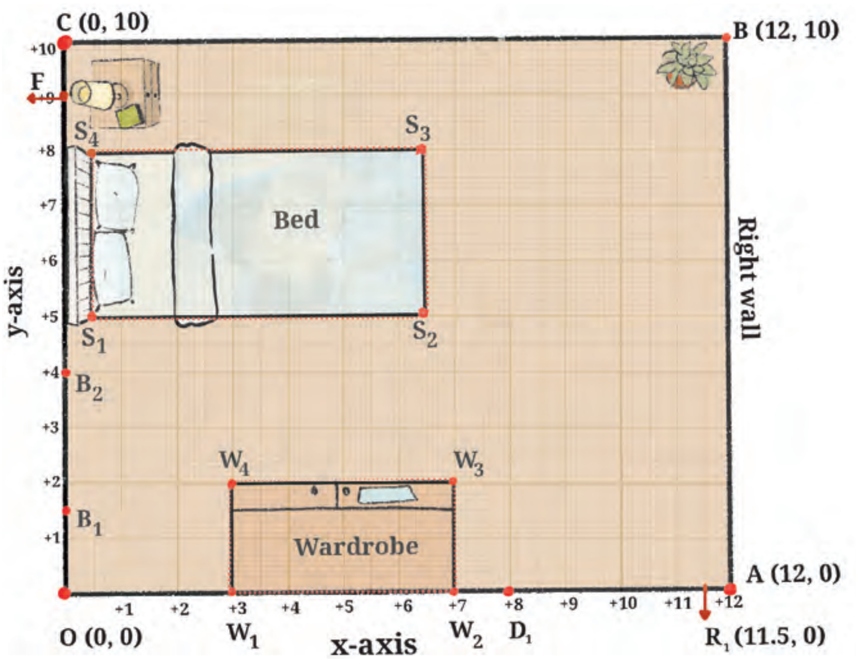

Fig. 1.3

Referring to Fig. 1.3, answer the following questions:

(i) If D_1R_1 represents the door to Reiaan's room, how far is the door from the left wall (the y-axis) of the room? How far is the door from the x-axis?

(ii) What are the coordinates of D_1?

(iii) If R_1 is the point (11.5, 0), how wide is the door? Do you think this is a comfortable width for the room door? If a person in a wheelchair wants to enter the room, will he/she be able to do so easily?

(iv) If B_1 (0, 1.5) and B_2 (0, 4) represent the ends of the bathroom door, is the bathroom door narrower or wider than the room door?

Think and Reflect

1. What are the standard widths for a room door? Look around your home and in school.
2. Are the doors in your school suitable for people in wheelchairs?

So far, we have only considered points on the two coordinate axes. What can you say about the coordinates of points that are not on either axes? The plane in which the axes are situated is called the **Cartesian plane**, the **coordinate plane** or the **xy-plane**. The axes divide the plane into four parts, called **quadrants**. They are numbered as shown in Fig. 1.4.

Using the conventions that we have stated earlier:

(i) Points in Quadrant I have both x- and y-coordinates positive.

(ii) Points in Quadrant II have negative x-coordinate and positive y-coordinate.

(iii) Points in Quadrant III have both x- and y-coordinates negative.

(iv) Points in Quadrant IV have positive x-coordinate and negative y-coordinate.

Can you now see the meaning of the coordinates of a point in 2-D space? In general, the coordinates of a point P in 2-D space are represented by (x, y). Here, x represents the perpendicular distance of P from the y-axis, measured along the x-axis, and y is the perpendicular distance of P from the x-axis, measured along the y-axis. x is the x-coordinate and y is the y-coordinate of the point (x, y).

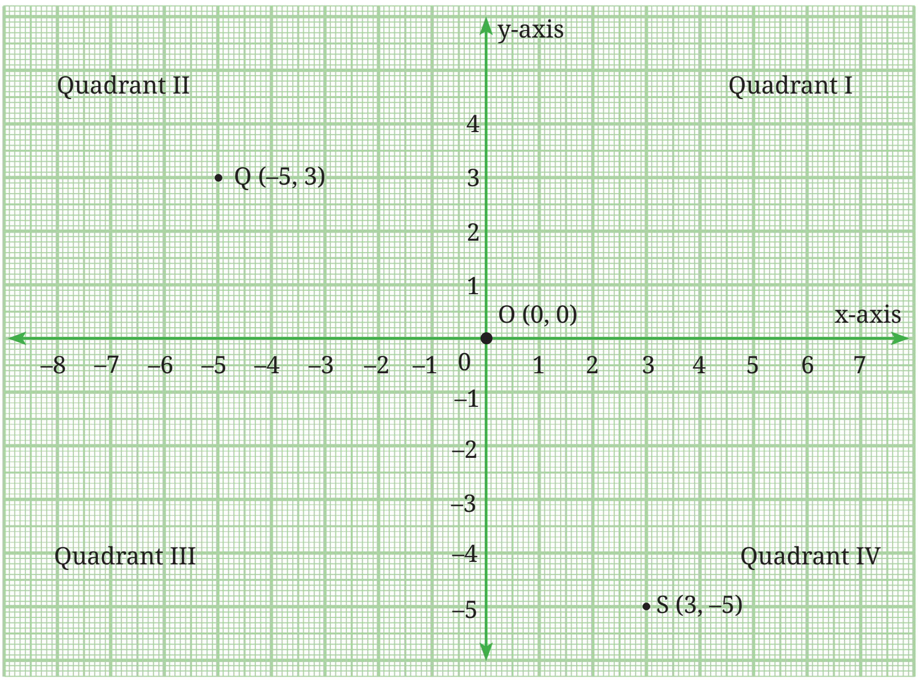

Fig. 1.4

For example, in Fig. 1.4, point S (3, –5) is in Quadrant IV, the x-coordinate is 3 units and the y-coordinate is –5 units. What about point Q (–5, 3)? It is in the second quadrant, with x-coordinate –5 and y-coordinate 3. Copy Fig. 1.4 and mark S and Q in your diagram. Mark any point P in Quadrant I and any point R in Quadrant III, and write down their coordinates.

Think and Reflect

1. What is the x-coordinate of a point on the y-axis?
2. Is there a similar generalisation for a point on the x-axis?
3. Does point Q (*y*, *x*) ever coincide with point P (*x*, *y*)? Justify your answer.
4. If $x \neq y$, then $(x, y) \neq (y, x)$; and $(x, y) = (y, x)$ if and only if $x = y$. Is this claim true?

Exercise Set 1.2

On a graph sheet, mark the x-axis and y-axis and the origin O. Mark points from (–7, 0) to (13, 0) on the x-axis and from (0, –15) to (0, 12) on the y-axis. (Use the scale 1 cm = 1 unit.) Using Fig. 1.5, answer the given questions.

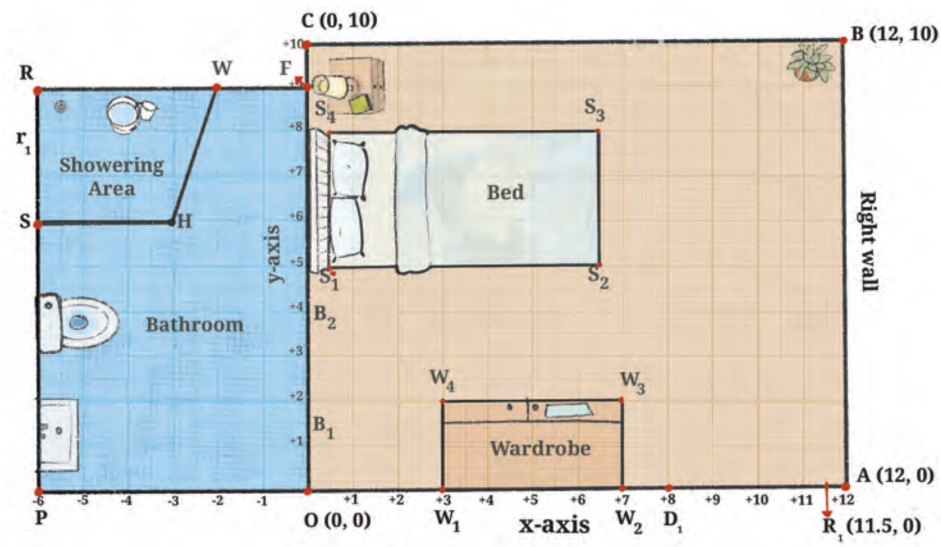

Fig. 1.5

1. Place Reiaan's rectangular study table with three of its feet at the points (8, 9), (11, 9) and (11, 7).
   - (i) Where will the fourth foot of the table be?
   - (ii) Is this a good spot for the table?
   - (iii) What is the width of the table? The length? Can you make out the height of the table?
2. If the bathroom door has a hinge at B_1 and opens into the bedroom, will it hit the wardrobe? Are there any changes you would suggest if the door is made wider?
3. Look at Reiaan's bathroom.
   - (i) What are the coordinates of the four corners O, F, R, and P of the bathroom?
   - (ii) What is the shape of the showering area SHWR in Reiaan's bathroom? Write the coordinates of the four corners.
   - (iii) Mark off a 3 ft × 2 ft space for the washbasin and a 2 ft × 3 ft space for the toilet. Write the coordinates of the corners of these spaces.
4. Other rooms in the house:
   - (i) Reiaan's room door leads from the dining room which has the length 18 ft and width 15 ft. The length of the dining room extends from point P to point A. Sketch the dining room and mark the coordinates of its corners.
   - (ii) Place a rectangular 5 ft × 3 ft dining table precisely in the centre of the dining room. Write down the coordinates of the feet of the table.

1.4 Distance Between Two Points in the 2-D Plane

We know how to find the distance between two points if they are on the axes or if they form a line segment parallel to the axes. For example, in Fig. 1.5 we can find the distances W_1W_2 and S_1S_2. What should we do if the segment joining the points is not parallel to either axis? We can use the Baudhāyana–Pythagoras theorem which you have studied in Grade 8. We can use this result to find the distance between any two points in the xy-plane.

Look at triangle ADM in Fig. 1.6.

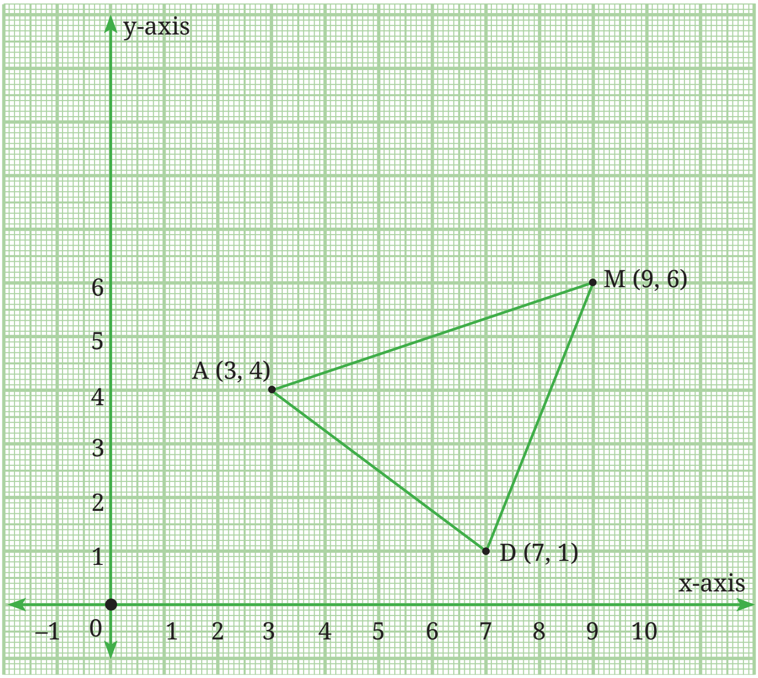

Fig. 1.6

Triangle ADM is an acute angled triangle in the first quadrant. How do we find the lengths of its sides AD, DM and MA?

Think and Reflect

1. In moving from A (3, 4) to D (7, 1), what distance has been covered along the x-axis? What about the distance along the y-axis?
2. Can these distances help you find the distance AD?

Fig. 1.7 gives us a clue. The distance moved along the x-axis is given by CD.

CD = x-coordinate of D – x-coordinate of A = 7 – 3 = 4.

The distance moved along the y-axis is given by AC.

AC = y-coordinate of A – y-coordinate of D = 4 – 1 = 3.

Using the Baudhāyana–Pythagoras Theorem, we get the distance

$$AD = \sqrt{4^2 + 3^2} = 5 \text{ units.}$$

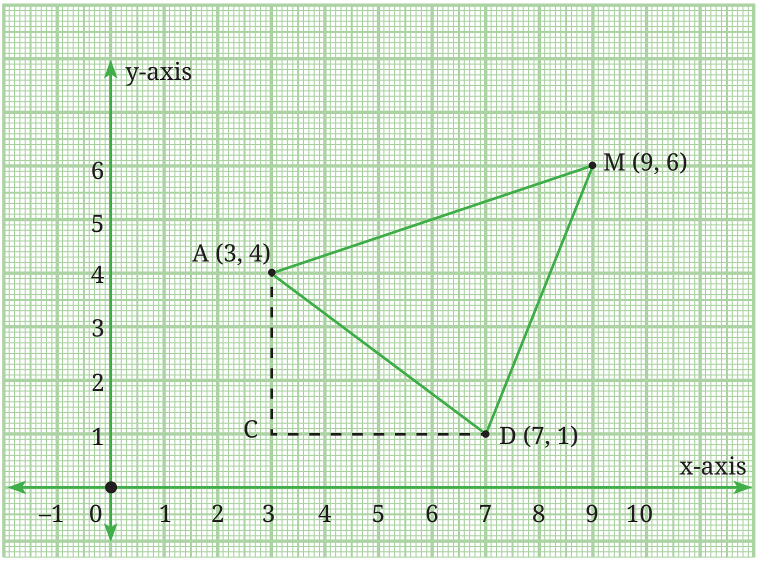

Fig. 1.7

We similarly find the distances DM and MA:

$$DM = \sqrt{2^2 + 5^2} = \sqrt{29} \text{ units.}$$

$$MA = \sqrt{6^2 + 2^2} = \sqrt{40} \text{ units.}$$

In general, the distance between the points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and is calculated as shown in Fig. 1.8.

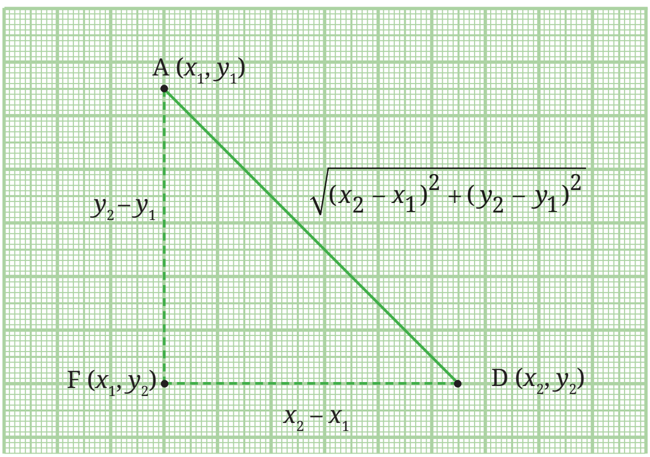

Fig. 1.8

It makes no difference whether $(x_2 - x_1)$ and $(y_2 - y_1)$ are positive or negative, as we are simply measuring the shifts along the two axes.

What if, x_1, x_2, y_1, y_2 take negative values? In Fig. 1.9, triangle AMD is reflected in the y-axis. What are the coordinates of the images of points A, M, and D?

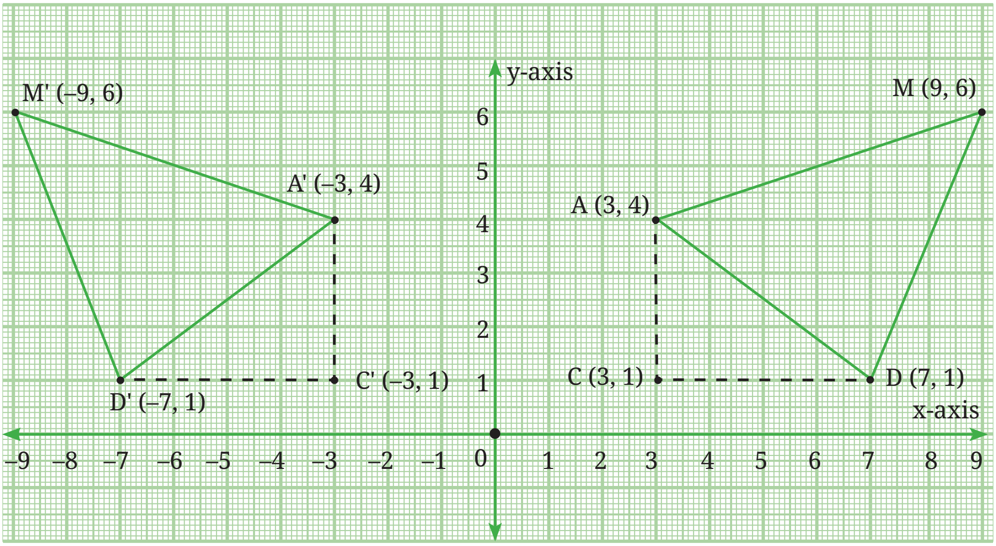

Fig. 1.9

C'D' = x-coordinate of A' – x-coordinate of D' = –3 – (–7) = 4.

A'C' = y-coordinate of A' – y-coordinate of D' = 4 – 1 = 3.

Using the Baudhāyana–Pythagoras Theorem, we get AD = $\sqrt{4^2 + 3^2} = 5$ units.

You can similarly calculate both D'M' and M'A':

$$D'M' = \sqrt{(-2)^2 + 5^2} = \sqrt{29} \text{ units}$$

$$M'A' = \sqrt{(-6)^2 + 2^2} = \sqrt{40} \text{ units.}$$

We see that reflection has preserved the lengths of the sides of the triangles.

Think and Reflect

1. What has remained the same and what has changed with this reflection?
2. Would these observations be the same if ΔADM is reflected in the x-axis (instead of the y-axis)?

End-of-Chapter Exercises

1. What are the x-coordinate and y-coordinate of the point of intersection of the two axes?

2. Point W has x-coordinate equal to -5. Can you predict the coordinates of point H which is on the line through W parallel to the y-axis? Which quadrants can H lie in?

3. Consider the points R (3, 0), A (0, -2), M (-5, -2) and P (-5, 2). If they are joined in the same order, predict:
   (i) Two sides of RAMP that are perpendicular to each other.
   (ii) One side of RAMP that is parallel to one of the axes.
   (iii) Two points that are mirror images of each other in one axis. Which axis will this be?

   Now plot the points and verify your predictions.

4. Plot point Z (5, -6) on the Cartesian plane. Construct a right-angled triangle IZN and find the lengths of the three sides.

   (**Comment:** Answers may differ from person to person.)

5. What would a system of coordinates be like if we did not have negative numbers? Would this system allow us to locate all the points on a 2-D plane?

*6. Are the points M (-3, -4), A (0, 0) and G (6, 8) on the same straight line? Suggest a method to check this without plotting and joining the points.

*7. Use your method (from Problem 6) to check if the points R (-5, -1), B (-2, -5) and C (4, -12) are on the same straight line. Now plot both sets of points and check your answers.

*8. Using the origin as one vertex, plot the vertices of:
   (i) A right-angled isosceles triangle.
   (ii) An isosceles triangle with one vertex in Quadrant III and the other in Quadrant IV.

*9. The following table shows the coordinates of points S, M and T. In each case, state whether M is the midpoint of segment ST. Justify your answer.

S	M	T	Is M the midpoint of ST? Yes or No	Reason for your answer
(–3, 0)	(0, 0)	(3, 0)		
(2, 3)	(3, 4)	(4, 5)		
(0, 0)	(0, 5)	(0, –10)		
(–8, 7)	(0, –2)	(6, –3)		

When M is the mid-point of ST, can you find any connection between the coordinates of M, S and T?

*10. Use the connection you found to find the coordinates of B given that M (–7, 1) is the midpoint of A (3, – 4) and B (x, y).

*11. Let P, Q be points of trisection of AB, with P closer to A, and Q closer to B. Using your knowledge of how to find the coordinates of the midpoint of a segment, how would you find the coordinates of P and Q? Do this for the case when the points are A (4, 7) and B (16, –2).

*12. (i) Given the points A (1, – 8), B (– 4, 7) and C (–7, – 4), show that they lie on a circle K whose center is the origin O (0, 0). What is the radius of circle K?

(ii) Given the points D (–5, 6) and E (0, 9), check whether D and E lie within the circle, on the circle, or outside the circle K.

*13. The midpoints of the sides of triangle ABC are the points D, E, and F. Given that the coordinates of D, E, and F are (5, 1), (6, 5), and (0, 3), respectively, find the coordinates of A, B and C.

14. A city has two main roads which cross each other at the centre of the city. These two roads are along the North–South (N–S) direction and East–West (E–W) direction. All the other streets of the city run parallel to these roads and are 200 m apart. There are 10 streets in each direction.

(i) Using 1 cm = 200 m, draw a model of the city in your notebook. Represent the roads/streets by single lines.

(ii) There are street intersections in the model. Each street intersection is formed by two streets — one running in the N–S direction and another in the E–W direction. Each street intersection is referred to in the following manner: If the second street running in the N–S direction and 5th street in the E–W direction meet at some crossing, then we call this street intersection (2, 5). Using this convention, find:

(a) how many street intersections can be referred to as (4, 3).

(b) how many street intersections can be referred to as (3, 4).

15. A computer graphics program displays images on a rectangular screen whose coordinate system has the origin at the bottom-left corner. The screen is 800 pixels wide and 600 pixels high. A circular icon of radius 80 pixels is drawn with its centre at the point A (100, 150). Another circular icon of radius 100 pixels is drawn with its centre at the point B (250, 230). Determine:

(i) whether any part of either circle lies outside the screen.

(ii) whether the two circles intersect each other.

16. Plot the points A (2, 1), B (–1, 2), C (–2, –1), and D (1, –2) in the coordinate plane. Is ABCD a square? Can you explain why? What is the area of this square?

Chapter Summary

- To locate the position of an object or a point in a plane, we require two perpendicular lines — One of them is horizontal, and the other is vertical.
- The plane is called the **cartesian plane**, the **coordinate plane** or the **xy-plane** and the lines are called the coordinate axes.
- The horizontal line is called the **x-axis** and the vertical line is called the **y-axis**.
- The coordinate axes divide the plane into four parts called **quadrants**.
- The point of intersection of the axes is called the **origin**.
- The distance of a point from the y-axis is its **x-coordinate** and the distance of the point from the x-axis is its **y-coordinate**. If the x-coordinate of a point is x, and the y-coordinate is y, then (x, y) are called the **coordinates** of the point.

- The coordinates of a point on the x-axis are of the form $(x, 0)$, and those of the points on the y-axis are of the form $(0, y)$.
- The coordinates of the origin are $(0, 0)$.
- The coordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.
- If $x = y$, then $(x, y) = (y, x)$. If $x \neq y$, then $(x, y) \neq (y, x)$.
- The distance between points (x_1, y) and (x_2, y) is the absolute value $|x_2 – x_1|$ of the difference between x_1 and x_2.
- The distance between points (x, y_1) and (x, y_2) is the absolute value $|y_2 – y_1|$ of the difference between y_1 and y_2.
- By the Baudhāyana–Pythagoras Theorem, the distance between points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.