

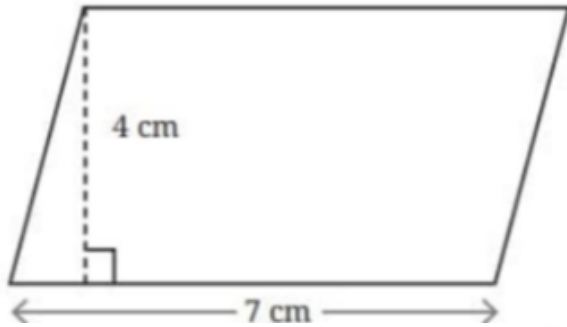
\* Answer The Following Questions In One Sentence.[1 Marks Each]

[15]

1. Can any polygon be divided into triangles?

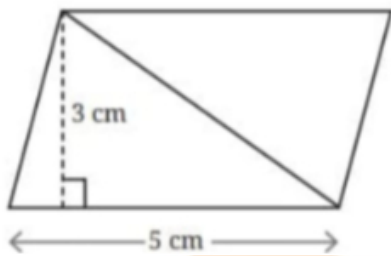
Ans. : self

2. Find the area of the parallelogram:



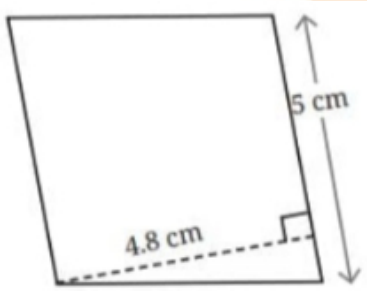
Ans. : self

3. Find the area of the parallelogram:



Ans. : self

4. Find the area of the parallelogram:



Ans. : self

5. Find the area of the parallelogram:



Ans. : self

6. Find the area of a rhombus whose diagonals are 20 cm and 15 cm.

**Ans. :** Given, first diagonal = 20 cm

second diagonal = 15 cm

The area of a rhombus =  $\frac{1}{2} \times$  (Product of diagonals)

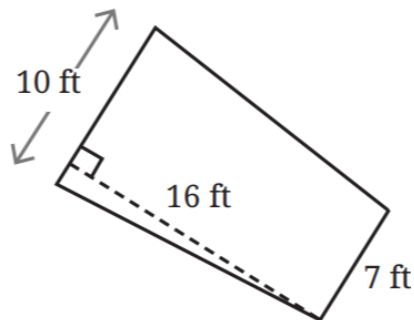
=  $\frac{1}{2} \times$  First diagonal  $\times$  second diagonal

=  $\frac{1}{2} \times 20\text{cm} \times 15\text{cm}$

=  $150\text{cm}^2$

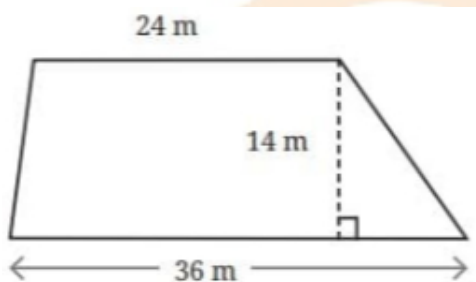
Thus, the area of a rhombus is  $150\text{cm}^2$

7. Find the area of the given figure:



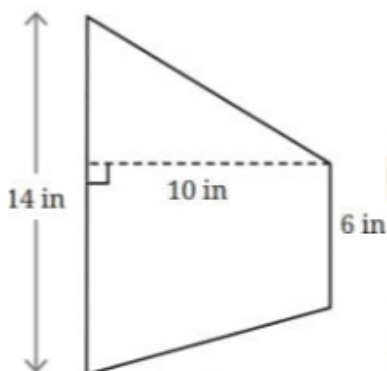
**Ans. :** self

8. Find the area of the given figure:



**Ans. :** self

9. Find the areas of the given figure:



**Ans. :** self

10. What do you think is the area of the tabletop that you use at school or at home?

You could perhaps try to visualise how many A4 sheets can fit on your table.

**Ans. :** self

11. How many  $cm^2$  is  $1in^2$ ?

Ans. : self

12. How many  $cm^2$  is  $10in^2$ ?

Ans. : self

13. Convert  $161.29cm^2$  to  $in^2$ .

Ans. : self

14. What do you think is the area of your school? Make an estimate and compare it with the actual data.

Ans. : self

15. Find out the local unit of area measurement in your region.

Ans. : self

\* Questions With Calculation.[2 Marks Each]

[26]

16. Why do we count the number of unit squares to assign measures for area? Couldn't we have just used the perimeter of a region as a measure of its area?

Ans. : self

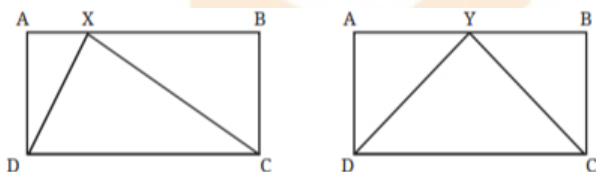
17. If two regions have the same perimeter, can't we conclude that they have the same area? Or, if one region has a larger perimeter than another, can't we conclude that it also has a larger area?

Ans. : self

18. Also, give an example of two regions of other shapes where the region with the larger perimeter has the smaller area! This property should be visually clear in your example.

Ans. : self

19. In the given figure, which triangle has a greater area:  $\triangle XDC$  or  $\triangle YDC$ , if both the rectangles are identical?



Ans. : self

20. Give a method to obtain a rectangle whose area is twice that of a given triangle. What are the different methods that you can think of?

Ans. : Given: Triangle with area A

Required: Rectangle with area = A

Method 1: If the triangle has base b and height h



$$\text{Area of triangle} = \frac{1}{2} \times b \times h = A$$

To get a rectangle with an area of  $2A$ .

Take length =  $b$ , width =  $h$

$$\text{Area of a rectangle} = b \times h$$

$$= 2 \times \left(\frac{1}{2} \times b \times h\right)$$

$$= 2A$$

Steps:

1. Measure the base and height of the given triangle.
2. Construct a rectangle with these measurements as length and width.

Method 2: Scaling method:

1. Take the rectangle.
2. Create a rectangle with base = (base of triangle) and height = height of triangle.
3. This rectangle automatically has twice the area of the rectangle.

21. [Śulba-Sūtras] Give a method to obtain a rectangle of the same area as a given triangle.

**Ans. :** Given: A triangle with base  $b$  and height  $h$ .

Required: Rectangle with the same area

$$\text{Area of triangle} = \frac{1}{2}bh$$

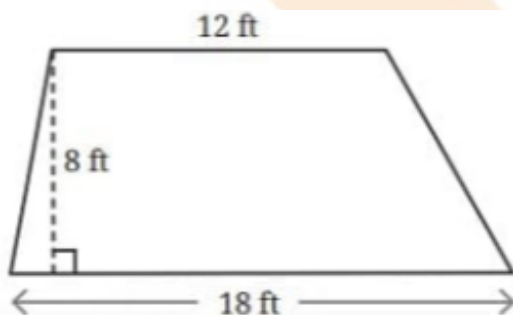
To get a rectangle with the same area

$$\text{Rectangle length} = \frac{b}{2} \text{ (half the triangle's base)}$$

$$\text{Rectangle width} = h \text{ (same as the triangle's height)}$$

$$\text{Area} = \frac{b}{2} \times h = \frac{1}{2} \times b \times h$$

22. Find the areas of the given figure:



**Ans. :** self

23. What do you think is the area of an A4 sheet? Its sidelengths are 21 cm and 29.7 cm. Now find its area.

**Ans. :** We know that the size of an  $A_4$  sheet is rectangular.

$$\therefore \text{The area of an } A_4 \text{ sheet} = \text{Length} \times \text{Breadth}$$

$$= 21 \text{ cm} \times 29.7 \text{ cm}$$

$$= 623.7 \text{ cm}^2$$

24. Express the following lengths in centimeters:

i. 5 in

ii. 7.4 in

**Ans. :** We know that,

$$1 \text{ in} = 2.54 \text{ cm}$$

$$\text{i. } 5 \text{ in} = 5 \times 2.54 \text{ cm} = 12.7 \text{ cm}$$

$$\text{ii. } 7.4 \text{ in} = 7.4 \times 2.54 \text{ cm} = 18.796 \text{ cm}$$

25. Express the following lengths in inches:

i. 5.08 cm

ii. 11.43 cm

**Ans. :** We know that,

$$2.54 \text{ cm} = 1 \text{ in}$$

$$\therefore 1 \text{ cm} = \frac{1}{2.54} \text{ in}$$

$$\text{i. } 5.08 \text{ cm} = 5.08 \times \frac{1}{2.54} \text{ in} = 2 \text{ in}$$

$$\text{ii. } 11.43 \text{ cm} = 11.43 \times \frac{1}{2.54} \text{ in} = 4.5 \text{ in}$$

26. How many  $m^2$  is a  $km^2$  ?

**Ans. :** We know that,

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ km}^2 = 1000 \text{ m} \times 1000 \text{ m}$$

$$= 1000000 \text{ m}^2$$

$$= 10^6 \text{ m}^2$$

$$\text{Thus, } 1 \text{ km}^2 = 1000000 \text{ m}^2$$

27. What do you think is the area of your village/town/city? Make an estimate and compare it with the actual data.

Larger areas are measured in  $km^2$ .

**Ans. :** self

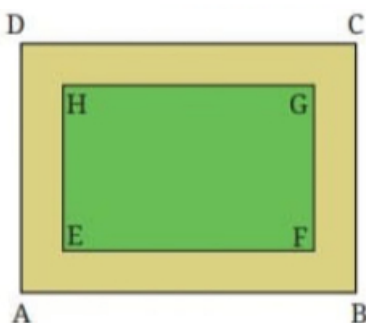
28. Find the city with the smallest area in (i) India, and (ii) the world.

**Ans. :** self

\* Questions With Calculation.[3 Marks Each]

[45]

29. The figure shows a path (the shaded portion) laid around a rectangular park EFGH.



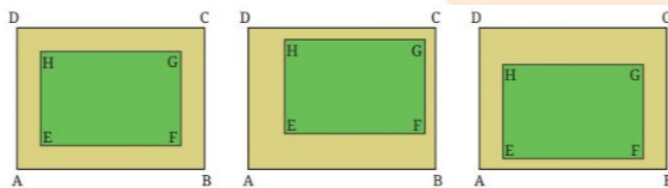
- i. What measurements do you need to find the area of the path? Once you identify the lengths to be measured, assign possible values of your choice to these measurements and find the area of the path. Give a formula for the area.

An example of a formula — Area of a rectangle = length  $\times$  width. [Hint: There is a relation between the areas of EFGH, the path, and ABCD.]

- ii. If the width of the path along each side is given, can you find its area? If not, what other measurements do you need? Assign values of your choice to these measurements and find the area of the path. Give a formula for the area using these measurements.

[Hint: Break the path into rectangles.]

- iii. Does the area of the path change when the outer rectangle is moved while keeping the inner rectangular park EFGH inside it, as shown?



**Ans. :** i. Measurements needed:

Length of outer rectangle ABCD = A

Width of outer rectangle ABCD = B

Length of inner rectangle EFGH = a

Width of inner rectangle EFGH = b

Let A = 10, B = 8, a = 6, b = 4.

Calculation: Area of path = Area of ABCD – Area of EFGH

$$= 10 \times 8 - 6 \times 4$$

$$= 80 - 24$$

$$= 56m^2$$

Formula:

$$\text{Area of path} = (A \times B) - (a \times b)$$

- ii. Yes, if the width of the path is uniform along each side, we can find its area, but we also need the dimensions of either the outer or inner rectangle.

If the width of the path  $d = 2$  m (Uniform on all sides)

Length of inner path EFGH =  $l = 16$  m

Width of inner path EFGH =  $w = 11$  m

Length of outer rectangle =  $l + 2d$

$$= 16 + 2(2)$$

$$= 20 \text{ m}$$

Width of outer rectangle =  $w + 2d$

$$= 11 + 2(2)$$

$$= 15 \text{ m}$$

Breaking the path into rectangles

Now there are 4 rectangles

Left rectangle =  $w \times d$

$$= 11 \times 2$$

$$= 22m^2$$

Right rectangle =  $w \times d$

$$= 11 \times 2$$

$$= 22m^2$$

Top rectangle =  $(l + 2d) d$

$$= 20 \times 2$$

$$= 40m^2$$

Bottom rectangle =  $(l + 2d) d$

$$= 20 \times 2$$

$$= 40m^2$$

Total area of path =  $22 + 22 + 40 + 40 = 124m^2$

Formula:

Area of path =  $2d(l + w) + 4d$

where  $d$  - width of path

$l$  - length of inner path

$w$  - width of inner path

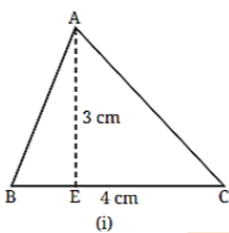
iii. No, the area of the path does not change.

Reason: The area of the path depends only on:

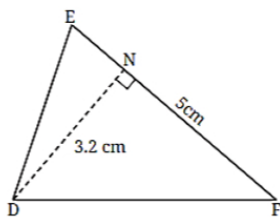
Area of outer rectangle ABCD.

Area of inner rectangle EFGH.

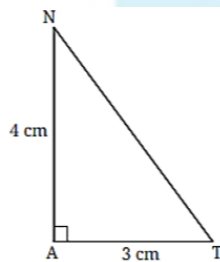
30. Find the areas of the following triangles:



(i)



(ii)



(iii)

Ans. : (i) Area of triangle  $ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times BC \times AE$$

$$= \frac{1}{2} \times 4cm \times 3cm$$

$$= 6cm^2$$

Thus, the area of the triangle,  $ABC = 6cm^2$

(ii) Area of triangle  $DEF = \frac{1}{2} \times EF \times ND$

$$= \frac{1}{2} \times 5cm \times 3.2cm$$

$$= 5 \times 1.6cm^2$$

$$= 8cm^2$$

Thus, the area of the triangle  $DEF = 8\text{cm}^2$

(iii) Area of triangle  $= \frac{1}{2} \times \text{base} \times \text{height}$

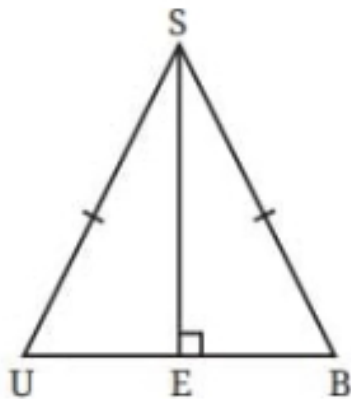
Area of  $\triangle NAT = \frac{1}{2} \times AT \times NA$

$$= \frac{1}{2} \times 3\text{cm} \times 4\text{cm}$$

$$= 6\text{cm}^2$$

Thus, the area of the triangle,  $NAT = 6\text{cm}^2$

31. Find the area of  $\triangle SUB$ , given that it is isosceles,  $SE$  is perpendicular to  $UB$ , and the area of  $\triangle SEB$  is 24 sq. units.



**Ans. :** Given,

The area of  $\triangle SEB = 24$  sq. units

Given that  $\triangle SUB$  is an isosceles triangle.

$SU$  and  $SB$  are equal sides, and  $UB$  is the base.

$\therefore SE$  is perpendicular to  $UB$

$$\Rightarrow UE = EB$$

$SE$  is the common base of  $\triangle SUE$  and  $\triangle SEB$ .

$$\therefore \text{Area of } \triangle SEB = 24 \text{ sq. units} = \text{Area of } \triangle SEU$$

$$\therefore \text{The area of triangle } SUB = \text{Area of } \triangle SEU + \text{Area of } \triangle SEB$$

$$= 24 + 24$$

$$= 48 \text{ sq. units}$$

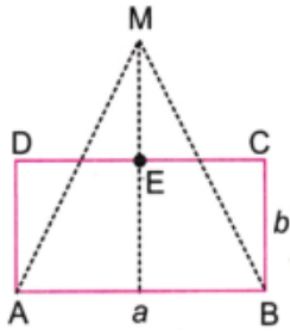
Thus, the area of  $\triangle SUB$  is 48 sq. units.

In the Sulba-Sutras, which are ancient Indian geometric texts that deal with the construction of altars, we can find many interesting problems on the topic of areas. When altars are built, they must have the exact prescribed shape and area. This gives rise to problems of the kind where one has to transform a given shape into another of the same area. The Sulba-Sutras give solutions to many such problems. Such problems are also posed and solved in Euclid's Elements. Here are two problems of this kind.

32. [Śulba-Sūtras] Give a method to transform a rectangle into a triangle of equal area.



Ans. : 1. Let us take a rectangle ABCD, with length  $a$  and breadth  $b$ .



2. Now mark the midpoint  $E$  of side  $CD$ .

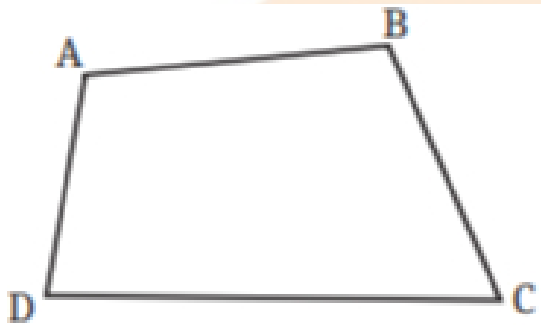
Draw a line perpendicular to  $CD$  passing through  $E$ .

Mark a point  $M$  on it such that  $ME = b$ .

3. Draw a triangle using base =  $AB$  (same as the rectangle's length) and join  $M$  to  $A$  and  $B$ .

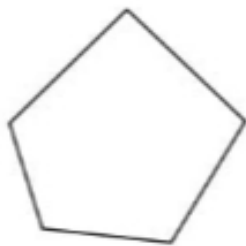
33. How do we find the area of this quadrilateral? What measurements do we need for this?

If we join  $BD$ , the quadrilateral  $ABCD$  gets divided into two triangles. By finding their areas, we can find the area of  $ABCD$ .



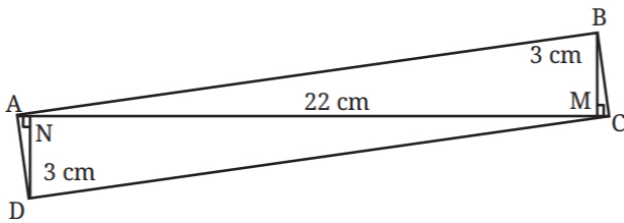
Ans. : self

34. How do we find the area of this pentagon?



Ans. : self

35. Find the area of the quadrilateral  $ABCD$  given that  $AC = 22$  cm,  $BM = 3$  cm,  $DN = 3$  cm,  $BM$  is perpendicular to  $AC$ , and  $DN$  is perpendicular to  $AC$ .



**Ans. :** Area of the quadrilateral ABCD = Area of triangle CAD + Area of triangle ACB

$$\text{Area of } \triangle ACB = \frac{1}{2} \times AC \times BM$$

$$= \frac{1}{2} \times 22\text{cm} \times 3\text{cm}$$

$$= 33\text{cm}^2$$

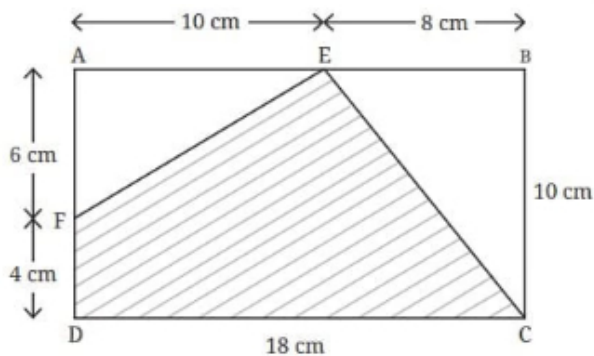
$$\text{Area of } \triangle CAD = \frac{1}{2} \times AC \times DN$$

$$= \frac{1}{2} \times 22\text{cm} \times 3\text{cm}$$

$$= 33\text{cm}^2$$

$$\therefore \text{The area of the quadrilateral } ABCD = 33\text{cm}^2 + 33\text{cm}^2 = 66\text{cm}^2$$

36. Find the area of the shaded region given that ABCD is a rectangle.



**Ans. :** The area of the shaded region = Area of the rectangle ABCD - (Area of triangle AEF + Area of triangle EBC)

Area of the rectangle ABCD = Length  $\times$  Breadth

$$= AB \times AD$$

$$= 18\text{cm} \times 10\text{cm} [\because AB = AE + EB = 10\text{cm} + 8\text{cm} = 18\text{cm}; AD =$$

$$AF + FD = 6\text{cm} + 4\text{cm} = 10\text{cm}]$$

$$= 180\text{cm}^2$$

$$\text{Area of the triangle } AEF = \frac{1}{2} \times AE \times AF$$

$$= \frac{1}{2} \times 10\text{cm} \times 6\text{cm}$$

$$= 30\text{cm}^2$$

$$\text{Area of the triangle } EBC = \frac{1}{2} \times EB \times BC$$

$$= \frac{1}{2} \times 8\text{cm} \times 10\text{cm}$$

$$= 40\text{cm}^2$$

$$\therefore \text{The area of the shaded region} = 180\text{cm}^2 - (30\text{cm}^2 + 40\text{cm}^2)$$

$$= 180\text{cm}^2 - 70\text{cm}^2$$

$$= 110\text{cm}^2$$

37. What measurements would you need to find the area of a regular hexagon?

**Ans. :** Minimum measurement needed

Side length ( $l$ ) of the hexagon

$$\text{Area of regular hexagon} = \frac{3\sqrt{3}a^2}{2}$$

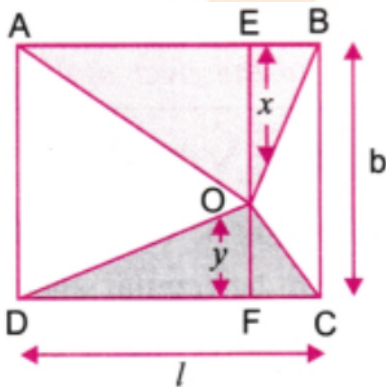
where  $l$  is the side length.



38. What fraction of the total area of the rectangle is the area of the blue region?



**Ans. :** Let  $l$  be the length and  $b$  be the breadth of the rectangle ABCD.



Total area of rectangle, ABCD = DC  $\times$  BC =  $l \times b$  sq. units

$$\text{Area of } \triangle AOB = \frac{1}{2} \times AB \times OE = \frac{1}{2} \times l \times x \text{ sq. units}$$

$$\text{Area of } \triangle DOC = \frac{1}{2} \times DC \times OF = \frac{1}{2} \times l \times y \text{ sq. units}$$

$\therefore$  The area of the red region = Area of  $\triangle AOB$  + Area of  $\triangle DOC$

$$= \frac{1}{2} \times l \times x + \frac{1}{2} \times l \times y$$

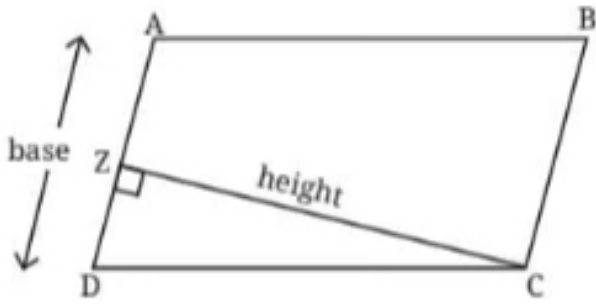
$$= \frac{1}{2} \times l \times (x + y) \text{ sq. units}$$

$$= \frac{1}{2} \times l \times b \text{ sq. units } [ \because x + y = b ]$$

$\therefore$  Area of red region =  $\frac{1}{2} \times$  Area of rectangle

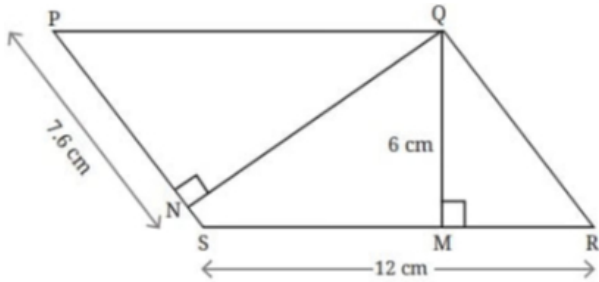
Thus, the required fraction is  $\frac{1}{2}$ .

39. Can the area of the parallelogram be determined by taking another side as the base and its corresponding height?



Ans. : self

40. Find QN.



Ans. : In  $\triangle PNQ$ ,  $\angle PNQ = 90^\circ$

$$PN = 7.6\text{cm}$$

$$PQ = 12\text{cm}$$

By Pythagoras theorem

$$PQ^2 = PN^2 + NQ^2$$

$$\Rightarrow (12)^2 = (7.6)^2 + NQ^2$$

$$\Rightarrow QN^2 = (12)^2 - (7.6)^2$$

$$\Rightarrow QN^2 = 144 - 57.76$$

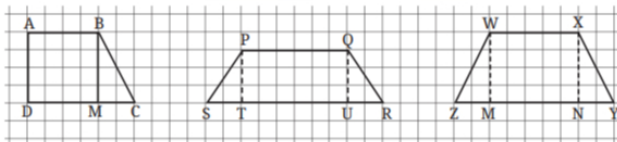
$$\Rightarrow QN^2 = 86.24$$

$$\Rightarrow QN = 9.28\text{cm}$$

41. Simplify the expression to show that we get the same formula for the area of a rhombus in terms of its diagonals.

Ans. : self

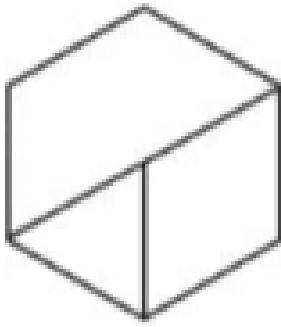
42. Find the areas of the following trapeziums by breaking them into figures whose areas can be computed.



One way of finding the area of a trapezium is by breaking it into a rectangle and triangles.

Ans. : self

43. A regular hexagon is divided into a trapezium, an equilateral triangle, and a rhombus, as shown. Find the ratio of their areas.



**Ans. :** Here total area of hexagon =  $6 \times \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2$

Equilateral triangle:

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

Rhombus:

$$\text{Area} = 2 \times \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{2}a^2$$

Trapezium:

Remaining Area = Total area - Triangle area - Rhombus area

$$= \frac{3\sqrt{3}}{4}a^2 - \frac{\sqrt{3}}{4}a^2 - \frac{\sqrt{3}}{2}a^2$$

$$= \frac{3\sqrt{3}}{4}a^2$$

Ratio of areas = Triangle : Rhombus : Trapezium

$$= \frac{\sqrt{3}}{4}a^2 : \frac{\sqrt{3}}{2}a^2 : \frac{3\sqrt{3}}{4}a^2$$

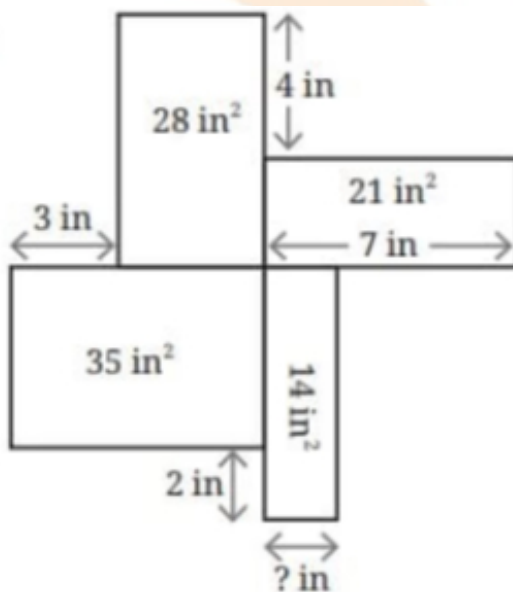
$$= 1 : 2 : 3$$

**\* Questions With Calculation.[5 Marks Each]**

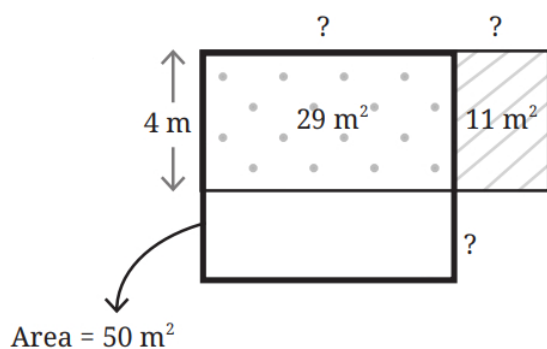
[110]

44. Identify the missing sidelengths.

i.



ii.



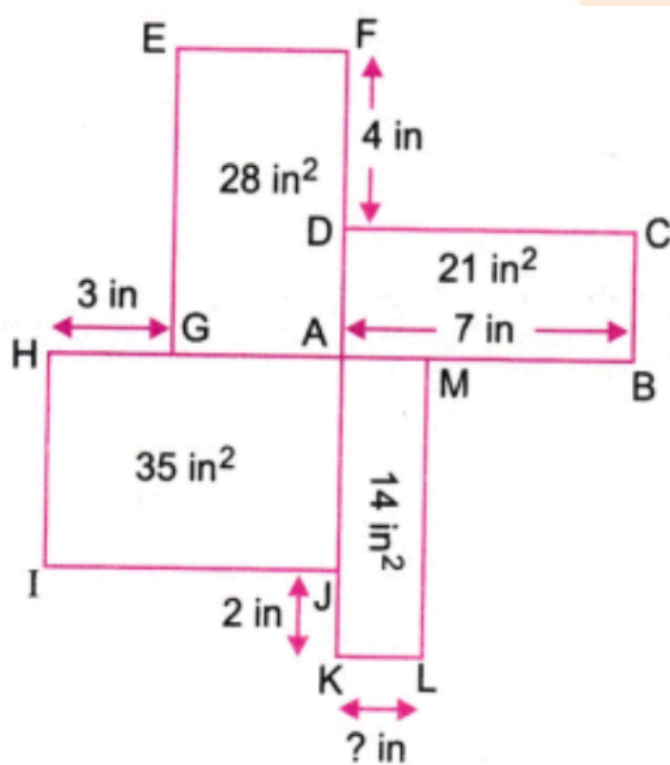
**Ans. :** i. After naming the figure

In rectangle ABCD,

Area of rectangle = Length  $\times$  Breadth

$$7 \times BC = 21$$

$$\Rightarrow BC = 3 \text{ in}$$



$$\therefore AF = AD + DF$$

$$= 3 \text{ in} + 4 \text{ in}$$

$$= 7 \text{ in}$$

In the rectangle EFAG,

$$EF \times AF = 28 \text{ in}^2$$

$$\Rightarrow EF \times 7 \text{ in} = 28 \text{ in}^2$$

$$\Rightarrow EF = 4 \text{ in}$$

$$\therefore HA = HG + GA$$

$$= 3 \text{ in} + 4 \text{ in}$$

$$= 7 \text{ in}$$

In rectangle HIJA,

$$\text{Area} = HA \times AJ$$

$$\Rightarrow 35\text{in}^2 = 7\text{in} \times AJ$$

$$\Rightarrow AJ = 5\text{in}$$

$$\therefore AK = AJ + JK$$

$$= 5\text{in} + 2\text{in}$$

$$= 7\text{in}$$

In rectangle KLMA,

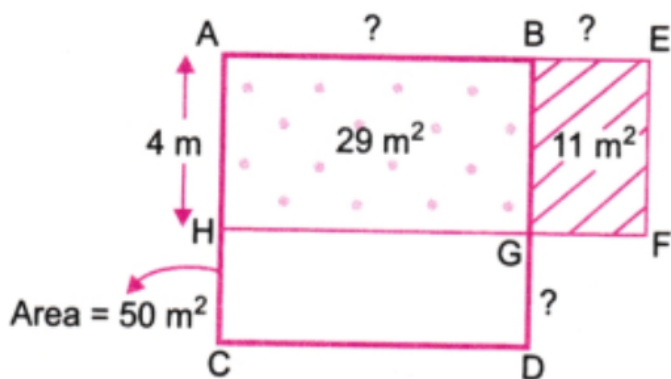
$$\text{Area} = KL \times LM$$

$$\Rightarrow x\text{in} \times 7\text{in} = 14\text{in}^2$$

$$\Rightarrow x = 2\text{in}$$

Thus, the missing sidelength = 2 in

ii. After naming the figure,



In rectangle ABGH,

$$\text{Area} = AB \times AH$$

$$AB \times 4\text{m} = 29\text{m}^2$$

$$AB = \frac{29}{4}\text{m} \text{ or } 7.25\text{ m}$$

Area of rectangle HGDC = Area of rectangle ABDC - Area of rectangle ABGH

$$= 50\text{m}^2 - 29\text{m}^2$$

$$= 21\text{m}^2$$

In rectangle HGDC,

$$CD \times GD = 21\text{m}^2$$

$$\Rightarrow \frac{29}{4}\text{m} \times GD = 21\text{m}^2$$

$$\Rightarrow GD = \frac{84}{29}\text{m} \text{ or } 2.9\text{ m}$$

In rectangle BEFG,

$$BG \times BE = 11\text{m}^2$$

$$\Rightarrow 4\text{m} \times M = 11\text{m}^2$$

$$\Rightarrow BE = \frac{11}{4}\text{m} \text{ or } 2.75\text{ m}$$

$$\text{Thus, } AB = \frac{29}{4}\text{m}; BE = \frac{11}{4}\text{m} \text{ and } GD = \frac{84}{29}\text{m}$$

45. The figure shows a plot with sides 14 m and 12 m, and with a crosspath. What other measurements do you need to find the area of the crosspath? Once you

identify the lengths to be measured, assign some possible values of your choice and find the area of the path. Give a formula for the area based on the measurements you choose.



**Ans. :** Measurements needed:

Length of plot = 14 m

Width of plot = 12 m

Width of horizontal path =  $x_1$

Width of vertical path =  $x_2$

Assign values:

Let  $x_1 = 2m$

$x_2 = 2m$

Now area of horizontal path =  $14 \times 2 = 28 m$

Area of vertical path =  $12 \times 2 = 24 m$

Area of overlapping square =  $2 \times 2 = 4m^2$

$\therefore$  Area of cross path =  $28 + 24 - 4 = 48m^2$

Formula = Area of cross path =  $(L \times w_1) + (W \times w_2) - (w_1 \times w_2)$

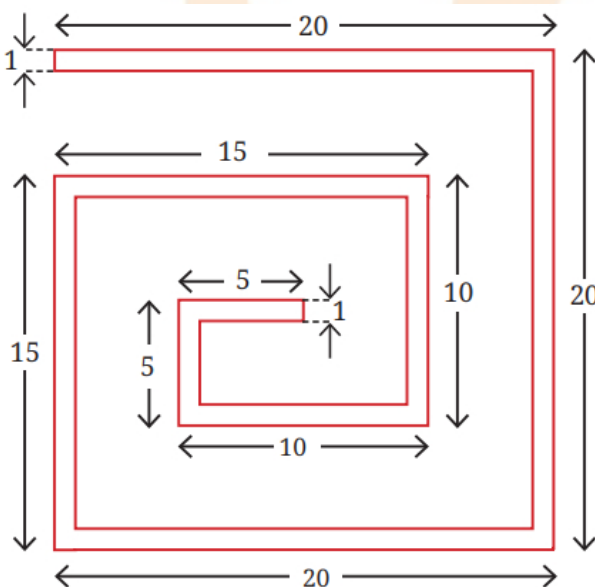
Here, L = length of plot

W = Width of plot

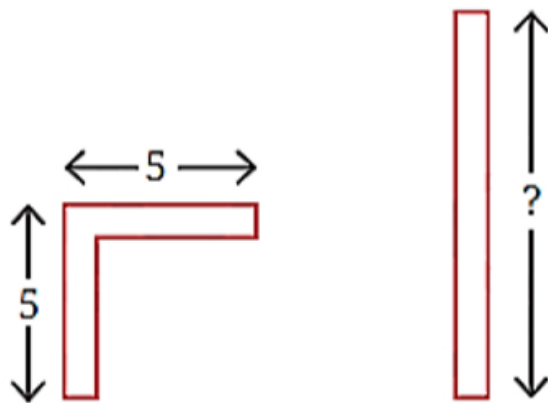
$W_1$  = Width of horizontal path

$W_2$  = Width of vertical path

46. Find the area of the spiral tube shown in the figure. The tube has the same width throughout.

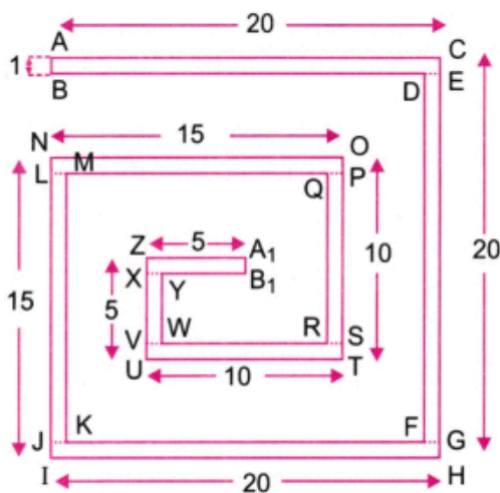


[Hint: There are different ways of finding the area. Here is one method.]



What should be the length of the straight tube if it is to have the same area as the bent tube on the left?

Ans. : After naming the figure,



The area of the spiral tube = Area of the rectangle, ABCE + Area of the rectangle, DEGF + Area of the rectangle, GHIJ + Area of the rectangle, JKML + Area of the rectangle, NOPL + Area of the rectangle, PQRS + Area of the rectangle, STUV + Area of the rectangle, VWYX + Area of the rectangle,  $XZA_1B_1$

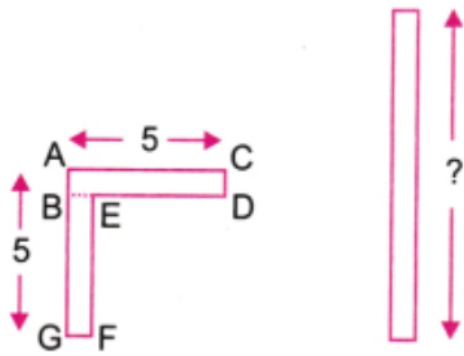
$$\begin{aligned}
 &= AC \times AB + EG \times DE + IH \times JI + LJ \times LM + NO \times NL + PQ \times PS + UT \times ST + VX \times VW + ZA_1 \times A_1B_1 \\
 &= 20 \times 1 + 18 \times 1 + 20 \times 1 + 13 \times 1 + 15 \times 1 + 8 \times 1 + 10 \times 1 + 3 \times 1 + 5 \times 1 \\
 &= 20 + 18 + 20 + 13 + 15 + 8 + 10 + 3 + 5 \\
 &= 112 \text{ sq. units}
 \end{aligned}$$

Thus, the area of the spiral tube is 112 sq. units.

Let the length of the straight tube be  $x$ .

The area of the bent tube on the left = Area of rectangle, BACD + Area of rectangle, BGFE

$$\begin{aligned}
 &= AC \times CD + BG \times BE \\
 &= 5 \times 1 + 4 \times 1 \\
 &= 9 \text{ sq. units}
 \end{aligned}$$



Area of straight tube =  $x \times 1$

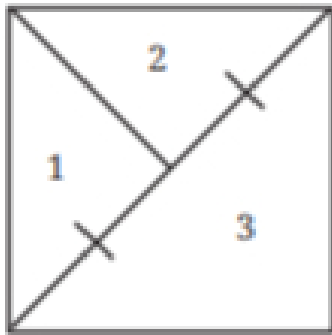
Area of bent tube = 9 sq. units

According to the question, both are the same.

So  $x \times 1 = 9$

$\Rightarrow x = 9$  units.

47. In this figure, if the sidelength of the square is doubled, what is the increase in the areas of the regions 1, 2 and 3? Give reasons.



**Ans. :** Let 'a' be the side of the square.

Area of triangle, DCB (region 3)

$$= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times DC \times BC$$

$$= \frac{1}{2} \times a \times a = \frac{a^2}{2} \text{ sq. units}$$

In right angled DCB,

$$DB = \sqrt{DC^2 + BC^2} = \sqrt{a^2 + a^2} = \sqrt{2}a \text{ units}$$

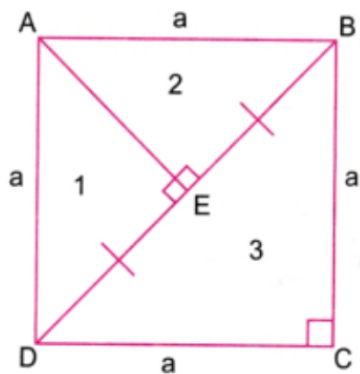
$$\therefore AE = DE = EB = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{\sqrt{2} \times \sqrt{2}}{2 \times \sqrt{2}} a$$

$$= \frac{2}{2\sqrt{2}} a = \frac{a}{\sqrt{2}} \text{ units}$$

Area of triangle, AED (region 1)

$$= \frac{1}{2} \times DE \times AE = \frac{1}{2} \times \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}}$$

$$= \frac{a^2}{4} \text{ sq. units}$$

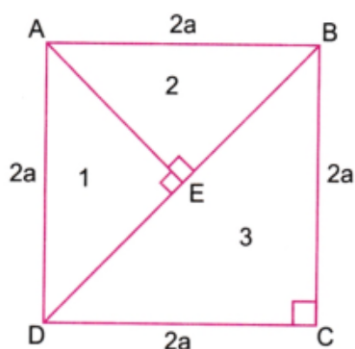


Area of triangle,  $AEB$  (region 2)

$$= \frac{1}{2} \times EB \times AE = \frac{1}{2} \times \frac{a}{\sqrt{2}} \times \frac{a}{\sqrt{2}} = \frac{a^2}{2} \times \frac{1}{2}$$

$$= \frac{a^2}{4} \text{ sq. units}$$

If the side of the square is doubled  $= 2a$



Area of triangle,  $BCD$  (region 3)

$$= \frac{1}{2} \times DC \times BC$$

$$= \frac{1}{2} \times 2a \times 2a = 2a^2 \text{ sq. units}$$

In right angled triangle,  $BCD$ .

By Pythagoras theorem,

$$BD = \sqrt{DC^2 + BC^2}$$

$$= \sqrt{(2a)^2 + (2a)^2}$$

$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}a \text{ units}$$

$$\therefore AE = ED = EB = \frac{BD}{2} = \frac{2\sqrt{2}a}{2} = \sqrt{2}a \text{ units}$$

Area of triangle,  $AED$  (region 1)

$$= \frac{1}{2} \times DE \times AE$$

$$= \frac{1}{2} \times \sqrt{2}a \times \sqrt{2}a = \frac{1}{2} \times 2a^2 = a^2 \text{ sq. units}$$

Area of triangle,  $AEB$  (region 2)

$$= \frac{1}{2} \times EB \times AE = \frac{1}{2} \times \sqrt{2}a \times \sqrt{2}a$$

$$= a^2 \text{ sq. units}$$

The increase in area of region 1

$$= \frac{a^2}{a^2/4} = 4 \text{ times}$$

The increase in area of region 2

$$= \frac{a^2}{a^2/4} = 4 \text{ times}$$

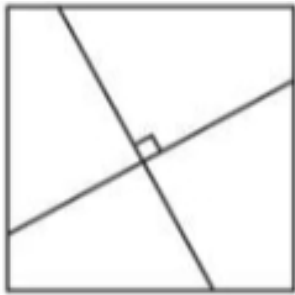
The increase in area of region 3

$$= \frac{2a^2}{a^2/2} = \frac{2a^2}{a^2} \times 2 = 4 \text{ times}$$

Thus, the increase in the areas of regions 1, 2, and 3 is 4 times.

Reason: If the sidelength of the square is doubled, then the area becomes 4 times.

48. Divide a square into 4 parts by drawing two perpendicular lines inside the square as shown in the figure. Rearrange the pieces to get a larger square, with a hole inside.



**Ans. :** 1. Let us take a square of cardboard (8 cm × 8 cm).

2. Draw two perpendicular lines inside the square (not through the center), dividing it into 4 rectangular pieces.

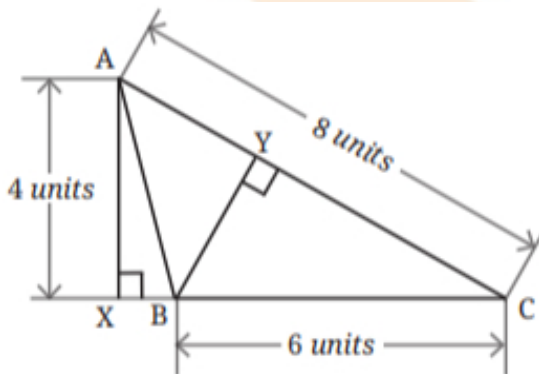
3. Cut along these lines to get 4 pieces.

4. Rearrange these 4 pieces.

Place them at the four corners of a larger imaginary square.

The pieces should be arranged so that they form a square with a hole in the middle.

49. Find the length of the altitude BY.



**Ans. :** Area of  $\triangle AXC = \frac{1}{2} \times XC \times AX$

$$= \frac{1}{2} \times (XB + 6) \times 4$$

$$= 2 \times (XB + 6)$$

$$= 2 \times XB + 12 \text{ sq. units}$$

Area of  $\triangle AXB = \frac{1}{2} \times XB \times AX$

$$= \frac{1}{2} \times XB \times 4$$

$$= 2 \times XB \text{ sq. units}$$

Area of  $\triangle ABC = \frac{1}{2} \times AC \times BY$

$$= \frac{1}{2} \times 8 \times BY$$

$$= 4BY$$

$\therefore$  Area of  $\triangle AXC =$  Area of  $\triangle AXB +$  Area of  $\triangle ABC$

$$2XB + 12 = 2XB + 4BY$$

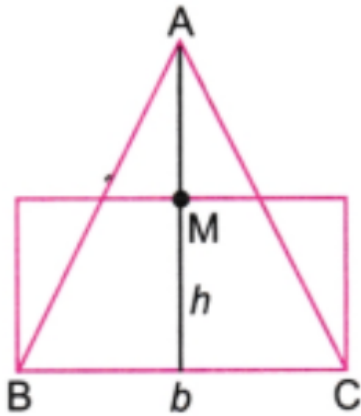
$$\Rightarrow 4BY = 12$$

$$\Rightarrow BY = 3 \text{ units}$$

Thus, the length of the altitude BY is 3 units.

50. [Śulba-Sūtras] Give a method to transform a triangle into a rectangle of equal area.

**Ans. :** 1. Take a triangle ABC with base b and height h.



2. Find the midpoint M of the height.

3. Draw a line parallel to the base through M.

This line intersects the sides of the triangle.

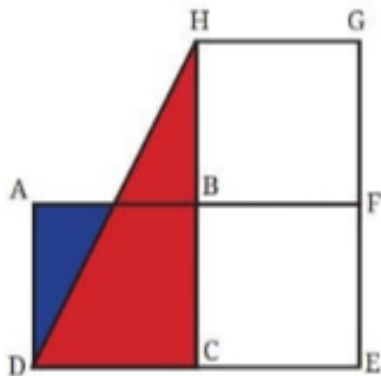
4. Create a rectangle using.

Length = Same as the triangle's base = b

Width = Half of the triangle's height =  $\frac{h}{2}$

51. ABCD, BCEF, and BFGH are identical squares.

- i. If the area of the red region is 49 sq. units, then what is the area of the blue region?
- ii. In another version of this figure, if the total area enclosed by the blue and red regions is 180 sq. units, then what is the area of each square?



**Ans. :** Given that ABCD, BCEF, and BFGH are identical.

The area of the red region ( $\triangle HBI + IBCD$ ) = 49 sq. units.

Let the side of each square be a.

$$\therefore IB = \frac{AB}{2} = \frac{a}{2} \text{ units}$$

Let 'a' units be the side of the square.

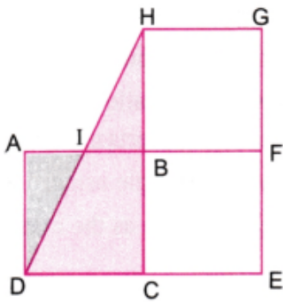
i. Area of the red region  $\triangle HDC = \frac{1}{2} \times DC \times HC$

$$(\because HC = HB + BC = a + a = 2a)$$

$$= \frac{1}{2} \times a \times 2a$$

$$\Rightarrow \frac{1}{2} \times 2a^2 = 49$$

$$\Rightarrow a = 7 \text{ units}$$



$$\therefore \text{The area of the black region, } \triangle IAD = \frac{1}{2} \times AI \times AD$$

$$= \frac{1}{2} \times \frac{7}{2} \times 7$$

$$= \frac{49}{4} \text{ sq. units}$$

$$= 12.25 \text{ sq. units}$$

Thus, the area of the black region is 12.25 square units.

ii. Given, the total area enclosed by the black and red regions = Area of  $\triangle HDC +$

Area of  $\triangle AID = 180 \text{ sq. units}$

Let 'a' be the side of the square.

$$\frac{1}{2} \times DC \times HC + \frac{1}{2} \times AI \times AD$$

$$= 180 \text{ sq. units}$$

$$\frac{1}{2} \times a \times 2a + \frac{1}{2} \times \frac{a}{2} \times a = 180$$

$$\Rightarrow a^2 + \frac{a^2}{4} = 180$$

$$\Rightarrow \frac{4a^2 + a^2}{4} = 180$$

$$\Rightarrow \frac{5a^2}{4} = 180 \Rightarrow 5a^2 = 180 \times 4$$

$$\Rightarrow a^2 = \frac{180 \times 4}{5} = 36 \times 4 = 144$$

$$\Rightarrow a^2 = (12)^2$$

$$\Rightarrow a = 12 \text{ sq. units}$$

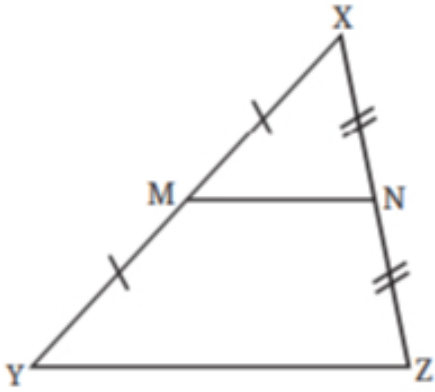
$$\therefore \text{The area of each square} = a^2$$

$$= (12)^2$$

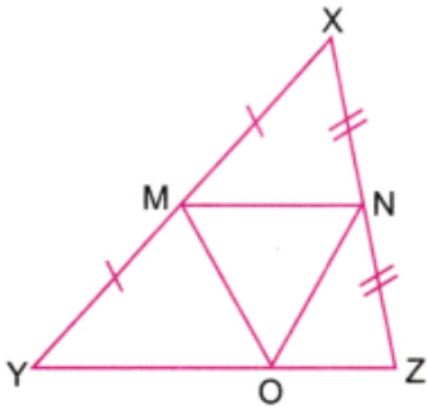
$$= 144 \text{ sq. units}$$

Thus, the area of each square is 144 sq. units

52. If M and N are the midpoints of XY and XZ, what fraction of the area of  $\triangle XYZ$  is the area of  $\triangle XMN$ ?



Ans. : Let O be the midpoint of YZ, then join M to O and N to O.



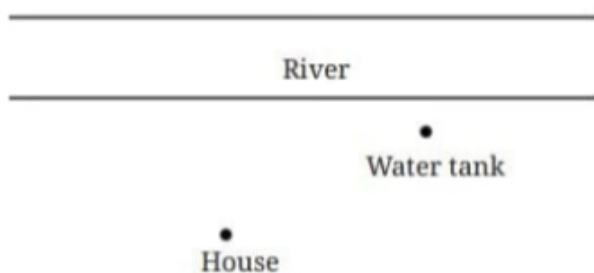
According to mid point theorem,

$$MN = \frac{1}{2}YZ, \text{ and } MN \text{ is parallel to } YZ.$$

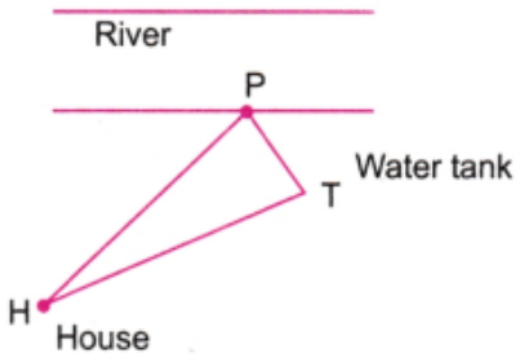
The triangle XYZ is divided into four equal triangles.

$$\text{So, Area of } \triangle XMN = \frac{1}{4} \times \text{Area of } \triangle XYZ.$$

53. Gopal needs to carry water from the river to his water tank. He starts from his house. What is the shortest path he can take from his house to the river and then to the water tank? Roughly recreate the map in your notebook and trace the shortest path.



Ans. : Let P be the point on the bank of the river, then the shortest path from H to P to T is created by the point P such that  $\triangle HPT$  has minimum area.



Shortest path

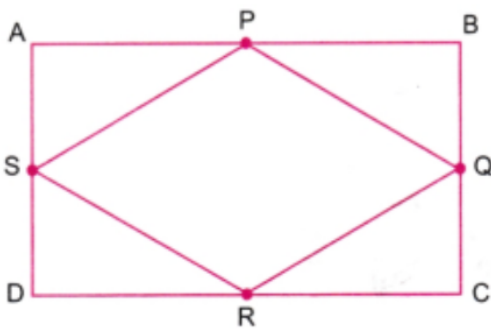
House ( $H$ )  $\rightarrow$   $P$  (Point on river)  $\rightarrow$  Water tank

54. Give a method to obtain a quadrilateral whose area is half that of a given quadrilateral.

**Ans. :** Let ABCD be a given quadrilateral.

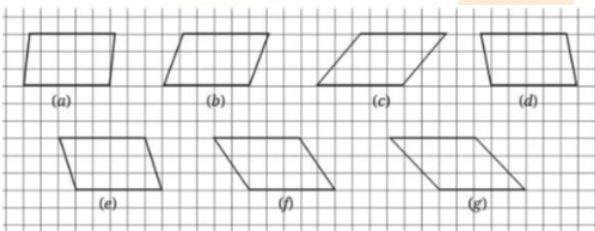
Mark mid points of AB, BC, CD, and DA as P, Q, R, and S.

Join midpoints, then PQRS is the required quadrilateral with half the area of the given quadrilateral ABCD.



55. Observe the parallelograms in the figure below.

- i. What can we say about the areas of all these parallelograms?
- ii. What can we say about their perimeters? Which figure appears to have the maximum perimeter, and which has the minimum perimeter?



**Ans. :** i. (a) Area of parallelogram = base  $\times$  height

$$= 5 \times 3$$

$$= 15 \text{ sq. units}$$

$$(b) \text{ Area of parallelogram} = 5 \times 3 = 15 \text{ sq. units}$$

$$(c) \text{ Area of parallelogram} = 5 \times 3 = 15 \text{ sq. units}$$

$$(d) \text{ Area of parallelogram} = 5 \times 3 = 15 \text{ sq. units}$$

$$(e) \text{ Area of parallelogram} = 5 \times 3 = 15 \text{ sq. units}$$

$$(f) \text{ Area of parallelogram} = 5 \times 3 = 15 \text{ sq. units}$$

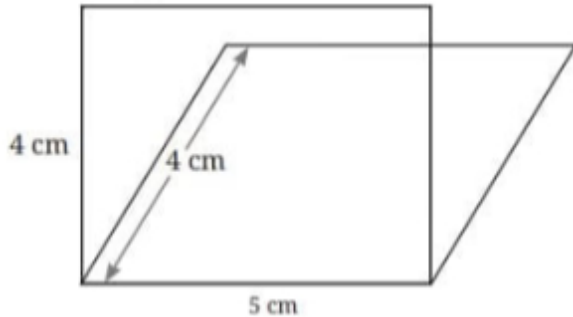
(g) Area of parallelogram =  $5 \times 3 = 15$  sq. units

All parallelograms have equal areas.

ii. The perimeters of these parallelograms are different even though their areas are the same.

Figure (d) has the minimum perimeter, and Figure (g) has the maximum perimeter.

56. Consider a rectangle and a parallelogram of the same sidelengths: 5 cm and 4 cm. Which has the greater area?



**Ans. :** For rectangle:

$l = 5, w = 4$ , all angles =  $90^\circ$

$\therefore$  Area =  $l \times w$

=  $5 \times 4$

=  $20\text{cm}^2$

For parallelogram:

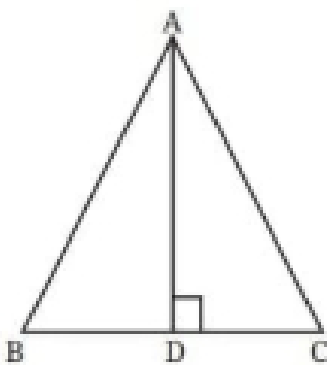
Base (b) = 5 cm, one slanted side = 4 cm

Height will be less than 4 cm because the side is slanted.

Area =  $5 \times 4 < 20\text{cm}^2$

Hence, the rectangle has a greater area than the parallelogram.

57. [Śulba-Sūtras] An isosceles triangle can be converted into a rectangle by dissection in a simpler way. Can you find out how to do it?



**Ans. :** Given: Isosceles triangle ABC, where  $AB = AC$ , and AD is the altitude from A to BC.

Method: Since the triangle is isosceles:

AD is perpendicular to BC. D is the midpoint of BC (property of an isosceles triangle).

AD bisects the triangle into two congruent right triangles:  $\triangle ADB$  and  $\triangle ADC$ .

Dissection Process:

Step 1: The altitude AD divides the isosceles triangle into two congruent right triangles  $\triangle ADB$  and  $\triangle ADC$ .

Step 2: Each of these right triangles can be made into half of a rectangle.

Step 3: Assembly:

Take triangle  $\triangle ADB$

Take triangle  $\triangle ADC$

Rotate one triangle  $180^\circ$

Arrange them so that:

The two equal sides ( $AB$  and  $AC$ ) form opposite sides of a rectangle.

The altitude AD appears twice, forming the other pair of opposite sides.

Step 4: The resulting figure is a rectangle with:

Length = BC (base of the isosceles triangle)

Width =  $\frac{AD}{2}$  (half the altitude) or alternatively.

Length = AB (= AC, the equal sides)

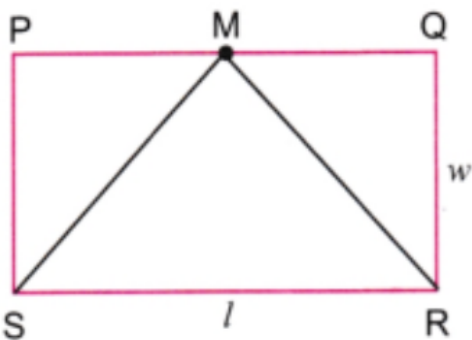
Width related to the base.

58. [Śulba-Sūtras] Give a method to convert a rectangle into an isosceles triangle by dissection.

**Ans. :** This is the reverse of Question 7.

Method:

Given: Rectangle PQRS with length  $l$  and width  $w$ .



Required: Isosceles triangle with the same area

Dissection Steps:

Step 1: Take a rectangle PQRS with  $PQ = l$  and  $PS = w$ .

Step 2: Mark the midpoint M of side PQ.

Step 3: From M, draw lines to the bottom corners R and S.

Step 4: Cut the rectangle into three pieces:

Triangle PMS (left); Triangle QMR (right); Central region (if any)

Step 5: Rearrange:

Flip the triangle PMS and attach it along MS to form one half of the triangle.

Flip triangle QMR and attach it along MR to form the other half.

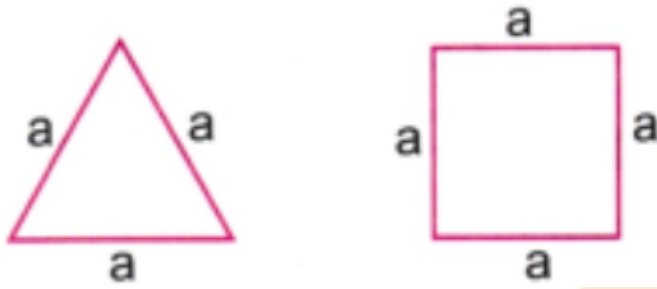
These create an isosceles triangle.

59. Which has greater area - an equilateral triangle or a square of the same side length as the triangle? Which has greater area - two identical equilateral triangles

together or a square of the same side length as the triangle? Give reasons.

**Ans. :** Area of equilateral triangle =  $\frac{\sqrt{3}}{4}a^2$

Area of square =  $a^2$



$$\Rightarrow \frac{\sqrt{3}}{4}a^2 < a^2$$

So, the area of a square is greater than the area of an equilateral triangle of the same side length.

Area of two identical equilateral triangles =  $\frac{\sqrt{3}}{4}a^2 + \frac{\sqrt{3}}{4}a^2$

$$= \frac{2\sqrt{3}}{4}a^2$$

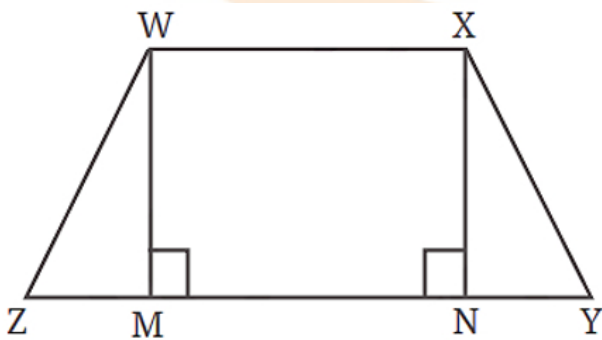
$$= \frac{\sqrt{3}}{2}a^2$$

Area of square of side length  $a = a^2$

Clearly,  $\frac{\sqrt{3}}{2}a^2 < a^2$

So, the area of a square is greater than the area of two identical equilateral triangles.

60. Consider a trapezium WXYZ with  $WX \parallel ZY$ . Find its area.



Construct  $WM \perp ZY$ , and  $XN \perp ZY$  (  $WM$  and  $XN$  perpendicular to  $ZY$  ).

**Ans. :** self

61. Give a method to convert a rectangle into a rhombus of equal area using dissection.

**Ans. :** This is the reverse of the rhombus to rectangle dissection.

Method:

Given: Rectangle PQRS with length  $l$  and width  $w$

Required: Rhombus with the same area =  $l \times w$

Dissection Process:

Step 1: The rhombus will have diagonals  $d_1$  and  $d_2$  such that:  $\frac{1}{2} \times d_1 \times d_2 = I \times w$

So,  $d_1 \times d_2 = 2lw$ .

Step 2: Choose convenient diagonal lengths:

Let  $d_1 = 2l$  (twice the rectangle length)

Then  $d_2 = w$  (same as rectangle width)

Check:  $\frac{1}{2} \times 2l \times w = lw$

Or

Let  $d_1 = 2w$  (twice the rectangle width)

Then  $d_2 = l$  (same as rectangle length)

Step 3: Dissection process (reverse of textbook method):

Divide the rectangle into two halves

Mark the center point O.

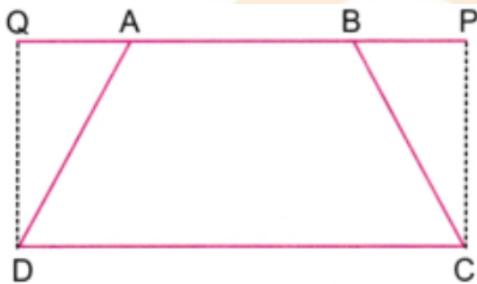
Cut and rotate pieces to form two isosceles triangles.

Arrange these triangles to share a common diagonal.

This creates a rhombus.

62. [Śulba-Sūtras] Give a method to convert an isosceles trapezium to a rectangle using dissection.

**Ans. :** An isosceles trapezium has special properties that make dissection simpler.



Properties of Isosceles Trapezium ABCD:

$AB \parallel CD$  (parallel sides)

$AD = BC$  (non-parallel sides are equal)

$\angle A = \angle B$  and  $\angle D = \angle C$  (base angles are equal)

Dissection Method:

Step 1: Draw perpendiculars from C and D to AB, meeting at points P and Q, respectively.

This creates rectangle PQDC in the middle.

Two congruent right triangles:  $\triangle APD$  and  $\triangle BQC$ .

Step 2: Since the trapezium is isosceles:

$\triangle APD \cong \triangle BQC$  (congruent triangles)

$AP = BQ$

Step 3: Rearrangement:

Cut triangle  $\triangle APD$

Rotate it and attach it to the right side (next to  $\triangle BQC$ )

The two triangles together form a rectangle with a width = height of the trapezium

Step 4: Combine: The central rectangle PQDC  
 The rectangle formed from the two triangles.  
 These can be joined to form one large rectangle.

Resulting Rectangle:

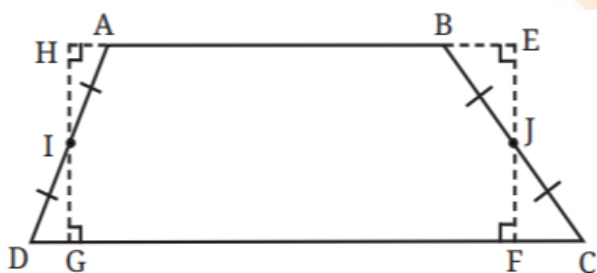
$$\begin{aligned} \text{Length} &= CD + AP \\ &= CD + (AB - CD)/2 \\ &= (AB + CD)/2 \end{aligned}$$

Width =  $h$  (height of trapezium)

$$\text{Area} = (AB + CD)/2 \times h = \frac{1}{2}h(AB + CD)$$

This matches the trapezium area formula!

63. Here is one of the ways to convert trapezium ABCD into a rectangle EFGH of equal area. Given the trapezium ABCD, how do we find the vertices of the rectangle EFGH?



**Ans. :** Given: Trapezium ABCD with  $AB \parallel CD$

Required: Find the positions of the vertices E, F, G, and H to form a rectangle EFGH with equal area

Using the Hint: If  $\triangle AHI \cong \triangle DGI$  and  $\triangle BEJ \cong \triangle CFJ$ , then areas are equal.

Method:

Step 1: The rectangle EFGH should have:

EF as one side (top side)

GH is the opposite parallel side (bottom side)

EH and FG are the other pair of sides.

Step 2: Position the rectangle such that:

Points I and J are strategically chosen on the trapezium.

$\triangle AHI$  (part outside rectangle on left) is cut and moved to fill  $\triangle DGI$ .

$\triangle BEJ$  (part outside rectangle on right) is cut and moved to fill  $\triangle CFJ$ .

Step 3: For congruency:

Mark I on side AD

Mark J on side BC

Choose positions such that:

$HI = GI$  (making  $\triangle AHI \cong \triangle DGI$  possible)

$EJ = FJ$  (making  $\triangle BEJ \cong \triangle CFJ$  possible)

Step 4: The height of the rectangle =  $h$  (height of the trapezium)

Step 5: The length of rectangle =  $(a + b)/2$

where  $a$  and  $b$  are parallel sides

This ensures: Area of rectangle =  $h \times (a + b)/2$  = Area of trapezium

Practical construction:

Draw the trapezium ABCD

Calculate required rectangle length =  $(AB + CD)/2$

Mark points H and E on AB such that the central portion has this length.

Draw perpendiculars to get rectangle EFGH.

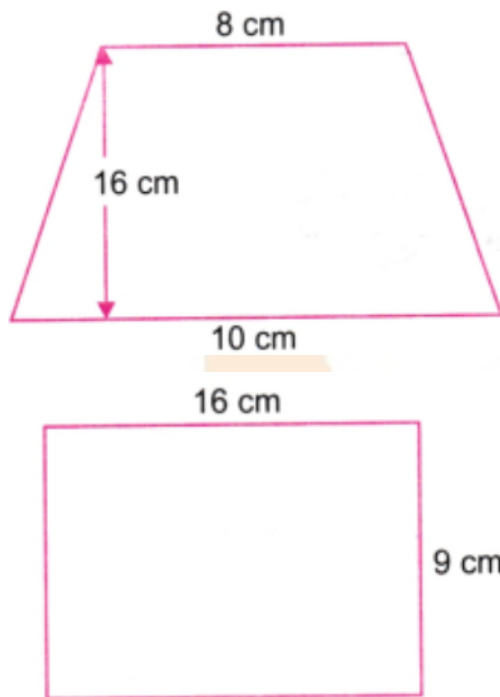
Verify that the triangular pieces outside match those inside.

This construction beautifully demonstrates area conservation through dissection!

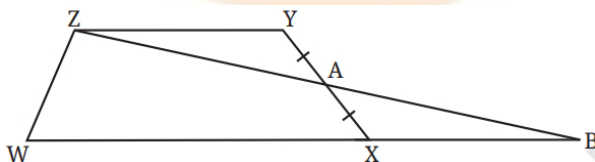
64. Using the idea of converting a trapezium into a rectangle of equal area, and vice versa, construct a trapezium of area  $144\text{cm}^2$ .

**Ans. :** Area of the trapezium =  $\frac{1}{2} \times (10 + 8) \times 16 = 144\text{cm}^2$

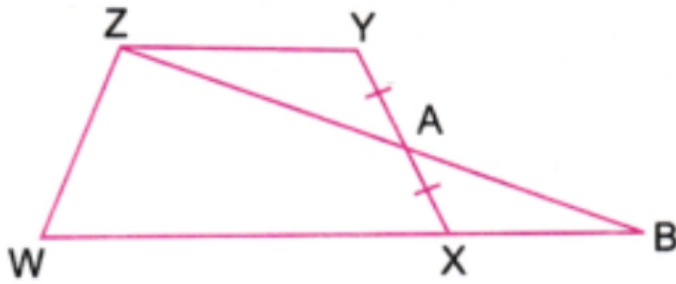
Area of square =  $16 \times 9 = 144\text{cm}^2$



65.  $ZYXW$  is a trapezium with  $ZY \parallel WX$ .  $A$  is the midpoint of  $XY$ . Show that the area of the trapezium  $ZYXW$  is equal to the area of  $\triangle ZWB$ .



Ans. :  $\angle ZAY = \angle BAX$  (Vertically opposite angles)



$AY = AX$  ( $\because$  A is mid point of  $XY$ )

$\angle YZB = \angle XBZ$  ( $\because$   $ZY \parallel XB$  alternate interior angles are equal)

$\angle ZYA = \angle BXA$  ( $\because$  they are alternate interior angles )

So  $\triangle ZAY \cong \triangle BAX$

By the AAA congruence.

So Area of  $\triangle ZAY =$  Area of  $\triangle BAY$

Thus, Area of trapezium  $ZYXW =$  Area of triangle  $ZWB$

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Student Bro