* Questions With Calculation.[2 Marks Each] [18]

1. How would you change this game to make the final answer 3? What about 5?

Ans. : (a) We can understand such tricks through algebra.
Step 1: Think of a number $= x$
Step 2: Triple it $= 3x$
Step 3: Add $9 = 3x + 9$
Step 4: Divide by $3 = \frac{3x+9}{3} = x + 3$
Step 5: Subtract the original number you thought of $(x+3) - x = 3$.
For Example:
Consider a number 23.
Triple it = 3 × 23 = 69
Add 9 = 69 + 9 = 78
Divide by $3 = 78 \div 3 = 26$
∴ 26 – 23 = 3
(b) To make the final answer 5
Step 1: Think of a number = x
Step 2: Double it = 2 x
Step 3: Add 10; $2x + 10$
Step 4: Divide by $2 = \frac{2x+10}{2} = x + 5$
Step 5: Subtract the original number $= x + 5 - x = 5$

2. Can you come up with more complicated steps that always lead to the same final value?

Ans. : Yes. Here is an example.
We can understand such tricks through algebra.
Step 1: Think of a number $= x$
Step 2. 5 times it $= 5x$
Step 3: Add $25 = 5x + 25$
Step 4: Divide by $5 = \frac{5x+25}{5} = x + 5$
Step 5: Subtract the original number you thought of $(x+5) - x = 5$
For Example:
Consider a number 18.
5 times 18 = 90
Add 25 = 90 + 25 = 115
Divide by 5 = 115 ÷ 5 = 23
∴ 23 – 18 = 5

3. Mukta thinks of another date, follows the same steps, and reports her answer as 1390. What date did Mukta start with this time?

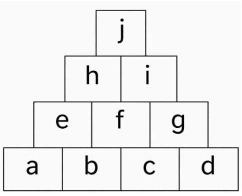
Ans. : self

4. Write an expression for the topmost row of a pyramid with 4 rows in terms of the values in the bottom row.

Ans. : Let a, b, c, d, e, f, g, h, i, and j be the elements of the pyramid.

Here, bottom row =

a	b	c	d

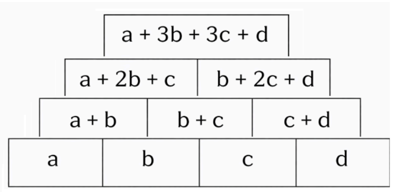


$\therefore$ e = a + b, f = b + c, g = c + d

h = e + f = (a + b) + (b + c) = a + 2b + c

i = f + g = (b + c) + (c + d) = b + 2c + d

j = h + i = (a + 2b + c) + (b + 2c + d) = a + 3b + 3c + d



Thus, the expression for the top row is (a + 3b + 3c + d).

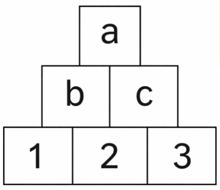
5. If the first three Virahāṅka-Fibonacci numbers are written in the bottom row of a number pyramid with three rows, fill in the rest of the pyramid. What numbers appear in the grid? What is the number at the top? Are they all Virahāṅka-Fibonacci numbers?

Ans. : We know that the first three Virahanka-Fibonacci number sequence = 1, 2, 3

Here, the bottom row

1	2	3

Let a, b, and c be the missing numbers.

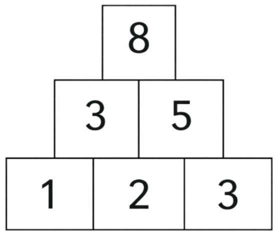


b = 1 + 2 = 3

c = 2 + 3 = 5

and a = b + c = 3 + 5 = 8

The complete pyramid is:



The numbers appear in the grid = 1, 2, 3, 3, 5, 8

∴ The number at the top = 8

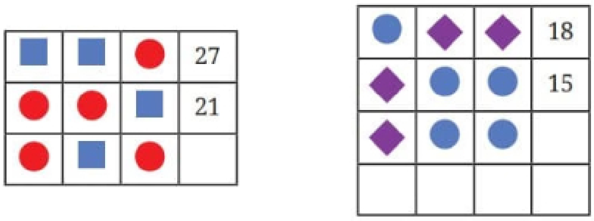
Yes, 1, 2, 3, 3, 5, 8 are Virahanka-Fibonacci numbers.

6. If the bottom row of an n row pyramid contains the first n Virahāṅka-Fibonacci numbers, what can we say about the numbers in the pyramid? What can we say about the number at the top?

Ans. : When the bottom row uses the first n Virahanka-Fibonacci numbers = (2n – 1)th

∴ The number at the top of the pyramid = (2n – 1)th Virahanka-Fibonacci number.

7. In the following grids, find the values of the shapes and fill in the empty squares:



Ans. : (a) We have,

$■ + ■ + ● = 27$ ....(i)

$● + ● + ■ = 21$ ....(ii)

Adding equations (i) and (ii), we get:

$3 \times ■ + 3 \times ● = 48$

$\Rightarrow ■ + ● = 16$ ....(iii)

From equations (i) and (iii), we get:

$■ + 16 = 27 \Rightarrow ■ = 27 - 16 = 11$

Put ■ = 11 in equation (*iii*), we get:

$11 + ● = 16 \Rightarrow ● = 16 - 11 = 5$

$\therefore$ ■ = 11 and ● = 5

Thus, | ● | ■ | ● | = 5+11+5=21.

(b)

●	◆	◆	18
◆	●	●	15
◆	●	●	

we have

● + ◆ + ◆ = 18 ...(*i*)

◆ + ● + ● = 15 ...(*ii*)

Adding equations (i) and (ii), we get:

3 × ● + 3 × ◆ = 33

⇒ ● + ◆ = 11 ....(iii)

From equations (i) and (iii), we get:

11 + ◆ = 18 ⇒ ◆ = 18 – 11 = **7**

Put ◆ = 7 in equation (*iii*), we get:

● + 7 = 11 ⇒ ● = 11 – 7 = **4**

∴ ◆ = 7 and ● = 4

Thus,

●	◆	◆	18
◆	●	●	15
◆	●	●	**15**
18	**15**	**15**	**12**

8. Fill the digits 1, 3, and 7 in __ __ x __to make the largest product possible.

Ans. : There are six ways to place three digits:
We can fill the first box with 1, 3, or 7.
For each of these choices, we have 2 ways of filling the remaining 2 digits.
The six choices are:

$\boxed{3}\ \boxed{7} \times \boxed{1}$ and $\boxed{7}\ \boxed{3} \times \boxed{1}$

$\boxed{1}\ \boxed{7} \times \boxed{3}$ and $\boxed{7}\ \boxed{1} \times \boxed{3}$

$\boxed{1}\ \boxed{3} \times \boxed{7}$ and $\boxed{3}\ \boxed{1} \times \boxed{7}$

In each pair, the one with the larger multiplicand generates the larger product, so we can reduce the comparison to these three expressions.

$\boxed{7}\ \boxed{3} \times \boxed{1}$ and $\boxed{7}\ \boxed{1} \times \boxed{3}$ $\boxed{3}\ \boxed{1} \times \boxed{7}$

It is clear that $\boxed{7}\ \boxed{1} \times \boxed{3}$ is greater than

$\boxed{7}\,\boxed{3} \times \boxed{1}$,so we only need to compare

$\boxed{7}\,\boxed{1} \times \boxed{3}$ and $\boxed{3}\,\boxed{1} \times \boxed{7}$.

Let us expand these two:

$\boxed{7}\,\boxed{1} \times \boxed{3} = (70 + 1) \times 3 = 70 \times 3 + 1 \times 3$

$= 10 \times 7 \times 3 + 1 \times 3 = 213$

$\boxed{3}\,\boxed{1} \times \boxed{7} = (30 + 1) \times 7 = 30 \times 7 + 1 \times 7$

$= 10 \times 3 \times 7 + 1 \times 7$

Thus, the first term in both expressions is equal.

The second term shows that $\boxed{3}\,\boxed{1} \times \boxed{7}$ = 277 it is the largest.

9. Fill the digits 3, 5, and 9 in _ _ x_ to make the largest product possible.

Ans. : There are six ways to place three digits:

We can fill the first box with 3, 5, or 9.

For each of these choices, we have 2 ways of filling the remaining 2 digits.

The six choices are:

$\boxed{5}\,\boxed{9} \times \boxed{3}$ and $\boxed{9}\,\boxed{5} \times \boxed{3}$

$\boxed{3}\,\boxed{9} \times \boxed{5}$ and $\boxed{9}\,\boxed{3} \times \boxed{5}$

$\boxed{3}\,\boxed{5} \times \boxed{9}$ and $\boxed{5}\,\boxed{3} \times \boxed{9}$

In each pair, the one with the larger multiplicand generates the larger product, so we can reduce the comparison to these three expressions.

$\boxed{9}\,\boxed{5} \times \boxed{3}$, $\boxed{9}\,\boxed{3} \times \boxed{5}$ and $\boxed{5}\,\boxed{3} \times \boxed{9}$

It is clear that $\boxed{9}\,\boxed{3} \times \boxed{5}$ is greater than $\boxed{9}\,\boxed{5} \times \boxed{3}$, so we only need to compare $\boxed{9}\,\boxed{3} \times \boxed{5}$ and $\boxed{5}\,\boxed{3} \times \boxed{9}$.

Let us expand these two:

$\boxed{9}\,\boxed{3} \times \boxed{5} = (90 + 3) \times 5 = 90 \times 5 + 3 \times 5 = 9{\times}10 \times 5 + 3 \times 5 = 465$

$\boxed{5}\,\boxed{3} \times \boxed{9} = (50 + 3) \times 9 = 50 \times 9 + 3 \times 9 = 5{\times}10 \times 9 + 3 \times 9 = 477$

Thus, the first term in both expressions is equal.

The second term shows that $\boxed{5}\,\boxed{3} \times \boxed{9}$ = 477 it is the largest.

* **Questions With Calculation.[3 Marks Each]** **[33]**

10. Find the dates if the final answers are the following:

i. 1269

ii. 394

iii. 296

Ans. : i. 1269

1269 = 100 M + 165 + D

Here, M = Month, D = Day

1269 – 165 = 100 M + D

$\Rightarrow 1104 = 100M + D$

$\Rightarrow 1100 + 04 = 100M + D$

$\therefore$ M = 11, D = 04

Thus, the date is 4th of November, i.e., 04/11.

ii. 394

394 = 100 M + 165 + D

$\Rightarrow 394 - 165 = 100M + D$

$\Rightarrow 229 = 100M + D$

$\Rightarrow 200 + 29 = 100M + D$

$\therefore$ M = 02, D = 29

Thus, the date is 29th of February, i.e., 29/02.

iii. 296 = 100 M + 165 + D

$\Rightarrow 296 - 165 = 100M + D$

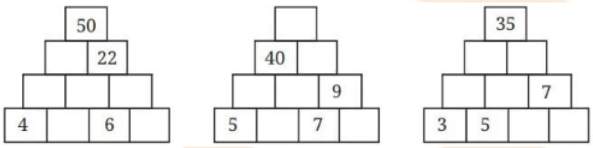
$\Rightarrow 131 = 100M + D$

$\Rightarrow 100 + 31 = 100M + D$

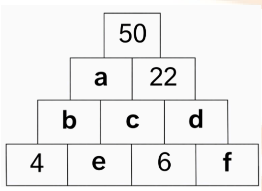
$\therefore$ M = 01, D = 31

Thus, the date is 31st of January, i.e., 31/01.

11. Fill the following pyramids:



Ans. : i. Let a, b, c, d, e, and f be the missing numbers.



a + 22 = 50

$\Rightarrow a = 50 - 22$

$\Rightarrow a = 28$

b + c = 28 .....(i)

c + d = 22 .....(ii)

Adding equations (i) and (ii), we get

b + d + 2c = 50 ......(iii)
Also, 4 + e = b .....(iv)
e + 6 = c ...(v)
6 + f = d ....(vi)
Adding equations (iv) and (v), we get
10 + 2e = b + c
10 + 2e = 28 [From (i)]
2e = 18
$\Rightarrow e = 9$
4 + 9 = b
$\Rightarrow b = 13$
9 + 6 = c
$\Rightarrow c = 15$
Putting c = 15 in equation (ii), we get
d = 22 – 15 = 7
$\Rightarrow d = 7$
Putting d = 7 in equation (vi), we get
6 + f = 7
$\Rightarrow f = 1$

50			
28	22		
13	15	7	
4	9	6	1

∴ a = 28, b = 13, c = 15, d = 7, e = 9 and f = 1.
ii. Let a, b, c, d, e, and f be the missing numbers.

a			
40	b		
c	d	9	
5	e	7	f

40 + b = a ...(i)
c + d = 40 ...(ii)
d + 9 = b ...(iii)
5 + e = c ...(iv)
e + 7 = d ...(v)
7 + f = 9
$\Rightarrow f = 2$
Adding equations (iv) and (v), we get
2e + 12 = c + d
$\Rightarrow 2e + 12 = 40$ [From (ii)]

$\Rightarrow 2e = 28$

$\Rightarrow e = 14$

c = 5 + 14 = 19

$\Rightarrow c = 19$[From (iv)]

14 + 7 = d

$\Rightarrow d = 21$

b = 21 + 9 = 30

$\Rightarrow b = 30$

a = 40 + 30 = 70

$\Rightarrow a = 70$

	70		
	40	30	
19	21	9	
5	14	7	2

∴ a = 70, b = 30, c = 19, d = 21, e = 14, and f = 2.

iii. Let a, b, c, d, e, and f be the missing numbers.

	35		
	a	b	
c	d	7	
3	5	e	f

a + b = 35 ...(i)

c + d = 40 ...(ii)

d + 7 = b ...(iii)

Adding equations (ii) and (iii), we get

c + d + d +7 = a + b

$\Rightarrow c + 2d = 35 - 7 = 28$[From (i)]

$\Rightarrow c + 2d = 28 \ldots (iv)$

Also, 3 + 5 = c

$\Rightarrow c = 8$

Putting c = 8 in equation (iv), we get

8 + 2d = 28

$\Rightarrow 2d = 28 - 8 = 20$

$\Rightarrow d = 10$

8 + 10 = a

$\Rightarrow a = 18$

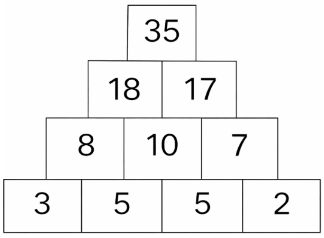
10 + 7 = b

$\Rightarrow b = 17$

5 + e = 10

$\Rightarrow e = 10 - 5 = 5$

$\Rightarrow e = 5$



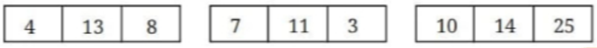
5 + f = 7

$\Rightarrow f = 7 - 5 = 2$

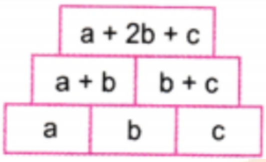
$\Rightarrow f = 2$

∴ a = 18, b = 17, c = 8, d = 10, e = 5, and f = 2.

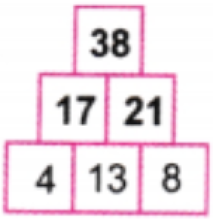
12. Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.



Ans. : We know that,



(a) Given, bottom row:



The number in the topmost row = 4 + 2 × 13 + 8

= 4 + 26 + 8

= 38

(b) Given, bottom row:

The number in the topmost row = 7 + 2 × 11 + 3

= 7 + 22 + 3

= 32

(c) Given, bottom row:

The number in the topmost row = 10 + 2 × 14 + 25

= 10 + 28 + 25
= 63

13. Without building the entire pyramid, find the number in the topmost row given the bottom row in each of these cases.

8	19	21	13

7	18	19	6

9	7	5	11

Recall the Virahāṅka-Fibonacci number sequence 1, 2, 3, 5,... where each number is the sum of the two numbers before it.

Ans. : (a) If a, b, c, and d are the bottom row, then the expression of the topmost row of the pyramid is a + 3b + 3c + d.
Given, bottom row;

8	19	21	13

Here, a = 8, b = 19, c = 21, and d = 13
∴ The number in the topmost row = a + 3b + 3c + d
= 8 + 3(19) + 3(21) + 13
= 8 + 57 + 63 + 13
= 141
Thus, the number in the topmost row is 141.
(b) Given, bottom row:

7	18	19	6

Here, a = 7, b = 18, c = 19 and d = 6
∴ The number in the topmost row = a + 3b + 3c + d
= 7 + 3(18) + 3(19) + 6
= 7 + 54 + 57 + 6
= 124
Thus, the number in the topmost row is 124.
(c) Given, bottom row:

9	7	5	11

Here, a = 9, b = 1,c = 5, and d = 11
∴ The number in the topmost row = a + 3b + 3c + d
= 9 + 3(7)+ 3(5) + 11
= 9 + 21 + 15 + 11
= 56
Thus, the number in the topmost row is 56.

14. What can you say about the numbers in the pyramid and the number at the top in the following cases?

i. The first four Virahāṅka-Fibonacci numbers are written in the bottom row of a four row pyramid.

ii. The first 29 Virahāṅka-Fibonacci numbers are written in the bottom row of a 29 row pyramid.

Ans. : i. We know that,

The first four Virahanka-Fibonacci numbers = 1, 2, 3, 5

Here, the bottom row

1	2	3	5

Let a, b, c, d, e, and f be the missing numbers.

	a		
	b	c	
d	e	f	
1	2	3	5

d = 1 + 2 = 3

e = 2 + 3 = 5

f = 3 + 5 = 8

b = d + e = 3 + 5 = 8

c = e + f = 5 + 8 = 13

and a = b + c = 8 + 13 = 21

	21		
	8	13	
3	5	8	
1	2	3	5

The numbers in the pyramid are 1, 2, 3, 5, 8, 8, 13, 21,....

We can say that the numbers are a Virahanka-Fibonacci sequence.

∴ The number at the top = 21

ii. From the above solution, we get

The number at the top = 2 × (Total number of digits present at the bottom) – 1

$= 2x - 1$

= 2 × 29 – 1

= 58 – 1

= 57th

Fibonacci numbers.

15. In the trick given above, what is the quotient when you divide by 9? Is there a relationship between the two numbers and the quotient?

Ans. : Let ab be the two-digit number. $(b > a)$

$\therefore ba > ab$

The difference is $(10b + a) - (10a + b) = 10b + a - 10a - b$

$= 9b - 9a$

$= 9(b - a)$, is divisible by 9.

When 9(b – a) is divided by 9, then the quotient is (b – a).

16. In the trick given above, instead of finding the difference of the two 2-digit numbers, find their sum. What will happen? Can we justify this claim using algebra?
    - We start with 31. After reversing we get 13. Adding 31 and 13, we get 44.
    - We start with 28. After reversing we get 82. Adding 28 and 82, we get 110.
    - We start with 12. After reversing we get 21. Adding 12 and 21, we get 33.

    Observe that all these numbers are divisible by 11. Is this always true? Can we justify this claim using algebra?

Ans. : 44, 110, 33 are divisible by 11.

Yes, it is always true.

44 = 4 – 4 = 0, divisible by 11.

110 = (1 + 0) – 1 = 0, divisible by 11.

33 = 3 – 3 = 0, divisible by 11.

Using Algebra

Original number = 10a + b

Reversed number = 10b + a

Sum = 10a + b + 10b + a = 11(a + b)

Hence, the sum is always divisible by 11.

17. Consider any 3-digit number, say abc (100a + 10 b + c) . Make two other 3-digit numbers from these digits by cycling these digits around, yielding bca and cab. Now add the three numbers. Using algebra, justify that the sum is always divisible by 37. Will it also always be divisible by 3?

Ans. : abc = 100a + 10b + c

bca = 100b + 10c + a

cab = 100c + 10a + b

Sum of abc + bca + cab = 111a + 111b + 111c

= 111(a + b + c)

= 37 × 3(a + b + c), is always divisible by 37.

111 = 1 + 1 + 1 = 3, is always divisible by 3.

For example:

Consider a number 153.

Other two numbers = 531 and 315

Sum = 153 + 531 + 315 = 999

999 = 37 × 27, which is divisible by 37.

999 = 9 + 9 + 9 = 27, which is also divisible by 3.

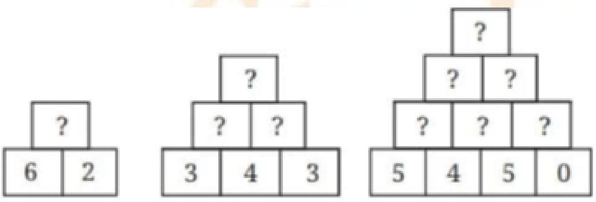
18. A mother is 5 times her daughter's age. In 6 years' time, the mother will be 3 times her daughter's age. How old is the daughter now?

Ans. : Let the present age of the daughter = x years
And the present age of her mother = y years
According to the question,
$5(x) = y$
$\Rightarrow 5x = y \ldots\ldots(i)$
In 6 years,
$3(x+6) = y+6$
$\Rightarrow 3x+18 = y+6$
$\Rightarrow 3x+18-6 = y$
$\Rightarrow 3x+12 = y \ldots\ldots(ii)$
From equations (i) and (ii), we get
$3x+12 = 5x$
$\Rightarrow 5x-3x = 12$
$\Rightarrow 2x = 12$
$\Rightarrow x = 6$
The present age of the daughter = 6 years

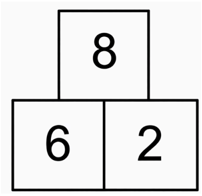
19. Two friends, Gauri and Naina, are cowherds. One day, they pass each other on the road with their cows. Gauri says to Naina, "You have twice as many cows as I do". Naina says, "That's true, but if I gave you three of my cows, we would each have the same number of cows". How many cows do Gauri and Naina have?

Ans. : Let x, y be the number of cows of Gauri and Naina.
According to the question,
$2x = y \ldots$ (i)
Also, $x+3 = y-3$
$x-y = -3-3 = -6$.....(ii)
From equations (i) and (ii), we get
$x-2x = -6$
$\Rightarrow -x = -6$
$\Rightarrow x = 6$
Putting x = 6 in equation (i), we get,
y = 2 × 6 = 12
Thus, Gauri and Naina have 6 and 12 cows, respectively.

20. Use the same rule to fill these pyramids:

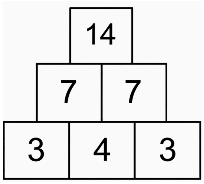


Ans. : Each number is the sum of the two numbers directly below it.
i.



8 = 6 + 2

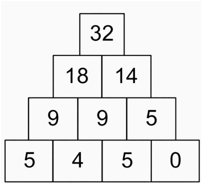
ii.



3 + 4 = 7; 4 + 3 = 7 and 7 + 7 = 14

iii. 5 + 4 = 9; 4 + 5 = 9 and 5 + 0 = 5;

9 + 9 = 18; 9 + 5 = 14 and 18 + 14 = 32.



* Questions With Calculation.[5 Marks Each] [35]

21. Consider any 3-digit number, say abc. Make it a 6-digit number by repeating the digits, that is abcabc. Divide this number by 7, then by 11, and finally by 13. What do you get? Try this with other numbers. Figure out why it works.

Ans. : Given that abc is a 3-digit number.

abc = 100a + 10b + c

Make it a 6-digit number = abcabc

= 100000a + 10000b + 1000c + 100a + 10b + c

= 100100a + 10010b + 1001c

= 1001(100a + 10b + c)

The smallest number, divisible by 7, 11, and 13 = LCM (7, 11, 13)

= 7 × 11 × 13

= 1001

∴ abcabc = 1001(100a + 10b + c), is divisible by 7, 11, and 13.

Consider 836 a 3-digit number.

Make it 6-digit number = 836836 = 1001 × 836

∴ 836836 is divisible by 7, 11, and 13.

This works because 10001 = 7 × 11 × 13, and repeating a 3-digit number creates a multiple of 1001.

22. There are 3 shrines, each with a magical pond in the front. If anyone dips flowers into these magical ponds, the number of flowers doubles. A person has some flowers. He dips them all in the first pond and then places some flowers in shrine

1. Next, he dips the remaining flowers in the second pond and places some flowers in shrine 2. Finally, he dips the remaining flowers in the third pond and then places them all in shrine 3. If he placed an equal number of flowers in each shrine, how many flowers did he start with? How many flowers did he place in each shrine?

Ans. : Let x be the initial number of flowers, and k be the equal number of flowers placed in each of the three shrines.

In shrine 1, the remaining flowers $= 2x - k$

In shrine 2, the remaining flowers $= 2(2x - k) - k$

$= 4x - 2k - k$

$= 4x - 3k$

In shrine 3 , the remaining flowers $= 2(4x - 3k) - k$

$= 8x - 6k - k$

$= 8x - 7k$

$\therefore 8x - 7k = 0$

$\Rightarrow 8x = 7k$

$\Rightarrow x = \frac{7k}{8}$

For the minimum possible number of flowers, we use the smallest positive integer k , which is $k = 8$.

$\therefore x = \frac{7\times8}{8} = 7$

Thus, the person started with 7 flowers and placed 8 flowers in each shrine.

23. A farm has some horses and hens. The total number of heads of these animals is 55 and the total number of legs is 150. How many horses and how many hens are on the farm?

Ans. : Method 1: Using Algebra

Let x and y be the number of horses and hens, respectively.

According to the questions,

$x + y = 55 \ldots\ldots\ldots$ (i)

And, $4x + 2y = 150$

$\Rightarrow 2x + y = 75$...(ii)

Subtracting (i) from (ii), we get

$2x + y - x - y = 75 - 55$

⇒ x = 20

Putting $x = 20$ in equation (i), we get

$20 + y = 55$

$\Rightarrow y = 55 - 20 = 35$

Thus, the number of horses = 20 and the number of hens = 35.

Method 2: (without letter numbers)

If all 55 animals were hens

Total legs would be 55 × 2 = 110 legs

But actual legs = 150

Difference = 150 – 110 = 40 legs

Each time we replace a hen with a horse.

We remove 2 legs (hen) and add 4 legs (horse).

Net increase = 2 legs

Number of horses needed $= 40 \div 2 = 20$

Number of hens = 55 – 20 = 35

24. I run a small dosa cart and my expenses are as follows:

- Rent for the dosa cart is ₹5000 per day.
- The cost of making one dosa (including all the ingredients and fuel) is ₹10.

i. If I can sell 100 dosas a day, what should be the selling price of my dosa to make a profit of ₹2000?

ii. If my customers are willing to pay only ₹ 50 for a dosa, how many dosas should I aim to sell in a day to make a profit of ₹ 2000?

Ans. : Given, rent for the dosa cart = ₹ 5000/day.

The total cost of making one dosa = ₹ 10

i. Given,

Number of dosas = 100

∴ The cost of making 100 dosas = 100 × ₹ 10 = ₹ 1000

Total cost price = Rent for the dosa cart + The cost of making 100 dosas

= ₹ 5000 + ₹ 1000

= ₹ 6000

Profit = ₹ 2000

∴ Total selling price = ₹ 6000 + ₹ 2000 = ₹ 8000

The selling price of one dosa $= \frac{8000}{100} =$ ₹80

ii. Let n be the number of dosa.

Then total cost price = n × ₹ 10 + ₹ 5000

Total selling price = n × ₹ 50

Profit = ₹ 2000

S.P = C.P + profit

$\Rightarrow 50n = 10n + 5000 + 2000$

$\Rightarrow 50n - 10n = 5000 + 2000$

$\Rightarrow 40n = 7000$

$\Rightarrow n = \frac{7000}{40}$

$\Rightarrow n = 175$

So, I should sell 175 dosa.

25. Evaluate the following sequence of fractions:

$\frac{1}{3}, \frac{(1+3)}{(5+7)}, \frac{(1+3+5)}{(7+9+11)}$

What do you observe? Can you explain why this happens?

Ans. : $\frac{1}{3} = \frac{1}{3}$,

$\frac{1+3}{5+7} = \frac{4}{12} = \frac{1}{3}$,

$\frac{1+3+5}{7+9+11} = \frac{9}{27} = \frac{1}{3}$

Thus, the given sequences are equivalent fractions.

We know that the sum of the first n odd numbers is n2.

Numerators:

1 = 12 = 1

1 + 3 = 22 = 4

1 + 3 + 5 = 32 = 9

Denominators:

3 = 3 × 12

5 + 7 = 12 = 3 × 22

7 + 9 + 11 = 27 = 3 × 32

Thus, each fraction is $\frac{n^2}{3n^2} = \frac{1}{3}$

26. Karim was taking a nap under a tree. He had a dream about a magical lamp and a genie. He heard a voice saying, "I have come to serve you, Oh master". He woke up and to his surprise, it was a genie!

"Do you want to make money?", asked the genie. Karim nodded dumbly in bewilderment. The genie continued, "Do you see the banyan tree over there? All you have to do is go around it once. The money in your pocket will double".

Karim immediately started towards the tree, only to be stopped by the genie. "One moment!", said the genie. "Since I am bringing you great riches, you should share some of your gains with me. You must give me 8 coins each time you go around the tree."

Thinking that was a trifling amount, Karim readily agreed.

He went around the tree once. Just as the genie had said, the number of coins in his pocket doubled! He gave 8 coins to the genie. He made another round. Again the number of coins doubled. He gave 8 more coins to the genie. He went around the tree for the third time. The number of coins doubled again, but to his horror, he was left with only 8 coins, exactly the number of coins he owed the genie!

As Karim began to wonder how the genie tricked him, the genie let out a loud laugh and disappeared.

i. How many coins did Karim initially have?

ii. For what cost per round should Karim agree to the deal, if he wants to increase the number of coins he has?

iii. Through its magical powers, the genie knows the number of coins that Karim has. How should the genie set the cost per round so that it gets all of Karim's coins?

Ans. : i. Let Karim have n coins initially

No. of coins after 1st round = 2n – 8

No. of coins after 2nd round = [2(2n – 8)] – 8

= 4n – 16 – 8

= 4n – 24

No. of coins after 3rd round = 2(4n – 24) – 8 = 0

$\Rightarrow 2(4n - 24) - 8 = 0$

$\Rightarrow 8n - 48 - 8 = 0$

$\Rightarrow 8n = 56$

$\Rightarrow n = 7$

So, Karim initially had 7 coins.

ii. Let c = cost per round (coins to give genie)

Starting with 7 coins:

After round 1: 2(7) – c = 14 – c

After round 2: 2(14 – c) – c = 28 – 3c

After round 3: 2(28 – 3c) – c = 56 – 7c

For Karim to increase his coins:

$56 - 7c > 7$

$\Rightarrow 56 - 7 > 7c$

$\Rightarrow 49 > 7c$

$\Rightarrow c < 7$

The cost per round should be less than 7 coins.

For example, if c = 6:

After 3 rounds: 56 – 7(6) = 56 – 42 = 14 coins (doubled his money!)

iii. Let Karim start with n coins, and let c = cost per round.

After 3 rounds, Karim has: 8n – 7c coins

For the genie to get all coins:

8n – 7c = 0

$\Rightarrow 7c = 8n$

$\Rightarrow c = 8n/7$

The genie should charge ($8n$)/7 coins per round, where n is Karim's starting amount.

For this to be a whole number, n must be a multiple of 7.

27. Create your own calendar trick. For instance, choose a grid of a different size and shape.

Ans. : i. Add the 5 numbers in this grid and tell the sum.

	1	
7	8	9
	15	

1 + 7 + 8 + 9 + 15 = 40

Let 'a' represent the topmost number.

	a	
a+6	a+7	a+8
	a+14	

My own calendar trick.

Sum = a + (a + 6) + (a + 7) + (a + 8) + (a + 14) = 5a + 35.

Consider a 3 × 3 grid. Add the 9 numbers.

10	11	12
17	18	19
24	25	26

(10 + 11 + 12) + (17 + 18 + 19) + (24 + 25 + 26) = 33 + 54 + 75 = 162

Let 'a' represent the top left number.

a	a + 1	a + 2
a + 7	a + 8	a + 9
a + 14	a + 15	a + 16

My own calendar trick.

Sum = a + (a + 1) + (a + 2) + (a + 7) + (a + 8) + (a + 9) + (a + 14) + (a + 15) + (a + 16)

= 9a + 72

= 9(a + 8)

Consider a 1 × 3 grid.

Add the 3 numbers.

28	29	30

28 + 29 + 30 = 87

Let ‘a’ represent the left number.

a	$a+1$	$a+2$

My own calendar trick.

Sum = a + (a + 1) + (a + 2)

= 3a + 3

= 3(a + 1)

ii.

		3		
		13		
21	22	23	24	25
		33		
		43		

Let ‘a’ represent the topmost number.

		a		
		a+10		
a+18	a+19	a+20	a+21	a+22
		a+30		
		a+40		

My own trick.

18	19	
	29	30

Let ‘a’ represent the top left number.

a	a+1	
	a+11	a+12

This trick works.

	27	
	37	
46	47	48

Let ‘a’ be the top-most number.

	a	
	a+10	
a+19	a+20	a+21

This trick works.

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