*** Answer The Following Questions In One Sentence.[1 Marks Each]** [3]

1. Is $\sqrt{2}$ less than or greater than 1?

   Ans. : self

2. Is $\sqrt{2}$ less than or greater than 2?

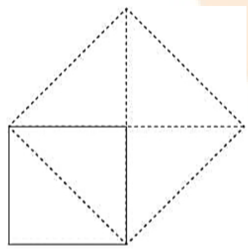
   Ans. : self

3. Every Baudhāyana triple is either a primitive triple or a scaled version of a primitive triple.
   a. True
   b. False

   Ans. : Let (a, b, c) be a Baudhayana triple.
   Then HCF (a, b, c) = 1 or HCF (a, b, c) $\neq$ 1
   (3, 4, 5): HCF is 1
   (10, 24, 26): HCF = 2
   If HCF is 1, then the triple is primitive.
   If HCF is a number other than 1, then the triple is scaled.

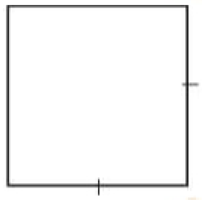
*** Questions With Calculation.[2 Marks Each]** [26]

4. Why does the new dotted square have double the area of the original square?



   Ans. : self

5. Now suppose we are given a square, and we want to construct a square whose area is half that of the original square. How would you do it?



   Ans. : self

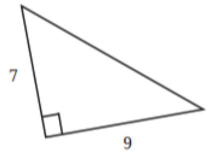
6. What is the value of $\sqrt{2}$?

   Ans. : self

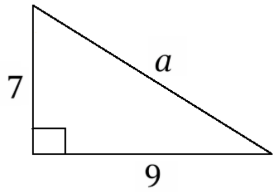
7. Is (5, 12, 13) a primitive Baudhāyana triple? What are the other primitive Baudhāyana triples with numbers less than or equal to 20?

Ans. : self

8. Find the missing sidelengths in the right triangle:



Ans. :



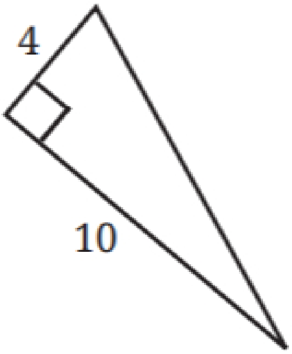
$a^2 = 7^2 + 9^2$

$= 49 + 81$

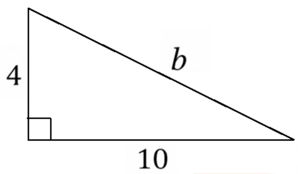
$= 130$

$\Rightarrow a = \sqrt{130}$

9. Find the missing sidelengths in the right triangle:



Ans. :



$b^2 = 4^2 + 10^2$

$= 16 + 100$

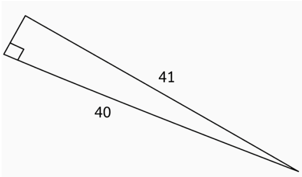
$= 116$

$\Rightarrow b = \sqrt{116}$

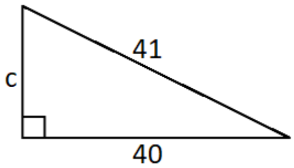
$= \sqrt{2 \times 2 \times 29}$

$= 2\sqrt{29}$

10. Find the missing sidelengths in the right triangle:



Ans. :



$40^2 + c^2 = 41^2$

$\Rightarrow 1600 + c2 = 1681$

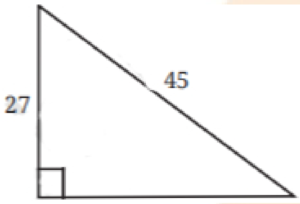
$\Rightarrow c^2 = 1681 - 1600$

$\Rightarrow c^2 = 81$

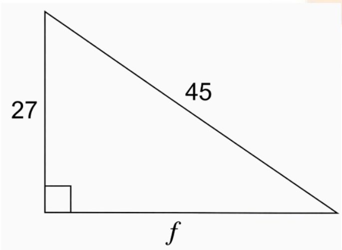
$\Rightarrow c = \sqrt{81}$

$\Rightarrow c = 9$

11. Find the missing sidelengths in the right triangle:



Ans. :



$27^2 + f^2 = 45^2$

$\Rightarrow 729 + f^2 = 2025$

$\Rightarrow f^2 = 2025 - 729$

$\Rightarrow f^2 = 1296$

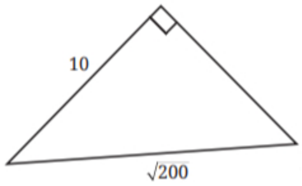
$\Rightarrow f = \sqrt{1296}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

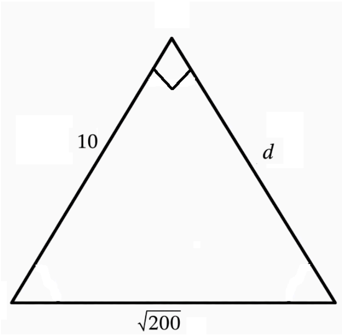
$= 2 \times 2 \times 3 \times 3$

$= 36$

12. Find the missing sidelengths in the right triangle:



Ans. :



$10^2 + d^2 = (\sqrt{200})^2$

$\Rightarrow 100 + d^2 = 200$

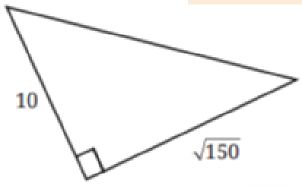
$\Rightarrow d^2 = 200 - 100$

$\Rightarrow d^2 = 100$

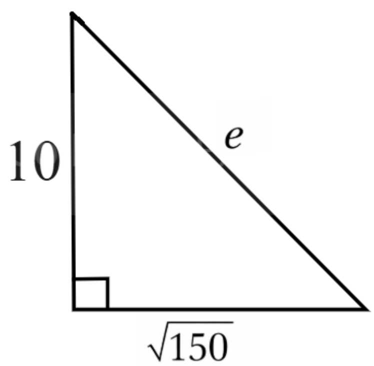
$\Rightarrow d = \sqrt{100}$

$\Rightarrow d = 10$

13. Find the missing sidelengths in the right triangle:



Ans. :



$e^2 = 10^2 + (\sqrt{150})^2$

$\Rightarrow e^2 = 100 + 150$

$\Rightarrow e^2 = 250$

$\Rightarrow e = \sqrt{250}$

$\Rightarrow e=\sqrt{5\times5\times5\times2}$

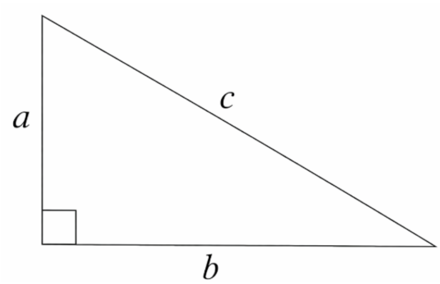
$\Rightarrow e=5\sqrt{10}$

14. Suppose the grid extends indefinitely. What are the possible integer-valued areas of squares you can create in this manner?

Ans. : Let the given value of area be x , where ' x ' is an integer. Then, $x=a^2+b^2$, where ' a ' and ' b ' are integers or x is a perfect square, we can create squares with vertices as dots of the grid.

15. Let a, b and c denote the length of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in a = 9, c = 15.

Ans. :



Here, $15^2=9^2+b^2$

$b^2=15^2-9^2$

$=225-81$

$=144$

$\Rightarrow b=\sqrt{144}$

$=\sqrt{2\times2\times2\times2\times3\times3}$

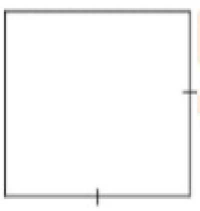
$=12$

16. Find the hypotenuse of an isosceles right triangle whose equal sides have length 12.

Ans. : self

*** Questions With Calculation.[3 Marks Each]** **[36]**

17. How can one construct a square having double the area of a given square?
A first guess might be to simply double the length of each side of the square. Will this new square have double the area of the original square?



Ans. : self

18. Can $\sqrt{2}$ be expressed as a fraction m/n, where m and n are counting numbers?

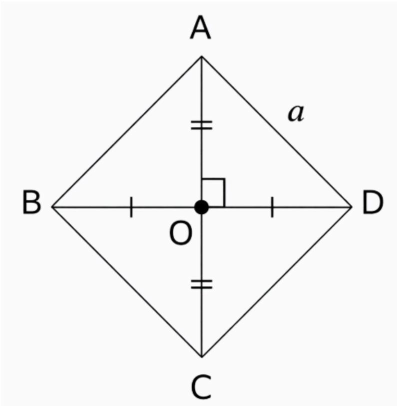
Ans. : self

19. Is there an unending sequence of Baudhāyana triples?

Ans. : self

20. Find the sidelength of a rhombus whose diagonals are of length 24 units and 70 units.

Ans. :



$OA = \frac{1}{2} \times 24 = 12$

$OD = \frac{1}{2} \times 70 = 35$

In $\triangle AOD$,

$a^2 = 12^2 + 35^2$

$= 144 + 1225$

= 1369

$a = \sqrt{1369} = 37$

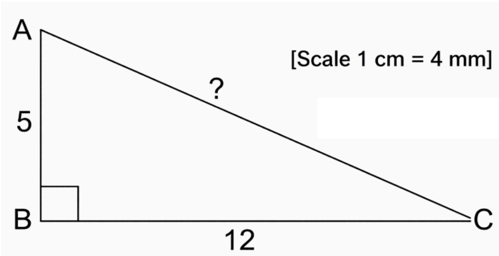
$\therefore$ The side of the rhombus is 37 units.

21. Does this method yield non-primitive Baudhāyana triples?

Ans. : (24, 7, 25)
HCF of 24, 7, 25 is 1.
(40, 9, 41)
HCF of 40, 9, 41 is 1, etc.
The triples generated by the above method are primitive in nature.

22. If a right-angled triangle has shorter sides of lengths 5 cm and 12 cm, then what is the length of its hypotenuse? First draw the right-angled triangle with these sidelengths and measure the hypotenuse, then check your answer using Baudhāyana's Theorem.

Ans. : Given AB = 5 cm
BC = 12 cm
AC = 13 cm (by measurement)



Now, using Baudhayana's Theorem

$AC^2 = AB^2 + BC^2$

$= 5^2 + 12^2$

$= 25 + 144$

= 169

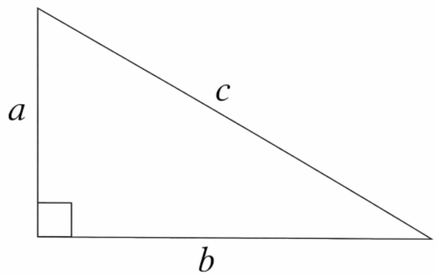
$= 13^2$

$\therefore AC = 13cm$

Therefore, the hypotenuse of the right-angled triangle is 13 cm, according to Baudhayana's Theorem.

23. Let a, b and c denote the length of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in a = 5, b = 7.

Ans. :



Now, $c^2 = 5^2 + 7^2$

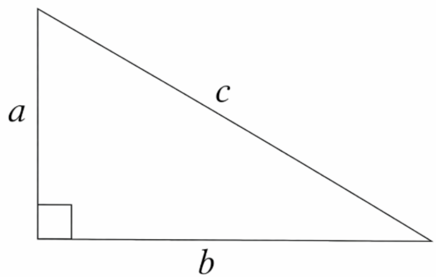
$= 25 + 49$

= 74

$\Rightarrow c = \sqrt{74}$

24. Let a, b and c denote the length of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in a = 8, b = 12.

Ans. :



Now, $c^2 = 8^2 + 12^2$

= 64 + 144

= 208

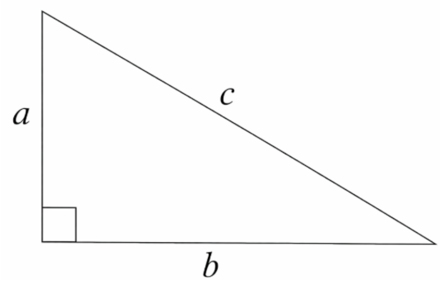
$\Rightarrow c = \sqrt{208}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 13}$

$= 4\sqrt{13}$

25. Let a, b and c denote the length of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in a = 7, b = 12.

Ans. :



$c^2 = 7^2 + 12^2$

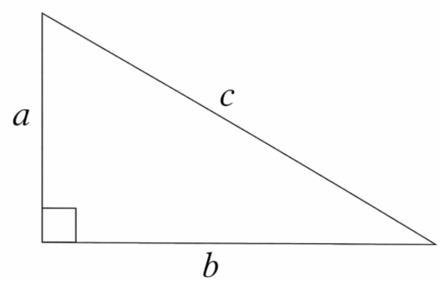
$= 49 + 144$

= 193

$\Rightarrow c = \sqrt{193}$

26. Let a, b and c denote the length of the sides of a right triangle, with c being the length of the hypotenuse. Find the missing sidelength in a = 1.5, b = 3.5.

Ans. :



$c^2 = 1.5^2 + 3.5^2$

$= 2.25 + 12.25$

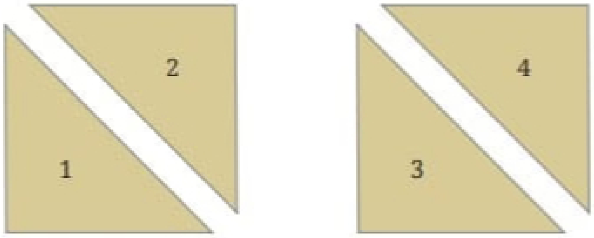
$= 14.5$

$\Rightarrow c = \sqrt{14.5}$

27. If the hypotenuse of an isosceles right triangle is $\sqrt{72}$, find its other two sides.

Ans. : self

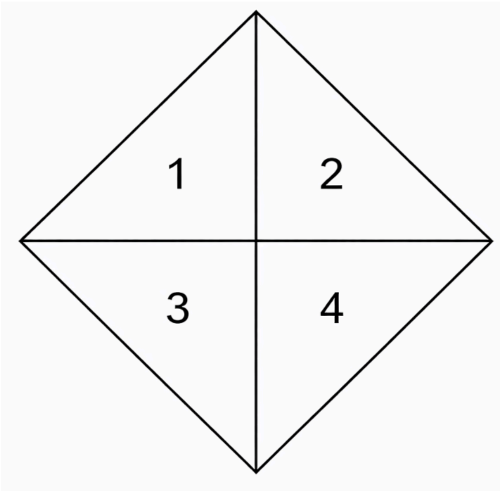
28. Earlier, we saw a method to create a square with double the area of a given square paper. There is another method to do this in which two identical square papers are cut in the following way.



Can you arrange these pieces to create a square with double the area of either square?"

Ans. : Two identical squares

These two are cut diagonally, forming two equal triangles, as shown in the figure.



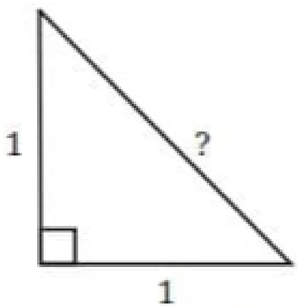
Thus, we have four identical triangles from two identical squares.

Area of two triangles = Area of the given square

Area of four triangles = double the area of the given square.

*** Questions With Calculation.[5 Marks Each]** **[75]**

29. Find the hypotenuse of this isosceles right triangle.



Ans. : self

30. List down all the Baudhāyana triples with numbers less than or equal to 20.

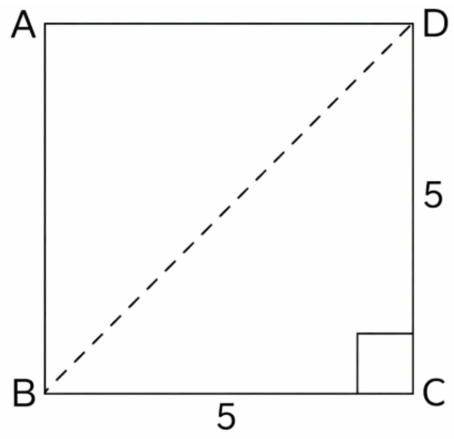
Ans. : self

31. Generate 5 scaled versions of each of these primitive triples. Are these scaled versions primitive?

Ans. : self

32. Find the diagonal of a square with sidelength 5 cm.

Ans. :



$BD^2 = 5^2 + 5^2$

$= 25 + 25$

= 50

$BD = \sqrt{50}$

$= \sqrt{5 \times 5 \times 2}$

$= 5\sqrt{2}$

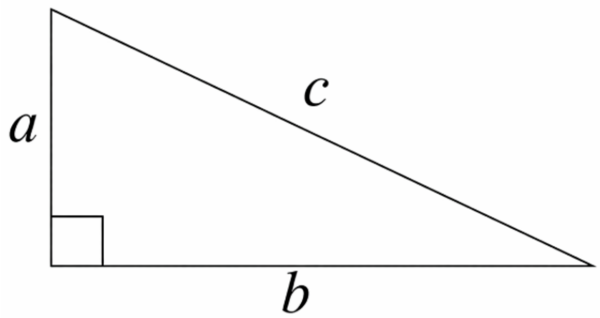
$= 5 \times 1.414$

= 7.07 (~ 7.1)

Hence, the length of the diagonal is $5\sqrt{2}(7.1cm)$ approx.

33. Is the hypotenuse the longest side of a right triangle? Justify your answer.

Ans. :



$c^2 = a^2 + b^2$

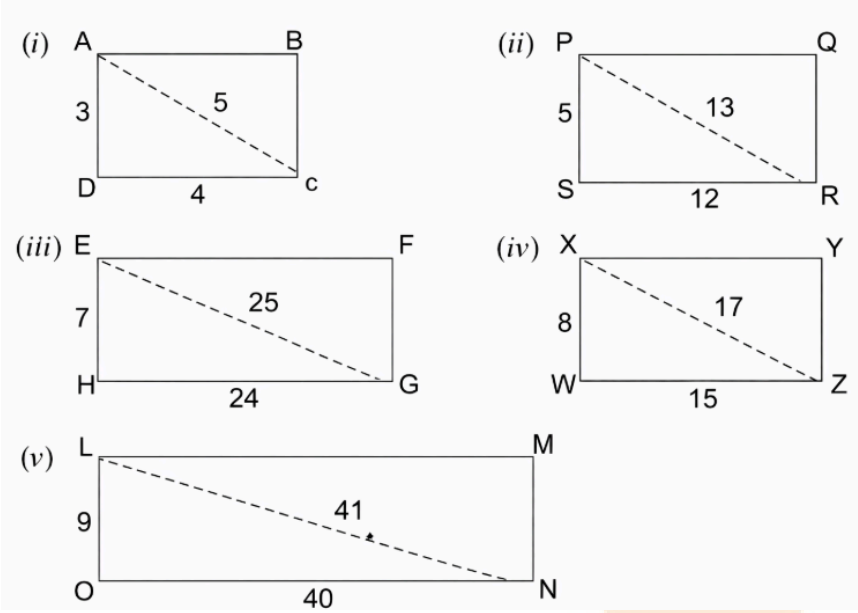
$\therefore c^2 > a^2$ and $c^2 > b^2$

or $c > a$ and $c > b$

Hence, 'c' is the longest side of the right triangle.

34. Give 5 examples of rectangles whose sidelengths and diagonals are all integers.

Ans. :

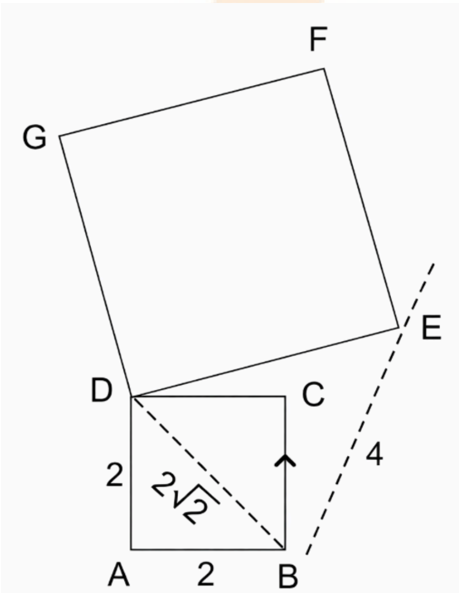


35. Construct a square whose area is equal to the difference of the areas of squares of sidelengths 5 units and 7 units.

Ans. : Area of square $= 7^2 - 5^2$

$= 49 - 25$

= 24 sq. units



1. Construct a square $ABCD$ with a side of 2 cm .

Then $DB = 2\sqrt{2}$ units

2. Draw $BE \perp DB$ at B such that $BE = 4$ units

3. Join DE

$DE^2 = (2\sqrt{2})^2 + 4^2$

$= 8 + 16$

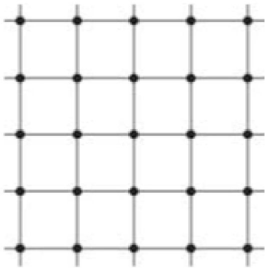
$= 24$

$\Rightarrow DE = \sqrt{24}$

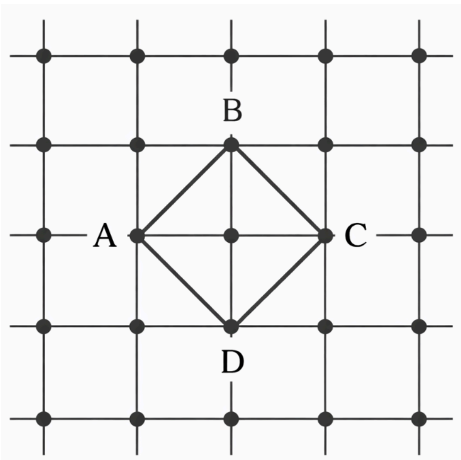
4. Draw a square with side DE (DEFG).

Area of $DFFG = (\sqrt{24})^2 = 24$ square units.

36. Using the dots of a grid as the vertices, can you create a square that has an area of (a) 2 sq. units, (b) 3 sq. units, (c) 4 sq.units, and (d) 5 sq. unit?



Ans. : (i) (a) $2 = 1^2 + 1^2$



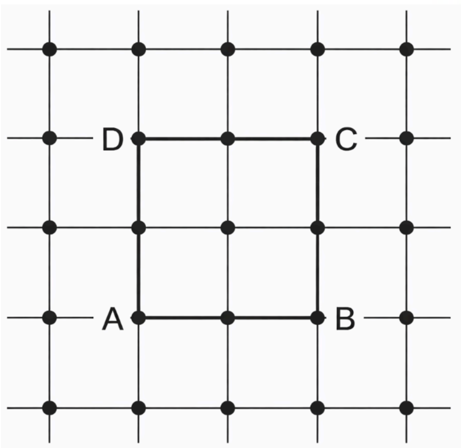
Mark dots A, B, C, and D as shown.

Join AB, BC, CD, and DA.

Then ABCD is a square and area ABCD = 2 sq. units

(b) Square with area 3 units is not possible as $3 \neq a^2 + a^2$ for any integer ‘a’.

(c) $4 = 2 \times 2$



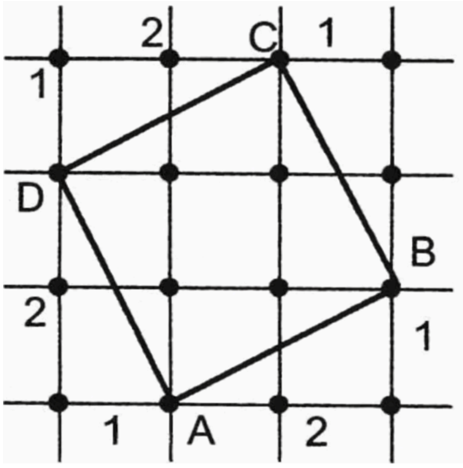
Mark dots A, B, C, D as shown.

Join AB, BC, CD, DA.

Then ABCD is a square.

and ar ABCD = $2 \times 2 = 4$ sq. units

(d) (i)



Mark dots A, B, C, and D as shown.

Join A, B, C, and D

$AB^2 = 2^2 + 1^2 = 5$

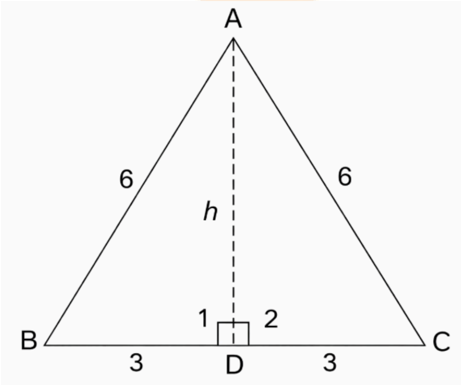
$AB = \sqrt{5}$ units

Hence, ABCD is a square with an area of 5 sq units.

37. Find the area of an equilateral triangle with sidelength 6 units.

Ans. : Let $\triangle ABC$ be an equilateral triangle.

$AB = BC = CA = 6cm$



Let AD be perpendicular to BC.

Then $\angle 1 = \angle 2$ (each $= 90^\circ$)

AB = AC (each = 6 cm)

AD = AD (common)

$\triangle ADB \cong \triangle ADC (RHS)$

BD = DC (CPCT)

$\therefore BD = DC = \frac{1}{2} \times 6cm = 3cm$

In $\triangle ADC$,

$h^2 + 3^2 = 6^2$ (Baudhayana's triple)

$\Rightarrow h^2 = 36 - 9 = 27$

$\Rightarrow h = \sqrt{3 \times 3 \times 3}$

$\Rightarrow h = 3\sqrt{3}cm$

$ArABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 6 \times 3\sqrt{3}$ sq. units

$= 9\sqrt{3}$ sq. units

38. Find 5 more Baudhāyana triples using this idea.

Ans. : $(1 + 3 + 5 + \ldots + 47) + 49 = 25^2$

$24^2 + 7^2 = 25^2 (24, 7, 25)$

$(1 + 3 + 5 + \ldots + 79) + 81 = 41^2$

$40^2 + 9^2 = 41^2 (40, 9, 41)$

$(1 + 3 + 5 + \ldots + 119) + 121 = 61^2$

$60^2 + 11^2 = 61^2 (60, 11, 61)$

$(1 + 3 + 5 + \ldots + 167) + 169 = 85^2$

842 + 132 = $852 (84, 13, 85)$

$(1 + 3 + 5 + \ldots + 223) + 225 = 113^2$

$112^2 + 15^2 = 113^2 (112, 15, 113)$

39. Are there primitive triples that cannot be obtained through this method? If yes, give examples.

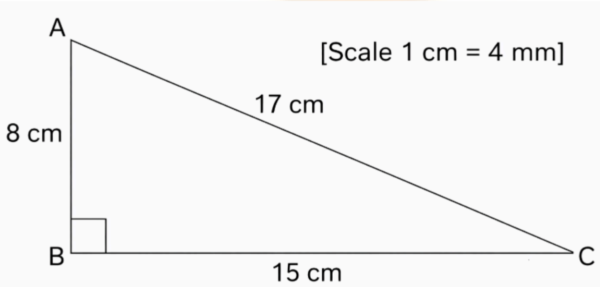
Ans. : In each of the above case we have taken the sum of the first 'n' odd numbers where 'n' is a perfect square.
We observe that the smallest number in the triple is always odd.
Consider triples such as (8, 15, 17), (16, 63, 65), etc.
Such triples cannot be generated by this method.

40. If a right-angled triangle has a short side of length 8 cm and hypotenuse of length 17 cm, what is the length of the third side? Again, try drawing the triangle and measuring, and then check your answer using Baudhāyana's Theorem.

Ans. : Here, $AB = 8cm$

$AC = 17cm$

BC = 15 cm (by measurement)



Now, using Baudhayana's Theorem

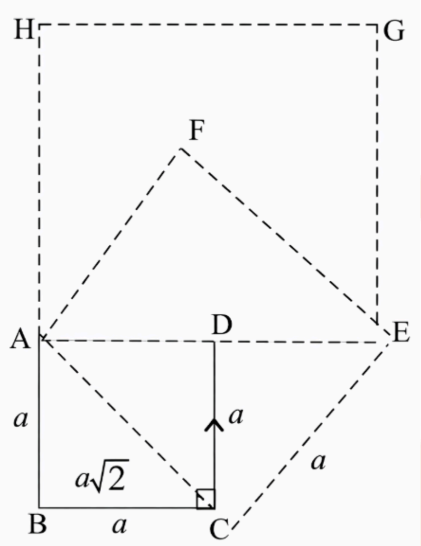
$AB^2 + BC^2 = AC^2$

$8^2 + BC^2 = 17^2$

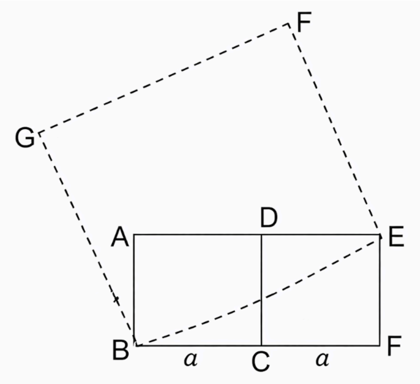
$BC^2 = 289 - 64$
$= 225$
$= 15^2$
$\therefore BC = 15 cm$
Therefore, the other side of the right-angled triangle is 15 cm , which satisfies Baudhayana's Theorem.

41. Using the constructions you have now seen, how would you construct a square whose area is triple the area of a given square? Five times the area of a given square? (Baudhāyana's Śulba-Sūtra, Verse 1.10)

Ans. : (a) $ABCD$ is a square with side a.
$AC = a\sqrt{2}$
ACEF is a rectangle with sides $a\sqrt{2}$ and a .
Now $AE = a\sqrt{3}$



AEGH is a square with a side of av3
Then Ar AEGH $= 3a^2$
Ar $ABCD = a^2$
$\therefore$ Area of AEGH $= 3\times$ Area of ABCD
(b) $ABCD$ and CFED are squares with side ' a '.
BE is a diagonal of the rectangle ABFE .
In rectangle ABFE
$EF = a$ and $BF = BC + CF = a + a = 2a$



Using Baudhayana's Theorem

$BE^2 = EF^2 + BF^2$

$= a^2 + (2a)^2$

$= a^2 + 4a^2$

$= 5a^2$

$BE = \sqrt{5}a$

BEFG is a square with side BE .

Area $BEFG = BE^2$

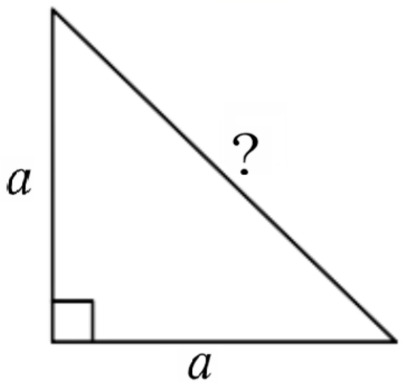
$= (\sqrt{5}a)^2$

$= 5a^2$

Then the area of BEFG $= 5\times$ the area of $ABCD$

42. The length of the two equal sides of an isosceles right triangle is given. Find the length of the hypotenuse. Find bounds on the length of the hypotenuse such that they have at least one digit after the decimal point.
    i. 3
    ii. 4
    iii. 6
    iv. 8
    v. 9

Ans. : If a is the length of two equal sides of an isosceles right triangle, then hypotenuse $= \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$



(i) $a = 3$

Hypotenuse $= a\sqrt{2} = 3\sqrt{2} = \sqrt{3 \times 3 \times 2} = \sqrt{18}$

$\sqrt{18}$ lies between $\sqrt{16}$ and $\sqrt{25}$

$\sqrt{16} < \sqrt{18} < \sqrt{25}$

$\Rightarrow 4 < 3\sqrt{2} < 5$

In one decimal point

$4.1^2 = 16.81, 4.2^2 = 17.64, 4.3^2 = 18.49$

So, $4.2 < 3\sqrt{2} < 4.3$

(ii) $a = 4$

Hypotenuse $= a\sqrt{2} = 4\sqrt{2} = \sqrt{32}$

$\sqrt{25} < \sqrt{32} < \sqrt{36}$

$\Rightarrow 5 < 4\sqrt{2} < 6$

In one decimal point

$5.1^2 = 26.01, 5.2^2 = 27.04, 5.3^2 = 28.09, 5.4^2 = 29.16, 5.5^2 = 30.25, 5.6^2 = 31.36, 5.7^2 = 32.49$

So, $5.6 < 4\sqrt{2} < 5.7$

(iii) $a = 6$

Hypotenuse $= a\sqrt{2} = 6\sqrt{2} = \sqrt{6 \times 6 \times 2} = \sqrt{72}$

$\sqrt{64} < \sqrt{72} < \sqrt{81}$

$\Rightarrow 8 < 6\sqrt{2} < 9$

In one decimal point

$8.1^2 = 65.61, 8.2^2 = 67.24, 8.3^2 = 68.89, 8.4^2 = 70.56, 8.5^2 = 72.25$

So, $8.4 < 6\sqrt{2} < 8.5.$

(iv) a = 8

Hypotenuse $= a\sqrt{2} = 8\sqrt{2} = \sqrt{128}$

$\sqrt{121} < \sqrt{128} < \sqrt{144}$

$\Rightarrow 11 < 8\sqrt{2} < 12$

In one decimal point

$11.1^2 = 123.21, 11.2^2 = 125.44, 11.3^2 = 127.69, 11.4^2 = 129.96$

So, $11.3 < 8\sqrt{2} < 11.4.$

(v) $a = 9$

Hypotenuse $= a\sqrt{2} = 9\sqrt{2} = \sqrt{162}$

$\sqrt{144} < \sqrt{162} < \sqrt{169}$

$\Rightarrow 12 < 9\sqrt{2} < 13$

In one decimal point

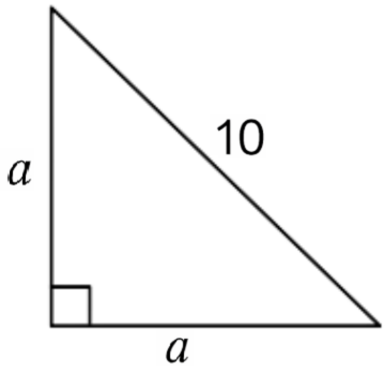
$12.1^2 = 146.41, 12.2^2 = 148.84, 12.3^2 = 151.29, 12.4^2 = 153.76, 12.5^2 = 156.25, 12.6^2 = 158.76, 12.7^2 = 161.29,$

$12.8^2 = 163.84$

So, $12.7 < 9\sqrt{2} < 12.8$

43. The hypotenuse of an isosceles right triangle is 10. What are its other two sidelengths?

Ans. :



If a is the length of two equal sides of an isosceles right triangle, then

$a^2 + a^2 = 102$

$\Rightarrow 2a^2 = 100$

$\Rightarrow a^2 = 50$

$\Rightarrow a = \sqrt{50}$

$\Rightarrow a = 5\sqrt{2}$

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