* Answer The Following Questions In One Sentence.[1 Marks Each] [1]

1. Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p.

Ans. : If p is the greatest of five consecutive numbers, then the other four numbers are (p – 1), (p – 2), (p – 3), and (p – 4).

* Questions With Calculation.[2 Marks Each] [28]

2. For statement given below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
The sum of two even numbers is a multiple of 3.

Ans. : Let the two even numbers be $2a+2b$
$\text{Sum } = 2a+2b = 2(a+b)$
For $2(a+b)$ to be a multiple of $3, (a+b)$ must be multiple of 3 .
Example :
$2+4=6 \rightarrow \text{ divisible by 3}$
$2+8=10 \rightarrow$ not divisible by 3
Conclusion : Sometimes true.

3. For statement given below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
If a number is not divisible by 18, then it is also not divisible by 9.

Ans. : If a number is divisible by 18 , then it is also divisible by 9 because 9 is a factor of 18 . $18a \div 9 = 2a \rightarrow$ divisible by 9 .
But if a number is divisible by 9 , it is not always divisible by 18 . $9b \div 18 = b/2 \rightarrow$ not divisible by 9 .
Example : 9 is divisible by 9 but not divisible by 18 .
27 is divisible by 9 but not 18 .
Conclusion : Sometimes true.

4. For statement given below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
If two numbers are not divisible by 6, then their sum is not divisible by 6.

Ans. : Let the two numbers be a and b.
Not divisible by 6 means they do not satisfy 6|a or 6|b.
But their sum can still be divisible by 6 .
Example : 2 and $4 \rightarrow$ both not divisible by 6 .

But, $2+4=6 \rightarrow$ divisible by 6 .

Conclusion : Sometimes true.

5. For statement given below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
   The sum of a multiple of 6 and a multiple of 9 is a multiple of 3.

Ans. : Let the multiple of 6 be 6 a , the multiple of 9 be 9 b .

Sum : $6a+9b=3(2a+3b) \rightarrow$ clearly divisible by 3 .

Example :

$6+9=15 \rightarrow$ divisible by 3 .

$12+18=30 \rightarrow$ divisible by 3 .

Conclusion : Always true.

6. For statement given below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.
   The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

Ans. : Let multiple of 6 be 6 a, multiple of 3 be 3b.

Sum : $6a+3b=3(2a+b)$.

For it to divisible by $9, 2a+b$ must be divisible by 3 .

Example :

$6(6\times1)+3(3\times1)=9 \rightarrow$ divisible by 9

$6+6=12 \rightarrow$ not divisible by 9

Conclusion : Sometimes true.

7. Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.

Ans. : L.C.M of 3 and $4=12$.

All such numbers are given by the expression $=12a+2$.

Examples :

(i) $12\times1+2=12+2=14$.

(ii) $12\times2+2=24+2=26$.

(iii) $12\times3+2=36+2=38$.

8. When divided by 7 , the number 661 leaves a remainder of 3 , and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 7 ? Show the solution both algebraically and visually.
   (i) $4779+661$
   (ii) $4779-661$

Ans. : (i) $4779 + 661$
= Remainder 5 + Remainder 3
= Remainder 8
8 divided by $7 \rightarrow$ remainder 1 .
(ii) $4779 - 661$
= Remainder 5 - Remainder 3
= Remainder 2

9. Find the smallest multiple of 9 with no odd digits.

Ans. : self

10. Find the multiple of 9 that is closest to the number 6000.

Ans. : self

11. How many multiples of 9 are there between the numbers 4300 and 4400?

Ans. : self

12. The digital root of an 8-digit number is 5. What will be the digital root of 10 more than that number?

Ans. : self

13. Write any number. Generate a sequence of numbers by repeatedly adding 11. What would be the digital roots of this sequence of numbers? Share your observations.

Ans. : self

14. What will be the digital root of the number 9a + 36b + 13?

Ans. : self

15. Make conjectures by examining if there are any patterns or relations between
(i) the parity of a number and its digital root.
(ii) the digital root of a number and the remainder obtained when the number is divided by 3 or 9.

Ans. : self

*** Questions With Calculation.[3 Marks Each]** **[18]**

16. The sum of four consecutive numbers is 34. What are these numbers?

Ans. : Let four consecutive numbers be $x, (x+1), (x+2)$ and $(x+3)$.

$x + (x+1) + (x+2) + (x+3) = 34$
$x + x + 1 + x + 2 + x + 3 = 34$
$4x + 6 = 34$
$4x = 34 - 6$
$4x = 28$
$x = 28/7 = 7$

So, $(x+1)=7+1=8$
$(x+2)=7+2=9$
$(x+3)=7+3=10$
Therefore, the given four consecutive numbers are $7,8,9$, and 10 .

17. Tathagat has written several numbers that leave a remainder of 2 when divided by $6. He$ claims, "If you add any three such numbers, the sum will always be a multiple of $6.$" Is Tathagat's claim true?

Ans. : A number that leaves remainder of 2 when divided by 6 can be written as $6k+2$.
Three such numbers are: $(6a+2),(6b+2),(6c+2)$.
$(6a+2)+(6b+2)+(6c+2)=6(a+b+c)+6=6(a+b+c+1)$
This sum is divisible by 6 .
So yes, Tathagat's claim is always true.
Example :
Take $20,26,32 \rightarrow$ sum $=78 \rightarrow$ divisible by 6 .
Take $2,8,14 \rightarrow$ sum $=24 \rightarrow$ divisible by 6 .

18. Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

Ans. : A number that leaves a remainder of 2 when divided by 3 is $=3x+2$
A number that leaves a remainder of 3 when divided by 4 is $=4x+3$
A number that leaves a remainder of 4 when divided by 5 is $=5x+4$
L.C.M of 3,4 , and $5=60$
All the numbers are the same, so $4x+3=3x+2$
$4x-3x=2-3$
$x=-1$
Each remainder is 1 less than the divisor.
Hence, the number is 1 less than the L.C.M $=(60-1)=59$.
So, 59 is the smallest number that satisfies all the given conditions.

19. If 3p7q8 is divisible by 44, list all possible pairs of values for p and q.

Ans. : self

20. Write five multiples of 36 between 45,000 and 47,000. Share your approach with the class.

Ans. : self

21. Choose any 3 numbers. When is their sum divisible by 3? Explore all possible cases and generalise.

Ans. : self

[30]

*** Questions With Calculation.[5 Marks Each]**

22. Find, without dividing, whether the following numbers are divisible by 9.
(i) 123 (ii) 405 (iii) 8888 (iv) 93547 (v) 358095

Ans. : self

23. Find three consecutive numbers such that the first number is a multiple of 2, the second number is a multiple of 3, and the third number is a multiple of 4.
Are there more such numbers? How often do they occur?

Ans. : self

24. Deepak claims, "There are some multiples of 11 which, when doubled, are still multiples of 11. But other multiples of 11 don't remain multiples of 11 when doubled". Examine if his conjecture is true; explain your conclusion.

Ans. : self

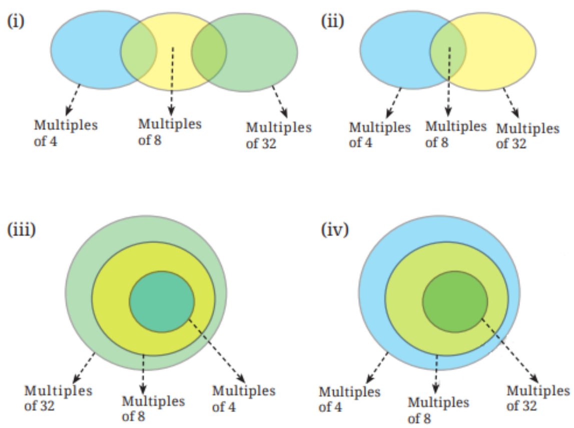
25. Is the product of two consecutive integers always multiple of 2? Why? What about the product of these consecutive integers? Is it always a multiple of 6? Why or why not? What can you say about the product of 4 consecutive integers? What about the product of five consecutive integers?

Ans. : self

26. Solve the cryptarithms -
(i) $EF \times E = GGG$
(ii) $WOW \times 5 = MEOW$

Ans. : self

27. Which of the following Venn diagrams captures the relationship between the multiples of 4, 8, and 32?



Ans. : self

*** Questions With Calculation.[4 Marks Each]** [36]

28. "I hold some pebbles, not too many, When I group them in 3's, one stays with me. Try pairing them up - it simply won't do, A stubborn odd pebble remains in

my view. Group them by 5 , yet one's still around, But grouping by seven, perfection is found. More than one hundred would be far too bold, Can you tell me the number of pebbles I hold?"

Ans. : Grouped in 3's leaves 1.
Pairing (2's) leaves 1.
Grouped by 5 leaves 1.
Grouped by 7 is perfect.
Number ≤ 100.
L.C.M of 2,3 , and $5=30$.
In all those cases, when we group them, 1 pebble remains.
So, the actual number of pebbles must be $=30+1=31$, but 31 is not divisible by 7 .
The next multiple of 30 is $2 \times 30=60$.
So, $60+1=61$, but this is also not divisible by 7 .
Similarly, the next number is $90+1=91$.
And 91 is divisible by 7 .
Hence, the number of pebbles I hold $=91$.

29. If $31 z 5$ is a multiple of 9 , where z is a digit, what is the value of z ? Explain why there are two answers to this problem.

Ans. : self

30. "I take a number that leaves a remainder of 8 when divided by $12 . I$ take another number which is 4 short of a multiple of 12 . Their sum will always be a multiple of 8 ", claims Snehal. Examine his claim and justify your conclusion.

Ans. : self

31. When is the sum of two multiples of 3 , a multiple of 6 and when is it not? Explain the different possible cases, and generalise the pattern.

Ans. : self

32. Sreelatha says, "I have a number that is divisible by 9. If I reverse its digits, it will still be divisible by 9 ".
(i) Examine if her conjecture is true for any multiple of 9.

(ii) Are any other digit shuffles possible such that the number formed is still a multiple of 9 ?

Ans. : self

33. If $48a23b$ is a multiple of 18 , list all possible pairs of values for a and b.

Ans. : self

34. The middle number in the sequence of 5 consecutive even numbers is 5p. Express the other four numbers in sequence in terms of p.

Ans. : self

35. Write a 6-digit number that it is divisible by 15, such that when the digits are reversed, it is divisible by 6.

Ans. : self

36. Determine whether the statements below are 'Always True', 'Sometimes True', or 'Never True'. Explain your reasoning.
(i) The product of a multiple of 6 and a multiple of 3 is a multiple of 9 .
(ii) The sum of three consecutive even numbers will be divisible by 6 .
(iii) If abcdef is a multiple of 6 , then badcef will be a multiple of 6 .
(iv) $8(7b-3)-4(11b+1)$ is a multiple of 12 .

Ans. : self

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