* **Fill In The Blanks With Correct Alternative.[1 Marks Each]** [50]

1. Find the squares of the : 1^2 = ________

Ans. : 1

2. Find the squares of the : 2^2 = ________

Ans. : 4

3. Find the squares of the : 3^2 = ________

Ans. : 9

4. Find the squares of the : 4^2 = ________

Ans. : 16

5. Find the squares of the : 5^2 = ________

Ans. : 25

6. Find the squares of the : 6^2 = ________

Ans. : 36

7. Find the squares of the : 7^2 = ________

Ans. : 49

8. Find the squares of the : 8^2 = ________

Ans. : 64

9. Find the squares of the : 9^2 = ________

Ans. : 81

10. Find the squares of the : 10^2 = ________

Ans. : 100

11. Find the squares of the : 11^2 = ________

Ans. : 121

12. Find the squares of the : 12^2 = ________

Ans. : 144

13. Find the squares of the : 13^2 = ________

Ans. : 169

14. Find the squares of the : 14^2 = ________

Ans. : 196

15. Find the squares of the : 15^2 = ________

Ans. : 225

16. Find the squares of the : 16^2 = ________

Ans. : 256

17. Find the squares of the : 17^2 = ________

Ans. : 289

18. Find the squares of the : 18^2 = ________

Ans. : 324

19. Find the squares of the : 19^2 = ________

Ans. : 361

20. Find the squares of the : 20^2 = ________

Ans. : 400

21. Find the squares of the : 21^2 = ________

Ans. : 441

22. Find the squares of the : 22^2 = ________

Ans. : 484

23. Find the squares of the : 23^2 = ________

Ans. : 529

24. Find the squares of the : 24^2 = ________

Ans. : 576

25. Find the squares of the : 25^2 = ________

Ans. : 625

26. Find the squares of the : 26^2 = ________

Ans. : 676

27. Find the squares of the : 27^2 = ________

Ans. : 729

28. Find the squares of the : 28^2 = ________

Ans. : 784

29. Find the squares of the : 29^2 = ________

Ans. : 841

30. Find the squares of the : 30^2 = ________

Ans. : 900

31. 1^3 = ________

Ans. : 1

32. 2^3 = ________

Ans. : 8

33. 3^3 = ________

Ans. : 27

34. 4^3 = ________

Ans. : 64

35. 5^3 = ________

Ans. : 125

36. 6^3 = ________

Ans. : 216

37. 7^3 = ________

Ans. : 343

38. 8^3 = ________

Ans. : 512

39. 9^3 = ________

Ans. : 729

40. 10^3 = ________

Ans. : 1000

41. 11^3 = ________

Ans. : 1331

42. 12^3 = ________

Ans. : 1728

43. 13^3 = ________

Ans. : 2197

44. 14^3 = ________

Ans. : 2744

45. 15^3 = ________

Ans. : 3375

46. 16^3 = ________

Ans. : 4096

47. 17^3 = ________

Ans. : 4913

48. 18^3 = ________

Ans. : 5832

49. 19^3 = ________

Ans. : 6859

50. 20^3 = ________

Ans. : 8000

*** Answer The Following Questions In One Sentence.[1 Marks Each]** **[16]**

51. Write the locker numbers that remain open.

Ans. : 10 lockers with square locker numbers, i.e, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, will remain open.

52. Which are these five lockers?

Ans. : The lockers that are toggled twice are the prime numbers, since each prime number has 1 and the number itself as factors. So, the code is 2-3-5-7-11.

53. What patterns do you notice? Share your observations and make conjectures.

Ans. : All perfect square numbers end with 0, 1, 4, 5, 6, or 9, and none of them end with 2, 3, 7, or 8.

54. Write 5 numbers such that you can determine by looking at their unit digit that they are not squares.

Ans. : 478, 1072, 7543, 9047, and 1257.

55. Which of the following numbers have the digit 6 in the units place?
38^2

Ans. : 38^2
$38 \rightarrow$ Units digit $\rightarrow 8$
$8 \times 8 = 64$ (ends in 4)
So, 38^2 does not end in 6 .

56. Which of the following numbers have the digit 6 in the units place?

34^2

Ans. : 34^2

34 $\rightarrow$ Units digit $\rightarrow 4$

$4 \times 4 = 16$ (ends in 6)

So, 34^2 ends in 6 .

57. Which of the following numbers have the digit 6 in the units place?

46^2

Ans. : 46^2

$46 \rightarrow$ Units digit $\rightarrow 6$

$6 \times 6 = 36$ (ends in 6)

So, 46^2 ends in 6 .

58. Which of the following numbers have the digit 6 in the units place?

56^2

Ans. : 56^2

$56 \rightarrow$ Units digit $\rightarrow 6$

$6 \times 6 = 36$ (ends in 6)

So, 56^2 ends in 6 .

59. Which of the following numbers have the digit 6 in the units place?

74^2

Ans. : 74^2

$74 \rightarrow$ Units digit $\rightarrow 4$

$4 \times 4 = 16$ (ends in 6)

So, 74^2 ends in 6 .

60. Which of the following numbers have the digit 6 in the units place?

82^2

Ans. : 82^2

$82 \rightarrow$ Units digit $\rightarrow 2$

$2 \times 2 = 4$ (ends in 4)

So, 82^2 ends in 4 .

61. What can you say about the parity of a number and its square?

Ans. : The square of an even number is always even, and that of an odd number is always odd.

62. We know that 0, 1, 4, 5, 6, 9 are the only last digits possible for squares. What are the possible last digits of cubes?

Ans. : The last digits of cubes can be any digit from 0 to 9.

63. Can you tell what this sum is without doing the calculation?
91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109.

Ans. : This series has 10 consecutive odd numbers, and their sum is $10^3 = 1000$.

64. Find the cube roots of these numbers : $\sqrt[3]{64} =$

Ans. : $\sqrt[3]{64}$

$$64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2^3 \times 2^3$$
$$= (2 \times 2)^3 = 4^3$$
$$\therefore \sqrt[3]{64} = 4$$

65. Find the cube roots of these numbers : $\sqrt[3]{512} =$

Ans. : $\sqrt[3]{512}$

$$512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2^3 \times 2^3 \times 2^3$$
$$= (2 \times 2 \times 2)^3 = 8^3$$
$$\therefore \sqrt[3]{512} = 8$$

66. Find the cube roots of these numbers : $\sqrt[3]{729} =$

Ans. : $\sqrt[3]{729}$

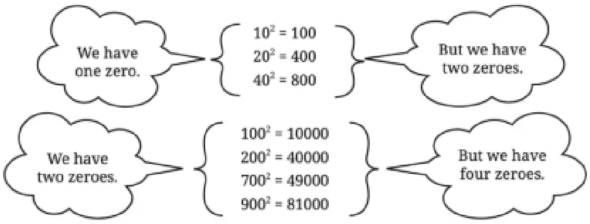
$$729 = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$$
$$= 3^3 \times 3^3$$
$$= (3 \times 3)^3 = 9^3$$
$$\therefore \sqrt[3]{729} = 9.$$

* Questions With Calculation.[2 Marks Each] [24]

67. If a number ends in 0, 1, 4, 5, 6, or 9, is it always a square?

Ans. : We cannot determine if a number is a square just by looking at the digit in the units place. But the unit digit can tell us when a number is not a square. If a number ends with 2, 3, 7, or 8, then we can say that it is not a square. For example, 26 ends in 6 but is not a perfect square.

68. Consider the following numbers and their squares.



If a number contains 3 zeros at the end, how many zeros will its square have at the end?

Ans. : The number of zeros at the end of the square of a number is always double the number of zeros at the end of the original number. Therefore, if a number contains 3 zeros at the end, then its square will have 6 zeros.

69. Find how many numbers lie between two consecutive perfect squares. Do you notice a pattern?

Ans. : There are exactly ' $2n$ ' numbers between n^2 and $(n+1)^2$. For example: Between $3^2(n)$ and $4^2(n+1)$ there are $2 \times 3 = 6(2n)$ numbers.

70. How many square numbers are there between 1 and 100? How many are between 101 and 200? Using the table of squares you filled earlier, enter the values below, tabulating the number of squares in each block of 100. What is the largest square less than 1000?

1 – 100	101 – 200	201 – 300	301 – 400	401 – 500
___	___	___	___	___
501 – 600	601 – 700	701 – 800	801 – 900	901 – 1000
___	___	___	___	___

Ans. :

The largest square less than 1000 is $31^2 = 961$.

1 – 100	101 – 200	201 – 300	301 – 400	401 – 500
10	4	3	2	3
501 – 600	601 – 700	701 – 800	801 – 900	901 – 1000
3	2	3	2	2

71. Given $125^2 = 15625$, what is the value of 126^2 ?

(i) $15625 + 126$

(ii) $15625 + 26^2$

(iii) $15625 + 253$

(iv) $15625 + 251$

(v) $15625 + 25^2$

Ans. : $125^2 = 15625$

This means 15625 is the sum of the first 125 odd numbers.

$126^2 = 15625 + 127^{\text{th}}$ odd number

127^{th} odd number $= (2 \times 127) - 1 = 252 - 1 = 251$.

$\therefore 126^2 = 15625 + 251$

Therefore, (iv) $15625 + 251$ is the correct answer.

72. Find the length of the side of a square whose area is $441\ m^2$.

Ans. : Area of the square $=$ side $\times$ side $= 441\ m^2$.

$$(\text{side})^2 = 441$$
$$\text{side} = \sqrt{441}$$
$$\text{side} = \pm 21\ m$$

Since the side of a square cannot be negative, the length of the side of the square is 21 m .

73. Find the smallest square number that is divisible by each of the following numbers : 4, 9, and 10.

Ans. : The L.C.M. of 4, 9, and 10 is 180.

So, the smallest number divisible by 4,9 , and 10 is 180 .

$$180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$$
$$180 = 2^2 \times 3^2 \times 5$$

Here, 5 has no pair.

So, 180 is not a perfect square. To make it a perfect square, we multiply it by 5.

Hence, the required smallest square number is $180 \times 5 = 900$.

74. How many numbers lie between the squares of the following numbers?
(i) 16 and 17 (ii) 99 and 100

Ans. : There are $2n$ numbers between n^2 and $(n+1)^2$.

(i) 16^2 and 17^2

Here, $n = 16$ and $(n+1) = 17$

Therefore, the numbers between 16^2 and $17^2 = 2n = 2 \times 16 = 32$.

(ii) 99^2 and 100^2

Here, $n = 99$ and $(n+1) = 100$

Therefore, the numbers between the squares 99^2 and $100^2 = 2n = 2 \times 99 = 198$.

75. In the following pattern, fill in the missing numbers :

$$1^2 \times 2^2 \times 2^2 = 3^2$$
$$2^2 \times 3^2 \times 6^2 = 7^2$$
$$3^2 \times 4^2 \times 12^2 = 13^2$$
$$4^2 \times 5^2 \times 20^2 = (\ )^2$$
$$9^2 \times 10^2 \times (\ )^2 = (\ )^2$$

Ans. :

$$1^2 \times 2^2 \times 2^2 = 3^2$$
$$2^2 \times 3^2 \times 6^2 = 7^2$$
$$3^2 \times 4^2 \times 12^2 = 13^2$$
$$4^2 \times 5^2 \times 20^2 = (\underline{21})^2$$
$$9^2 \times 10^2 \times (\underline{90})^2 = (\underline{91})^2$$

76. Can a cube end with exactly two zeroes (00)? Explain.

Ans. : No. A cube cannot end with exactly two zeros because zeros in a cube occur in multiples of three. If a number ends in one zero, its cube ends in three zeros.

77. The next two taxicab numbers after 1729 are 4104 and 13832. Find the two ways in which each of these can be expressed as the sum of two positive cubes.

Ans. : 4104 :

$$2^3+16^3=8+4096=4104$$
$$9^3+15^3=729+3375=4104$$

13832 :

$$2^3+24^3=8+13824=13832$$
$$18^3+20^3=5832+8000=13832$$

78. What number will you multiply by 1323 to make it a cube number?

Ans. : Prime factorization of $1323=\underline{3\times3\times3}\times7\times7$

Here, there is no triplet of 7.

So, 1323 is not a perfect cube. To make it a cube number, we multiply it by 7 .

$1323\times7=\underline{3\times3\times3}\times\underline{7\times7\times7}=9261$ which is a perfect square.

Hence, the number by which 1323 needs to be multiplied to make it a cube number is 7 .

*** Questions With Calculation.[3 Marks Each]** **[18]**

79. Does every number have an even number of factors?

Ans. : Not every number has an even number of factors. Only perfect squares have an odd number of factors, because they each have one factor which, when multiplied by itself, equals the number.

Number	Factors	Number of factors
1	1	1 (odd)
2	1, 2	2 (even)
3	1, 3	2 (even)
4	1, 2, 4	3 (odd)
5	1, 5	2 (even)
6	1, 2, 3, 6	4 (even)
7	1, 7	2 (even)
8	1, 2, 4, 8	4 (even)
9	1, 3, 9	3 (odd)
10	1, 2, 5, 10	4 (even)

80. Can you use this insight to find more numbers with an odd number of factors?

Ans. :

Number	Factors	Number of factors
1	1	1 (odd)
4	1, 2, 4	3 (odd)
9	1,3,9	3 (odd)
16	1, 2, 4, 8, 16	5 (odd)
25	1, 5, 25	3 (odd)
36	1, 2, 3, 4, 6, 9, 12, 18, 36	9 (odd)
49	1, 7, 49	3 (odd)
64	1, 2, 4, 8, 16, 32, 64	7 (odd)
81	1, 3, 9, 27, 81	5 (odd)
100	1, 2, 4, 5, 10, 20, 25, 50, 100	9 (odd)

81. Find whether 1156 and 2800 are perfect squares using prime factorisation.

Ans. : (i) $1156=(2\times 2)\times(17\times 17)$

$=2^2\times 17^2$
$=(2\times 17)^2$
$=(34)^2$
$\therefore \sqrt{1156}=34$

(ii) $2800=(2\times 2)\times(2\times 2)\times(5\times 5)\times 7$

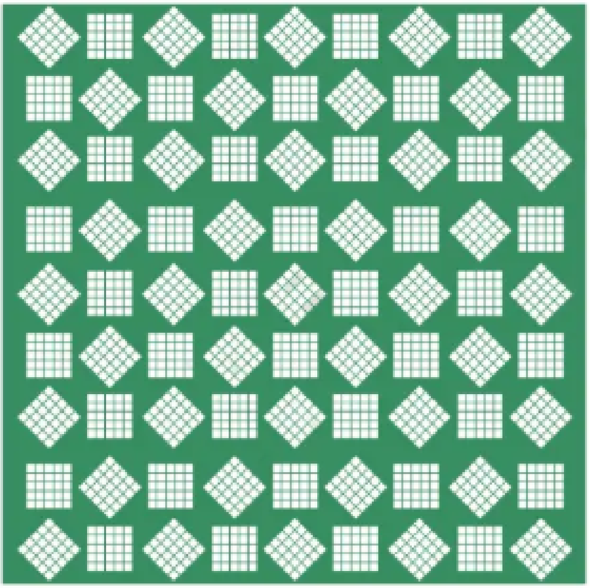
$=2^2\times 2^2\times 5^2\times 7$
Since the factors cannot be paired
$\therefore 2800$ is not a perfect square.

82. Find the smallest number by which 9408 must be multiplied so that the product is a perfect square. Find the square root of the product.

Ans. : $9408=\underline{2\times 2}\times\underline{2\times 2}\times\underline{2\times 2}\times 3\times\underline{7\times 7}$
$9408=2^2\times 2^2\times 2^2\times 7^2\times 3$
Here, 3 has no pair.
So, 9408 is not a perfect square. To make it a perfect square, we multiply it by 3 .
Therefore, $9408\times 3=28224$, which is a perfect square.

$\sqrt{28224}=\sqrt{2\times 2\times 2\times 2\times 2\times 2\times 3\times 3\times 7\times 7}$
$\sqrt{28224}=\sqrt{2^2\times 2^2\times 2^2\times 3^2\times 7^2}$
$\therefore \sqrt{28224}=2\times 2\times 2\times 3\times 7=8\times 21=168$

83. How many tiny squares are there in the following picture? Write the prime factorisation of the number of tiny squares.



Ans. : Number of squares in a row $= 9$.
Number of squares in a column $= 9$.
Total number of squares in the picture $= 9 \times 9 = 81$.
Number of tiny squares in a square $= 5 \times 5 = 25$.
Total number of tiny squares in the picture $= 25 \times 81 = 2025$.
Prime factorization of 2025 :

$$2025 = (3 \times 3) \times (3 \times 3) \times (5 \times 5)$$
$$2025 = 3^2 \times 3^2 \times 5^2$$
$$2025 = 3^4 \times 5^2$$

84. Similar to squares, can you find the number of cubes with 1 digit, 2 digits, and 3 digits? What do you observe?

Ans. : 1-digit cubes : $1^3 = 8, 2^3 = 8$
Count : 2 cubes $(1, 8)$

2-digit cubes : $3^3 = 27, 4^3 = 64$
Count : 2 cubes $(27, 64)$

3-digit cubes : $5^3 = 125, 6^3 = 216, 7^3 = 343, 8^3 = 512, 9^3 = 729$
Count : 5 cubes ($125, 216, 343, 512, 729$)

The number of perfect cubes increases as the numbers get larger.
Unlike squares, cubes grow more quickly, so fewer cubes fit into smaller ranges.

*** Questions With Calculation.[5 Marks Each]** **[10]**

85. State true or false. Explain your reasoning.
(i) The cube of any odd number is even.
(ii) There is no perfect cube that ends with 8.

(iii) The cube of a 2-digit number may be a 3-digit number.
(iv) The cube of a 2-digit number may have seven or more digits.
(v) Cube numbers have an odd number of factors.

Ans. : (i) False because the cube of an odd number is always odd.

For example: $3^3 = 27, 5^3 = 125, 7^3 = 343$, all are odd.

(ii) False because some cubes do end with 8.

For example: $2^3 = 8, 12^3 = 1728$, both end with 8 .

(iii) False, because the cube of a 2-digit number can range from 4-digit to 6-digit numbers.

For example: $10^3 = 1000, 99^3 = 970299$.

(iv) False because the largest 2-digit number ' $99'$ has a cube that is a 6 -digit number, i.e. $99^3 = 970299$.

So, a 2-digit whole number will always have a cube with at most 6 digits.

(v) False because only perfect squares have an odd number of factors.

For example :

Factors of $8 = 1, 2, 4, 8 \rightarrow 4$ factors (even).

Factors of $27 = 1, 3, 9, 27 \rightarrow 4$ factors (even).

86. You are told that 1331 is a perfect cube. Can you guess without factorisation what its cube root is? Similarly, guess the cube roots of 4913, 12167, and 32768.

Ans. : (i) Yes, the cube root of 1331 can be guessed without factorization.

$10^3 = 1000 \text{ and } 20^3 = 8000$

So, $10 < \sqrt[3]{1331} < 20$

Since 1331 ends in 1, its cube root ends in 1.

The number between 10 and 20 that ends in 1 is 11 .

$\therefore \sqrt[3]{1331} = 11.$

(ii) $\sqrt[3]{4913}$

$10^3 = 1000 \text{ and } 20^3 = 8000$

So, $10 < \sqrt[3]{4913} < 20$

Since 4913 ends in 3 , its cube root ends in 7.

The number between 10 and 20 that ends in 7 is 17.

$\therefore \sqrt[3]{4913} = 17.$

(iii) $\sqrt[3]{12167}$

$20^3 = 8000 \text{ and } 30^3 = 27000$

$\text{So, } 20 < \sqrt[3]{12167} < 30$

Since 12167 ends in 7 , its cube root ends in 3.

The number between 20 and 30 that ends in 3 is 23 .

$\therefore \sqrt[3]{12167} = 23.$

(iv) $\sqrt[3]{32768}$

$30^3 = 27000$ and $40^3 = 64000$

So, $30 < \sqrt[3]{32768} < 40$

Since 32768 ends in 8 , its cube root ends in 2.

The number between 30 and 40 that ends in 2 is 32 .

$\sqrt[3]{32768} = 32.$

*** Questions With Calculation.[4 Marks Each]** **[20]**

87. Before the process begins, Khoisnam realises that he already knows which lockers will be open at the end. How did he figure out the answer?

Ans. : He noticed that the number of toggles a locker receives equals the number of factors of that locker's number. A locker toggled an odd number of times will be open at the end; a locker toggled an even number of times will be closed. Only perfect squares have an odd number of factors, so exactly the lockers numbered with perfect squares (1, 4, 9, 16, ...) remain open.

Locker Number	Factors	Number of factors/Number of locker toggles
1	1(o)	1 (odd)
2	1(o), 2(c)	2 (even)
3	1(o), 3(c)	2 (even)
4	1(o), 2(c), 4(o)	3 (odd)
5	1(o), 5(c)	2 (even)
6	1(o), 2(c), 3(o), 6(c)	4 (even)
7	1(o), 7(c)	2 (even)
8	1(o), 2(c), 4(o), 8(c)	4 (even)
9	1(o), 3(c), 9(o)	3 (odd)
10	1(o), 2(c), 5(o), 10(c)	4 (even)

88. Which of the following numbers are not perfect squares?

(i) 2032

(ii) 2048

(iii) 1027

(iv) 1089

Ans. : A perfect square ends in 0, 1, 4, 5, 6, or 9 at its unit's place.

(i) 2032 ends in 2 at the unit's place. So, it is not a perfect square.

(ii) 2048 ends in 8 at the unit's place. So, it is not a perfect square.
(iii) 1027 ends in 7 at the unit's place. So, it is not a perfect square.
(iv) 1089 ends in 9 at the unit's place. So, it is a perfect square.

89. Which one among $64^2, 108^2, 292^2, 36^2$ has the last digit 4?

Ans. : (i) 64^2
The unit's digit of 64 is 4
$4 \times 4 = 16$ (last digit 6)

(ii) 108^2
The unit's digit of 108 is 8
$8 \times 8 = 64$ (last digit 4)

(iii) 292^2
The unit's digit of 292 is 2
$2 \times 2 = 4$ (last digit 4)

(iv) 36^2
The unit's digit of 36 is 6
$6 \times 6 = 36$ (last digit 6)

Therefore, 108^2 and 292^2 have 4 as their last digits.

90. Find the cube roots of 27000 and 10648.

Ans. : (i) $\sqrt[3]{2}27000$
Prime factorization of $27000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$27000 = 2^3 \times 3^3 \times 5^3$
$27000 = (2 \times 3 \times 5)^3$
$27000 = 30^3$
$\therefore \sqrt[3]{27000} = 30$

(ii) $\sqrt[3]{10648}$
Prime factorisation of $10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

$10648 = 2^3 \times 11^3$
$10648 = (2 \times 11)^3$
$10648 = 22^3$
$\therefore \sqrt[3]{10648} = 22$

91. Which of the following is the greatest? Explain your reasoning.
(a) $67^3 - 66^3$

(b) $43^3 - 42^3$

(c) $67^2 - 66^2$

(d) $43^2 - 42^2$

Ans. : Using, $n^3 - (n-1)^3 = 3n^2 - 3n + 1;$

$n^2 - (n-1)^2 = 2n - 1.$

(a) $67^3 - 66^3 = 3 \times 67^2 - 3 \times 67 + 1$

$= 3 \times 4489 - 201 + 1$

$= 13467 - 200 = 13267.$

(b) $43^3 - 42^3 = 3 \times 43^2 - 3 \times 43 + 1$

$= 3 \times 1848 - 129 + 1$

$= 5547 - 128 = 5419$

(c) $67^2 - 66^2 = 2 \times 67 - 1$

$= 134 - 1 = 133.$

(d) $43^2 - 42^2 = 2 \times 43 - 1$

$= 86 - 1 = 85.$

Thus, (i) $67^3 - 66^3$ is the greatest.

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