*** Answer The Following Questions In One Sentence.[1 Marks Each]** **[2]**

1. Find the HCF and LCM of the following (state your answers in the form of prime factorisations):
   $3 \times 3 \times 5 \times 7 \times 7$ and $12 \times 7 \times 11$

Ans. : Here $3 \times 3 \times 5 \times 7 \times 7$ and $12 \times 7 \times 11 = 2 \times 2 \times 3 \times 7 \times 11$
$\therefore$ HCF = $3 \times 7 = 21$
$\therefore$ LCM = $2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7 \times 11$.

2. Find the HCF and LCM of the following (state your answers in the form of prime factorisations):
   45 and 36

Ans. : Here $45 = 3 \times 3 \times 5$
$36 = 2 \times 2 \times 3 \times 3$
HCF (45, 36) = 3×3
LCM (45, 36) = $2 \times 2 \times 3 \times 3 \times 5$

*** Questions With Calculation.[2 Marks Each]** **[36]**

3. List all the factors of the following numbers:
   90

Ans. : 90
Here

2	90
3	45
3	15
5	5
	1

$\therefore$ $90 = 2 \times 3 \times 3 \times 5$
Hence, factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, and 90.
Total factors = 12

4. Find the HCF of the following numbers:
   24, 180

Ans. : Given 24, 180

3	81
3	27
3	9
3	3
	1

and

3	243
3	81
3	27
3	9
3	3
	1

Clearly, the common prime factors are 2 and 3.

∴ HCF (24, 180) = 2 × 2 × 3 = 4 × 3 = 12

5. Find the HCF of the following numbers:
   42, 75, 24

Ans. : Now

$$42 = 2 \times 7 \times \boxed{3}$$
$$75 = 5 \times 5 \times \boxed{3}$$
$$24 = 2 \times 2 \times \boxed{3} \times 2$$

Clearly only common prime factor is 3.

∴ HCF (42, 75, 24) = 3.

6. Find the HCF of the following numbers:
   240, 378

Ans. : Here 240 and 378

2	240
2	120
2	60
2	30
3	15
5	5
	1

2	378
3	189
3	63
3	21
7	7
	1

∴ 240 = 2 × 2 × 2 × 2 × 3 × 5

and 378 = 2 × 3 × 3 × 3 × 7

Hence, HCF(240, 378) = 2 × 3 = 6

7. Find the HCF of the following numbers:
   400, 2500

Ans. : Here 400 and 2500

2	400
2	200
2	100
2	50
5	25
5	5
	1

2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$\therefore$ $400 = \boxed{2 \times 2} \times 2 \times 2 \times \boxed{5 \times 5}$

and $2500 = \boxed{2 \times 2} \times \boxed{5 \times 5} \times 5 \times 5$

∴ HCF(400, 2500) = 2 × 2 × 5 × 5 = 100.

8. Find the HCF of the following numbers:
   300, 800

Ans. : Here 300 and 800

2	300
2	150
3	75
5	25
5	5
	1

and

2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$\therefore$ $300 = \boxed{2 \times 2} \times 3 \times \boxed{5 \times 5}$

$800 = \boxed{2 \times 2} \times 2 \times 2 \times 2 \times \boxed{5 \times 5}$

∴ HCF(300, 800) = 2 × 2 × 5 × 5 = 100.

9. List all the factors of the following number:
   105

Ans. : 105

Here

3	105
5	35
7	7
	1

∴ 105 = 3 × 5 × 7

Hence, factors of 105 are 1, 3, 5, 7, 15, 21, 35, and 105.

Total factors = 8

10. List all the factors of the following number:
    132

Ans. : 132

Here

2	132
2	66
3	33
11	11
	1

∴ 132 = 2 × 2 × 3 × 11

Hence, factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132.

Total factors = 12

11. List all the factors of the following number:
360 (this number has 24 factors)

Ans. : 360

Here

2	360
2	180
2	90
3	45
3	15
5	5
	1

∴ 360 = 2 × 2 × 2 × 3 × 3 × 5

Hence, factors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360.

Total factors = 24

12. List all the factors of the following number:
840 (this number has 32 factors)

Ans. : 840

Here

2	840
2	420
2	210
3	105
5	35
7	7
	1

∴ 840 = 2 × 2 × 2 × 3 × 5 × 7

Hence, factors of 840 are 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840.

Total factors = 32

13. Find the LCM of the following numbers:
 30, 72

Ans. : Here 30 = 2 × 3 × 5
(One occurrence of 2, one occurrence of 3, and one occurrence of 5)
and 72 = 2 × 2 × 2 × 3 × 3
(Three occurrences of 2s and two occurrences of 3s)
∴ LCM (30, 72) = 2 × 2 × 2 × 3 × 3 × 5
(Three occurrences of 2s and two occurrences of 3s, and one occurrence of 5)
= 8 × 9 × 5
= 360

14. Find the LCM of the following numbers:
 36, 54

Ans. : Here 36 = 2 × 2 × 3 × 3
(Two occurrences of 2s and two occurrences of 3s)
and 54 = 2 × 3×3×3
(One occurrence of 2, and three occurrences of 3s)
∴ LCM(36, 54) = 2 × 2 × 3 × 3 × 3 = 4 × 27 = 108
(One occurrence of 2, and three occurrences of 3s)
∴ LCM(36, 54) = 2 × 2 × 3 × 3 × 3 = 4 × 27 = 108

15. Find the LCM of the following numbers:
 105, 195, 65

Ans. : Here 105, 195, and 65
Now 105 = 3 × 5 × 7
195 = 3 × 5 × 13
65 = 5 × 13
∴ LCM(105, 195, 65) = 3 × 5 × 7 × 13 = 1365

16. Find the LCM of the following numbers:
222, 370

Ans. : Here 222, 370

2	222
3	111
37	37
	1

2	370
5	185
37	37
	1

222 = 2 × 3 × 37 and 370 = 2 × 5 × 37
∴ LCM(222, 370) = 2 × 3 × 5 × 37 = 1110.

17. Is 5 × 7 × 11 × 11 a multiple of 5 × 7 × 7 × 11 × 2?

Ans. : Here let a = 5 × 7 × 11 × 11 and b = 5 × 7 × 7 × 11 × 2
For a number a to be a multiple of number b, all prime factors of b must be present in a with at least the same power.
Number a does not have the prime factor 2.
Number a has 71, while number b has 7.
Hence, a is not a multiple of b.

18. Is 5 × 7 × 11 × 11 a factor of 5 × 7 × 7 × 11 × 2?

Ans. : Let a = 5 × 7 × 11 × 11
6 = 5 × 7 × 7 × 11 × 2
For a to be a factor of b, b must contain all the prime factors of a, in equal or higher powers.
But here b has only one 11 (a has two 11s), so b does not include all factors of a.
Hence, 5 × 7 × 11 × 11 is not a factor of 5 × 7 × 7 × 11 × 2.

19. Tick the correct statement(s). The LCM of two different prime numbers (m, n) can be:
(a) Less than both numbers
(b) In between the two numbers
(c) Greater than both numbers
(d) Less than m × n
(e) Greater than m × n

Ans. : LCM of two different prime numbers, m and n, is their product of m × n.
Since both m and n are prime numbers, their only common factor is 1.
Hence, LCM is always greater than both individual numbers.
Hence, option (c) is correct.

20. A dog is chasing a rabbit that has a head start of 150 feet. It jumps 9 feet every time the rabbit jumps 7 feet. In how many leaps does the dog catch up with the rabbit?

Ans. : Here, distance gained per leap = Dog's leap distance – Rabbit's leap distance
= 9 – 7
= 2
The dog needs to close a 150-foot head start.

No. of leaps $= \frac{\text{Head Start Distance}}{\text{Distance Gained Per Leap}}$

$= \frac{150}{2}$

= 75 leaps

* Questions With Calculation.[3 Marks Each] [51]

21. Find the common factor and the HCF of the following number:
50, 60

Ans. : Here, 50

2	50
5	25
	5

$\therefore 50 = \boxed{2} \times 5 \times \boxed{5}$

and 60

2	60
2	30
3	15
5	5
	1

$\therefore 60 = \boxed{2} \times 2 \times 3 \times \boxed{5}$

∴ Common factors of 50 and 60 are 2, 5, and HCF (50, 60) = 2 × 5 = 10.

22. Find the common factor and the HCF of the following number:
140, 275

Ans. : Here 140 and 275

2	140
2	70
5	35
7	7
	1

5	275
5	55
11	11
	1

∴ 140 = 2 × 2 × 5 × 7 and 275 = 5 × 5 × 11
∴ Common factor of 140 and 275 = 5, and HCF of 140 and 275 = 5.

23. Find the common factor and the HCF of the following number:
77, 725

Ans. : Here, 77 and 725

7	77
11	11
	1

and

5	725
5	145
29	29
	1

∴ 77 = 7 × 11 and 725 = 5 × 5 × 29

∴ Common factor = 1 and HCF (77, 725) = 1 (as there are no common prime factors)

24. Find the common factor and the HCF of the following number:
 370, 592

Ans. : Here, 370 and 592

2	370
5	185
37	37
	1

and

2	592
2	296
2	148
2	74
37	37
	1

∴ 370 = 2 × 5 × 37 and 592 = 2 × 2 × 2 × 2 × 37

∴ Common factors = 1, 2, 37, and HCF (370, 592) = 2 × 37 = 74

25. Find the common factor and the HCF of the following number:
 81, 243

Ans. : Here 81 and 243

3	81
3	27
3	9
3	3
	1

and

3	243
3	81
3	27
3	9
3	3
	1

∴ 81 = 3 × 3 × 3 × 3 and 243 = 3 × 3 × 3 × 3 × 3

∴ Common factors = 3 × 3 × 3 × 3 and HCF (81, 243) = 3 × 3 × 3 × 3 = 81.

26. Consider the numbers 72 and 144. Suppose they are factorised into composite numbers as: 72 = 6 × 12 and 144 = 8 × 18. Seeing this, can one say that these two numbers have no common factor other than 1? Why not?

Ans. : No, one cannot say that 72 and 144 have no common factor other than 1 because their factorisations have composite numbers.

Here 72 = 6 × 12

144 = 8 × 18

These are not prime factorisations.

Both 6 and 12 are composite numbers, as are 8 and 18.
Prime factorisation of 72 = 2 × 2 × 2 × 3 × 3
Prime factorisation of 144 = 2 × 2 × 2 × 2 × 3 × 3
∴ HCF(72, 144) = 2 × 2 × 2 × 3 × 3 = 8 × 9 = 72
Since the HCF is 72, which is greater than 1, the numbers have common factors other than 1.

27. The LCM of 3 and 24 is 24 (it is one of the two given numbers):
Make a general statement about such numbers. Describe such number pairs using algebra.

Ans. : General Statement: For any two positive integers, let's call them a and b, the Least Common Multiple (LCM) will be equal to one of the numbers (specifically, the larger number) if and only if the smaller number is a factor (or a divisor) of the larger number.
In the original example, the LCM of 3 and 24 is 24 because 3 is a factor of 24 (24 ÷ 3 = 8)
Algebraic Description: Let the two positive integers be a and b, where a The LCM of a and b will be b if and only if b is a multiple of a.
This can be expressed using algebra as: LCM(a, b) = b if and only if b = k × a, where k is a positive integer.

28. Make a general statement about the LCM for the following pairs of numbers. You could consider examples before coming up with these general statements. Look for possible explanations of why they hold.
Two multiples of 3

Ans. : Two Multiples of 3
Examples:
(i) (6, 9)
LCM (6, 9) = 18
(ii) (9, 12)
LCM (9, 12) = 36
(iii) (12, 18)
LCM (12, 18) = 36
Observation: The LCM of two multiples of 3 is also a multiple of 3.
Reason: Since both numbers are divisible by 3, their common multiples will also be divisible by 3.
Hence, the LCM must include 3 as a factor.
General Statement: The LCM of two multiples of 3 is always a multiple of 3.

29. Make a general statement about the LCM for the following pairs of numbers. You could consider examples before coming up with these general statements. Look for possible explanations of why they hold.
Two consecutive even numbers

Ans. : Two Consecutive Even Numbers

Examples:

(i) (2, 4)

LCM (2, 4) = 4

(ii) (6, 8)

LCM (6, 8) = 24

(iii) (10, 12)

LCM (10, 12) = 60

Observation: The LCM of two consecutive even numbers is half of their product.

Reason: Consecutive even numbers always share a common factor of 2, but not more.

Therefore, when finding the LCM, one factor of 2 overlaps, so the LCM becomes smaller than their product.

General Statement: The LCM of two consecutive even numbers 2n and 2n + 2 is always equal to half of their product.

or LCM $(2n, 2n+2) = \frac{2n \times (2n+2)}{2}$

= n(2n + 2)

$2n^2 + 2n$

30. Make a general statement about the LCM for the following pairs of numbers. You could consider examples before coming up with these general statements. Look for possible explanations of why they hold.

    Two consecutive numbers

Ans. : Two Consecutive Numbers

Examples:

(i) (7, 8)

LCM (7, 8) = 56

(ii) (9, 10)

LCM (9, 10) = 90

(iii) (10, 11)

LCM (10, 11) = 110

Observation: The LCM of two consecutive numbers is equal to their product.

Reason: Consecutive numbers have no common factors other than 1, so their product is the smallest number divisible by both.

General Statement: The LCM of two consecutive numbers is their product.

31. Make a general statement about the LCM for the following pairs of numbers. You could consider examples before coming up with these general statements. Look for possible explanations of why they hold.

    Two co-prime numbers

Ans. : Two Co-prime Numbers

Examples:

(i) (4, 9)
LCM (4, 9) = 36
(ii) (5, 8)
LCM (5, 8) = 40
(iii) (7, 10)
LCM (7, 10) = 70
Observation: The LCM of two co-prime numbers is equal to their product.
Reason: Co-prime numbers do not share any common factors except 1, so the smallest number that contains both is simply their product.
General Statement: The LCM of two co-prime numbers is equal to their product.
Note: Co-prime numbers are any two natural numbers that have no common factor other than 1.

32. In the two rows below, colours repeat as shown. When will the black stars meet next?

Ans. : The positions of the black star in the first row are 4 and 10.
The difference in position = 10 – 4 = 6.
So, the next positions of the black star in the first row are: 4, 10, 16, 22, 28, ...
The positions of the black star in the second row are 4 and 8.
The difference in position = 8 – 4 = 4.
So, the next positions of the black star in the second row are: 4, 8, 12, 16, 20, ...
The 16th position is common in both rows.
Hence, the black stars meet again at the 16th position.

33. Find two numbers whose HCF is 1, and LCM is 66.

Ans. : For two numbers a and b, the product of the numbers is equal to the product of their LCM and HCF.
∴ a × b = HCF(a, b) × LCM(a, b)
Here HCF = 1, LCM = 66
⇒ a × b = 1 × 66 = 66
∴ Pairs of factors of 66 are (1, 66), (2, 33), (3, 22), (6, 11)
Any of the following pairs of numbers will satisfy the conditions: (1, 66), (2, 33), (3, 22), (6, 11).

34. A cowherd took all his cows to graze in the fields. The cows can go to a crossing with 3 gates. An equal number of cows passed through each gate. Later, at another crossing with 5 gates again an equal number of cows passed through each gate. The same happened at the third crossing with 7 gates. If the cowherd had fewer than 200 cows, how many cows did he have? (Based on the folklore mathematics from Karnataka).

Ans. : No. of cows is divisible by 3, 5, 7.

This means the number of cows is a multiple of the LCM(3, 5, 7).

The total number of cows is also less than 200.

LCM (3, 5, 7) = 3 × 5 × 7 = 105

No. of cows must be a multiple of 105.

Multiples of 105 are 105, 210, 315.

The problem states that the cowherd had fewer than 200 cows.

Only multiples of 105 that are less than 200 are 105.

Hence cowherd had 105 cows.

35. Among the numbers below, which is the largest number that perfectly divides both 306 and 36?

(a) 36 (b) 612 (c) 18 (d) 3 (e) 2 (f) 360

Ans. : Longest number that perfectly divides (306, 36) = HCF (306, 36)

Now

2	306
3	153
3	51
17	17
	1

$$\therefore 306 = \boxed{2} \times \boxed{3 \times 3} \times 17$$
$$36 = \boxed{2} \times 2 \times \boxed{3 \times 3}$$

HCF (306, 36) = 2 × 3 × 3 = 18.

∴ (c) is the correct option.

36. Find the smallest number that is divisible by 3, 4, 5, and 7, but leaves a remainder of 10 when divided by 11.

Ans. : LCM(3, 4, 5,7) = 3 × 4 × 5 × 7 = 420

The number must be a multiple of 420, so the number can be written as 420k, where k is a whole number.

N = 420 × 1 = 420

When divided by 11 leaves a remainder of 2

11 × 38 + 2 = 420

But we require 10 as a remainder

∴ 2k = 10

⇒ k = 5

Hence number = 420 × 5 = 2100

37. Here is a problem posed by the ancient Indian Mathematician Mahaviracharya (850 C.E.). Add together $\frac{8}{15}, \frac{1}{20}, \frac{7}{36}, \frac{11}{63}$ and $\frac{1}{21}$ What do you get? How can we find this sum efficiently?

Ans. : Here $\frac{8}{15}, \frac{1}{20}, \frac{7}{36}, \frac{11}{63}, \frac{1}{21}$

Now 15 = 3 × 5

20 = 2 × 2 × 5

36 = 2 × 2 × 3 × 3

63 = 3 × 3 × 7

21 = 3 × 7

LCM of denominators = 2 × 2 × 3 × 3 × 5 × 7

= 4 × 9 × 5 × 7

= 1260

$$\text{Now } \frac{8}{15} + \frac{1}{20} + \frac{7}{36} + \frac{11}{63} + \frac{1}{21}$$

$$= \frac{8 \times 84}{1260} + \frac{1 \times 63}{1260} + \frac{7 \times 35}{1260}$$

$$= \frac{11}{1260} \times 20 + \frac{1 \times 60}{1260}$$

$$= \frac{1260}{1260} = 1$$

*** Questions With Calculation.[5 Marks Each]** **[15]**

38. The LCM of 3 and 24 is 24 (it is one of the two given numbers):
Find more such number pairs where the LCM is one of the two numbers.

Ans. : Here are more number pairs where the LCM of the two numbers is one of the given numbers:

(i) LCM of 2 and 4: The LCM is 4.

Prime factorization of 2: 2

Prime factorization of 4: 2 × 2

LCM (2, 4) = 2 × 2 = 4

(ii) LCM of 5 and 10: The LCM is 10.

Prime factorization of 5: 5

Prime factorization of 10: 2 × 5

LCM (5, 10) = 2 × 5 = 10

(iii) LCM of 6 and 12: The LCM is 12.

Prime factorization of 6: 2 × 3

Prime factorization of 12: 2 × 2 × 3

LCM (6, 12) = 2 × 2 × 3 = 12

(iv) LCM of 7 and 49: The LCM is 49.

Prime factorization of 7: 7

Prime factorization of 49: 7 × 7

LCM (7, 49) = 49

(v) LCM of 10 and 100: The LCM is 100.

Prime factorization of 10: 2 × 5

Prime factorization of 100: 2 × 2 × 5 × 5
LCM (10, 100) = 2 × 2 × 5 × 5 = 100

39. The length, width, and height of a box are 12 cm, 18 cm, and 36 cm, respectively. Which of the following-sized cubes can be packed in this box without leaving gaps?
(a) 9 cm (b) 6 cm (c) 4 cm (d) 3 cm (e) 2 cm

Ans. : Here, dimensions of box: Length = 12 cm, Width = 18 cm, Height = 36 cm
The size of the largest cube that can exactly fit (without gaps)
That means the side of the cube must exactly divide all three dimensions of the box.
So, we need to find the HCF (Highest Common Factor) of 12, 18, and 36.
Prime factorisation 12 = 2 × 2 × 3, 18 = 2 × 3 × 3, and 36 = 2 × 2 × 3 × 3
Common factors = 2 × 3 = 6
HCF = 6 cm
The cube must have a side length equal to a factor of the HCF.
From the options:
(a) 9 cm ✗ (9 doesn't divide 12 evenly)
(b) 6 cm ✓ (divides 12, 18, and 36 exactly)
(c) 4 cm ✗ (doesn't divide 18 evenly)
(d) 3 cm ✓ (also divides all)
(e) 2 cm ✓ (also divides all)
Hence, the largest possible cube that fits without gaps is (b) 6 cm.

40. Children are playing 'Fire in the Mountain.' When the number 6 was called out, no one got out. When the number 9 was called out, no one got out. But when the number 10 was called out, some people got out. How many children could have been playing initially?
(a) 72 (b) 90 (c) 45 (d) 3 (e) 36 (f) None of these

Ans. : Interpretation is that when a number k is called, the children are grouped into rows of size k, and "no one got out" means the children formed complete rows with no remainder.
So: "No one got out" when 6 was called ⇒ the total number N is divisible by 6.
"No one got out" when 9 was called ⇒ N is divisible by 9.
"Some people got out" when 10 was called ⇒ N is not divisible by 10.
If N is divisible by both 6 and 9, then it must be divisible by their LCM:
LCM (6, 9) = 18.
So N is a multiple of 18, but not a multiple of 10.
Now check the options:
(a) 72 = 18 × 4 – divisible by 18 and not by 10 → possible.
(b) 90 = 18 × 5 – divisible by 18 but is divisible by 10 → not possible.
(c) 45 – not divisible by 18 → not possible.
(d) 3 – not divisible by 18 → not possible.

(e) 36 = 18 × 2 – divisible by 18 and not by 10 → possible.
So the numbers that could have been playing are 36 and 72.

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