Work and Energy

5.1 CONSERVATIVE AND NON-CONSERVATIVE FORCES

Recollect these common occurrences:

- A ball thrown upward comes back down to your hand.
- A stretched rubber band returns to its original shape.
- A sliding book on a table finally stops.

Now think carefully:

- Why does the ball come back?
- Why does the rubber band regain its shape?
- Why does the book stop moving?

In all these cases, forces are acting. But are all these forces the same?

Conservative Forces

- If the work done by the force doesn't depend on the path.
- Work done by the force on the closed path is always zero.
- For a conservative force, the work done by the force equals the decrease in potential energy: $W = -\Delta U = -(U\_final - U\_initial) = U\_initial - U\_final$. Equivalently, $\Delta U = U\_final - U\_initial = -W$.

Examples: Gravitational force (Earth pulling objects downward) and Spring force (stretched or compressed spring)

Non-Conservative Forces: If the work done by the force depends on the path taken.

Examples: Friction (solid and drag)

When you slide a book across a table, it eventually stops because friction converts its kinetic energy into heat. This lost energy cannot be fully recovered.

That is why friction is a non-conservative force.

Reflect on the following:

- If there were no friction, would a moving object ever stop?
- Why do pendulums slowly stop after some time?
- Why do machines require lubrication?

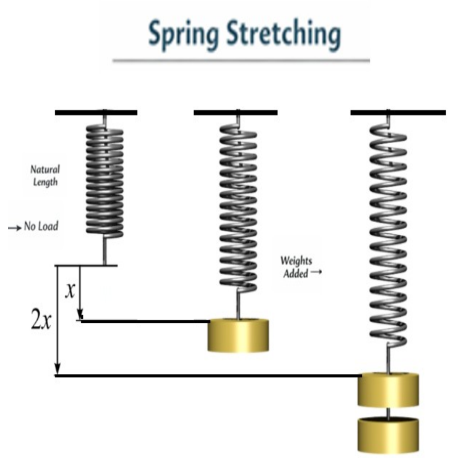
Quick Check

1. Define a conservative force with one example.
2. Why is gravitational force called a conservative force?
3. Why is friction called a non-conservative force?
4. What happens to energy when a non-conservative force acts on an object?
5. If there were no friction on Earth, how would motion be different? Explain.

5.2 Potential Energy of a Spring

Activity 5.2:

Collect the following items: A spring, a stand, a weight hanger, slotted weights, a ruler.



1. Suspend a spring vertically from a rigid support.
2. Attach a weight hanger to the free end of the spring and note the initial length of the spring.
3. Add a known weight to the hanger and measure the extension produced in the spring.
4. Increase the weight gradually and note the corresponding extension each time.
5. Repeat the experiment using springs made of different materials or thickness.

Observation:

- As more weight is added, the extension of the spring increases.

Different springs show different extensions for the same applied weight.

Conclusion:
The extension of a spring is directly proportional to the applied force (weight), provided the elastic limit is not exceeded. This relationship can be expressed as:

$$F = kx$$

where
F= applied force,
x= extension produced,
k= spring constant, which depends on the nature of the spring.

This law is called Hooke's law and is mathematically stated as F=−kx. The negative sign indicates the force is a restoring force acting against the direction of displacement (elongation or compression), aiming to return the spring to its original length.

Its unit is N m^{-1}. The spring is said to be stiff if k is large and soft if k is small.

Derivation

- The spring obeys Hooke's Law

$$F \propto x$$
$$F = kx$$

Prepare a graph between Force and extension in the spring with the help of data observed in the activity by taking

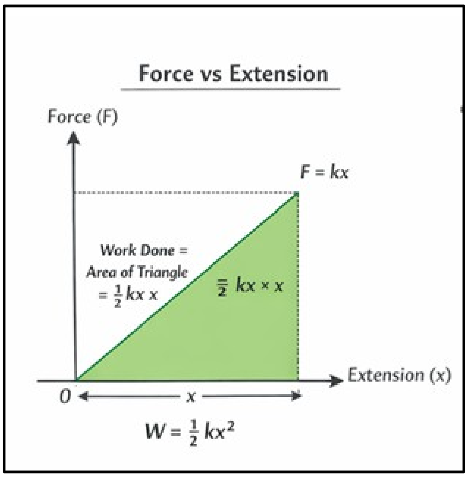
- X-axis → Extension (m or cm)
- Y-axis → Force (N)

Observation Table

These values give a straight-line graph passing through origin.

Sample Values for k= 100 N/m

Force (F) in N	Extension (x) (in cm)	Extension (x) (in m)
0	0 cm	0 m
20	20 cm	0.2 m
40	40 cm	0.40 m
60	60 cm	0.60 m
80	80 cm	0.80 m
100	100 cm	1 m



Calculation of Average Force:

For a spring stretched from 0 to maximum force:

$$Average\ Force = \frac{F_{initial} + F_{final}}{2}$$

When the spring is stretched gradually from zero extension to a maximum extension x, the force acting on it does not remain constant.

- At the beginning, force = 0
- At extension x, force = kx

So, the average force (spring force changes linearly from 0 to maximum as extension increases.) acting on the spring is given by:

$$F_{average} = \frac{0 + kx}{2} = \frac{kx}{2}$$

Work done in stretching the spring = Average force × Extension

$$W = \frac{kx}{2} \times x = \frac{1}{2}kx^2$$

Conclusion

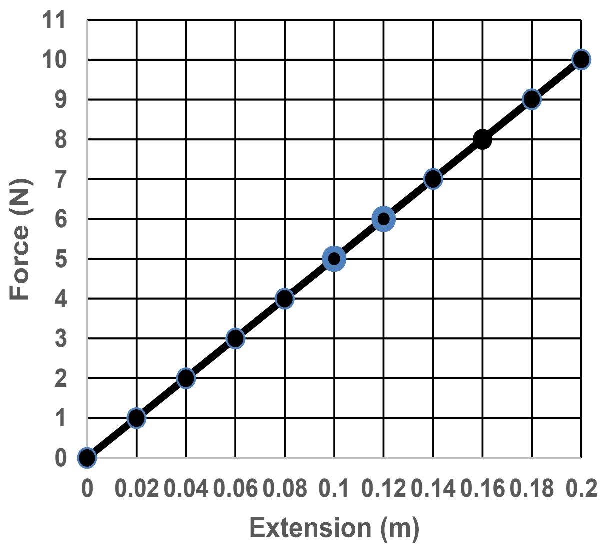
The work done in stretching the spring is stored in it as elastic potential energy.

$$U = \frac{1}{2}kx^2$$

Example: A spring obeys Hooke's law with a spring constant of 30 N m^{-1}. If a force of 100 N is applied to the spring, calculate the extension produced in the spring. The Force-extension graph of a spring of spring constant 100 N m^{-1} is given in figure:

(a) Using the graph, determine the work done in stretching the spring from 2 cm to 6 cm.

(b) If the spring is released from the stretched position of 6 cm, calculate the maximum speed of a body of mass 0.5 kg attached to the spring, assuming no loss of energy.



Solution:

(a) Work done = area under force–extension graph

$$W = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$W = \frac{1}{2} \times 100 \times (0.06^2 - 0.02^2)$$

$$W = 50 \times (0.0036 - 0.0004)$$

$$W = 50 \times 0.0032 = 0.16\,\text{J}$$

(b) $k = 100$ N/m, $x = 0.06$ m. Elastic PE = $\frac{1}{2} \times 100 \times 0.06^2 = 0.18$ J. At maximum speed, all PE converts to KE: $0.18 = \frac{1}{2} \times 0.5 \times v^2$, so $v^2 = 0.72$, $v \approx 0.85$ m/s.

Check Your Understanding

1) Explain the conversion of potential energy to kinetic energy when a ball is thrown upward.
2) Why is gravitational potential energy considered a conservative force?
3) Calculate the potential energy of a 5 kg object kept on the top of a 30m high building. (Considering potential energy to be zero at the base of the building.)
4) What is the increment in its potential energy?
5) A 10 kg weight is hung from a 5 m wire, causing it to stretch by 1 mm. Calculate the energy stored.
6) Calculate the work done by an external force to lift a 2 m long rod from a horizontal to a vertical position.

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