The Geometry of Power- Advanced Simple Machines

4.1 Introduction

Welcome to the study of Mechanical Advantage. While simple machines such as levers and inclined planes form the foundational concepts of physics, the machines that shape our modern world—like cranes, trucks, and bicycles—apply these same principles in more advanced and integrated ways through systems of wheels, axles, and pulleys.

In this chapter, we will examine how the principles of geometry and force distribution allow a relatively small input force to be transformed into a much larger output force.

Activity 4.1:

- A truck driver turning a massive vehicle using only two hands.
- A crane lifting heavy concrete beams smoothly.
- A cyclist moving very fast by pedaling lightly.

Now think carefully:

- Is the driver extremely strong?
- Does the crane create extra force?
- Does the cyclist get “free” speed?

In all these cases, machines are helping us multiply force or increase speed.

This multiplication is called **Mechanical Advantage (MA).**

4.2 Wheel and Axle – The Steering Mastery

Activity 4.2: Think and Answer

“Think about a steering wheel and axle (steering column) and their respective radius.”

- The steering wheel is large. The steering column connected to it is small. Why is this so?
- Why not make both of equal size?

A wheel and axle consist of:

- a large wheel
- a smaller axle fixed at the center

Both rotate together. When effort is applied on the wheel, torque increases at the axle.

The Mechanical Advantage is calculated using the formula:

$$M.A = \frac{Radius\ of\ wheel}{Radius\ of\ Axle}$$

Example

A driver needs to maneuver the truck on muddy ground, requiring a resistance force of 1,200 N to turn the steering axle. The steering wheel has a radius of 30cm, and the steering axle has a radius of 3cm.

a) Calculate the Mechanical Advantage (MA) of the steering system.

b) How much effort (E) must the driver apply to the rim of the steering wheel to turn the truck?

Solution:

a) $M.A = \frac{Radius\ of\ wheel}{Radius\ of\ Axle}$

$= \frac{30\ cm}{3\ cm} = 10$

b) $Effort\ (E) = \frac{load}{M.A}$

$= \frac{1200N}{10} = 120N$

The driver needs to apply an effort of 120N to the rim of the steering wheel to turn the truck.

Note: In practice, some input work is lost to friction within the machine. The efficiency of a machine is defined as η = (useful output work / total input work) × 100%. A real machine always has η < 100%. Mechanical Advantage as calculated here assumes an ideal (frictionless) machine.

Quick Check

In a mechanical watch, a single power source (a spring or motor) must move three different hands at three different speeds. This is achieved through a **Gear Train**, where the "output" of one gear becomes the "input" for the next. The seconds-to-minutes gear ratio is 60:1 and the minutes-to-hours ratio is 60:1;

1. If the seconds gear is 2 mm, how large would the hour gear be in meters?
2. Which of the three hands gear should be directly connected to the motor? Why?

4.3 Tension

Activity 4.3:

Hang a thread from an iron stand as shown in figure. Observe its natural length.

Now attach a small bob to the lower end of the thread. Notice how the thread stretches slightly.

Replace the small bob with a heavier bob. Does the stretch increase or decrease?

You will observe that the thread stretches more when a heavier bob is attached. This shows that a greater pulling force is acting on the thread.

Further,

Pass the thread over a pulley. Attach a weight to one side and observe.

Now attach equal weights (equal bobs) on both sides of the pulley. Does the rope move, or does it only stretch?

Replace one of the equal bobs with a heavier bob on the left side. Observe carefully the direction in which the system moves.

When both sides have equal weights, the system remains at rest because the forces are balanced. When one side is heavier, the system moves toward the heavier side.

Tension

Tension is the pulling force/ stretch force that travels through a stretched string, thread, rope, or cable. When you hang an object using a thread, the object pulls the thread downward because of its weight. In response, the thread pulls the object upward. This pulling force inside the thread is called tension. If the weight attached to the string increases, the tension in the string also increases.

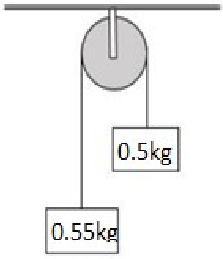
It always acts along the length of the string and pulls away from the object to which it is attached.

S.I unit of tension is Newton.

When the forces acting on an object are balanced (for example, the upward tension is equal to the downward weight), the object remains at rest or moves with constant speed. This state is called **equilibrium.** In a pulley system, if equal weights are placed on both sides, the tensions balance and the system does not move. But if one side is heavier, the forces become unbalanced, and the system moves toward the heavier side. This is by Newton's First Law of Motion.

Let us Calculate: Tension and acceleration are produced when two unequal masses are connected over a pulley.

1. Do the setup of weights, string and simple pulley as shown.



2. Since 0.55 kg > 0.5 kg, the 0.55 kg mass will move downward. The 0.5 kg mass will move upward. Both masses will move with the same acceleration because they are connected by the same string.
3. For 0.55 kg mass (moving downward):
   a. Downward force = Weight = ___________
   b. Upward force = Tension (T)
4. Net force: $0.55g - T = 0.55a$
5. For 0.5 kg mass (moving upward):
   a. Downward force = Weight = ___________
   b. Upward force = Tension (T)
6. Net force: $T - 0.5g = 0.5a$
7. Add both equations: ________________________________
8. Acceleration of the system: ________________
9. Find Tension: __________________

Examples:

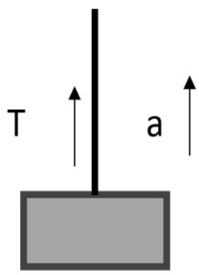
1. A 5kg object is suspended stationary from a rope. Calculate the tension.

   Ans: The weight of the object is: $T = m \times g = 5 \times 9.8 = 49\,N$

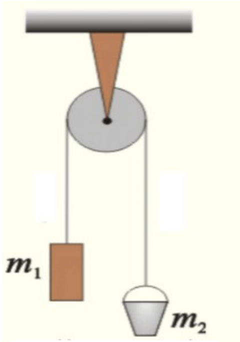
2. A 4 kg mass is lifted upward with an acceleration of 2 m/s². Calculate the tension.

   Ans: Using Newton's Second Law

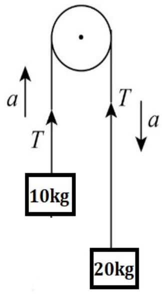
   $T - mg = maT = m(g + a)T = 4(9.8 + 2)T = 47.2\,N$



Check Your Understanding :



1. Show the direction of weight and tension for both objects m_1 and m_2.
2. An 8 kg mass hangs freely from a single fixed pulley. The system is at rest. Find the tension in the rope.
3. Observe the given diagram. Find out in which direction the rope will move? What will be the net downward force?



4. A 6 kg mass hangs freely from a single fixed pulley. The system is at rest. Find the tension in the rope.
5. Two objects having masses 2 kg and 6 kg are connected over a frictionless pulley with the help of rope. Find acceleration and tension in the rope.

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