Capacitor

Question1

A capacitor of capacitance 5μ F is charged by a battery of emf 10 V . At an instant of time, the potential difference across the capacitors is 4 V and the time rate of change of potential difference across the capacitor is $0.6 \mathrm{Vs}^{-1}$. Then, the time rate at which energy is stored the capacitor at $\geq$ instant is

KCET 2024

Options:

A. 12μ W

B. 3μ W

C. zero

D. 30μ W

Answer: A

Solution:

Given, battery emf, $e=10$ V

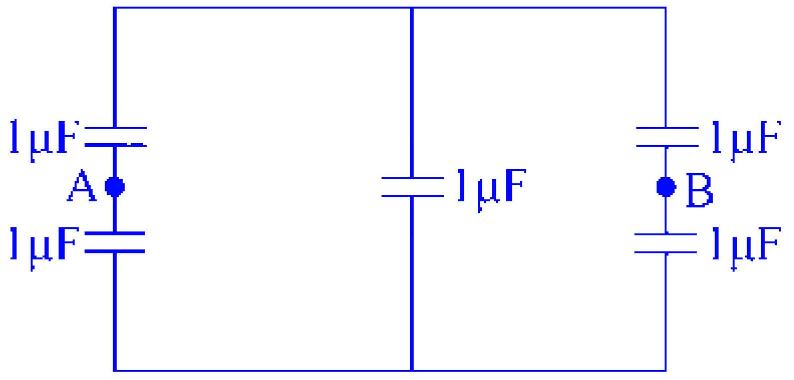
$C=5\mu \mathrm{~F}=5\times10^{-6}$ F.

We know that stored energy,

$$\begin{aligned} U &= \frac{1}{2} C V^2 \\ \therefore \quad \frac{dU}{dt} &= \frac{1}{2} C \cdot 2V \frac{dV}{dt} = CV \frac{dV}{dt} \\ &= 5 \times 10^{-6} \times 4 \times 0.6 \\ &= 12 \times 10^{-6} \mathrm{~W} = 12\mu \mathrm{~W} \end{aligned}$$

Question2

Five capacitance each of value $1\ \mu\text{F}$ are connected as shown in the figure. The equivalent capacitance between A and B is



KCET 2023

Options:

A. $3\ \mu$ F

B. $1\ \mu$ F

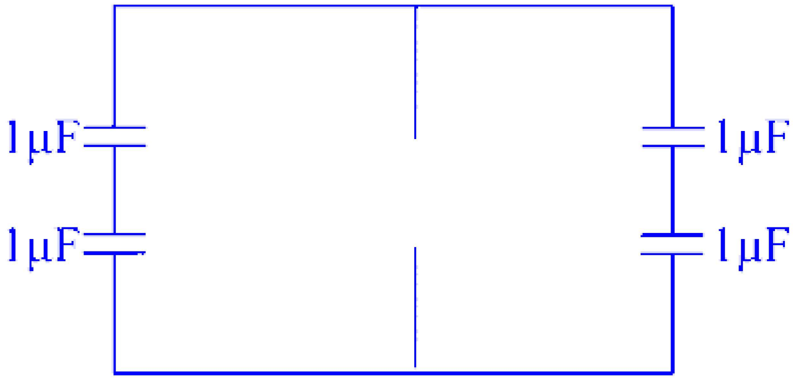
C. $2\ \mu$ F

D. $5\ \mu$ F

Answer: B

Solution:

Since, the middle branch conducts no current, therefore the middle capacitor represents open circuit and hence reduced circuit is shown.



$$\begin{aligned} \text{Now} \quad \frac{1}{C_{\text{series}}} &= \frac{1}{1} + \frac{1}{1} \\ C_{\text{series}} &= 0.5\mu\text{F} \\ C_{\text{parallel}} &= 0.5 + 0.5 \\ \text{or} \quad C_{AB} &= 1\mu\text{F} \end{aligned}$$

Question3

A parallel plate capacitor of capacitance C_1 with a dielectric slab in between its plates is connected to a battery. It has a potential difference V_1 across its plates. When the dielectric slab is removed, keeping the capacitor connected to the battery, the new capacitance and potential difference are C_2 and V_2 respectively, Then

KCET 2023

Options:

A. $V_1 = V_2, C_1 < C_2$

B. $V_1 > V_2, C_1 > C_2$

C. $V_1 < V_2, C_1 > C_2$

D. $V_1 = V_2, C_1 > C_2$

Answer: D

Solution:

To solve this problem, let's break it down step by step.

First, recall the relationship between the capacitance and the presence of a dielectric material. The capacitance C of a parallel plate capacitor with a dielectric is given by:

$C = \kappa C_0$

where:

- κ is the dielectric constant of the material
- C_0 is the capacitance without the dielectric

Given that:

- C_1 is the capacitance with the dielectric.
- C_2 is the capacitance without the dielectric.

Since the dielectric constant κ is greater than 1, it follows that:

$C_1 > C_2$

When the capacitor is connected to a battery, the potential difference across its plates is determined by the battery. So, if we keep the capacitor connected to the battery while removing the dielectric slab, the potential difference across the plates remains unchanged.

Therefore:

$V_1 = V_2$

Given the above analysis, the correct relationship is:

$C_1 > C_2$ and $V_1 = V_2$

Thus, the correct option is:

Option D

$V_1 = V_2, C_1 > C_2$

Question4

A parallel place capacitor is charged by connecting a 2 V battery across it. It is then disconnected from the battery and a glass slab is introduced between plates. Which of the following pairs of quantities decrease?

KCET 2022

Options:

A. Potential difference and energy stored

B. Energy stored and capacitance

C. Capacitance and charge

D. Charge and potential difference

Answer: A

Solution:

When a charged capacitor is disconnected from the battery and a glass slab (dielectric material) is introduced between the plates, then charge (Q), capacitance (C), electric potential (V) and stored energy (U) is given as

$Q = Q_0, C = KC_0, V = \frac{V_0}{K}$ and $U = \frac{U_0}{K}$

where, $K =$ dielectric constant

$V =$ electric potential

$U =$ potential energy

$V_0 =$ initial potential of charged capacitor

$U_0 =$ Initial energy of charged capacitor

From above, we see that V and U decreases after introducing glass plate of dielectric constant k whereas capacitance increases and charge remains same because battery is disconnected after introducing dielectric material.

Question5

If a slab of insulating material (conceptual). 4×10^{-3} m thick is introduced between the plates of a parallel plate capacitor, the separation between the plates has to be increased by 3.5×10^{-3} m to restore the capacity to original value. The dielectric constant of the material will be

KCET 2021

Options:

A. 6

B. 8

C. 10

D. 12

Answer: B

Solution:

Let t be the thickness of the dielectric slab and K is the dielectric constant.

So, the increase in the distance of separation between the plates due to dielectric is given as

$$x = t - \frac{t}{K}$$
$$= t\left(1 - \frac{1}{k}\right)$$

Given, $x = 3.5 \times 10^{-3}$ m, $t = 4 \times 10^{-3}$ m

Substituting the given values in the above equation, we get

$$1 - \frac{1}{k} = \frac{x}{t} = \frac{3.5 \times 10^{-3}}{4 \times 10^{-3}} = \frac{3.5}{4}$$
$$\text{or} \quad \frac{1}{k} = 1 - \frac{3.5}{4} = \frac{0.5}{4}$$
$$\text{or} \quad k = 8$$

Question6

Eight drops of mercury of equal radii combine to form a big drop. The capacitance of a bigger drop as compared to each smaller drop is

KCET 2021

Options:

A. 2 times

B. 8 times

C. 4 times

D. 16 times

Answer: A

Solution:

Let R and r be the radii of bigger and each smaller drop, respectively.

Volume of bigger drop $= 8\times$ Volume of smaller drop

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

or $\quad R = 2r \quad$ .... (i)

As, capacitance of a smaller spherical drop is

$$C = 4\pi\varepsilon_0 r$$
$$\Rightarrow \quad C \propto r$$

So, $\frac{C_{\text{bigger}}}{C_{\text{smaller}}} = \frac{r_{\text{bigger}}}{r_{\text{smaller}}} = \frac{R}{r} = \frac{2r}{r} = 2$ [using Eq. (i)]

So, capacitance of bigger drop is two times as compared to each smaller drop.

Question7

An electrician requires a capacitance of $6\mu\text{F}$ in a circuit across a potential difference of 1.5 kV. A large number of $2\mu\text{F}$ capacitors which can withstand a potential difference of not more than 500 V are available. The minimum number of capacitors required for the purpose is

KCET 2021

Options:

A. 3

B. 9

C. 6

D. 27

Answer: D

Solution:

Number of capacitors that can be connected in each row,

$$m = \frac{\text{Desired voltage}}{\text{Voltage across each capacitor}}$$
$$= \frac{1.5\text{kV}}{500\text{ V}} = \frac{1500}{500} = 3$$

Effective capacitance when m capacitors are connected in n rows is given as

$C_{\text{eff}} = \frac{nC}{m}$

Here, $m = 3, C = 2\mu\text{F}, C_{\text{eff}} = 6\mu\text{F}$

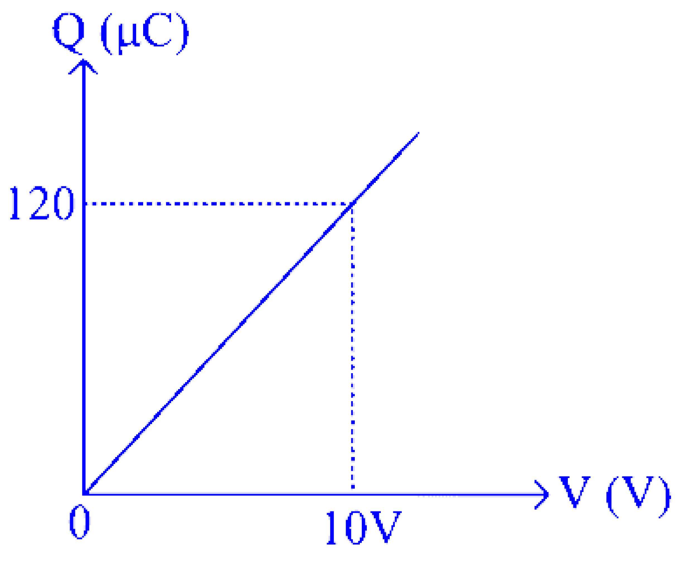
$\Rightarrow \quad n \times \frac{2}{3} = 6$

or $\quad n = \frac{18}{2} = 9$

$\therefore$ Total number of capacitors required, $N = mn = 3 \times 9 = 27$

Question8

In figure, charge on the capacitor is plotted against potential difference across the capacitor. The capacitance and energy stored in the capacitor are respectively.



KCET 2021

Options:

A. $12\mu\text{F}, 1200\mu\text{J}$

B. $12\mu\text{F}, 600\mu\text{J}$

C. $24\mu\text{F}, 600\mu\text{J}$

D. $24 \mu F, 1200 \mu J$

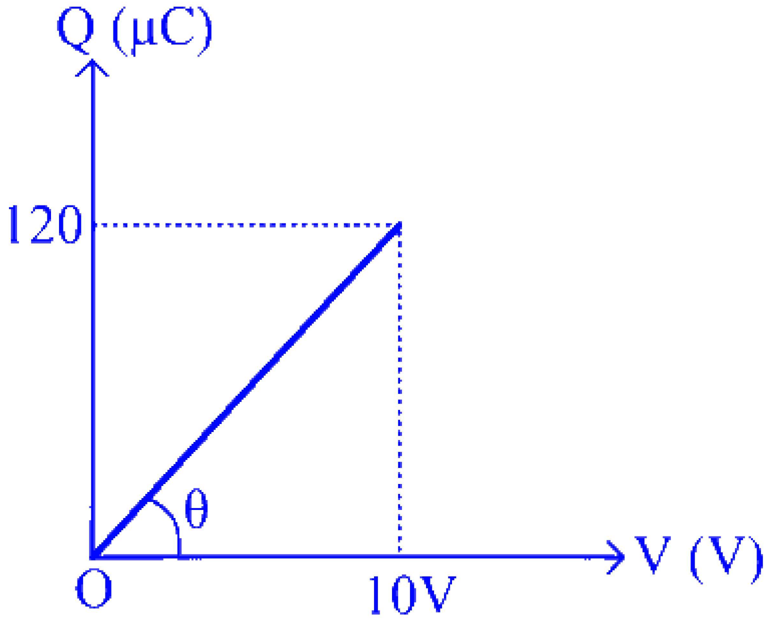
Answer: B

Solution:

Energy stored in a capacitor is given as

$U=\frac{1}{2} C V^2$ .... (i)

where, C is the capacitance. From the given graph, we get



Slope $=\tan \theta=\frac{Q}{V}=\frac{120 \mu C}{10 \mathrm{~V}}$ .... (ii)

As we know, capacitance is given as

$C=\frac{Q}{V}$

Substituting the value of $\frac{Q}{V}$ from Eq. (ii) in the above relation, we get

$$C=\frac{120 \mu C}{10 \mathrm{~V}}=\frac{120 \times 10^{-6}}{10}=12 \times 10^{-6} \mathrm{~F}$$
$$=12 \mu F$$

Now, substituting the values of C and V in Eq: (i), we get

$$U=\frac{1}{2} \times 12 \times(10)^2$$
$$=600 \mu J$$

Question9

The difference between equivalent capacitances of two identical capacitors connected in parallel to that in series is $6 \mu \mathrm{F}$. The value of capacitance of each capacitor is

KCET 2020

Options:

A. $2 \mu \mathrm{F}$

B. $3 \mu \mathrm{F}$

C. $4 \mu \mathrm{F}$

D. $6 \mu \mathrm{F}$

Answer: C

Solution:

Given, $C_1=C_2=C$ (identical capacitors)

When both capacitors are connected in parallel, then equivalent capacitance

$$
\begin{aligned}
& C_p=C_1+C_2=C+C \\
& C_p=2C \quad \text{.... (i)}
\end{aligned}
$$

When both capacitors are connected in series, then equivalent capacitance,

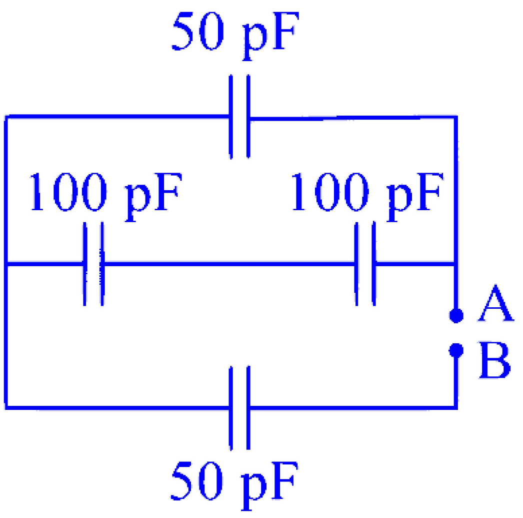
$$
\begin{aligned}
& \frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{C}+\frac{1}{C} \\
\Rightarrow \quad & \frac{1}{C_s}=\frac{2}{C} \Rightarrow C_s=\frac{C}{2}
\end{aligned}
$$

According to question,

$$
\begin{aligned}
& C_p-C_s=6 \mu \mathrm{F} \\
\Rightarrow \quad & 2C-\frac{C}{2}=6 \mu \mathrm{F} \quad \Rightarrow \quad \frac{3C}{2}=6 \mu \mathrm{F} \\
\Rightarrow \quad & C=4 \mu \mathrm{F}
\end{aligned}
$$

Question10

The equivalent capacitance between A and B is



KCET 2019

Options:

A. 50 pF

B. $\frac{100}{3}$ pF

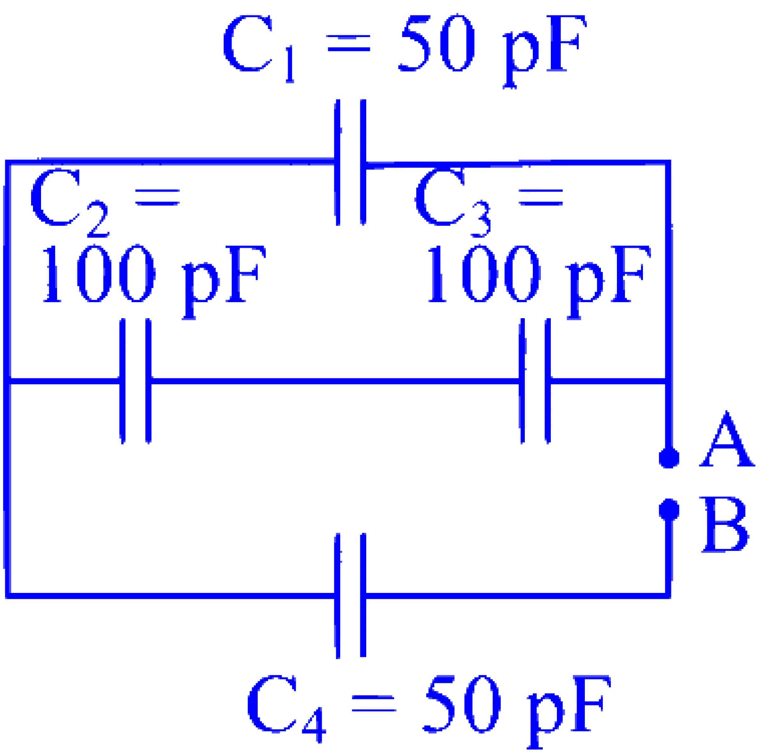
C. 150 pF

D. 300 pF

Answer: B

Solution:

The circuit diagram is given as



C_2 and C_3 are in series,

Hence, $C_{23}=\frac{C_2\times C_3}{C_2+C_3}=\frac{100\times 100}{100+100}=50\text{ pF}$

C_1 is parallel to C_{23},

$\therefore C_{123}=C_1+C_{23}=50+50=100\text{ pF}$

Finally, C_{123} is series with C_4

$$C_{AB}=C_{1234}=\frac{C_{123}\times C_4}{C_{123}+C_4}=\frac{100\times 50}{100+50}=\frac{100\times 50}{150}$$
$$=\frac{100}{3}\text{pF}$$

Question11

A capacitor of capacitance C charged by an amount Q is connected in parallel with an uncharged capacitor of capacitance $2C$. The final charges on the capacitors are

KCET 2019

Options:

A. $\frac{Q}{2}, \frac{Q}{2}$

B. $\frac{Q}{4}, \frac{3Q}{4}$

C. $\frac{Q}{3}, \frac{2Q}{3}$

D. $\frac{Q}{5}, \frac{4Q}{5}$

Answer: C

Solution:

A capacitor of capacitance C charged by an amount Q is connected in parallel with a uncharged capacitor of capacitance $2C$, If q_1 and q_2 be charge on capacitors of capacitance C and $2C$ respectively, after connection, then

$q_1 + q_2 = Q$ ..... (i)

when both capacitors are connected in parallel, then $V_1 = V_2$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{C}{2C}$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{1}{2}$$

$$q_2 = 2q_1 \quad .... \text{(ii)}$$

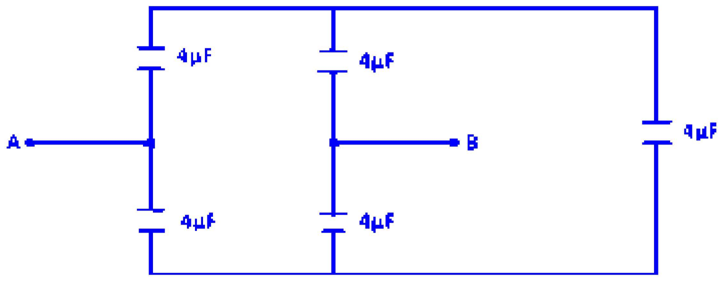
From Eqs. (i) and (ii), we have,

$$q_1 + 2q_1 = Q \Rightarrow q_1 = \frac{Q}{3}$$

$$\therefore \quad q_2 = \frac{2Q}{3}$$

Question12

For the arrangement of capacitors as shown in the circuit, the effective capacitance between the points A and B is (capacitance of each capacitor is 4μ F)



KCET 2018

Options:

A. 4μ F

B. 2μ F

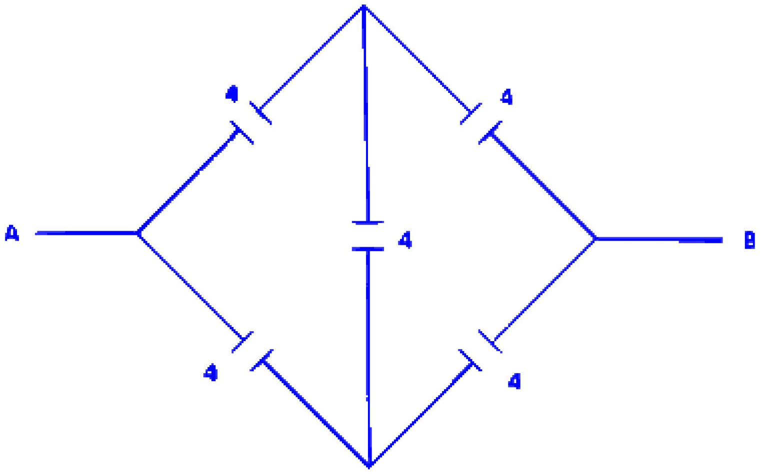
C. 1μ F

D. 8μ F

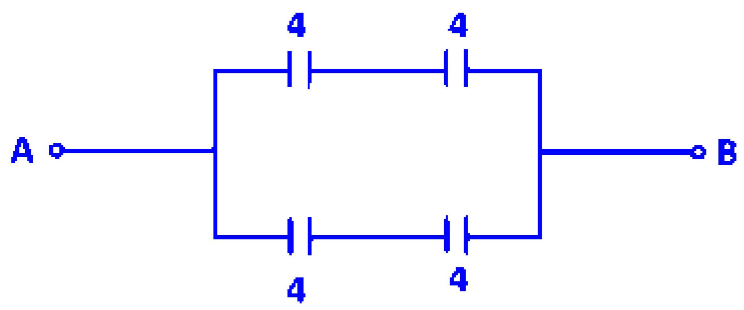
Answer: A

Solution:

The circuit can be redrawn as



This is a balanced Wheatstone bridge. So, the dircuit can be redrawn as



$\therefore \quad C_{AB} = \frac{4\times4}{4+4} + \frac{4\times4}{4+4} = 4\mu\text{ F}$

Question13

Two capacitors of 3μ F and 6μ F are connected in series and a potential difference of 900 V is applied across the combination. They are then disconnected and reconnected in parallel. The potential difference across the combination is

KCET 2018

Options:

A. zero

B. 200 V

C. 100 V

D. 400 V

Answer: B

Solution:

Let's work through the problem step by step.

Series connection:

The two capacitors are 3 μF and 6 μF. When connected in series, the equivalent capacitance is given by

$C_S = \frac{C_1C_2}{C_1+C_2} = \frac{3\mu F\times6\mu F}{3\mu F+6\mu F} = \frac{18}{9} = 2\mu F.$

With a potential difference of 900 V applied across the series combination, the same charge is stored on each capacitor. The charge is:

$Q = C_S \times V = 2\mu F \times 900V = 1800\mu C.$

Voltage across individual capacitors:

Voltage across the 3 µF capacitor:

$V_1 = \frac{Q}{C_1} = \frac{1800\mu C}{3\mu F} = 600V.$

Voltage across the 6 µF capacitor:

$V_2 = \frac{Q}{C_2} = \frac{1800\mu C}{6\mu F} = 300V.$

Notice that the sum, $600V + 300V = 900V$, matches the applied voltage.

Reconnection in parallel:

After disconnecting from the supply, the charges on the capacitors remain unchanged.

When reconnected in parallel (with like plates connected together), the total charge in the system is the sum of the individual charges:

$Q_{total} = 1800\mu C + 1800\mu C = 3600\mu C.$

In a parallel combination, the total capacitance is the sum of the individual capacitances:

$C_{total} = 3\mu F + 6\mu F = 9\mu F.$

The common potential difference across the parallel combination is:

$V = \frac{Q_{total}}{C_{total}} = \frac{3600\mu C}{9\mu F} = 400V.$

Thus, the potential difference across the combination when reconnected in parallel is 400 V.

Answer: Option D (400 V).

Question14

The minimum value of effective capacitance that can be obtained by combining 3 capacitors of capacitances 1pF, 2pF and 4 pF

KCET 2017

Options:

A. $\frac{4}{7}$pF

B. 1 pF

C. 2 pF

D. $\frac{7}{4}$pF

Answer: A

Solution:

The minimum value of effective capacitance is achieved when the capacitors are connected in series.

To find the equivalent capacitance for capacitors in series, use the formula:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For capacitors with capacitances 1 pF, 2 pF, and 4 pF, the equivalent capacitance calculation is:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4}$$

This combines to:

$$\frac{1}{C_{\text{eq}}} = \frac{4+2+1}{4} = \frac{7}{4}$$

Solving for C_{eq} gives:

$$C_{\text{eq}} = \frac{4}{7}\text{ pF}$$

Thus, the minimum effective capacitance obtainable by connecting the three capacitors in series is $\frac{4}{7}$ pF.

Question15

A system of two capacitors of capacitance 2μ F and 4μ F is connected in series across a potential difference of 6 V . The electric charge and energy stored in a system are

KCET 2017

Options:

A. 36μC and 108μ J

B. 8μC and 24μ J

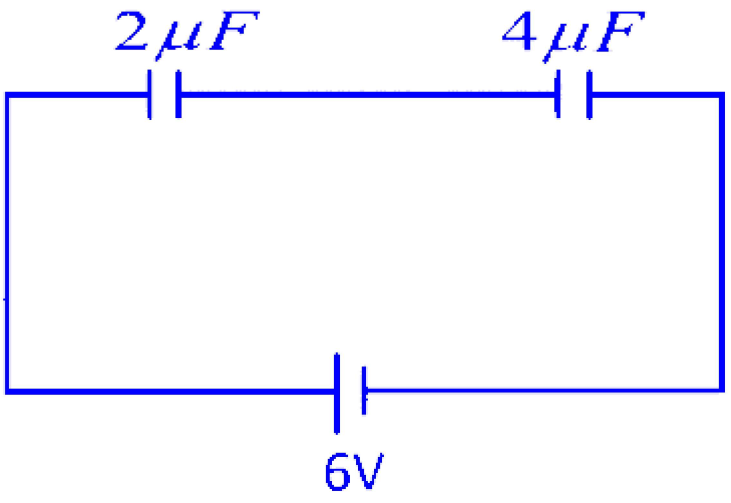
C. 1μC and 3μ J

D. 10μC and 30μ J

Answer: B

Solution:

According to the question, we can draw the following diagram.



The capacitor are connected in series combination,

$$So, \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4}$$

$$\Rightarrow \quad C_{eq} = \tfrac{4}{3}\mu \text{ F}$$

We know that,

$$Q = CV = \frac{4}{3} \times 6 = 8\mu\text{C}$$

$$Energy, \quad E = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{4}{3} \times (6)^2$$

$$= 24\mu \text{ J}$$
