Elasticity

Question1

Two wires A and B are made of same material. Their diameters are in the ratio of $1:2$ and lengths are in the ratio of $1:3$. If they are stretched by the same force, then increase in their lengths will be in the ratio of

KCET 2025

Options:

A. $3:4$

B. $2:3$

C. $3:2$

D. $4:3$

Answer: D

Solution:

The problem involves two wires, A and B, made from the same material. The ratio of their diameters is $1:2$, and the ratio of their lengths is $1:3$. When the same force is applied to both wires, we need to determine the ratio of their length increases.

Here's how we can solve it:

The formula for the increase in length ($\Delta\ell$) when a wire is stretched by a force F is given by:

$\Delta\ell = \frac{F\ell}{AY} = \frac{F\ell}{\pi r^2 Y}$

Where:

ℓ is the original length,

A is the cross-sectional area of the wire,

Y is Young's modulus,

r is the radius of the wire.

To find the ratio of the increase in lengths of wires A and B, we calculate:

$\frac{\Delta \ell_1}{\Delta \ell_2} = \left(\frac{\ell_1}{\ell_2}\right) \times \left(\frac{r_2}{r_1}\right)^2$

For wires A and B:

The length ratio $\ell_1 : \ell_2$ is $1 : 3$.

The diameter ratio ($\Rightarrow$ radius ratio) is $1 : 2$, so $\frac{r_2}{r_1} = \frac{2}{1}$.

Substitute these values into the equation:

$\frac{\Delta \ell_1}{\Delta \ell_2} = \frac{1}{3} \times \left(\frac{2}{1}\right)^2 = \frac{1}{3} \times 4 = \frac{4}{3}$

Therefore, the ratio of the increase in their lengths is $4 : 3$.

Question2

A thick metal wire of density ρ and length L is hung from a rigid support. The increase in length of the wire due to its own weight is ($Y =$ Young's modulus of the material of the wire)

KCET 2024

Options:

A. $\frac{\rho g L}{Y}$

B. $\frac{1}{2} \frac{\rho g L^2}{Y}$

C. $\frac{\rho g L^2}{Y}$

D. $\frac{1}{4Y} \rho g L^2$

Answer: B

Solution:

Weight of the metal wire, $w = mg$

$\therefore$ Longitudinal stress $= \frac{w}{A} = \frac{mg}{A}$

$\therefore \quad Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{\frac{mg}{A}}{\frac{\Delta l}{\left(\frac{l}{2}\right)}}$

[$\because$ Weight of the wire acts at its centre $\therefore L' = \frac{L}{2}$]

$$\Rightarrow \quad \Delta l = \frac{mgl}{2YA}$$
$$= \frac{m}{AL} \cdot \frac{L^2 g}{2Y}$$
$$= \frac{\rho L^2 g}{2Y}$$

Question3

A stretched wire of a material whose young's modulus $Y = 2 \times 10^{11}\ \text{Nm}^{-2}$ has poisson's ratio 0.25 . Its lateral strain $\varepsilon_l = 10^{-3}$. The elastic energy density of the wire is

KCET 2023

Options:

A. $16 \times 10^5\ \text{Jm}^{-3}$

B. $1 \times 10^5\ \text{Jm}^{-3}$

C. $4 \times 10^5\ \text{Jm}^{-3}$

D. $8 \times 10^5\ \text{Jm}^{-3}$

Answer: A

Solution:

Given, Young's modulus, $Y = 2 \times 10^{11}\ \text{Nm}^{-2}$

Poisson ratio, $\sigma = 0.25$

Lateral strain, $\varepsilon_1 = 10^{-3}$

Elastic potential energy density is given by, PE $= \frac{1}{2} \times Y \times (\text{ strain })^2$

Poisson ratio $= \frac{\text{Lateral strain}}{\text{longitudinal strain}} = \frac{\varepsilon_1}{\text{longitudinal strain}}$

$\Rightarrow \sigma = \frac{\varepsilon_1}{\text{longitudinal strain}}$

$\Rightarrow$ longitudinal strain $= \frac{\varepsilon_1}{\sigma} = \frac{10^{-3}}{0.25} = c \times 10^{-3}$

Elastic potential energy density $= \frac{1}{2} \times Y \times \left(\frac{E_1}{\sigma}\right)^2$

$= \frac{1}{2} \times 2 \times 10^{11} \times \left(4 \times 10^{-3}\right)^2$
$= 10^{11} \times 16 \times 10^{-6} = 16 \times 10^5\ \text{Jm}^{-3}$

Question4

A metallic rod breaks when strain produced is 0.2%. The Young's modulus of the material of the rod $7 \times 10^9\ \text{N/m}^2$. The area of crosssection to support a load of $10^4\ \text{N}$ is

KCET 2022

Options:

A. $7.1 \times 10^{-6}\ \text{m}^2$

B. $7.1 \times 10^{-4}\ \text{m}^2$

C. $7.1 \times 10^{-2}\ \text{m}^2$

D. $7.1 \times 10^{-8}\ \text{m}^2$

Answer: B

Solution:

Given, Youngs' modulus, $Y = 7 \times 10^9\ \text{N/m}^2$

Load, $F = 10^4\ \text{N}$

We know that,

$$Y = \frac{Fl}{A\Delta l}$$

$$\Rightarrow \quad A = \frac{Fl}{Y\Delta l} = \frac{F}{Y} \times \frac{1}{\left(\frac{\Delta l}{l}\right)}$$

$$= \frac{10^4}{7 \times 10^9} \times \frac{1}{0.2} = 7.1 \times 10^{-4}\,\mathrm{m}^2$$

Question5

Young's modulus of a perfect rigid body is

KCET 2020

Options:

A. zero

B. unity

C. infinity

D. Between (a) and (b)

Answer: C

Solution:

For a perfect rigid body, strain is always zero, for all values of stress.

As, Young's modulus, $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

So, its Young's modulus would be infinity.

Question6

A wire is stretched such that its volume remains constant. The poission's ratio of the material of the wire is

KCET 2019

Options:

A. 0.50

B. -0.50

C. 0.25

D. -0.25

Answer: B

Solution:

Volume of wire is given by $V = \pi r^2 l$ .... (i)

where, r = radius of wire

l = length of wire

Since, volume of wire is constant, hence

$dv = 0$ .... (ii)

From Eq (i)

$$\frac{dV}{V} = 2\frac{dr}{r} + \frac{dl}{l}$$

$$0 = 2\frac{dr}{r} + \frac{dl}{l}$$

$-\frac{dr}{r} = \frac{1}{2}\frac{dl}{l} \Rightarrow \frac{\frac{dr}{r}}{\frac{dl}{l}} = -\frac{1}{2}$

$\therefore$ Poission's ratio, $\sigma = \frac{dr/r}{dl/l} = -\frac{1}{2} = -0.5$

Question7

Two wires A and B are stretched by the same load. If the area of cross-section of wire A is double that of B, then the stress on B is

KCET 2018

Options:

A. equal to that on A

B. twice that on A

C. half that on A

D. four times that on A

Answer: B

Solution:

Stress in a wire is defined as the force applied per unit area, given by the formula:

$\text{Stress} = \frac{\text{Force}}{\text{Area}}$

Let's denote the load (force) by F and the area of cross-section of wire B by A_B. Since the area of wire A is double that of wire B, we have:

$A_A = 2A_B$

Now, compute the stress in each wire:

For wire A:

$\sigma_A = \frac{F}{A_A} = \frac{F}{2A_B}$

For wire B:

$\sigma_B = \frac{F}{A_B}$

To find the ratio of the stresses, divide σ_B by σ_A:

$\frac{\sigma_B}{\sigma_A} = \frac{\frac{F}{A_B}}{\frac{F}{2A_B}} = \frac{F}{A_B} \times \frac{2A_B}{F} = 2$

This shows that the stress on wire B is twice that on wire A. Therefore, the correct option is:

Option B: twice that on A.

Question8

'Young's modulus' is defined as the ratio of

KCET 2017

Options:

A. hydraulic stress and hydraulic strain

B. shearing stress and shearing strain

C. tensile stress and longitudinal strain

D. bulk stress and longitudinal strain

Answer: C

Solution:

Young's modulus is defined as the ratio of tensile stress to longitudinal strain. This relationship can be expressed in the following formula:

$$\text{Young's modulus} = \frac{\text{tensile stress}}{\text{longitudinal strain}}$$
