Gravitation

Question1

If r_p, v_p, L_p and r_a, v_a, L_a are radii, velocities and angular momenta of a planet at perihelion and aphelion of its elliptical orbit around the Sun respectively, then

KCET 2025

Options:

A. $r_p > r_a, v_p > v_a, L_a > L_p$

B. $r_p < r_a, v_p > v_a, L_a = L_p$

C. $\mathrm{r_p > r_a, v_p < v_a, L_a = L_p}$

D. $\mathrm{r_p < r_a, v_p < v_a, L_a < L_p}$

Answer: B

Solution:

Angular momentum is conserved in an elliptical orbit, meaning:

$L_p = L_a$

where L_p is the angular momentum at perihelion and L_a is the angular momentum at aphelion.

The perihelion is the point in the orbit where the planet is closest to the Sun. Therefore, the radius at perihelion (r_p) is less than the radius at aphelion (r_a):

$r_p < r_a$

Because angular momentum is conserved, we have:

$mv_p r_p = mv_a r_a$

From this, it follows that the velocity at perihelion (v_p) is greater than the velocity at aphelion (v_a):

$v_p > v_a$

Question2

The total energy of a satellite in a circular orbit at a distance $(R+h)$ from the centre of the Earth varies as [R is the radius of the Earth and h is the height of the oribit from Earth's surface]

KCET 2025

Options:

A. $-\frac{1}{(\mathrm{R+h})}$

B. $\frac{1}{(R+h)^2}$

C. $-\frac{1}{(R+h)^2}$

D. $\frac{1}{(\mathrm{R+h})}$

Answer: A

Solution:

The total (mechanical) energy E of a satellite in a circular orbit of radius $r = R + h$ is the sum of its kinetic and potential energies. You can show that

Gravitational potential energy:

$U = -\frac{GMm}{r}$

Kinetic energy (for a circular orbit):

$K = \frac{1}{2}\frac{GMm}{r}$

Total energy:

$E = K + U = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$

Since G, M and m are constants, E varies as

$E \propto -\frac{1}{r} = -\frac{1}{R+h}$.

Answer: Option A $-\frac{1}{R+h}$.

Question3

What is the value of acceleration due to gravity at a height equal to half the radius of the Earth, from its surface ?

KCET 2024

Options:

A. $4.4 \mathrm{~ms}^{-2}$

B. $6.5 \mathrm{~ms}^{-2}$

C. Zero

D. $9.8 \mathrm{~ms}^{-2}$

Answer: A

Solution:

Acceleration due to gravity at height h from the surface of the earth.

$g_h=\frac{g}{\left(1+\frac{h}{R}\right)^2}$

Given, $\quad h=\frac{R}{2}$

$$\begin{aligned} \therefore \quad g_h & =\frac{g}{\left(1+\frac{\frac{R}{2}}{R}\right)^2}=\frac{g}{\left(\frac{3}{2}\right)^2} \\ & =\frac{4}{9} g=\frac{4}{9} \times 9.8=4.4 \mathrm{~m} / \mathrm{s}^2\end{aligned}$$

Question4

When a planet revolves around the Sun, in general, for the planet

KCET 2023

Options:

A. linear momentum and linear velocity are constant.

B. linear momentum and areal velocity are constant.

C. kinetic and potential energy of the planet are constant.

D. angular momentum about the Sun and areal velocity of the planet are constant.

Answer: D

Solution:

When a planet revolves around the sun, net external torque is zero. Therefore its angular momentum is conserved. As a consequence, its areal velocity remains the same/constant.

Question5

Electrical as well as gravitational affects can be thought to be caused by fields. Which of the following is true for an electrical or gravitational field?

KCET 2022

Options:

A. Gravitational or electric field does not exist in the space around an object.

B. Fields are useful for understanding forces actin through a distance.

C. There is no way to verify the existence of a force field since it is just a concept.

D. The field concept is often used to describe contact forces.

Answer: B

Solution:

Concept of field is a model used to explain the influence that a massive body or charged particle extends into the space around itself, producing a force on another massive body or charged body placed in space. Therefore, force concept of field is used to explain gravitational and electrostatic phenomena.

Question6

Two bodies of masses 8 kg are placed at the vertices A and B of an equilateral triangle ABC. A third body of mass 2 kg is placed at the centroid G of the triangle. If $AG = BG = CG = 1 \text{ m}$, where should a fourth body of mass 4 kg be placed, so that the resultant force on the 2 kg body is zero?

KCET 2021

Options:

A. At C

B. At a point P on the line CG such that $PG = \frac{1}{\sqrt{2}} \text{ m}$

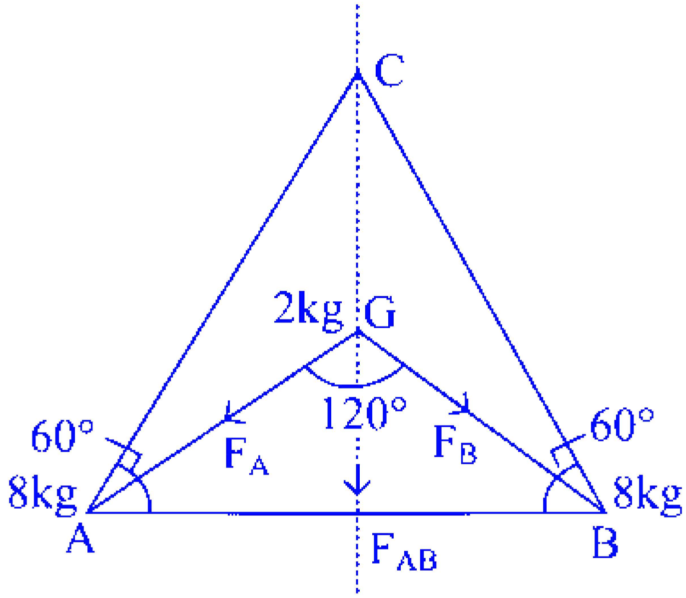
C. At a point P on the line CG such that $PG = 0.5 \text{ m}$

D. At a point P on the line CG such that $PG = 2 \text{ m}$

Answer: B

Solution:

According to the question, the arrangement of the masses is as shown below,



Gravitational force between two masses is given as

$F = \frac{Gm_1m_2}{r^2}$

where, $G =$ gravitational constant and $r =$ distance between them.

From the given values, we can say that force between masses at A and G.

$=$ Force between masses at B and G.

$$\Rightarrow \quad F_A = \frac{Gm_Am_G}{AG^2} \text{ and } F_B = \frac{Gm_Bm_G}{BG^2}$$
$$\text{or} \quad F_A = F_B = \frac{G\times8\times2}{1^2} = 16G \quad .... \text{(i)}$$

[$\because$ Given, $m_1 = m_A = m_B = 8$ kg, $m_2 = m_G = 2$ kg, $AG = BG = r = 1$ m]

From the figure, resultant of F_A and F_B is given as

$$F_{AB} = \sqrt{F_A^2 + F_B^2 + 2F_AF_B\cos\theta}$$
$$= \sqrt{2F_A^2 + 2F_A^2\cos 120^\circ}$$
$$= \sqrt{F_A^2} = \sqrt{(16G)^2} = 16G$$

[from Eq. (i)]

$$\Rightarrow \quad F_{AB} = F_A = F_B = 16G$$

For resultant force on **2kg** body to be zero, 4 kg body should be placed at a certain distance from G such that

$\mathbf{F}_{CG} = -\mathbf{F}_{AE}$

$$\Rightarrow \quad |\mathbf{F}_{CG}| = |\mathbf{F}_{AB}|$$
$$\text{or} \quad \frac{G \times m_Cm_G}{x^2} = G(16)$$

where, $x =$ distance between 4 kg and 2 kg body.

Here, $m_C = 4$ kg

$$\Rightarrow \quad \frac{G\times4\times2}{x^2} = G(16)$$

$$\Rightarrow \quad x^2 = \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{2}} \text{ m}$$

This means, 4 kg body should be placed at point P on line CG such that $PG = \frac{1}{\sqrt{2}}$ m.

Question7

The value of acceleration due to gravity at a height of 10 km from the surface of earth is x. At what depth inside the earth is the value of the acceleration due to gravity has the same value x ?

KCET 2020

Options:

A. 5 km

B. 20 km

C. 10 km

D. 15 km

Answer: B

Solution:

The value of acceleration due to gravity at height $(h = 10 \text{ km})$ is x.

$$\text{i.e.,} \quad g_h = x$$

$$\Rightarrow \quad \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = x$$

$$\Rightarrow \quad g\left(1-\frac{2h}{R_e}\right) = x \quad \text{.... (i)}$$

The value of acceleration due to gravity at depth d below the surface of earth,

$$g_d = x$$

$$\Rightarrow \quad g\left(1 - \frac{d}{R_e}\right) = x \quad .... \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\Rightarrow \quad g\left(1 - \frac{2h}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right)$$
$$\Rightarrow \quad 1 - \frac{2h}{R_e} = 1 - \frac{d}{R_e} \Rightarrow \frac{2h}{R_e} = \frac{d}{R_e}$$
$$\Rightarrow \quad d = 2h = 2 \times 10 = 20 \text{ km}$$

Question8

A satellite is orbiting close to the earth and has a kinetic energy K. The minimum extra kinetic energy required by it just overcome the gravitation pull of the earth is

KCET 2019

Options:

A. K

B. $2K$

C. $\sqrt{3}K$

D. $2\sqrt{2}K$

Answer: A

Solution:

Kinetic energy of satellite revolving close to the earth

$$K = \frac{1}{2}m \cdot v_0^2 = \frac{1}{2}m(\sqrt{gR})^2 \quad \left[v_0 = \sqrt{gR}\right]$$
$$K = \frac{1}{2}mgR$$

Escape energy near the earth surface

$$K_e = \frac{1}{2}m \cdot v_e^2 \quad [v_e \rightarrow \text{ escape velocity }]$$
$$= \frac{1}{2}m(\sqrt{2gR})^2 = mgR = 2 \times \frac{1}{2}mgR = 2K$$

$\therefore$ Minimum extra kinetic energy to overcome gravitational pull $= K_e - K = 2K - K = K$

Question9

A space station is at a height equal to the radius of the Earth. If ' v_E ' is the escape velocity on the surface of the Earth, the same on the space station is times v_E.

KCET 2018

Options:

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{3}}$

Answer: C

Solution:

To solve the problem, follow these steps:

The escape velocity is given by the formula:

$$v = \sqrt{\frac{2GM}{r}}$$

where:

G is the gravitational constant,

M is the mass of the Earth,

r is the distance from the center of the Earth.

On the Earth's surface, where $r = R$ (Earth's radius), the escape velocity is:

$$v_E = \sqrt{\frac{2GM}{R}}$$

For a space station at a height equal to the Earth's radius, the distance from the center is:

$r = R + R = 2R$

So, the escape velocity at the space station is:

$$v_{station} = \sqrt{\frac{2GM}{2R}} = \sqrt{\frac{GM}{R}}$$

The ratio of the escape velocities is:

$$\frac{v_{station}}{v_E} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Thus, the escape velocity at the space station is $\frac{1}{\sqrt{2}}$ times the escape velocity at the Earth's surface.

The correct option is:

Option C: $\frac{1}{\sqrt{2}}$

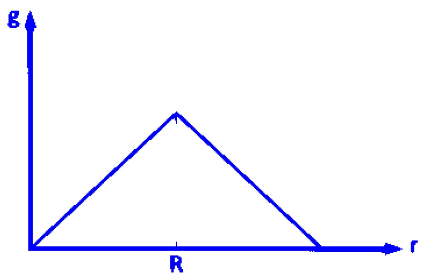
Question10

Which of the following graphs correctly represents the variation of g on the Earth?

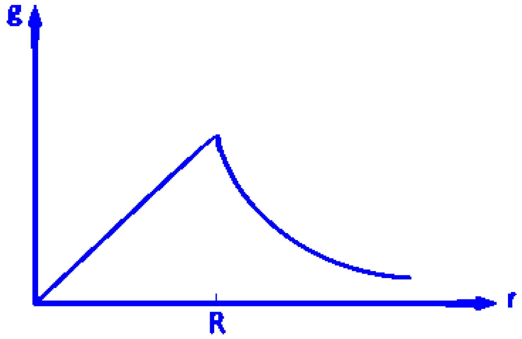
KCET 2018

Options:

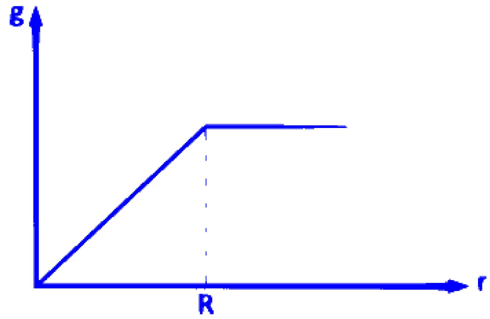
A.



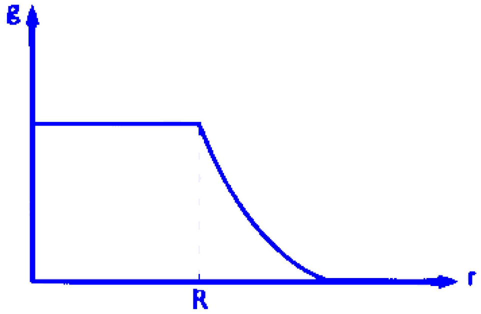
B.



C.



D.



Answer: B

Solution:

The acceleration due to gravity on the surface of Earth, $g=\frac{GM}{R^2}$

At a height h above the surface, $g=\frac{GM}{(R+h)^2}$

At a depth d below the surface, $g=\frac{Gm}{(R-d)^2}$

Question11

A mass m on the surface of the Earth is shifted to a target equal to the radius of the Earth. If R is the radius and M is the mass of the Earth, then work done in this process is

KCET 2018

Options:

A. $\frac{mgR}{2}$

B. mgR

C. 2 mgR

D. $\frac{mgR}{4}$

Answer: B

Solution:

The work done when moving a mass m from the surface of the Earth to a height equal to the Earth's radius R is equivalent to the change in gravitational potential energy.

The potential energy (PE) at a height h is given by:

$\text{PE} = mgh$

In this scenario, $h = R$. Therefore, the work done W is:

$W = mgR$

Question12

The value of acceleration due to gravity at a depth of 1600 km is equal to

KCET 2017

Options:

A. 4.9 ms^{-2}

B. 9.8 ms^{-2}

C. 7.35 ms^{-2}

D. 19.6 ms^{-2}

Answer: C

Solution:

To find the value of acceleration due to gravity at a depth of 1600 km, we can use the formula:

$g_d = g\left(1 - \frac{d}{R}\right)$

Where:

g is the acceleration due to gravity on the surface, approximately 9.8 ms^{-2}.

d is the depth, given as 1600 km.

R is the radius of the Earth, approximately 6400 km.

Substituting the values into the formula:

$g_d = 9.8\left(1 - \frac{1600}{6400}\right)$

Simplifying the expression inside the parentheses:

$g_d = 9.8\left(1 - \frac{1}{4}\right)$

Continue simplifying:

$g_d = 9.8\left(\frac{4-1}{4}\right) = 9.8 \times \frac{3}{4}$

Finally, calculating the result:

$g_d = 7.35 \text{ ms}^{-2}$

Thus, the value of acceleration due to gravity at a depth of 1600 km is 7.35 ms^{-2}.
