Differentiation

Question1

If $y = \frac{\cos x}{1+\sin x}$, then

(a) $\frac{dy}{dx} = \frac{-1}{1+\sin x}$

(b) $\frac{dy}{dx} = \frac{1}{1+\sin x}$

(c) $\frac{\mathrm{dy}}{\mathrm{dx}} = -\frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{\mathrm{x}}{2}\right)$

(d) $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{\mathrm{x}}{2}\right)$

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Options:

A. Only b is correct

B. Only a is correct

C. Both a and c are correct

D. Both b and d are correct

Answer: C

Solution:

$$y=\frac{\cos x}{1+\sin x}$$
$$y'=\frac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}=\frac{-(1+\sin x)}{(1+\sin x)^2}$$
$$y'=\frac{-1}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}=\frac{-1}{1+\sin x}$$
$$=\frac{-1}{2\cdot\left(\frac{1}{\sqrt{2}}\cdot\cos\frac{x}{2}+\frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)^2}=\frac{-1}{2\cdot\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$$
$$=\frac{-1}{2}\cdot\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

Question2

If $\mathrm{y}=\mathrm{a}\sin^3\mathrm{t}, \mathrm{x}=\mathrm{a}\cos^3\mathrm{t}$, then $\frac{\mathrm{dy}}{\mathrm{dx}}$ at $\mathrm{t}=\frac{3\pi}{4}$ is

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Options:

A. -1

B. $\frac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. 1

Answer: D

Solution:

$$y=a\sin^3 t, x=a\cos^3 t$$
$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3a\sin^2 t\cos t}{3a\cos^2 t(-8\pi nt)}=-\tan t$$
$$t=\frac{3\pi}{4}=1$$

Question3

The derivative of $\sin x$ with respect to $\log x$ is

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Options:

A. $\cos x$

B. $x \cos x$

C. $\frac{\cos x}{\log x}$

D. $\frac{\cos x}{x}$

Answer: B

Solution:

To find the derivative of $\sin x$ with respect to $\log x$, we need to use the concept of implicit differentiation and the chain rule.

Let $f(x) = \sin x$.

We want to find $\frac{d(\sin x)}{d(\log x)}$.

Using the chain rule, we have:

$$\frac{d(\sin x)}{d(\log x)} = \frac{d(\sin x)}{dx} \cdot \frac{dx}{d(\log x)}$$

First, we calculate each part separately:

$$\frac{d(\sin x)}{dx} = \cos x$$

To find $\frac{dx}{d(\log x)}$, consider that $\frac{d(\log x)}{dx} = \frac{1}{x}$, hence:

$$\frac{dx}{d(\log x)} = x$$

Putting it all together, we get:

$$\frac{d(\sin x)}{d(\log x)} = \cos x \cdot x = x \cos x$$

Therefore, the derivative of $\sin x$ with respect to $\log x$ is $x \cos x$.

Question4

If $y = 2x^{3x}$, then dy/dx at $x = 1$ is

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Options:

A. 2

B. 6

C. 3

D. 1

Answer: B

Solution:

$$\begin{aligned} y &= 2x^{3x} \\ \frac{dy}{dx} &= 2x^{3x}[3\log x + 3] \\ \text{So, } \left.\frac{dy}{dx}\right|_{x=1} &= 2 \cdot (1)^{3(1)}[3 \times 0 + 3] \\ &= 6 + 0 = 6 \end{aligned}$$

Question5

$\frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$ **is**

KCET 2024

Options:

A. $-\frac{3}{4}$

B. $-\frac{1}{2}$

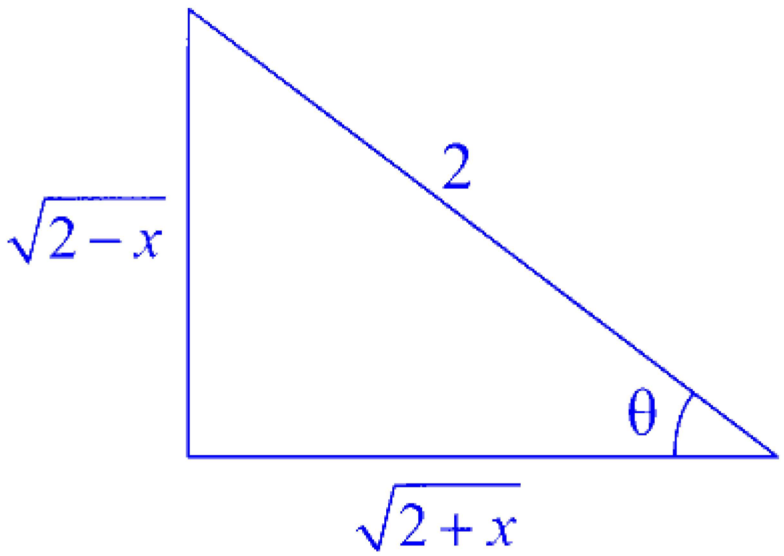
C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: D

Solution:

$$\because \frac{d}{dx}\left[\cos^2\left(\cot^{-1}\sqrt{\frac{2+x}{2-x}}\right)\right]$$
$$=\frac{d}{dx}\left(\cos\left(\cos^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\right)\right)^2$$
$$=\frac{d}{dx}\left(\frac{2+x}{4}\right)=\frac{1}{4}$$



$$\left(\because \cot\theta=\sqrt{\frac{2+x}{2-x}}\right.$$
$$\left.\therefore \cos\theta=\frac{\sqrt{2+x}}{2}\right)$$

Question6

If $y=a\sin x+b\cos x$, then $y^2+\left(\frac{dy}{dx}\right)^2$ is a

KCET 2023

Options:

A. function of y

B. function of x and y

C. constant

D. function of x

Answer: C

Solution:

Given, $y = a \sin x + b \cos x$

$\frac{dy}{dy} = a \cos x - b \sin x$

$y^2 + \left(\frac{dy}{dx}\right)^2 = (a \sin x + b \cos x)^2 + (a \cos x - b \sin x)^2$

$= a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x$

$\cdot \cos x + a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cdot \cos x$

$= a^2 \left(\sin^2 x + \cos^2 x\right) + b^2 \left(\sin^2 x + \cos^2 x\right)$

$= a^2 + b^2$

$\left[\because \sin^2 x + \cos^2 x = 1\right]$

$=$ constant

Question7

If $f(x) = 1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{6} x^3 + \ldots + x^n$, then $f^n(1)$ is equal to :

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Options:

A. $n(n-1)2^{n-2}$

B. $n(n-1)2^n$

C. 2^{n-1}

D. $(n-1)2^{n-1}$

Answer: A

Solution:

Given,

$f(x)=1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\ldots+x^n.$

The given equation can be written as $f(x)=(1+x)^n$

$$\because (1+x)^n=1+nx+\frac{n(n-1)}{2!}$$
$$x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots x^n$$

Now, $f'(x)=n(1+x)^{n-1}$

and, $f''(x)=n(n-1)(1+x)^{n-2}$

put $x=1$

$\therefore f''(1)=n(n-1)2^{n-2}$

Question8

If $f(x)$ and $g(x)$ are two functions with $g(x)=x-\frac{1}{x}$ and $f\circ g(x)=x^3-\frac{1}{x^3}$, then $f'(x)$ is equals to

KCET 2023

Options:

A. $3x^2+\frac{3}{x^4}$

B. $x^2-\frac{1}{x^2}$

C. $1-\frac{1}{x^2}$

D. $3x^2+3$

Answer: D

Solution:

$$\text{Here, } g(x) = x - \frac{1}{x}$$
$$f \circ g(x) = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right)$$
$$f \circ g(x) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$
$$\therefore f(x) = x^3 + 3x$$
$$f'(x) = 3x^2 + 3$$

Question9

If $y = \left(1 + x^2\right)\tan^{-1} x - x$, then $\frac{dy}{dx}$ is

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Options:

A. $2x \tan^{-1} x$

B. $\frac{\tan^{-1} x}{x}$

C. $x^2 \tan^{-1} x$

D. $x \tan^{-1} x$

Answer: A

Solution:

Given,

$y = \left(1 + x^2\right)\tan^{-1} x - x$

Differentiating the given function w.r.t. x

$$\frac{dy}{dx} = \left(1 + x^2\right)\frac{d}{dx}\left(\tan^{-1} x\right) + \tan^{-1} x \frac{d}{dx}\left(1 + x^2\right) - \frac{d}{dx}(x)$$
$$= \left(1 + x^2\right)\frac{1}{\left(1 + x^2\right)} + \tan^{-1} x(2x) - 1$$
$$= 1 + 2x\tan^{-1} x - 1 = 2x\tan^{-1} x$$

Question10

If $x = e^{\theta}\sin\theta, y = e^{\theta}\cos\theta$ where θ is a parameter, then $\frac{dy}{dx}$ at $(1, 1)$ is equal to

KCET 2022

Options:

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. $-\frac{1}{4}$

Answer: A

Solution:

Given, $x = e^{\theta}\sin\theta, y = e^{\theta}\cos\theta$

We have, $\frac{dx}{d\theta} = e^{\theta}\cos\theta + e^{\theta}\sin\theta$

$= e^{\theta}(\cos\theta + \sin\theta)$ .... (i)

$$\text{and } \frac{dy}{d\theta} = e^{\theta}(-\sin\theta) + e^{\theta}\cos\theta$$
$$= e^{\theta}(\cos\theta - \sin\theta) \quad \text{... (ii)}$$

On dividing Eq. (ii) by Eq (i), we get

$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{e^{\theta}(\cos\theta - \sin\theta)}{e^{\theta}(\cos\theta + \sin\theta)}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\cos\theta(1 - \frac{\sin\theta}{\cos\theta})}{\cos\theta(1 + \frac{\sin\theta}{\cos\theta})} = \frac{1 - \tan\theta}{1 + \tan\theta}$ .... (iii)

As we also have, $\frac{x}{y} = \frac{e^{\theta}\sin\theta}{e^{\theta}\cos\theta} \Rightarrow \frac{x}{y} = \tan\theta$

At $(x, y) = (1, 1) \Rightarrow \tan\theta = \frac{1}{1} = 1$

Putting $\tan\theta = 1$ into Eq. (iii), we get

$\frac{dy}{dx} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Question11

If $y = e^{\sqrt{x\sqrt{x\sqrt{x}\ldots}}}, x > 1$, then $\frac{d^2y}{dx^2}$ at $x = \log_e 3$ is

KCET 2022

Options:

A. 3

B. 5

C. 0

D. 1

Answer: A

Solution:

Given, $y = e^{\sqrt{x\sqrt{x\sqrt{x\ldots\ldots}}}}, x > 1$

$y = e^{\sqrt{x\sqrt{x\sqrt{x\ldots}}}}$

$\log_e y = (\sqrt{x\sqrt{x\sqrt{x}\ldots}}) \log_e e$

$$\Rightarrow \quad \log y = \sqrt{x\sqrt{x\sqrt{x\ldots\ldots}}} \Rightarrow \log y = \sqrt{x \cdot \log y}$$

$$\Rightarrow \quad (\log y)^2 = x \log y \Rightarrow \frac{(\log y)^2}{\log y} = x$$

$$\Rightarrow \quad \log y = x \quad \text{.... (i)}$$

Differentiating w.r.t. x, we get

$\frac{1}{y} \times \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = y$ .... (ii)

Differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = y$ [from Eq. (ii)] ..... (iii)

At $x = \log_e 3$, from Eq (i), we get

$\log y = \log 3 \Rightarrow y = 3$

Putting $y = 3$ into Eq. (iii), we get

$\frac{d^2y}{dx^2} = 3$

Question12

If $f(1) = 1, f'(l) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

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Options:

A. 10

B. 33

C. 35

D. 12

Answer: B

Solution:

Let $y = f(f(f(x))) + (f(x))^2$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) \cdot f'(x).$

At $x = 1$

$$\begin{aligned} \frac{dy}{dx} &= f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1) \cdot f'(1) \\ \frac{dy}{dx} &= f'(f(1)) \cdot f'(1) \cdot 3 + 2 \cdot 1 \cdot 3 = f'(1) \cdot 3 \cdot 3 + 6 \\ &= 9f'(1) + 6 = 9 \cdot 3 + 6 = 27 + 6 = 33 \end{aligned}$$

Question13

If $y = x^{\sin x} + (\sin x)^x$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

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Options:

A. $\frac{4}{\pi}$

B. $\pi \log \frac{\pi}{2}$

C. 1

D. $\frac{\pi^2}{2}$

Answer: C

Solution:

Given, $y = x^{\sin x} + (\sin x)^x$

Let $\quad u = x^{\sin -1}$ and $v = (\sin x)^x$

Now, $u = x^{\sin x}, \log u = \sin x \log x$

Differentiating w.r.t. x, we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \sin x \cdot \frac{1}{x} + \log x \times \cos x$$

$$\frac{du}{dx} = u\left(\frac{\sin x}{x} + \cos x \log x\right)$$

$$\frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \log x\right) \quad \text{.... (i)}$$

Now, $v = (\sin x)^x \Rightarrow \log v = x \log(\sin x)$

Differentiating w.r.t. x, we get

$$\frac{1}{vdv} = x\frac{\cos x}{\sin x} + \log(\sin x)$$

$$\frac{dv}{dx} = v(x \cot x + \log \sin x)$$

$$\frac{dv}{dx} = (\sin x)^x(x \cot x + \log \sin x) \quad \text{.... (ii)}$$

Adding Eqs. (i) and (ii), we get

$$\frac{du}{dx}+\frac{dv}{dx}=x^{\sin x}\left(\frac{\sin x}{x}+\cos x\log x\right)$$

$$+(\sin x)^{x}(x\cot x+\log\sin x)$$

$$\frac{dy}{dx}=x^{\sin x}\left(\frac{\sin x}{x}+\cos x\log x\right)$$

$$+(\sin x)^{x}(x\cot x+\log\sin x)$$

At $x=\frac{\pi}{2}$

$$\frac{dy}{dx}=\left(\frac{\pi}{2}\right)\left(\frac{1}{\pi/2}+0\log\frac{\pi}{2}\right)+(1)^{\frac{\pi}{2}}\left(\frac{\pi}{2}\cdot 0+\log(1)\right)$$

$$\frac{dy}{dx}=\left(\frac{\pi}{2}\right)\left(\frac{2}{\pi}+0\right)+1\cdot(0+0)=1$$

Question14

If $e^y+xy=e$ the ordered pair $\left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)$ at $x=0$ is equal to

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Options:

A. $\left(\frac{1}{e},\frac{1}{e^2}\right)$

B. $\left(\frac{-1}{e},\frac{-1}{e^2}\right)$

C. $\left(\frac{1}{e},\frac{-1}{e^2}\right)$

D. $\left(\frac{-1}{e},\frac{1}{e^2}\right)$

Answer: D

Solution:

Given, $e^y+xy=e$ .... (i)

Differentiating w.r.t. x, we get

$$e^y \frac{dy}{dx} + x\frac{dy}{dx} + 1 \cdot y = 0 \quad .... \text{(ii)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{(e^y + x)} \quad .... \text{(iii)}$$

Again, differentiating Eq. (ii) w.r.t. x, we get

$$e^y \frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\Rightarrow \quad (e^y + x)\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} = 0$$

Now, on putting $x = 0$ in Eq. (i), we get

$e^y + 0. y = e \Rightarrow e^y = e^1 \Rightarrow y = 1$

On putting $x = 0, y = 1$ in Eq. (iii), we get

$\frac{dy}{dx} = \frac{-1}{e+0} = -\frac{1}{e}$

Now, putting $x = 0, y = 1$ and $\frac{dy}{dx} = \frac{-1}{e}$ in Eq. (iv), we get

$$(e^1 + 0)\frac{d^2y}{dx^2} + e'\left(-\frac{1}{e}\right)^2 + 2\left(-\frac{1}{e}\right) = 0$$

$$\Rightarrow \quad e\frac{d^2y}{dx^2} + \frac{1}{e} - \frac{2}{e} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

Hence, $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$.

Question15

If a and b are fixed non-zero constants, then the derivative of $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$ is $ma + nb - p$, where

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Options:

A. $m = 4x^3, n = \frac{-2}{x^3}$ and $p = \sin x$

B. $m = \frac{-4}{x^5}, n = \frac{2}{x^3}$ and $p = \sin x$

C. $m = \frac{-4}{x^5}, n = \frac{-2}{x^3}$ and $p = \sin x$

D. $m = 4x^3, n = \frac{2}{x^3}$ and $p = -\sin x$

Answer: B

Solution:

Given, function is $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$.

Let $f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$

Differentiating w.r.t. x,

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2} + \cos x\right)$$
$$= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x$$
$$= ma + nb - P$$
$$m = -\frac{4}{x^5}, n = \frac{2}{x^3} \text{ and } p = \sin x.$$

Question16

If $y = \left(\cos x^2\right)^2$, then $\frac{dy}{dx}$ is equal to

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Options:

A. $-4x \sin 2x^2$

B. $-x \sin x^2$

C. $-2x \sin 2x^2$

D. $-x \cos 2x^2$

Answer: C

Solution:

$y=\left(\cos x^2\right)^2$

On differentiating w.r.t. x, we get

$$\begin{aligned}\frac{dy}{dx}&=2\cos x^2\frac{d}{dx}\left(\cos x^2\right)\\&=2\cos x^2\times\left(-\sin x^2\right)\frac{d}{dx}\left(x^2\right)\\&=-4x\cos x^2\sin x^2\\&=-2x\sin 2x^2\end{aligned}$$

Question17

For constant a, $\frac{d}{dx}\left(x^x+x^a+a^x+a^a\right)$ is

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Options:

A. $x^x(1+\log x)+ax^{a-1}$

B. $x^x(1+\log x)+ax^{a-1}+a^x\log a$

C. $x^x(1+\log x)+a^a(1+\log x)$

D. $x^x(1+\log x)+a^a(1+\log a)+ax^{a-1}$

Answer: B

Solution:

$$\begin{aligned}&\frac{d}{dx}\left(x^x+x^a+a^x+a^a\right)\\&\quad=\frac{d}{dx}\left(x^x\right)+\frac{d}{dx}\left(x^a\right)+\frac{d}{dx}\left(a^x\right)+\frac{d}{dx}\left(a^a\right)\quad \ldots.\text{(i)}\\&\quad=\frac{d}{dx}(y)+ax^{a-1}+a^x\log a+0\end{aligned}$$

$[\text{ let } x^x=y]$

$y=x^x$

Taking log on both sides,

$\log y=x\log x$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx}=x\times\frac{1}{x}+\log x$$
$$\Rightarrow \quad \frac{dy}{dx}=x^x(1+\log x)$$

Substitute the value of $\frac{dy}{dx}$ in Eq. (i),

$$\frac{d}{ax}(x^x+x^a+a^x+a^a)$$
$$=x^x(1+\log x)+ax^{a-1}+a^x\log a$$

Question18

Consider the following statements

Statement 1 : If $y=\log_{10}x+\log_e x$, then $\frac{dy}{dx}=\frac{\log_{10}e}{x}+\frac{1}{x}$

Statement 2 : If $\frac{d}{dx}(\log_{10}x)=\frac{\log x}{\log 10}$ and $\frac{d}{dx}(\log_e x)=\frac{\log x}{\log e}$

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Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is false, statement 2 is true.

C. Both statements 1 and 2 are true.

D. Both statements 1 and 2 are false.

Answer: A

Solution:

Statement 1,

$$y=\log_{10}x+\log_e x$$
$$\Rightarrow \quad y=\frac{\log_e x}{\log_e 10}+\log_e x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x \log_e 10} + \frac{1}{x}$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\log_{10} e}{x} + \frac{1}{x}$$

Statement 2,

$$\frac{d}{dx}\log_{10} x = \frac{d}{dx}\frac{\log_e x}{\log_e 10}$$
$$= \frac{1}{x \log_e 10}$$
$$\Rightarrow \quad \frac{d}{dx}(\log_e x) = \frac{d}{dx}\frac{\log_e x}{\log_e e} = \frac{1}{x}$$

Hence, statement 1 is true but statement 2 is false.

Question19

If the parametric equation of curve is given by $x = \cos\theta + \log\tan\frac{\theta}{2}$ and $y = \sin\theta$, then the points for which $\frac{dy}{dx} = 0$ are given by

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Options:

A. $\theta = \frac{n\pi}{2}, n \in Z$

B. $\theta = (2n+1)\frac{\pi}{2}, n \in Z$

C. $\theta = (2n+1)\pi, n \in Z$

D. $\theta = n\pi, n \in z$

Answer: D

Solution:

$x = \cos\theta + \log\tan\frac{\theta}{2}$

On differentiating w.r.t. θ, we get

$$\frac{dx}{d\theta} = -\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}$$

$$= -\sin\theta + \frac{1}{\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\sin\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \quad \text{..... (i)}$$

Now, $y = \sin\theta$

On differentiating w.r.t. θ

$\frac{dy}{d\theta} = \cos\theta$ .... (ii)

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{\cos\theta}{\frac{\cos^2\theta}{\sin\theta}} = \tan\theta$$

[from Eqs. (i) and (ii)]

If $\frac{dy}{dx} = 0$, then $\tan\theta = 0$.

Hence, $\theta = n\pi, n \in Z$.

Question20

If $y = (x-1)^2(x-2)^3(x-3)^5$, then $\frac{dy}{dx}$ at $x = 4$ is equal to

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Options:

A. 108

B. 54

C. 36

D. 516

Answer: D

Solution:

$y=(x-1)^2(x-2)^3(x-3)^5$

Taking log on th sides,

$$\Rightarrow \log y=\log\left[(x-1)^2(x-2)^3(x-3)^5\right]$$
$$\log y=2\log(x-1)+3\log(x-2)+5\log(x-3)$$

On both sides differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{2}{x-1}+\frac{3}{x-2}+\frac{5}{x-3}$$
$$\Rightarrow \frac{dy}{dx}=(x-1)^2(x-2)^3(x-3)^5\left[\frac{2}{(x-1)}+\frac{3}{(x-2)}+\frac{5}{(x-3)}\right]$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=4}=3^2\times 2^3\times 1^5\left[\frac{2}{3}+\frac{3}{2}+5\right]$$

$$=9\times 8\times\left(\frac{4+9+30}{6}\right)=516$$

Question21

If $2^x+2^y=2^{x+y}$, then $\frac{dy}{dx}$ is

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Options:

A. 2^{y-x}

B. -2^{y-x}

C. 2^{x-y}

D. $\frac{2^y-1}{2^x-1}$

Answer: B

Solution:

We have,

$2^x + 2^y = 2^{x+y}$ .... (i)

On differentiating Eq. (i) w.r.t. x, we get

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx}$$
$$= 2^{x+y} \log 2 \left(1 + \frac{dy}{dx}\right)$$
$$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y}\left(1 + \frac{dy}{dx}\right)$$
$$\Rightarrow 2^x - 2^{x+y} = \frac{dy}{dx}\left(2^{x+y} - 2^y\right)$$
$$\Rightarrow -2^y = \frac{dy}{dx}\left(2^x\right)$$
$$\Rightarrow \frac{dy}{dx} = -2^y/2^x = -2^{y-x}$$

Question22

If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2 \frac{d^2y}{dx^2}$ is

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Options:

A. $6n(n+1)y$

B. $n(n+1)y$

C. $x\frac{dy}{dx} + y$

D. y

Answer: B

Solution:

We have, $y = 2x^{n+1} + \dfrac{3}{x^n}$

$= 2x^{n+1} + 3x^{-n}$ ..... (i)

On differentiating Eq. (i) both sides w.r.t. x, we get

$\frac{dy}{dx} = 2(n+1)x^n + 3(-n)x^{-n-1}$

Again differentiate w.r.t. to x, then we get

$$\frac{d^2y}{dx^2} = 2(n+1)(n)x^{n-1} + 3(-n)(-n-1)x^{-n-2}$$
$$= n(n+1)\left(2x^{n-1} + 3x^{-n-2}\right)$$
$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} = n(n+1)\left(2x^{n+1} + \frac{3}{x^n}\right)$$
$$\Rightarrow \quad x^2\frac{dy}{dx} = n(n+1)y \quad \text{[using Eq. (i)]}$$

Question23

If $(xe)^y = e^y$, then $\frac{dy}{dx}$ is

KCET 2020

Options:

A. $\frac{\log x}{(1+\log x)^2}$

B. $\frac{1}{(1+\log x)^2}$

C. $\frac{\log x}{(1+\log x)}$

D. $\frac{e^x}{x(y-1)}$

Answer: A

Solution:

We have, $(xe)^y = e^x$

Taking log on both sides at base e, we get

$$y\log(xe) = x\log e$$
$$\Rightarrow \quad y(\log x + \log e) = x \; (\because \log_e e = 1)$$
$$\Rightarrow \quad y = \frac{x}{\log x + 1}$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(\log x + 1) - x\left(\frac{1}{x} + 0\right)}{(\log x + 1)^2}$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2}$$

Question24

If $[x]$ represents the greatest integer function and $f(x) = x - [x] - \cos x$, then $f'\left(\frac{\pi}{2}\right) =$

KCET 2019

Options:

A. 2

B. 0

C. does not exist

D. 1

Answer: A

Solution:

We have, $f(x) = x - [x] - \cos x$

$\therefore f(x) = x - 1 - \cos x$

$\left(\because \text{ In surrounding of } \frac{\pi}{2}, [x] = 1\right)$

$\Rightarrow f'(x) = 1 + \sin x \quad \therefore f'\left(\frac{\pi}{2}\right) = 1 + \sin\frac{\pi}{2}$

$= 1 + 1 = 2$

Question25

If $x = a\sec^2\theta$ & $y = a\tan^2\theta$, then $\frac{d^2y}{dx^2} =$

KCET 2019

Options:

A. 0

B. 2a

C. 4

D. 1

Answer: A

Solution:

If $x = a\sec^2\theta$ and $y = a\tan^2\theta$, then to find $\frac{d^2y}{dx^2}$:

First, compute the first derivative:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \cdot 2\tan\theta\sec^2\theta}{a \cdot 2\sec\theta \cdot \sec\theta \cdot \tan\theta} = 1$$

Since the derivative $\frac{dy}{dx}$ is a constant (1), the second derivative is zero:

$$\Rightarrow \frac{d^2y}{dx^2} = 0$$

Question26

$\sqrt[3]{y}\sqrt{x} = \sqrt[6]{(x+y)^5}$, **then** $\frac{dy}{dx} =$

KCET 2019

Options:

A. $x - y$

B. $\frac{x}{y}$

C. $\frac{y}{x}$

D. $x+y$

Answer: C

Solution:

We have, $y^{1/3} \cdot x^{1/2} = (x+y)^{5/6}$

Taking log on both sides, we get

$\frac{1}{3}\log y + \frac{1}{2}\log x = \frac{5}{6}\log(x+y)$

On differentiating both sides w.r.t. x, we get

$$\frac{1}{3} \cdot \frac{1}{y}\frac{dy}{dx} + \frac{1}{2x} = \frac{5}{6}\frac{1}{(x+y)}\left(1 + \frac{dy}{dx}\right)$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{5}{6(x+y)} - \frac{1}{3y}\right] = \frac{1}{2x} - \frac{5}{6(x+y)}$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{15y-6x-6y}{18y(x+y)}\right] = \frac{6x+6y-10x}{12x(x+y)}$$
$$\Rightarrow \frac{dy}{dx}\left[\frac{9y-6x}{3y}\right] = \frac{6y-4x}{2x}$$
$$\Rightarrow \frac{dy}{dx} = \frac{3y-2x}{x} \times \frac{y}{3y-2x} = \frac{y}{x}$$

Question27

If $\cos y = x\cos(a+y)$ with $\cos a \neq \pm 1$, then $\frac{dy}{dx}$ is equal to

KCET 2018

Options:

A. $\frac{\sin a}{\cos^2(a+y)}$

B. $\frac{\cos^2(a+y)}{\sin a}$

C. $\frac{\cos a}{\sin^2(a+y)}$

D. $\frac{\cos^2(a+y)}{\cos a}$

Answer: B

Solution:

To solve for $\frac{dy}{dx}$ given the equation $\cos y = x\cos(a+y)$ with $\cos a \neq \pm 1$, we start by rearranging the equation:

$$\frac{\cos y}{\cos(a+y)} = x$$

Next, we differentiate both sides with respect to x:

$$\frac{d}{dx}\left(\frac{\cos y}{\cos(a+y)}\right) = \frac{d}{dx}(x)$$

Applying the quotient rule on the left side, we have:

$$\frac{\cos(a+y)(-\sin y)+\cos y\sin(a+y)\cdot\frac{d(a+y)}{dx}}{\cos^2(a+y)}$$

Recognizing that $\frac{d(a+y)}{dx} = \frac{dy}{dx}$, the equation simplifies to:

$$\frac{\cos(a+y)(-\sin y)+\cos y\sin(a+y)\cdot\frac{dy}{dx}}{\cos^2(a+y)} = 1$$

Multiply both sides by $\cos^2(a+y)$ to eliminate the denominator:

$$\cos(a+y)(-\sin y) + \cos y\sin(a+y)\cdot\frac{dy}{dx} = \cos^2(a+y)$$

Reorganize to isolate $\frac{dy}{dx}$:

$$\cos y\sin(a+y)\cdot\frac{dy}{dx} = \cos^2(a+y) + \cos(a+y)\sin y$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\cos^2(a+y)+\cos(a+y)\sin y}{\cos y\sin(a+y)}$$

Since $\sin(a+y)\cos y - \cos(a+y)\sin y = \sin a$, the expression simplifies to:

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

Thus, $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

Question28

If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{6}\right)$ is equal to

KCET 2018

Options:

A. $-\frac{1}{2}(1+\sqrt{3})$

B. $\frac{1}{2}(1+\sqrt{3})$

C. $-\frac{1}{2}(1-\sqrt{3})$

D. $\frac{1}{2}(1-\sqrt{3})$

Answer: A

Solution:

We have,

$f(x) = |\cos x - \sin x|$

$$f(x) = \begin{cases} \cos x - \sin x; & \theta < x \leq \frac{\pi}{4} \\ \sin x - \cos x & \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$$

$$f'(x) = \begin{cases} -\sin x - \cos x; & 0 < x \leq \frac{\pi}{4} \\ \cos x + \sin x; & \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$$

$$f'\left(\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right)$$
$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}$$
$$= -\frac{(\sqrt{3}+1)}{2}$$

Question29

If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is equal **to**

KCET 2018

Options:

A. $\frac{1}{y^2-1}$

B. $\frac{1}{2y+1}$

C. $\frac{2y}{y^2-1}$

D. $\frac{1}{2y-1}$

Answer: D

Solution:

We begin with the expression:

$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$

This implies:

$y = \sqrt{x + y}$

Squaring both sides, we get:

$y^2 = x + y$

Rearranging terms, we find:

$y^2 - y = x$

Next, we differentiate both sides with respect to x:

$\frac{d}{dx}(y^2 - y) = \frac{d}{dx}(x)$

Applying the chain rule, we have:

$(2y - 1)\frac{dy}{dx} = 1$

Solving for $\frac{dy}{dx}$, we obtain:

$\frac{dy}{dx} = \frac{1}{2y-1}$

Question30

If $\sin x = \frac{2t}{1+t^2}$, $\tan y + \frac{2t}{1-t^2}$, then $\frac{dy}{dx}$ is equal to

KCET 2017

Options:

A. 0

B. 2

C. 1

D. -1

Answer: C

Solution:

To find $\frac{dy}{dx}$, we start with the given equations:

$\sin x = \frac{2t}{1+t^2}$

This implies:

$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$

Now, using a trigonometric identity, we transform this to:

$x = 2\tan^{-1} t \quad$ ...(i)

Similarly, we are given:

$\tan y = \frac{2t}{1-t^2}$

This implies:

$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$

Again, using the identity related to $\tan^{-1}$, we rewrite this as:

$y = 2\tan^{-1} t \quad$ ...(ii)

From equations (i) and (ii), it follows that:

$y = x$

Thus, differentiating both sides with respect to x, we find:

$\frac{dy}{dx} = 1$

So, the value of $\frac{dy}{dx}$ is 1.

Question31

If $y = \log(\log x)$, then $\frac{d^2y}{dx^2}$ is equal to

KCET 2017

Options:

A. $\frac{-(1+\log x)}{x^2 \log x}$

B. $\frac{-(1+\log x)}{(x \log x)^2}$

C. $\frac{(1+\log x)}{(x \log x)^2}$

D. $\frac{(1+\log x)}{x^2\log x}$

Answer: B

Solution:

To find the second derivative of $y=\log(\log x)$, we start by taking the first derivative:

First Derivative:

$y=\log(\log x)$

Using the chain rule, the derivative is:

$\frac{dy}{dx}=\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)=\frac{1}{\log x}\cdot\frac{1}{x}=\frac{1}{x\log x}$

Second Derivative:

Differentiate $\frac{1}{x\log x}=(x\log x)^{-1}$ with respect to x:

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left((x\log x)^{-1}\right)$

Apply the power rule and the chain rule:

$\frac{d^2y}{dx^2}=-(x\log x)^{-2}\cdot\left(\log x\cdot 1+x\cdot\frac{1}{x}\right)$

Simplify the expression:

$=-(x\log x)^{-2}(\log x+1)$

$=\frac{-(1+\log x)}{(x\log x)^2}$

Thus, the second derivative $\frac{d^2y}{dx^2}$ is $\frac{-(1+\log x)}{(x\log x)^2}$.

Question32

If $y=\tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

KCET 2017

Options:

A. $1/2$

B. 0

C. $\pi/4$

D. 1

Answer: D

Solution:

We begin with the function:

$y = \tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$

This expression can be rewritten by applying the tangent addition formula:

$y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$

Recognizing this as the formula for the tangent of the sum of two angles, we have:

$y = \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)$

This simplifies to:

$y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right)$

Since $\tan^{-1}\tan(\theta) = \theta$, assuming θ is in the principal range of the inverse tangent function, we get:

$y = \frac{\pi}{4}+x$

Differentiating with respect to x, we obtain:

$\frac{dy}{dx} = 1$

Question33

The derivative of $\cos^{-1}\left(2x^2-1\right)$ w.r.t. $\cos^{-1}x$ is

KCET 2017

Options:

A. $1-x^2$

B. $\frac{2}{x}$

C. $\frac{-1}{2\sqrt{1-x^2}}$

D. 2

Answer: D

Solution:

Let $u = \cos^{-1}\left(2x^2 - 1\right)$ and $v = \cos^{-1} x$

Now, $u = \cos^{-1}\left(2x^2 - 1\right)$

Put $x = \cos\theta$

$$\begin{aligned} \therefore \quad u &= \cos^{-1}\left(2\cos^2\theta - 1\right) \\ &= \cos^{-1}(\cos 2\theta) \\ &= 2\theta = 2\cos^{-1} x \end{aligned}$$

$$\begin{aligned} Again, \quad \frac{du}{dv} &= \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} \\ &= \frac{\frac{d}{dx}\left(2\cos^{-1} x\right)}{\frac{d}{dx}\left(\cos^{-1} x\right)} \\ &= \frac{\left(-\frac{2}{\sqrt{1-x^2}}\right)}{\left(\frac{-1}{\sqrt{1-x^2}}\right)} = 2 \end{aligned}$$

Question34

If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, then $\frac{dy}{dx}$ is equal to

KCET 2017

Options:

A.

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

B.

$$\begin{vmatrix} l & m & n \\ f(x) & g(x) & h(x) \\ a & b & c \end{vmatrix}$$

C.

$$\begin{vmatrix} f'(x) & l & a \\ g'(x) & m & b \\ h'(x) & n & c \end{vmatrix}$$

D.

$$\begin{vmatrix} l & m & n \\ a & b & c \\ f'(x) & g'(x) & h'(x) \end{vmatrix}$$

Answer: A, C, D

Solution:

We have,

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$$

$$\therefore \frac{dy}{dx} = \begin{vmatrix} \frac{d}{dx}f(x) & \frac{d}{dx}g(x) & \frac{d}{dx}h(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$$

$$+ \begin{vmatrix} f(x) & g(x) & h(x) \\ \frac{d}{dx}1 & \frac{d}{dx}m & \frac{d}{dx}n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ \frac{1}{d} & m & n \\ \frac{d}{dx}a \cdot \frac{d}{dx}b & \frac{d}{dx}C & \end{vmatrix}$$

$$= \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix}$$

$$+ \begin{vmatrix} f(x) & g(x) & h(x) \\ 1 & m & n \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 1 & m & n \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & m & n \\ a & b & c \\ f'(x) & g'(x) & h'(x) \end{vmatrix} = \begin{vmatrix} f'(x) & 1 & a \\ g'(x) & m & b \\ h'(x) & n & c \end{vmatrix}$$
