Functions

Question1

Domain of the function f, given by $f(x)=\frac{1}{\sqrt{(x-2)(x-5)}}$ is

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Options:

A. $(-\infty, 2] \cup [5, \infty)$

B. $(-\infty, 2) \cup (5, \infty)$

C. $(-\infty, 3) \cup [5, \infty)$

D. $(-\infty, 3] \cup (5, \infty)$

Answer: B

Solution:

$$f(x)=\frac{1}{\sqrt{(x-2)(x-5)}}$$
$$\Rightarrow (x-2)(x-5)>0$$
$$\Rightarrow x \in (-\infty, 2) \cup (5, \infty)$$

Question2

If $f(x)=\sin\left[\pi^2\right]x-\sin\left[-\pi^2\right]x$, where $[x]=$ greatest integer $\leq x$, then which of the following is not true?

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Options:

A. $f(0)=0$

B. $f\left(\frac{\pi}{2}\right)=1$

C. $f\left(\frac{\pi}{4}\right)=1+\frac{1}{\sqrt{2}}$

D. $f(\pi)=-1$

Answer: D

Solution:

$$
\begin{aligned}
& f(x)=\sin\left[\pi^2\right]x-\sin\left[-\pi^2\right]x \\
& =\sin 9x-\sin(-10)x \\
& =\sin 9x+\sin 10x \\
& f(\pi)=\sin 9\pi+\sin 10\pi=0
\end{aligned}
$$

Question3

Let the functions " f " and " g " be $\mathrm{f}:\left[0, \frac{\pi}{2}\right] \rightarrow \mathrm{R}$ given by $\mathrm{f}(\mathrm{x})=\sin \mathrm{x}$ and $\mathrm{g}:\left[0, \frac{\pi}{2}\right] \rightarrow \mathrm{R}$ given by $g(x)=\cos x$, where R is the set of real numbers

Consider the following statements:

Statement (I): f and g are one-one

Statement (II): $\mathrm{f}+\mathrm{g}$ is one-one

Which of the following is correct?

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Options:

A. Statement (I) is true, statement (II) is false

B. Statement (I) is false, statement (II) is true

C. Both statements (I) and (I) are true

D. Both statements (I) and (II) are false

Answer: A

Solution:

$$\mathrm{f}: \text{ one } - \text{ one } \left[0, \frac{\pi}{2}\right] \rightarrow \mathrm{R.\,f(x)} = \sin \mathrm{x}$$

$$\mathrm{g}: \text{ one } - \text{ one } \left[0, \frac{\pi}{2}\right] \rightarrow \mathrm{R.\,f(x)} = \cos \mathrm{x}$$

Statement I is true

$$\left.\begin{array}{l} (\mathrm{f}+\mathrm{g}):\left[0, \frac{\pi}{2}\right] \rightarrow \mathrm{R} \\ \mathrm{f}+\mathrm{g}(\mathrm{x})=\sin \mathrm{x}+\cos \mathrm{x} \\ (\mathrm{f}+\mathrm{g})(0)=0 \\ (\mathrm{f}+\mathrm{g})(\pi / 2)=0 \end{array}\right\} \Rightarrow \mathrm{f}+\mathrm{g} \text{ is not one } - \text{ one}$$

Question4

If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then

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Options:

A. $x \in [3, 4]$

B. $x \in [2, 4)$

C. $x \in [2, 3]$

D. $x \in (2, 3]$

Answer: B

Solution:

$$
\begin{aligned}
& [x]^2-5[x]+6=0 \\
\Rightarrow \quad & [x]^2-3[x]-2[x]+6=0 \\
\Rightarrow \quad & [x]([x]-3)-2([x]-3)=0 \\
\Rightarrow \quad & ([x]-3)([x]-2)=0 \\
\Rightarrow \quad & [x]=2 \text{ or } 3
\end{aligned}
$$

$[x]=2$ or $[x]=3$

$x=[2,3)$ or $x=[3,4)$

Hence, $x \in [2,4)$.

Question5

Let $f: R \rightarrow R$ be defined by $f(x)=x^2+1$. Then, the pre images of 17 and -3 , respectively are

KCET 2024

Options:

A. $\phi,\{4,-4\}$

B. $\{3,-3\}, \phi$

C. $\{4,-4\}, \phi$

D. $\{4,-4\},\{2,-2\}$

Answer: C

Solution:

$\because$ For pre image of 17 , we have

$$
\begin{aligned}
& f(x)=x^2+1=17 \\
\Rightarrow \quad & x^2=16 \Rightarrow x= \pm 4
\end{aligned}
$$

So, $x \in\{-4,4\}$

For pre image of -3 , we have

$$
\begin{aligned}
& f(x)=x^2+1=-3 \\
\Rightarrow \quad & x^2=-4(\text{ not possible })
\end{aligned}
$$

Hence, no possible pre image.

Question6

Let $(g \circ f)(x) = \sin x$ and $f \circ g(x) = (\sin \sqrt{x})^2$. Then,

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Options:

A. $f(x) = \sin^2 x, g(x) = x$

B. $f(x) = \sin \sqrt{x}, g(x) = \sqrt{x}$

C. $f(x) = \sin^2 x, g(x) = \sqrt{x}$

D. $f(x) = \sin \sqrt{x}, g(x) = x^2$

Answer: C

Solution:

$\because g(f(x)) = \sin x$ and $f(g(x)) = (\sin \sqrt{x})^2$

(a) $f(x) = \sin^2 x$ and $g(x) = x$

Now, $f(g(x)) = f(x) = \sin^2 x$

and $g(f(x)) = g\left(\sin^2 x\right) = \sin^2 x$

(b) $f(x) = \sin \sqrt{x}$ and $g(x) = \sqrt{x}$

Now, $f(g(x)) = f(\sqrt{x}) = \sin \sqrt{\sqrt{x}} = \sin(x)^{\frac{1}{4}}$

and $g(f(x)) = g(\sin \sqrt{x}) = \sqrt{\sin \sqrt{x}}$

(c) $f(x) = \sin^2 x$ and $g(x) = \sqrt{x}$

Now, $f(g(x)) = f(\sqrt{x}) = \sin^2 \sqrt{x}$

and $g(f(x)) = g\left(\sin^2 x\right) = \sqrt{\sin^2 x} = |\sin x|$

(d) $f(x) = \sin \sqrt{x}$ and $g(x) = x^2$

Now, $f(g(x)) = f\left(x^2\right) = \sin |x|$

and $g(f(x)) = g(\sin \sqrt{x}) = (\sin \sqrt{x})^2 = \sin^2 \sqrt{x}$

Question7

Let the function satisfy the equation $f(x+y) = f(x)f(y)$ for all $x, y \in R$, where $f(0) \neq 0$. If $f(5) = 3$ and $f'(0) = 2$, then $f'(5)$ is

KCET 2024

Options:

A. 6

B. 0

C. 3

D. -6

Answer: A

Solution:

$\because f(x+y) = f(x) \cdot f(y)$

Put $x = 0, y = 5$, we get

$$\begin{aligned} & f(0+5) = f(0)f(5) \\ \Rightarrow \quad & f(5)[f(0) - 1] = 0 \\ \Rightarrow \quad & f(0) = 1 \end{aligned}$$

$$\begin{aligned} \text{Consider } f'(5) &= \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} \\ &= \lim_{h \to 0} \frac{f(5)f(h) - f(5)}{h} \\ &= \lim_{h \to 0} \frac{f(5)[f(h) - 1]}{h} = f(5) \lim_{h \to 0} \frac{f(h) - f(0)}{h} \\ &= f(5)f'(0) = 2 \times 3 = 6 \end{aligned}$$

Question8

If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are respectively

KCET 2023

Options:

A. $2, -3$

B. $0, 2$

C. $2, 3$

D. $-3, -1$

Answer: A

Solution:

Given, $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$

We have, $f(x) = ax + b$

Put, $x = -1$ in above equation, we get

$f(-1) = a(-1) + b$
$\quad -5 = -a + b \quad$ ..... (i)

Now, $f(3) = 3$

$3 = 3a + b$ .... (ii)

From Eqs. (i) and (ii), we get

$a = 2, b = a - 5 = 2 - 5 = -3$

Question9

$f : R \to R$ and $g : [0, \infty) \to R$ defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true?

KCET 2023

Options:

A. $(f \circ g)(-4) = 4$

B. $(f \circ g)(2) = 2$

C. $(g \circ f)(-2) = 2$

D. $(g \circ f)(4) = 4$

Answer: A

Solution:

$f \circ g(x) = f(g(x))$

As domain of $g(x) \to [0, \infty)$, so domain of $f \circ g(\lambda)$ also $[0, \infty)$

$$\begin{aligned} \therefore \quad & f(g(x)) = (\sqrt{x})^2 \\ & g \circ f(x) = g(f(x)) \\ & g \circ f(x) = \sqrt{x^2} = |x| \end{aligned}$$

Hence, $f \circ g(-4) = (\sqrt{-4})^2$ is not define.

Question10

Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$, then $g \circ f$ is

KCET 2023

Options:

A. $\frac{3x^2-5}{9x^4-6x^2+26}$

B. $\frac{3x^2}{x^4+2x^2-4}$

C. $\frac{3x^2}{9x^4+30x^2-2}$

D. $\frac{3x^2-5}{9x^4-30x^2+26}$

Answer: D

Solution:

Given that $f(x) = 3x^2 - 5$

and

$$g(x) = \frac{x}{x^2+1}$$
$$g \circ t = g\{f(x)\} = g\left(3x^2 - 5\right)$$
$$= \frac{3x^2-5}{\left(3x^2-5\right)^2+1} = \frac{3x^2-5}{9x^4-30x^2+25+1}$$
$$= \frac{3x^2-5}{9x^4-30x^2+26}$$

Question11

Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$ then $g(f(x))$ is invertible in the domain

KCET 2023

Options:

A. $x \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$

B. $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

C. $x \in \left[0, \frac{\pi}{4}\right]$

D. $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Answer: A

Solution:

$f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$

$g[f(x)] = \left[(\sin 2x + \cos 2x)^2 - 1\right]$

$g[f(x)] = \sin^2 2x + \cos^2 2x + \sin 4x - 1$

$g[f(x)] = \sin 4x$

Let $gf(x) = y$

$y = \sin 4x \quad [\because \text{Domain of } \sin x \text{ is } -\frac{\pi}{2}, \frac{\pi}{2}]$

$\therefore$ Domain of $\sin 4x = \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$

Question12

If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is

KCET 2023

Options:

A. $\frac{5}{2}$

B. $\frac{2}{5}$

C. $\frac{1}{2}$

D. $\frac{-5}{2}$

Answer: D

Solution:

$$\begin{aligned} y =& f(f(x)) = f\left(\frac{1}{2+x}\right) \\ &= \frac{1}{2+\frac{1}{2+x}} = \frac{2+x}{4+2x+1} = \frac{2+x}{5+2x} \end{aligned}$$

This function is not continuous at $x = -2$ (because $f(x)$ is not defined at this point) and $x = -\frac{5}{2}$ because $f(f(x))$ is not defined at this point.

Question13

The domain of the function $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

KCET 2022

Options:

A. $[-2, 0) \cap (0, 1)$

B. $[-2, 1)$

C. $[-2, 0)$

D. $[-2, 0) \cup (0, 1)$

Answer: D

Solution:

Given, function

$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$f(x)$ will be defined if,

$$\begin{aligned} & 1-x>0, \neq 1 \text{ and } x+2 \geq 0 \\ \text{or} \quad & 1>x, 1-x \neq 1 \text{ and } x \geq -2 \\ \text{or} \quad & x<1, x \neq 0 \text{ and } x \geq -2 \\ \Rightarrow \quad & x \in [-2, 0) \cup (0, 1) \end{aligned}$$

Question14

If $f: R \rightarrow R$ be defined by

$$f(x)=\begin{cases}2x: & x>3 \\ x^2: & 1<x \leq 3 \\ 3x: & x \leq 1\end{cases}$$

then $f(-1)+f(2)+f(4)$ is

KCET 2022

Options:

A. 5

B. 10

C. 9

D. 14

Answer: C

Solution:

$$f(x) = \begin{cases} 2x & : \quad x > 3 \\ x^2 & : \quad 1 < x \leq 3 \\ 3x & : \quad x \leq 1 \end{cases}$$

$f(-1) + f(2) + f(4) = 3(-1)(2)^2 + 2(4)$

$= -3 + 4 + 8 = 9$

Question15

Domain of $f(x) = \frac{x}{1-|x|}$ is

KCET 2021

Options:

A. $R - [-1, 1]$

B. $(-\infty, 1)$

C. $(-\infty, 1) \cup (0, 1)$

D. $R - \{-1, 1\}$

Answer: D

Solution:

Given function,

$f(x) = \frac{x}{1-|x|}$

$\because$ Denominator can't be zero.

$$\begin{aligned} \therefore \quad 1-|x| &\neq 0 \\ \Rightarrow \quad |x| &\neq 1 \\ \Rightarrow \quad x &\neq \pm 1 \end{aligned}$$

Hence, domain is $R-\{-1,1\}$.

Question16

$f: R \rightarrow R$ defined by $f(x)$ is equal to
$\begin{cases} 2x, x>3 \\ x^2, 1<x \leq 3, \\ 3x, x \leq 1 \end{cases}$ then $f(-2)+f(3)+f(4)$ is

KCET 2021

Options:

A. 14

B. 9

C. 5

D. 11

Answer: D

Solution:

Given, $f(x)=\begin{cases}2x, & x>3\\ x^2, & 1<x\leq 3\\ 3x, & x\leq 1\end{cases}$

$\therefore \quad f(-2)+f(3)+f(4)$
$=3(-2)+(3)^2+2(4)=-6+9+8=11$

Question17

Let $A=\{x : x\in R, x$ is not a positive integer) Define $f : A\rightarrow R$ as $f(x)=\frac{2x}{x-1}$, then f is

KCET 2021

Options:

A. injective but not surjective.

B. surjective but not injective.

C. bijective.

D. neither injective nor surjective.

Answer: A

Solution:

Given function,

$f(x)=\frac{2x}{x-1}$

On differentiating w.r.t. x, we get

$$f'(x)=\frac{(x-1)(2-2x(1-0)}{(x-1)^2}$$
$$=\frac{2x-2-2x}{(x-1)^2}=\frac{-2}{(x-1)^2}<0$$

Function is strictly decreasing.

Function is injective

$\frac{2x}{x-1}=y\Rightarrow 2x=yx-y$

$\Rightarrow \quad y = x(y-2)$

Let $x = \frac{y}{y-2} \notin \pi$ for $y = 2$

f is not surjective.

Question18

The function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval

KCET 2021

Options:

A. $\left[\frac{-\pi}{6}, \frac{\pi}{3}\right]$

B. $\left[\frac{\pi}{6}, \frac{-\pi}{3}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{6}, \frac{-\pi}{3}\right]$

Answer: A

Solution:

Given function,

$$f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$$
$$f(x) = 2\left(\sin 2x \cdot \frac{\sqrt{3}}{2} - \cos 2x \cdot \frac{1}{2}\right) + 4$$
$$= 2\left(\sin 2x \cdot \cos\frac{\pi}{6} - \cos 2x \cdot \sin\frac{\pi}{6}\right) + 4$$
$$= 2\left[\sin\left(2x - \frac{\pi}{6}\right)\right] + 4$$

f is one-one.

$$-\frac{\pi}{2} \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{2}$$
$$\Rightarrow \quad -\frac{\pi}{2} + \frac{\pi}{6} \leq 2x \leq \frac{\pi}{2} + \frac{\pi}{6}$$
$$\Rightarrow \quad \frac{-3\pi+\pi}{6} \leq 2x \leq \frac{4\pi}{6} \Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$$
$$\therefore \quad x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

Question19

Domain of the function

$f(x) = \frac{1}{\sqrt{[x^2]-[x]-6}},$

where $[x]$ is greatest integer $\leq x$ is

KCET 2021

Options:

A. $(-\infty, -2) \cup [4, \infty)$

B. $(-\infty, -2 \cup [3, \infty]$

C. $[-\infty, -2] \cup [4, \infty]$

D. $[-\infty, -2] \cup [3, \infty)$

Answer: A

Solution:

Given function,

$f(x) = \frac{1}{\sqrt{[x]^2-[x]-6}}$

where $[x]$ is greatest integer $\leq x$.

$$
\begin{aligned}
& [x]^2 - [x] - 6 > 0 \\
\Rightarrow \quad & [x]^2 - 3[x] + 2x] - 6 > 0 \\
\Rightarrow \quad & ([x] - 3)([x] + 2 > 0 \\
\Rightarrow \quad & ([x] - 3) > 0, [x] + 2 < 0 \\
\Rightarrow \quad & [x] > 3, [x] < -2 \\
\Rightarrow \quad & x \in [4, \infty), x \in (-\infty, -2) \\
\therefore \quad & x \in (-\infty, -2) \cup [4, \infty).
\end{aligned}
$$

Question20

Let $f:[2,\infty)\rightarrow R$ be the function defined $f(x)=x^2-4x+5$, then the ranges of f is

KCET 2020

Options:

A. $(-\infty,\infty)$

B. $[1,\infty)$

C. $(1,\infty)$

D. $[5,\infty)$

Answer: B

Solution:

We have,

$$f(x)=x^2-4x+5$$
$$f(x)=x^2-4x+4+1$$
$$f(x)=(x-2)^2+1$$

$\therefore$ Range of $f(x)$ is $[1,\infty)$.

Question21

$f:R\rightarrow R$ and $g:[0,\infty)\rightarrow R$ is defined by $f(x)=x^2$ and $g(x)=\sqrt{x}$. Which one of the following is not true?

KCET 2019

Options:

A. $f \circ g(2) = 2$

B. $g \circ f(4) = 4$

C. $g \circ f(-2) = 2$

D. $f \circ g(-4) = 4$

Answer: D

Solution:

Given, $f : R \rightarrow R$ and $g : [0, \infty] \rightarrow R$

Here, $g(-4)$ is not defined

$\therefore$ fog (-4) does not exist

Question22

If $|3x - 5| \leq 2$ then

KCET 2019

Options:

A. $1 \leq x \leq \frac{9}{3}$

B. $-1 \leq x \leq \frac{7}{3}$

C. $-1 \leq x \leq \frac{9}{3}$

D. $1 \leq x \leq \frac{7}{3}$

Answer: D

Solution:

Key Idea : Use $|x| \leq a (a > 0)$

$\Rightarrow -a \leq x \leq a$ and simplify

Since, $|3x - 5| \leq 2$

$\Rightarrow -2 \leq (3x-5) \leq 2 \Rightarrow -2+5 \leq 3x \leq 2+5$

$\Rightarrow 3 \leq 3x < 7 \Rightarrow 1 \leq x \leq \frac{7}{3}$

Question23

The value of $\sqrt{24.99}$ is

KCET 2019

Options:

A. 5.001

B. 4.999

C. 4.897

D. 4.899

Answer: B

Solution:

Consider, $f(x)=x^{1/2} \Rightarrow f'(x)=\frac{1}{2\sqrt{x}}$

let $\quad x=25, \Delta x=-0.01$

Now, $f(x+\Delta x) \cong f(x)+\Delta x f'(x)$

$=x^{1/2}+\Delta x \cdot \frac{1}{2\sqrt{x}}=(25)^{1/2}+(-0.01) \times \frac{1}{2\sqrt{25}}$

$=5-0.01 \times \frac{1}{10}=5-0.001=4.999$

Question24

The domain of the function $f: R \rightarrow R$ defined by $f(x)=\sqrt{x^2-7x+12}$ is

KCET 2019

Options:

A. $(-\infty, 3] \cap [4, \infty)$

B. $(-\infty, 3] \cup [4, \infty)$

C. $(3, 4)$

D. $(-\infty, 3] \cup (4, \infty)$

Answer: B

Solution:

We have, $f(x) = \sqrt{x^2 - 7x + 12}$

$f(x)$ will be defined if $x^2 - 7x + 12 \geq 0$

$\Rightarrow (x-3)(x-4) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [4, \infty)$

$\therefore$ domain $f(x) = (-\infty, 3] \cup [4, \infty)$

Question25

If $|x+5| \geq 10$, then

KCET 2018

Options:

A. $x \in (-15, 5]$

B. $x \in (-5, 5]$

C. $x \in (-\infty, -15] \cup [5, \infty)$

D. $x \in [-\infty, -15] \cup [5, \infty)$

Answer: C

Solution:

To solve the inequality $|x+5| \geq 10$, we begin by considering the definition of absolute value. The inequality $|x+5| \geq 10$ can be split into two separate cases:

$x+5 \leq -10$

$x+5 \geq 10$

We will solve each inequality separately:

For $x+5 \leq -10$:

$x+5 \leq -10$

Subtract 5 from both sides:

$x \leq -15$

For $x+5 \geq 10$:

$x+5 \geq 10$

Subtract 5 from both sides:

$x \geq 5$

Thus, the solution to the inequality $|x+5| \geq 10$ is:

$x \leq -15 \quad \text{or} \quad x \geq 5$

This can be written in interval notation as:

$x \in (-\infty, -15] \cup [5, \infty)$

Therefore, the set of possible values for x includes all x less than or equal to -15 and all x greater than or equal to 5.

Question26

Let $f(x) = x - \frac{1}{x}$, then $f(-1)$ is

KCET 2018

Options:

A. 0

B. 2

C. 1

D. -2

Answer: A

Solution:

We are given the function

$f(x) = x - \frac{1}{x}$.

To find $f(-1)$, follow these steps:

Substitute $x = -1$ into the function:

$f(-1) = (-1) - \frac{1}{-1}$.

Simplify the fraction:

$\frac{1}{-1} = -1$.

Substitute back into the equation:

$f(-1) = (-1) - (-1) = -1 + 1 = 0$.

Thus, $f(-1) = 0$.

The correct answer is Option A.

Question27

Let $f, g : R \rightarrow R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \forall x \in R$. Then $(f \circ g)(x)$ for $x < 0$ is

KCET 2018

Options:

A. 0

B. $4x$

C. $-4x$

D. $2x$

Answer: A

Solution:

To find $(f \circ g)(x)$ for $x < 0$, we first need to understand the functions $f(x)$ and $g(x)$:

$f(x) = |x| + x$

$g(x) = |x| - x$

Now, let's evaluate $(f \circ g)(x) = f(g(x))$:

Evaluate $g(x)$:

For $x < 0$, $|x| = -x$, so

$g(x) = |-x| - x = -x - x = -2x$

Evaluate $f(g(x)) = f(-2x)$:

Since $-2x > 0$ for $x < 0$, $|-2x| = 2x$, therefore

$f(-2x) = |-2x| + (-2x) = 2x - 2x = 0$

So, $(f \circ g)(x) = 0$ for all $x < 0$.

Question28

A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is

KCET 2018

Options:

A. 6!-6

B. $6^6 - 6$

C. $6^6 - 6\,!$

D. 6!

Answer: C

Solution:

To determine the number of distinct functions from set A to itself that are not bijections, we first need to understand the total possibilities.

Given that set A has 6 distinct elements, the total number of functions from A to A is calculated as 6^6.

Next, we find the number of bijective functions. A bijection is both one-to-one (injective) and onto (surjective), and for a set with 6 elements, the number of bijective functions is the number of permutations of the set, which is 6!.

Thus, the number of functions that are not bijections is the total number of functions minus the number of bijective functions:

$6^6 - 6!$

Therefore, the number of distinct functions from A to A that are not bijections is $6^6 - 6!$.

Question29

Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x; & x > 3 \\ x^2; & 1 < x \leq 3 \\ 3x; & x \leq 1 \end{cases}. \text{ Then}$$

$f(-1) + f(2) + f(4)$ is

KCET 2018

Options:

A. 9

B. 14

C. 5

D. 10

Answer: A

Solution:

Let's evaluate the function $f : \mathbb{R} \to \mathbb{R}$ defined as follows:

$$f(x) = \begin{cases} 2x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \leq 3 \\ 3x, & \text{if } x \leq 1 \end{cases}$$

We need to find the value of $f(-1) + f(2) + f(4)$.

Calculate $f(-1)$:

Since $x = -1$ and $-1 \leq 1$, we use the third case of the function definition:

$f(-1) = 3(-1) = -3$

Calculate $f(2)$:

Since $1 < x = 2 \leq 3$, we use the second case of the function definition:

$f(2) = 2^2 = 4$

Calculate $f(4)$:

Since $x = 4 > 3$, we use the first case of the function definition:

$f(4) = 2(4) = 8$

Summing these values:

$f(-1) + f(2) + f(4) = -3 + 4 + 8 = 9$

Thus, the sum is 9.

Question30

Let $f : R \rightarrow R$ be defined by $f(x) = x^4$, then

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Options:

A. f may be one-one and onto

B. f is neither one-one nor onto

C. f is one-one and onto

D. f is one-one but not onto

Answer: B

Solution:

We have,

$$\begin{aligned} & f(x) = x^4 \\ \text{Let} \quad & f(x_1) = f(x_2) \\ \Rightarrow \quad & x_1^4 = x_2^4 \\ \Rightarrow \quad & x_1 = \pm x_2 \end{aligned}$$

$\therefore f(x)$ is not one-one.

Again, co-domain of $f(x) = R$ and Range of $f(x) = [0, \infty)$

$\therefore$ Co-domain $\neq$ Range

$\therefore f(x)$ is not onto.

Question31

If $f(x) = 8x^3, g(x) = x^{1/3}$, then $f \circ g(x)$ is

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Options:

A. $8x$

B. $8^3 x$

C. $(8x)^{1/3}$

D. $8x^3$

Answer: A

Solution:

To find $f \circ g(x)$, we need to evaluate $f(g(x))$ using the given functions:

$f(x) = 8x^3 \quad \text{and} \quad g(x) = x^{1/3}$

Therefore, $f \circ g(x) = f(g(x)) = f(x^{1/3})$.

Substituting $g(x) = x^{1/3}$ into $f(x)$, we have:

$f(x^{1/3}) = 8\left(x^{1/3}\right)^3$

Since $\left(x^{1/3}\right)^3 = x$, we get:

$f(x^{1/3}) = 8x$

Thus, $f \circ g(x) = 8x$.

Question32

If $|x - 2| \leq 1$, then

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Options:

A. $x \in [1, 3]$

B. $x \in (-1, 3)$

C. $x \in [1, 3)$

D. $x \in (1, 3)$

Answer: A

Solution:

To solve the inequality $|x-2| \leq 1$, follow these steps:

Break down the absolute value inequality:

$|x-2| \leq 1 \implies -1 \leq x-2 \leq 1$

Solve the compound inequality by adding 2 to each part to isolate x:

$-1+2 \leq x-2+2 \leq 1+2$

Simplify the inequality:

$1 \leq x \leq 3$

Thus, x is within the interval $[1, 3]$.

Question33

Binary operation * on $R - \{-1\}$ defined by $a^*b = \frac{a}{b+1}$ is

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Options:

A. * is neither associative not commutative

B. * is associative but not commutative

C. * is commutative but not commutative

D. * is associative and commutative

Answer: A

Solution:

To determine the properties of the binary operation * defined on $R-\{-1\}$ by $a^{*}b=\frac{a}{b+1}$, we need to analyze whether this operation is commutative and associative.

Commutativity

For the operation to be commutative, it should satisfy the condition:

$a^{*}b=b^{*}a$

Using the operation's definition:

$a^{*}b=\frac{a}{b+1}$

$b^{*}a=\frac{b}{a+1}$

Clearly, since $\frac{a}{b+1}\neq\frac{b}{a+1}$ for arbitrary a and b, the operation * is not commutative.

Associativity

For the operation to be associative, it should satisfy the condition:

$a^{*}\left(b^{*}c\right)=\left(a^{*}b\right)^{*}c$

Calculate each side separately:

Calculate $a^{*}\left(b^{*}c\right)$:

$b^{*}c=\frac{b}{c+1}$

$a^{*}\left(b^{*}c\right)=a^{*}\left(\frac{b}{c+1}\right)=\frac{a}{\frac{b}{c+1}+1}=\frac{a(c+1)}{b+c+1}$

Calculate $\left(a^{*}b\right)^{*}c$:

$a^{*}b=\frac{a}{b+1}$

$\left(a^{*}b\right)^{*}c=\left(\frac{a}{b+1}\right)^{*}c=\frac{\frac{a}{b+1}}{c+1}=\frac{a}{(b+1)(c+1)}$

Comparing the two results:

$\frac{a(c+1)}{b+c+1}\neq\frac{a}{(b+1)(c+1)}$

Since $a^{*}\left(b^{*}c\right)\neq\left(a^{*}b\right)^{*}c$, the operation * is not associative.

Conclusion

Given that the binary operation * is neither commutative nor associative, we conclude that the operation has neither of these properties.

Question34

The range of the function $f(x) = \sqrt{9 - x^2}$ is

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Options:

A. $(0, 3]$

B. $(0, 3)$

C. $[0, 3]$

D. $[0, 3)$

Answer: C

Solution:

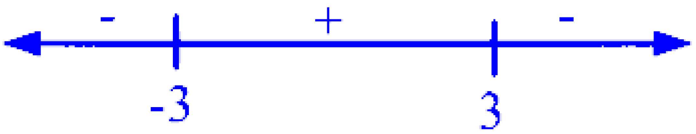
We have, $f(x) = \sqrt{9 - x^2}$

$Let \quad y = \sqrt{9 - x^2}$

$\Rightarrow \quad y^2 = 9 - x^2$
$\Rightarrow \quad x^2 = 9 - y^2$
$\Rightarrow \quad x = \sqrt{9 - y^2}$

Since, $f(x) \geq 0$

$\therefore \quad 9 - y^2 \geq 0$
$\Rightarrow \quad (3 - y)(3 + y) \geq 0$



$\therefore \quad y \in [-3, 3]$
But, $\quad f(x) = \sqrt{9 - x^2}$
$\therefore \quad f(x) \in [0, 3]$
