Gravitation

Question1

The ratio of the time periods of a simple pendulum at heights $2R_E$ and $3R_E$ from the surface of the Earth is (R_E is radius of the Earth)

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Options:

A.

1 : 2

B.

1 : 3

C.

3 : 4

D.

2 : 3

Answer: C

Solution:

Time Period of a Simple Pendulum:

The time period (how long it takes for one swing) of a simple pendulum is given by: $T = 2\pi\sqrt{\frac{l}{g}}$

This means T depends on $1/\sqrt{g}$, where g is gravity at the pendulum's location.

Gravity at a Height:

Gravity changes as you go away from the Earth's surface. At height h above Earth, gravity is: $g' = \frac{g}{\left(1+\frac{h}{R_E}\right)^2}$

where R_E is Earth's radius.

Time Period Ratio at Two Heights:

Let T_1 be the time period at height $2R_E$ and T_2 at $3R_E$.

So, $h_1 = 2R_E$ and $h_2 = 3R_E$.

The ratio is: $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

Calculate Gravity at Each Height:

At $h_1 = 2R_E$, gravity is $g_1 = \frac{g}{(1+2)^2} = \frac{g}{9}$

At $h_2 = 3R_E$, gravity is $g_2 = \frac{g}{(1+3)^2} = \frac{g}{16}$

Find the Ratio:

So, $\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{g/16}{g/9}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$

Therefore, the ratio of the time periods is 3 : 4.

Question2

If a body is projected vertically from the surface of the Earth with a speed of 8000 ms^{-1}, then the maximum height reached by the body is

(Radius of the Earth $= 6400$ km and acceleration due to gravity $= 10 \text{ ms}^{-2}$)

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Options:

A.

1600 km

B.

9600 km

C.

6400 km

D.

3200 km

Answer: C

Solution:

Applying conservation of energy

$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$

But $GM = gR^2$

$\therefore \quad \frac{1}{2}mv^2 - \frac{gR^2m}{R} = \frac{-gR^2m}{R+h}$

$$\Rightarrow \quad \frac{v^2}{2} - gR = -\frac{gR^2}{R+h}$$
$$\Rightarrow \quad h = R\left(\frac{v^2}{2gR - v^2}\right)$$
$$= 6.4 \times 10^6\left[\frac{(8000)^2}{2 \times 10 \times 6.4 \times 10^6 - (8000)^2}\right]$$
$$= 6.4 \times 10^6 \text{ m} = 6400 \times 10^3 \text{ m}$$
$$= 6400 \text{ km}$$

Question3

The range of gravitational forces is

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Options:

A. 10^{-15} m

B. 10^{-39} m

C. infinity

D. 10^{-2} m

Answer: C

Solution:

Gravitational force is one of the basic force in nature which is weakest force but has infinite range.

Question4

An object of mass m at a distance of $20R$ from the centre of a planet of mass M and radius R has an initity velocity u. The velocity with which the object hits the surface of the planet is

(G-Universal gravitational constant)

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Options:

A. $\left[u^2+\frac{19GM}{10R}\right]^{\frac{1}{2}}$

B. $\left[u^2+\frac{19Gm}{10R}\right]^{\frac{1}{2}}$

C. $\left[u^2-\frac{19GM}{10R}\right]^{\frac{1}{2}}$

D. $\left[u^2-\frac{19Gm}{10R}\right]^{\frac{1}{2}}$

Answer: A

Solution:

Initial energy at distance $20R$,

Kinetic energy, $\mathrm{KE}_i=\frac{1}{2}mu^2$

Gravitational potential energy,

$U_i=-\frac{GMm}{20R}$

Total initial energy, $E_i=K_i+U_i$

$=\frac{1}{2}mu^2-\frac{GMm}{20R}$

Final energy at the surface of the planet,

$E_f=\mathrm{KE}_f+U_f$

$$=\frac{1}{2}mu_f^2-\frac{GMm}{R}$$

According to conservation of energy,

$E_i = E_f$

$\frac{1}{2} m u^2 - \frac{GMm}{20R} = \frac{1}{2} m u_f^2 - \frac{GMm}{R}$

$m u_f^2 = m u^2 - \frac{GMm}{10R} + \frac{2GMm}{R}$

$m u_f^2 = m u^2 + \frac{19}{10} \frac{GMm}{R}$

$u_f^2 = u^2 + \frac{19}{10} \frac{GM}{R}$

$u_f = \left[u^2 + \frac{19}{10} \frac{GM}{R}\right]^{\frac{1}{2}}$

Question5

The ratio of the radii of a planet and the earth is $1 : 2$, the ratio of their mean densities is $4 : 1$. If the acceleration due to gravity on the surface of the earth is $9.8 \mathrm{~ms}^{-2}$, then the acceleration due to gravity on the surface of the planet is

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Options:

A. $4.9 \mathrm{~ms}^{-2}$

B. $29.4 \mathrm{~ms}^{-2}$

C. $8.9 \mathrm{~ms}^{-2}$

D. $19.6 \mathrm{~ms}^{-2}$

Answer: D

Solution:

To determine the acceleration due to gravity on the surface of a planet given certain ratios, let's start with the provided details:

The ratio of the radii of the planet and Earth is $\frac{r_1}{r_2} = \frac{1}{2}$.

The ratio of their mean densities is $\frac{\rho_1}{\rho_2} = \frac{4}{1}$.

The acceleration due to gravity on Earth is $g_2 = 9.8\,\mathrm{m/s}^2$.

The formula for gravitational acceleration g on the surface of a celestial body is:

$g = \frac{4}{3}\pi G\rho R$

where G is the universal gravitational constant.

Applying this formula:

For the planet, the gravitational acceleration g_1 is given by:

$g_1 = \frac{4}{3}\pi G\rho_1 r_1$

For Earth, its gravitational acceleration g_2 is:

$g_2 = \frac{4}{3}\pi G\rho_2 r_2$

To find the ratio of g_1 to g_2, we divide the equations:

$\frac{g_1}{g_2} = \frac{\rho_1 r_1}{\rho_2 r_2}$

Substitute the given ratios:

$\frac{g_1}{g_2} = \frac{4}{1} \times \frac{1}{2} = 2$

Since $g_2 = 9.8\,\mathrm{m/s}^2$, we calculate g_1:

$g_1 = 9.8 \times 2 = 19.6\,\mathrm{m/s}^2$

Thus, the acceleration due to gravity on the planet's surface is $19.6\,\mathrm{m/s}^2$.

Question6

Two stars of masses M and $2M$ that are at a distance d apart, are revolving one around another. The angular velocity of the system of two stars is (G-Universal gravitational constant)

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Options:

A. $\sqrt{\frac{4GM}{d^3}}$

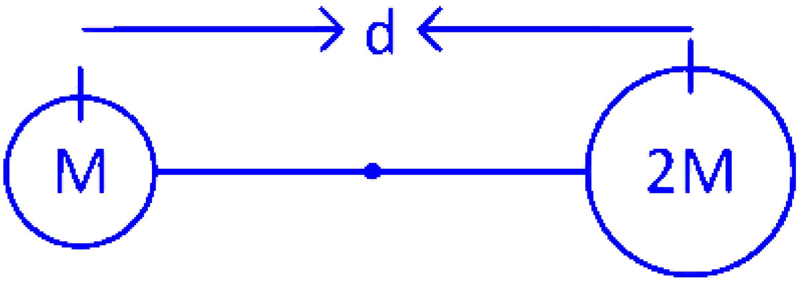
B. $\sqrt{\frac{2GM}{d^3}}$

C. $\sqrt{\frac{9GM}{d^3}}$

D. $\sqrt{\frac{3GM}{d^3}}$

Answer: D

Solution:



Mass of 1st star $m_1 = M$

Mass of 2nd star $m_2 = 2M$

Distance between two star $= d$

Let r_1 and r_2 be the orbital radius of 1 st and 2 nd star.

We know, $r_1 + r_2 = d$

$\Rightarrow \quad r_1 = d - r_2$

Gravitational force between two masses

$F_G = \frac{Gm_1m_2}{r^2}$

They revolve are each other due to centripetal force

$F_C = mr\omega^2$

From the centre of mass fomula,

$Mr_1 = 2Mr_2$

$$r_1 = 2(d - r_1)$$

$$r_1 = \frac{2}{3}d$$

Equate gravitation force acting on lst mass and centripetal force on lst mass

$$F_C = F_G$$

$$m_1r_1\omega^2 = \frac{Gm_1m_2}{d^2}$$

$$M\frac{2d}{3}\omega^2 = \frac{GM2M}{d^2}$$

$$\omega^2 = \frac{3GM}{d^3}$$

$$\omega = \sqrt{\frac{3GM}{d^3}}$$

Question7

The ratio of the accelerations due to gravity at heights 1280 km and 3200 km above the surface of the earth is(Radius of the earth $= 6400$ km)

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Options:

A. $25 : 16$

B. $5 : 2$

C. $1 : 1$

D. $25 : 4$

Answer: A

Solution:

Given, $h_1 = 1280$ km

$h_2 = 3200$ km

Acceleration due to gravity at height h,

$$g' = g\left(\frac{R_r}{R_t + h}\right)^2$$

$$\therefore g_1 = g\left(\frac{R_r}{R_e + h}\right)^2 = g\left(\frac{6400}{6400 + 1280}\right)^2$$

Similarly, $g_2 = g\left(\frac{6400}{6400+3200}\right)^2$

$$\therefore \frac{g_1}{g_2} = \frac{\left(\frac{6400}{6400+1280}\right)^2}{\left(\frac{6400}{6400+3200}\right)^2} = \frac{\left(\frac{1}{7680}\right)^2}{\left(\frac{1}{9600}\right)^2}$$

$$= \frac{9600 \times 9600}{7680 \times 7680}$$

$$= \frac{5 \times 5}{4 \times 4} = \frac{25}{16}$$

Question8

Regarding fundamental forces in nature, the correct statement is

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Options:

A. electromagnetic forces are always attractive

B. electromagnetic forces are always repulsive

C. gravitational forces are always attractive

D. strong nuclear forces are always repulsive

Answer: C

Solution:

Gravitational forces arise from the mass of objects and always attracts each other. This means that any two objects with mass will be drawn towards each other by gravity. This force is keeps planets in orbits around stars and object on earth's surface.

Question9

The energy required to take a body from the surface of the earth to a height equal to the radius of the earth is W. The energy required to take this body from the surface of the earth to a height equal to twice the radius of the earth is

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Options:

A. $\frac{W}{3}$

B. $\frac{2W}{3}$

C. W

D. $\frac{4W}{3}$

Answer: D

Solution:

The energy required to take a body from the surface of the earth to a height equal to the radius of the earth (W).

The gravitational potential energy,

$$W=\frac{GMm}{R}-\frac{GMm}{2R}$$

$$W=\frac{GMm}{2R} \quad \ldots \text{(i)}$$

To take body from the surface of the earth to a height to twice of the radius, the energy required (W') is

$$W'=\frac{GMm}{R}-\frac{GMm}{3R}$$

$$=\frac{2GMm}{3R} \quad \ldots \text{(ii)}$$

Comparing Eq. (i) and Eq. (ii)

$W'=\frac{4}{3}W$

So, the energy required to take the body from the surface of the earth to twice of the radius of the earth is 4/3 W times the energy required.

Question10

The ratio of the radii of two planets is r and the ratio of accelerations due to gravity on the planets is x. Then the ratio of the escape velocities from the planets is

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Options:

A. xr

B. $\sqrt{\frac{r}{x}}$

C. $\sqrt{\sqrt{x}}$

D. $\sqrt{\frac{x}{r}}$

Answer: C

Solution:

To determine the ratio of the escape velocities from two planets, consider the following:

Given Ratios:

Ratio of the radii of the two planets: r

Ratio of the accelerations due to gravity: x

Escape Velocity Formula:

The escape velocity, v, is given by the formula:

$v = \sqrt{2Rg}$

Where R is the radius of the planet and g is the acceleration due to gravity.

Applying to Two Planets:

For Planet 1:

$v_1 = \sqrt{2R_1g_1}$

For Planet 2:

$v_2 = \sqrt{2R_2g_2}$

Finding the Ratio of Escape Velocities:

The ratio of the escape velocities is:

$\frac{v_1}{v_2} = \sqrt{\frac{2R_1g_1}{2R_2g_2}}$

Substitute the Given Ratios:

Use the given ratios:

$r = \frac{R_1}{R_2}$

$x = \frac{g_1}{g_2}$

Final Calculation:

The ratio becomes:

$\frac{v_1}{v_2} = \sqrt{r \cdot x}$

Thus, the ratio of the escape velocities is $\sqrt{rx}$.

Question11

A ring has a mass M and radius R. The distance of the point on its geometric axis from its centre at which the gravitational field is strongest is

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Options:

A. $\frac{R}{2}$

B. $\frac{R}{4}$

C. $\frac{R}{\sqrt{3}}$

D. $\frac{R}{\sqrt{2}}$

Answer: D

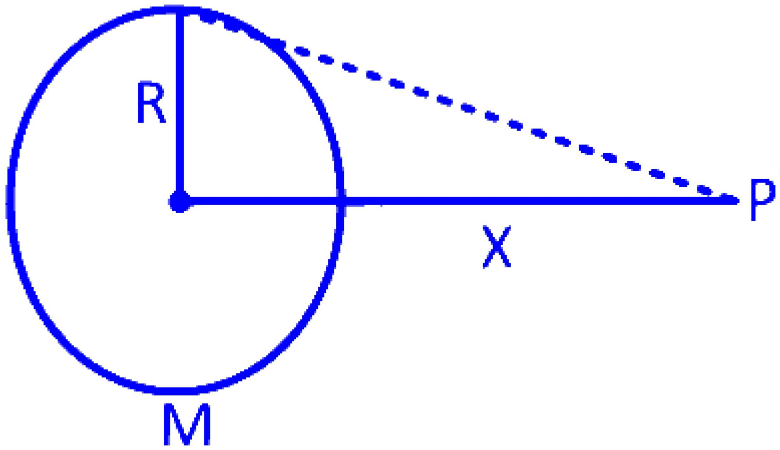
Solution:

Given,

Mass of ring $= m$

Radius $= R$

We have to calculate the distance from its geometrical axisGravitational field is given by at point P



.$E_P = \frac{GMx}{(R^2+x^2)^{3/2}}$

To get the maximum value of E_P

$$\frac{dE_p}{dx} = 0$$

$$\Rightarrow \quad \frac{GM}{(R^2+x^2)^{3/2}} - GMx \times \frac{3}{2(R^2+x^2)^{1/2}} \times 2x = 0$$

Solving above expression,

$$3x^2 = R^2 + x^2$$
$$2x^2 = R^2$$
$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}$$
