Functions

Question1

If $f : R - \{0\} \rightarrow R$ is defined by $3f(x) + 4f\left(\frac{1}{x}\right) = \frac{2-x}{x}$ then $f(3) =$

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

6

B.

12

C.

9

D.

3

Answer: D

Solution:

We have, $3f(x) + 4f\left(\frac{1}{x}\right) = \frac{2}{x} - 1$

$\therefore \quad 3f(3) + 4f\left(\frac{1}{3}\right) = \frac{2}{3} - 1 = \frac{-1}{3} \qquad \ldots(i)$

Also, $3f\left(\frac{1}{3}\right) + 4f(3) = 6 - 1 = 5 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$f(3) = 3$

Question2

The inverse of the function $y=\frac{10^x-10^{-x}}{10^x+10^{-x}}+1$ is $x=$

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$\log\left(\frac{y}{2-y}\right)$

B.

$\log_{10}\left(\frac{y}{2-y}\right)$

C.

$\frac{1}{10}\log\left(\frac{y}{1-y}\right)$

D.

$\frac{1}{2}\log_{10}\left(\frac{y}{2-y}\right)$

Answer: D

Solution:

We have, $y=\dfrac{10^x-10^{-x}}{10^x+10^{-x}}+1$

$\Rightarrow \quad y-1=\dfrac{10^x-10^{-x}}{10^x+10^{-x}}$

Let $\quad a=10^x$

$\therefore \quad 10^{-x}=\dfrac{1}{a}$

$\therefore \quad y-1=\dfrac{a-\frac{1}{a}}{a+\frac{1}{a}}=\dfrac{a^2-1}{a^2+1}$

$$\Rightarrow \quad (y-1)\left(a^2+1\right)=a^2-1 \Rightarrow (y-1)a^2+(y-1)=a^2-1$$
$$\Rightarrow \quad a^2(y-1-1)=-y \Rightarrow a^2(y-2)=-y$$
$$\Rightarrow \quad a^2=\frac{-y}{y-2} \Rightarrow 10^{2x}=\frac{-y}{y-2}$$
$$\Rightarrow \quad 2x=\log_{10}\left(\frac{-y}{y-2}\right) \Rightarrow 2x=\log_{10}\left(\frac{y}{2-y}\right)$$
$$\Rightarrow \quad x=\frac{1}{2}\log_{10}\left(\frac{y}{2-y}\right)$$

Question3

If $f(x)=\tan\left(\frac{\pi}{\sqrt{x+1}+4}\right)$ is a real valued function, then the range of f is

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Options:

A.

$[-1,1]$

B.

$(0,1]$

C.

$[-1,\infty)$

D.

R

Answer: B

Solution:

Given, $f(x)=\tan\left(\frac{\pi}{\sqrt{x+1}+4}\right)$

For $\sqrt{x+1}$ to be defined,

we have $x+1 \geq 0 \Rightarrow x \geq -1$

$$\therefore \quad \sqrt{x+1}+4 \geq 0+4=4$$
$$\Rightarrow \quad 0<\frac{1}{\sqrt{x+1}+4} \leq \frac{1}{4}$$
$$\Rightarrow \quad 0<\frac{\pi}{\sqrt{x+1}+4} \leq \frac{\pi}{4}$$
$$\therefore \quad f(x)=\tan \left(\frac{\pi}{\sqrt{x+1}}+4\right), \text{ where}$$
$$0<\frac{\pi}{\sqrt{x+1}+4} \leq \frac{\pi}{4}$$

So, the function $f(x)$ is strictly increasing in the interval $\left(0, \frac{\pi}{4}\right]$

And the range of the given function is $\left(\tan(0), \tan\left(\frac{\pi}{4}\right)\right]$, which is $(0,1]$

$\therefore$ The range of $f(x)$ is $(0,1]$

Question4

If $\frac{x^3+3}{(x-3)^3}=a+\frac{b}{x-3}+\frac{c}{(x-3)^2}+\frac{d}{(x-3)^3}$, then $(a+d)-(b+c)=$

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

49

B.

15

C.

-30

D.

-5

Answer: D

Solution:

Given,

$\frac{x^3+3}{(x-3)^3} = a + \frac{b}{x-3} + \frac{c}{(x-3)^2} + \frac{d}{(x-3)^3}$ $\quad\quad\quad\quad \ldots(i)$

Let $y = x - 3$

$\Rightarrow x = y + 3$

Then, $\frac{x^3+3}{(x-3)^3} = \frac{(y+3)^3+3}{y^3}$

$$= \frac{y^3 + 9y^2 + 27y + 27 + 3}{y^3}$$
$$= \frac{y^3 + 9y^2 + 27y + 30}{y^3}$$
$$= 1 + \frac{9}{y} + \frac{27}{y^2} + \frac{30}{y^3}$$
$$= 1 + \frac{9}{x-3} + \frac{27}{(x-3)^2} + \frac{30}{(x-3)^3}$$

Comparing this with Eq. (i), we get

$a = 1, b = 9, c = 27, d = 30$

So, $(a + d) - (b + c) = (1 + 30) - (9 + 27)$

$= 31 - 36 = -5$

Question5

$f(x) = ax^2 + bx + c$ is an even function and

$g(x) = px^3 + qx^2 + rx$ is an odd function.

If $h(x) = f(x) + g(x)$ and $h(-2) = 0$, then $8p + 4q + 2r =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $4a + 3b + 2c$

B. $a + b + c$

C. $4a + 2b + c$

D. $8a + 4b + 2c$

Answer: C

Solution:

Given two functions, $f(x) = ax^2 + bx + c$, which is even, and $g(x) = px^3 + qx^2 + rx$, which is odd, we need to analyze the combined function $h(x) = f(x) + g(x)$. We know that $h(-2) = 0$.

Properties of Even and Odd Functions:

Even Function: $f(x) = f(-x)$.

Odd Function: $g(x) = -g(-x)$.

Since $f(x)$ is even:

$$f(x) = f(-x) \quad \Rightarrow \quad f(2) = f(-2)$$

For the odd function $g(x)$:

$$g(-x) = -g(x) \quad \Rightarrow \quad g(-2) = -g(2)$$

Using the Given Information:

Given $h(-2) = 0$, we have:

$$h(-2) = f(-2) + g(-2) = 0$$

From the properties mentioned:

$$f(-2) = f(2) \quad \text{and} \quad g(-2) = -g(2)$$

Substitute these into $h(-2) = 0$:

$$f(2) - g(2) = 0$$

This implies:

$$f(2) = g(2)$$

Expressing $f(2)$ and $g(2)$:

Calculate $f(2)$:

$$f(2) = a(2)^2 + b(2) + c = 4a + 2b + c$$

And $g(2)$:

$g(2) = p(2)^3 + q(2)^2 + r(2) = 8p + 4q + 2r$

Equating the two expressions from $f(2) = g(2)$:

$4a + 2b + c = 8p + 4q + 2r$

Thus, the expression for $8p + 4q + 2r$ is:

$8p + 4q + 2r = 4a + 2b + c$

Question6

The range of the real valued function $f(x) = \log_3\left(5 + 4x - x^2\right)$ is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $(0, 2)$

B. $[0, 2]$

C. $(-\infty, 2]$

D. $[-1, 5]$

Answer: C

Solution:

To determine the range of the function $f(x) = \log_3(5 + 4x - x^2)$, we need to analyze the behavior of the quadratic expression $5 + 4x - x^2$.

Analysis of $5 + 4x - x^2$

The expression $5 + 4x - x^2$ is a quadratic function of the form $ax^2 + bx + c$, where $a = -1$, $b = 4$, and $c = 5$.

Determining the Range of the Quadratic

Since $a = -1 < 0$, the parabola opens downwards, and the quadratic function achieves its maximum value at its vertex. The maximum (or minimum for upward-opening parabolas) value of a quadratic function $ax^2 + bx + c$ is given by $-\frac{D}{4a}$, where $D = b^2 - 4ac$ is the discriminant.

Calculate the discriminant:

$D = 4^2 - 4 \times (-1) \times 5 = 16 + 20 = 36$

The maximum value of the quadratic $5 + 4x - x^2$ is:

$-\frac{D}{4a} = -\frac{36}{4\times(-1)} = 9$

Hence, the range of $5 + 4x - x^2$ is $(-\infty, 9]$.

Translating to the Logarithmic Function

Considering the function $f(x) = \log_3(5 + 4x - x^2)$, the possible values of $5 + 4x - x^2$ determine the input values for the logarithm.

Since $5 + 4x - x^2 \leq 9$, we take the logarithm base 3 of each side:

$\log_3(5 + 4x - x^2) \leq \log_3(9)$

Given that $\log_3(9) = 2$, we conclude:

$f(x) = \log_3(5 + 4x - x^2) \leq 2$

Thus, the range of $f(x)$ is $(-\infty, 2]$.

Question7

The sum of the maximum and minimum values of the function $f(x) = \frac{x^2-x+1}{x^2+x+1}$ is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $\frac{17}{4}$

B. $\frac{5}{2}$

C. $\frac{10}{3}$

D. 0

Answer: C

Solution:

To find the sum of the maximum and minimum values of the function $f(x) = \frac{x^2-x+1}{x^2+x+1}$, we start by setting:

$y = \frac{x^2-x+1}{x^2+x+1}$

Rewriting, we have:

$y(x^2+x+1)=x^2-x+1$

This simplifies to:

$(y-1)x^2+(y+1)x+(y-1)=0$

To ensure real values of x, the discriminant D of this quadratic in x must be non-negative. The discriminant is given by:

$(y+1)^2-4(y-1)^2 \geq 0$

Simplifying the inequality:

$(y+1-2(y-1))(y+1+2(y-1)) \geq 0$

$(3-y)(3y-1) \geq 0$

Reversing the inequality:

$(y-3)(3y-1) \leq 0$

This implies y lies within the interval:

$y \in \left[\frac{1}{3}, 3\right]$

Therefore, the sum of the maximum and minimum values of y is:

$\frac{1}{3}+3=\frac{10}{3}$

Question8

If f is a real valued function from A onto B defined by $f(x)=\frac{1}{\sqrt{|x-|x||}}$, then $A \cap B=$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. ϕ

B. $(-\infty, 0)$

C. $(0, \infty)$

D. $(-\infty, \infty)$

Answer: A

Solution:

Given,

$f(x)=\frac{1}{\sqrt{|x-|x||}}$

For

$x>0,|x|=x$

$x-|x|=0$

$|x-|x||=0$

$\therefore f(x)$ is not possible for $x>0$.

For

$x<0,|x|=-x$

$x-|x|=-2x$

$|x-|x||=2x$

$$f(x)=\frac{1}{\sqrt{-2x}}$$

$\therefore A$ is set of negative numbers and B is a set of positive numbers.

$\therefore A\cap B=\phi$

Question9

The domain of the real valued function $f(x)=\sqrt[3]{\frac{x-2}{2x^2-7x+5}}+\log\left(x^2-x-2\right)$ is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $(-\infty,-1)\cup\left(2,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

B. $R-\left\{1,\frac{5}{2}\right\}$

C. $(-\infty,-1)\cup(2,\infty)$

D. $(-1,2)$

Answer: A

Solution:

We have,

$$f(x)=\sqrt[3]{\frac{x-2}{2x^2-7x+5}}+\log\left(x^2-x-2\right)$$
$\therefore \quad 2x^2-7x+5\neq 0$ and $x^2-x-2>0$
$\Rightarrow (2x-5)(x-1)\neq 0$ and $(x+1)(x-2)>0$
$\Rightarrow x\neq 1,\frac{5}{2}$ and $x\in(-\infty,-1)\cup(2,\infty)$
$\Rightarrow x\in(-\infty,-1)\cup\left(2,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

$\therefore$ Domain of f is

$(-\infty,-1)\cup\left(2,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

Question10

f is a real valued function satisfying the relation $f\left(3x+\frac{1}{2x}\right)=9x^2+\frac{1}{4x^2}$. If $f\left(x+\frac{1}{x}\right)=1$, then x is equal to

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. ± 2

B. ± 1

C. ± 3

D. ± 6

Answer: D

Solution:

We have, $f\left(3x+\frac{1}{2x}\right)=9x^2+\frac{1}{4x^2}$

$\therefore \quad f\left(3x+\frac{1}{2x}\right)=\left(3x+\frac{1}{2x}\right)^2-3$

Now, $f\left(x+\frac{1}{x}\right)=1$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2-3=1 \Rightarrow \left(x+\frac{1}{x}\right)^2=4$$
$$\Rightarrow \quad x^2+\frac{1}{x^2}+2=4 \Rightarrow x^2+\frac{1}{x^2}-2=0$$
$$\Rightarrow \quad \left(x-\frac{1}{x}\right)^2=0 \Rightarrow x-\frac{1}{x}=0$$
$$\Rightarrow \quad x^2-1=0 \Rightarrow x^2=1$$
$$\Rightarrow \quad x=\pm 1$$

Question11

If $f(x)=\frac{2x-3}{3x-2}$ and $f_n(x)=$ (fofofo ....n times) (x), then $f_{32}(x)=$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{2x-3}{3x-2}$

B. x

C. $\frac{3x+2}{2x+3}$

D. $t_{23}(x)$

Answer: B

Solution:

We have, $f(x)=\dfrac{2x-3}{3x-2}$

$$f(f(x))=\frac{2f(x)-3}{3f(x)-2}=\frac{2\left(\frac{2x-3}{3x-2}\right)-3}{3\left(\frac{2x-3}{3x-2}\right)-2}$$

$$=\frac{4x-6-9x+6}{6x-9-6x+4}$$

$$=\frac{-5x}{-5}=x$$

$$\because f(f(x))=x$$

$$\Rightarrow f_{32}(x) = x$$

Question12

The domain of the real valued function $f(x) = \sqrt{\cos(\sin x)} + \cos^{-1}\left(\frac{1+x^2}{2x}\right)$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $(-1, 1)$

B. $[-1, 1]$

C. $R - (-1, 1)$

D. $\{-1, 1\}$

Answer: D

Solution:

We have,

$$f(x) = \sqrt{\cos(\sin x)} + \cos^{-1}\left(\frac{1+x^2}{2x}\right)$$

$$\Rightarrow \quad \cos(\sin x) > 0$$

$$\because \quad \sin x \in [-1, 1]$$

$$\therefore \quad \cos(\sin x) > 0, \forall x \in R$$

and $\quad -1 \leq \dfrac{1+x^2}{2x} \leq 1$

$$-1 \leq \frac{1+x^2}{2x} \text{ and } \frac{1+x^2}{2x} \leq 1$$

$$\Rightarrow \quad x \in \{-1, 1\}$$

Question13

The domain of the function $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $[-2, 0) \cup (0, 1)$

B. $[1, \infty) \cap [-2, 2]$

C. $[-2, -1] \cup [1, 2]$

D. $(-\infty, 1] \cap [-2, 2]$

Answer: C

Solution:

The given function is

$f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$

To find the domain of $f(x)$

Domain $(f(x))$ = Domain of

$\sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$

where, $\log_2\left(\frac{x^2}{2}\right) \in [-1, 1]$

$$\Rightarrow \quad -1 \leq \log_2\left(\frac{x^2}{2}\right) \leq 1$$
$$\Rightarrow \quad \frac{1}{2} \leq \frac{x^2}{2} \leq 2$$
$$\Rightarrow \quad 1 \leq x^2 \leq 4$$

For, $x^2 \leq 4 \Rightarrow -2 \leq x \leq 2$

$\Rightarrow x \in [-2, 2] \qquad \ldots (i)$

and also, $x^2 \geq 1$

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty) \qquad \ldots (ii)$

Now, from Eqs. (i) and (ii), taking intersection, we get

$x \in [-2, -1] \cup [1, 2]$

Question14

The range of the function $f(x) = -\sqrt{-x^2 - 6x - 5}$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $[0, 2]$

B. $[-2, 0]$

C. $[-2, 2]$

D. $(-\infty, 2]$

Answer: B

Solution:

Given function,

$$f(x) = -\sqrt{-x^2 - 6x - 5}$$
$$= -\sqrt{-(x^2 + 6x + 5)}$$

Let $y = f(x) = -\sqrt{-(x+5)(x+1)}$

Now, squaring on both sides, we get

$$y^2 = -(x^2 + 6x + 5)$$
$$\Rightarrow x^2 + 6x + (5 + y^2) = 0$$

Applying quadratic formula, we get

$$x=\frac{-6 \pm \sqrt{36-4\left(5+y^2\right)}}{2}$$
$$=\frac{-6 \pm \sqrt{16-4 y^2}}{2}$$

Clearly, $16-4 y^2 \geq 0 \Rightarrow y^2 \leq \frac{16}{4}=4$

$$\Rightarrow \quad y^2 \leq 4$$
$$\Rightarrow \quad -2 \leq y \leq 2$$

Clearly, $y<0$ because $\sqrt{-\left(x^2-6 x-5\right)}$ always positive.

So, $y \in[-2,0]$

Thus, range of given function is $[-2,0]$

Question15

If $f: R \rightarrow R$ is defined by $f(x)=2 x+\sin x, x \in R$, then f is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. one-one and onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto

Answer: A

Solution:

Given function is, $f(x)=2 x+\sin x, x \in R$

For one one $f\left(x_1\right)=f\left(x_2\right)$

$\Rightarrow 2x_1 + \sin x_1 = 2x_2 + \sin x_2$

This can be possible only, when $x_1 = x_2$

So, $f(x)$ is one-one.

For onto Range of

$f(x) = R =$ Co-domain

So, Range $=$ Co-domain

Thus, $f(x)$ is onto.

Question16

If t is a parameter, $A = (a \sec t, b \tan t)$, $B = (-a \tan t, b \sec t)$ and $O = (0, 0)$, then the locus of the centroid of $\triangle OAB$ is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $9xy = ab$

B. $xy = 9ab$

C. $x^2 - 9y^2 = a^2 - b^2$

D. $x^2 - y^2 = \frac{1}{9}\left(a^2 - b^2\right)$

Answer: A

Solution:

To find the locus of the centroid of triangle $\triangle OAB$ where the vertices are $A = (a \sec t, b \tan t)$, $B = (-a \tan t, b \sec t)$, and $O = (0, 0)$, we calculate the centroid (x, y) using the formula for the centroid of a triangle:

$x = \frac{0 + a \sec t - a \tan t}{3} = \frac{a(\sec t - \tan t)}{3}$

Similarly,

$y = \frac{0 + b \tan t + b \sec t}{3} = \frac{b(\sec t + \tan t)}{3}$

Next, we compute the product $(3x)(3y)$:

$3x = a(\sec t - \tan t) \quad \text{and} \quad 3y = b(\sec t + \tan t)$

So,

$(3x)(3y) = [a(\sec t - \tan t)][b(\sec t + \tan t)] = ab(\sec^2 t - \tan^2 t)$

Since $\sec^2 t - \tan^2 t = 1$ (using the identity $\sec^2 t - \tan^2 t = 1$), we have:

$9xy = ab \quad \text{or} \quad xy = \frac{ab}{9}$

Thus, the locus of the centroid of $\triangle OAB$ satisfies the equation $9xy = ab$.

Question17

If $f : [2, \infty) \to R$ is defined by $f(x) = x^2 - 4x + 5$, then the range of f is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. R

B. $[1, \infty)$

C. $[4, \infty)$

D. $[5, \infty)$

Answer: B

Solution:

Let's explore the function $f(x) = x^2 - 4x + 5$, which is defined on the interval $[2, \infty)$.

First, consider the discriminant Δ of the quadratic equation:

$\Delta = (-4)^2 - 4 \times 1 \times 5 = 16 - 20 = -4$

Since the discriminant Δ is negative, the quadratic equation does not have real roots, which means it doesn't intersect the x-axis and is always positive.

Next, find the vertex of the parabola, as this will help us determine its minimum value. In a quadratic function of the form $ax^2 + bx + c$, the x-coordinate of the vertex is given by:

$x_{\text{min}} = -\frac{b}{2a} = \frac{4}{2} = 2$

Since the vertex lies at $x = 2$ and the parabola is opening upwards (as the coefficient of x^2 is positive), $f(x)$ is increasing for $x \geq 2$.

Calculate $f(x)$ at $x = 2$:

$f(2) = 2^2 - 4 \times 2 + 5 = 4 - 8 + 5 = 1$

Therefore, $f(2) = 1$, which is the minimum value of the function on the given interval. As x approaches infinity, $f(x)$ also approaches infinity.

Thus, the range of $f(x)$ is $[1, \infty)$.

Question18

If $f(x) = -|x|$, then (fofof)(x) + (fofof)$(-x)$ =

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $-2f(x)$

B. $|f(x)|$

C. $2f(x)$

D. $-|f(x)|$

Answer: C

Solution:

To solve the problem, let's analyze the function $f(x) = -|x|$.

First, let's evaluate $f(-x)$:

$f(-x) = -|-x| = -|x|$

Next, consider the composition $f(f(x))$:

$f(f(x)) = -|f(x)| = -|-|x|| = -|x| = f(x)$

Now evaluate $f(f(f(x)))$:

$f(f(f(x))) = f(f(x)) = f(x)$

Thus, $f(f(f(x))) = f(x)$.

Similarly, since $f(x) = -|x|$, we have:

$$f(f(f(-x))) = f(f(-|x|)) = f(-|x|) = -|-|x|| = -|x| = f(x)$$

Therefore, the expression $(f \circ f \circ f)(x) + (f \circ f \circ f)(-x)$ simplifies to:

$$f(f(f(x))) + f(f(f(-x))) = f(x) + f(x) = 2f(x)$$

So, the result is:

$$2f(x)$$
