Ellipse

Question1

A line segment joining a point A on X-axis to a point B on Y-axis is such that $AB = 15$. If P is a point on AB such that $\frac{AP}{PB} = \frac{2}{3}$, then the locus of P is

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Options:

A.

$x = 9\cos\theta, y = 6\sin\theta$

B.

$x = 6\cos\theta, y = 9\sin\theta$

C.

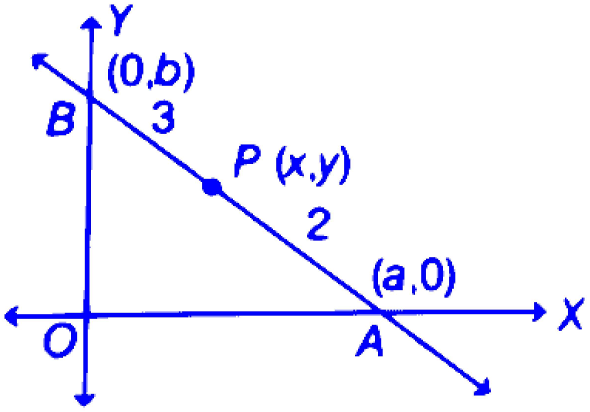
$x = 6\cos\theta, y = 6\sin\theta$

D.

$x = 9\cos\theta, y = 9\sin\theta$

Answer: A

Solution:



Here, $P \equiv \left(\frac{3a}{5}, \frac{2b}{5}\right) = (x, y)$

$$\Rightarrow \quad x = \frac{3a}{5}, y = \frac{2b}{5}$$

$$\Rightarrow \quad a = \frac{5x}{3}, b = \frac{5y}{2}$$

$$\therefore \quad a^2 + b^2 = (15)^2$$

$$\Rightarrow \quad \left(\frac{5x}{3}\right)^2 + \left(\frac{5y}{2}\right)^2 = 225$$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 9$$

$$\Rightarrow \quad \frac{x^2}{81} + \frac{y^2}{36} = 1$$

which represent equation of ellipse

$\therefore \quad x = 9\cos\theta$

And $\quad y = 6\sin\theta$

Question2

If any tangent drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ touches one of the circles $x^2 + y^2 = \alpha^2$, then the range of α is

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Options:

A.

$9 \leq \alpha \leq 16$

B.

$16 \leq \alpha \leq 25$

C.

$3 \leq \alpha \leq 4$

D.

$4 \leq \alpha \leq 6$

Answer: C

Solution:

Given equation of ellipse is

$\frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad \ldots (i)$

where, $a^2 = 16, b^2 = 9$

$\Rightarrow \quad a = 4, b = 3$

$\therefore$ Equation of tangent to the ellipse (i) is

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$
$$\Rightarrow \quad y = mx \pm \sqrt{16m^2 + 9}$$
$$\Rightarrow \quad y - mx \mp \sqrt{16m^2 + 9} = 0 \qquad \ldots (ii)$$

Given equation of circle $x^2 + y^2 = \alpha^2$

Centre $= (0, 0)$, radius $= \alpha$

Since, Eq. (ii) touches the given circle

$$\therefore \quad \alpha = \left| \frac{0 - 0 \mp \sqrt{16m^2+9}}{\sqrt{1+m^2}} \right|$$
$$\Rightarrow \quad \alpha = \left| \frac{\sqrt{16m^2+9}}{\sqrt{1+m^2}} \right|$$
$$\Rightarrow \quad \alpha^2 = \frac{16m^2+9}{1+m^2}$$
$$\text{Let} \quad f(m) = \frac{16m^2+9}{1+m^2}$$
$$\Rightarrow \quad f(m) = \frac{16m^2+16-16+9}{1+m^2}$$
$$\Rightarrow \quad f(m) = \frac{16(m^2+1)-7}{m^2+1}$$
$$= 16 - \frac{7}{m^2+1}$$

Here, $m^2 \geq 0$

$\Rightarrow \quad m^2 + 1 \geq 1$

$$\Rightarrow \quad 0 \leq \frac{1}{m^2+1} \leq 1$$
$$\Rightarrow \quad 0 \leq \frac{7}{m^2+1} \leq 7$$
$$\Rightarrow \quad 0 \geq -\frac{7}{m^2+1} \geq -7$$
$$\Rightarrow \quad 16 \geq 16-\frac{7}{m^2+1} \geq 16-7$$
$$\Rightarrow \quad 16 \geq 16-\frac{7}{m^2+1} \geq 9$$
$$\Rightarrow \quad 9 \leq f(m) \leq 16$$
$$\Rightarrow \quad 9 \leq \alpha^2 \leq 16$$
$$\Rightarrow \quad 3 \leq |\alpha| \leq 4$$

Question3

If S and S' are the foci of an ellipse $\frac{x^2}{169}+\frac{y^2}{144}=1$ and the point B lying on positive Y-axis is one end of its minor axis, then the incentre of the $\triangle SBS'$ is

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Options:

A.

$\left(0, \frac{10}{3}\right)$

B.

$\left(\frac{13}{3}, \frac{10}{3}\right)$

C.

$\left(\frac{10}{3}, \frac{13}{3}\right)$

D.

$\left(0, \frac{13}{3}\right)$

Answer: A

Solution:

The equation of ellipse is

$\frac{x^2}{169} + \frac{y^2}{144} = 1$

So, $a^2 = 169, b^2 = 144$

$$\begin{aligned} \Rightarrow \quad & a = 13, b = 12 \\ \therefore \quad & c^2 = a^2 - b^2 \Rightarrow 169 - 144 = 25 \\ \Rightarrow \quad & c = 5 \end{aligned}$$

So, the foci are at $(\pm c, 0)$. So, $S = (5, 0)$ and $S' = (-5, 0)$

Since, the point B is one end of the minor axis lying on the positive Y-axis.

So, $B = (0, \pm b) = (0, 12)$

$\therefore$ The vertices of the triangle are $S(5, 10), S'(-5, 0)$ and $B(0, 12)$

$$\begin{aligned} SS' &= \sqrt{(-5-5)^2 + (0-0)^2} \\ &= \sqrt{(-10)^2} = 10 \\ SB &= \sqrt{(0-5)^2 + (12-0)^2} \\ &= \sqrt{25+144} = \sqrt{169} = 13 \\ S'B &= \sqrt{(0-(-5))^2 + (12-0)^2} \\ &= \sqrt{25+144} = \sqrt{169} = 13 \end{aligned}$$

Since, $SB = S'B$, So SBS' is an isosceles triangle.

Now, the incenter (I_x, I_y) of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and opposite side lengths a, b and c is

$I_x = \frac{ax_1+bx_2+cx_3}{a+b+c}, I_y = \frac{ay_1+by_2+cy_3}{a+b+c}$

Here, $(x_1, y_1) = S(5, 0), (x_2, y_2) = S'(-5, 0), (x_3, y_3) = B(0, 12)$ and $a = S'B = 13$,
$b = SB = 13, C = SS' = 10$

$$\begin{aligned} \therefore \quad & I_x = \frac{13(5)+13(-5)+10(0)}{13+13+10} \\ \Rightarrow \quad & \frac{65-65+0}{36} = 0 \\ & I_y = \frac{13(0)+13(0)+10(12)}{13+13+10} \\ \Rightarrow \quad & \frac{0+0+120}{36} = \frac{10}{3} \end{aligned}$$

So, the incenter of the $\Delta SBS'$ is $\left(0, \frac{10}{3}\right)$.

Question4

One of the foci of an ellipse is $(2, -3)$ and its corresponding directrix is $2x + y = 5$. If the eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$, then the

coordinates of the other focus are

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Options:

A.

$(18, 5)$

B.

$(4, -2)$

C.

$(-2, -5)$

D.

$(-4, -6)$

Answer: C

Solution:

Let the given focus be $F_1(2, -3)$ and the corresponding directrix be $L_1 : 2x + y - 5 = 0$ and its eccentricity,

$e = \frac{\sqrt{5}}{3}$

Let the other focus be $F_2\left(x_2, y_2\right)$

Now, the distance from $F_1(2, -3)$ to $2x + y - 5 = 0$ is

$d_1 = \frac{|2(2)+(-3)-5|}{\sqrt{2^2+1^2}}$

$= \frac{|4-3-5|}{\sqrt{4+1}} = \frac{|-4|}{\sqrt{5}} = \frac{4}{5}$

Let $P(x, y)$ be any point on the ellipse.

So, $PF_1 = \sqrt{(x-2)^2 + (y+3)^2}$

And the distance from P to the directrix L_1 is

$$PM_1 = \frac{|2x + y - 5|}{\sqrt{2^2 + 1^2}}$$

$$\Rightarrow \quad \frac{|2x + y - 5|}{\sqrt{5}}$$

Since, $PF_1 = e \cdot PM_1$

$$\Rightarrow \sqrt{(x-2)^2+(y+3)^2} = \frac{\sqrt{5}}{3} \cdot \frac{|2x+y-5|}{\sqrt{5}}$$

$$\Rightarrow \frac{|2x+y-5|}{3}$$

Squaring on both sides, we get

$$(x-2)^2+(y+3)^2 = \frac{(2x+y-5)^2}{9}$$

$$\Rightarrow 9\left((x-2)^2+(y+3)^2\right)$$

$$= 4x^2+y^2+25+4xy-20x-10y$$

$$= 9x^2-36x+36+9y^2+54y+81$$

$$= 4x^2+y^2+25+4xy-20x-10y$$

$$\Rightarrow 5x^2+8y^2-4xy-16x+64y+92=0$$

This is equation of ellipse.

Now, slope of directrix is $m_D = -2$ and the line joining the two foci is perpendicular to the directrix.

So, $m_F = \frac{-1}{m_D} = \frac{-1}{-2} = \frac{1}{2}$

Let, the other focus be $F_2\left(x_2, y_2\right)$

$$\text{So, } \frac{y_2-(-3)}{x_2-2} = \frac{y_2+3}{x_2-2} = \frac{1}{2}$$

$$\Rightarrow 2\left(y_2+3\right) = x_2-2$$

$$\Rightarrow \quad x_2 = 2y_2+8$$

We know that $c = ae$ and the distance between the focus and directrix is

$$\frac{a}{e} - c = \frac{a}{e} - ae = \frac{a\left(1-e^2\right)}{e}$$

$$\Rightarrow \quad \frac{a\left(1-\left(\frac{\sqrt{5}}{3}\right)^2\right)}{\frac{\sqrt{5}}{3}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \quad \frac{a\left(1-\frac{5}{9}\right)}{\frac{\sqrt{5}}{3}} = \frac{4}{\sqrt{5}} \Rightarrow a\left(\frac{4}{9}\right) \times \frac{3}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \quad \frac{4a}{3\sqrt{5}} = \frac{4}{\sqrt{5}} \Rightarrow a = 3$$

$$\therefore \quad c = ae = 3 \times \frac{\sqrt{5}}{3} = \sqrt{5}$$

The distance between two foci $= 2c = 2\sqrt{5}$

Let $F_2\left(x_2, y_2\right)$. So, the distance

$$F_1F_2 = \sqrt{\left(x_2-2\right)^2+\left(y_2+3\right)^2} = 2\sqrt{5}$$

$$\Rightarrow \sqrt{(2y_2+8-2)^2+(y_2+3)^2} = 2\sqrt{5}$$
$(\because x_2 = 2y_2 + 8)$
$$\Rightarrow \quad \sqrt{4(y_2+3)^2+(y_2+3)^2} = 2\sqrt{5}$$
$$\Rightarrow \quad \sqrt{5(y_2+3)^2} = 2\sqrt{5}$$
$$\Rightarrow \quad \sqrt{5}\,|y_2+3| = 2\sqrt{5} \Rightarrow |y_2+3| = 2$$

So, $y_2 + 3 = 2$ or $y_2 + 3 = -2$

$\Rightarrow \quad y_2 = -1$ or $y_2 = -5$

$\therefore \quad x_2 = 2(-1) + 8 = 6$ and

$x_2 = 2(-5) + 8 = -2$

So, $F_2(6, -1)$ and $F_2(-2, -5)$

Line $2x + y - 5 = 0$

For $F_1(2, -3), 2(2) + (-3) - 5 = 4 - 8 = -4$

For $F_2(6, -1), 2(6) + (-1) - 5 = 12 - 6 = 6$

Since, -4 and 6 have opposite signs, so foci of an ellipse is $F_2(-2, -5)$.

Question5

If the focus of an ellipse is $(-1, -1)$, equation of its directrix corresponding to this focus is $x + y + 1 = 0$ and its eccentricity is $\frac{1}{\sqrt{2}}$, then the length of its major axis is

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Options:

A. 2

B. 1

C. 4

D. 3

Answer: A

Solution:

To solve the problem of finding the length of the major axis of the given ellipse, let's break it down step by step:

Given Information:

Focus of the ellipse: $S = (-1, -1)$

Equation of the directrix: $x + y + 1 = 0$

Eccentricity of the ellipse: $e = \frac{1}{\sqrt{2}}$

Equation of the Ellipse:

The equation connecting the focus, directrix, and eccentricity is:

$$PS = e \cdot PM$$

where PS is the distance from a point $P(x, y)$ on the ellipse to the focus S, and PM is the perpendicular distance from P to the directrix.

Distance PS:

$$PS = \sqrt{(x+1)^2 + (y+1)^2}$$

Perpendicular distance PM from P to the directrix:

$$PM = \frac{|x+y+1|}{\sqrt{2}}$$

Substitute into ellipse equation:

$$\sqrt{(x+1)^2 + (y+1)^2} = \frac{1}{\sqrt{2}} \cdot \frac{|x+y+1|}{\sqrt{2}}$$

This simplifies to:

$$\sqrt{(x+1)^2 + (y+1)^2} = \frac{x+y+1}{2}$$

Form an equation by solving and simplifying:

Upon squaring both sides and simplifying, the equation becomes:

$$3x^2 + 3y^2 - 2xy + 6x + 6y + 7 = 0$$

Finding the Center:

To determine the center of the ellipse, we take partial derivatives and solve:

Derivative with respect to x:

$$\frac{\partial f}{\partial x} = 6x - 2y + 6 = 0$$

Derivative with respect to y:

$$\frac{\partial f}{\partial y} = 6y - 2x + 6 = 0$$

Solving these equations simultaneously gives the center $O\left(-\frac{3}{2}, -\frac{3}{2}\right)$.

Determine the Length of Major Axis:

Distance OS (from center to the focus):

$OS = ae = \frac{1}{\sqrt{2}}$

Given:

$a \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

This implies:

$a = 1$

Length of the major axis:

$2a = 2 \times 1 = 2$

Hence, the length of the major axis of the ellipse is **2**.

Question6

If the normal drawn at the point $(2, -1)$ to the ellipse $x^2 + 4y^i = 8$ meets the ellipse again at (a, b), then $17a =$

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Options:

A. 23

B. 14

C. 37

D. 9

Answer: B

Solution:

To find where the normal drawn at the point $(2, -1)$ on the ellipse $x^2 + 4y^2 = 8$ meets the ellipse again, we start by determining the equation of the normal. The given ellipse is $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

The equation of the normal at the point $(2, -1)$ is derived as follows:

Calculate the slope of the normal at $(2, -1)$:

$\frac{8x}{2} - \frac{2y}{-1} = 8 - 2$

Simplify the equation to find the normal line:

$$4x + 2y = 6 \quad \Rightarrow \quad 2x + y = 3 \quad \Rightarrow \quad y = 3 - 2x$$

To find where this normal line meets the ellipse again, substitute $y = 3 - 2x$ into the ellipse equation:

$$x^2 + 4(3 - 2x)^2 = 8$$

Expand and simplify the equation:

$$x^2 + 4(9 - 12x + 4x^2) = 8$$

$$x^2 + 36 - 48x + 16x^2 = 8$$

Combine and equate:

$$17x^2 - 48x + 28 = 0$$

Factorize:

$$(17x - 14)(x - 2) = 0$$

The solutions for x are $x = 2$ and $x = \frac{14}{17}$.

Choosing $a = \frac{14}{17}$, we find:

$$17a = 14$$

Question7

If the locus of the centroid of the triangle with vertices $A(a, 0), B(a \cos t, a \sin t)$ and $C(b \sin, -b \cos t)$ (t is a parameter) is $9x^2 + 9y^2 - 6x\overline{\overline{x}}49$, then the area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is

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Options:

A. $\frac{49}{2}$

B. $\frac{7}{2}$

C. $\frac{1}{2}$

D. $\frac{47}{2}$

Answer: B

Solution:

To find the area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes, we can follow these steps:

Identify Intercepts:

The x-intercept is found by setting $y = 0$ in the equation $\frac{x}{a} + \frac{y}{b} = 1$. This gives $x = a$.

The y-intercept is found by setting $x = 0$ in the equation $\frac{x}{a} + \frac{y}{b} = 1$. This results in $y = b$.

Vertices of the Triangle:

The vertices of the triangle formed by this line with the axes are $(a, 0)$, $(0, b)$, and the origin $(0, 0)$.

Calculate the Area:

The area of a triangle formed by vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by the formula:

Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Substituting the vertices $(a, 0)$, $(0, b)$, and $(0, 0)$, we get:

Area $= \frac{1}{2}|a(0 - b) + 0(0 - 0) + 0(b - 0)| = \frac{1}{2}| - ab| = \frac{ab}{2}$

Substitute Known Values:

Since we've identified from the given equations that $a = 1$ and $b = 7$, substitute these values into the area formula:

Area $= \frac{1 \cdot 7}{2} = \frac{7}{2}$

Therefore, the area of the triangle formed by the line with the coordinate axes is $\frac{7}{2}$.

Question8

$S = (-1, 1)$ is the focus, $2x - 3y + 1 = 0$ is the directrix corresponding, to S and $\frac{1}{2}$ is the eccentricity of an ellipse, If (a, b) is the centre of the ellipse, then $3a + 2b$:

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Options:

A. $\frac{30}{13}$

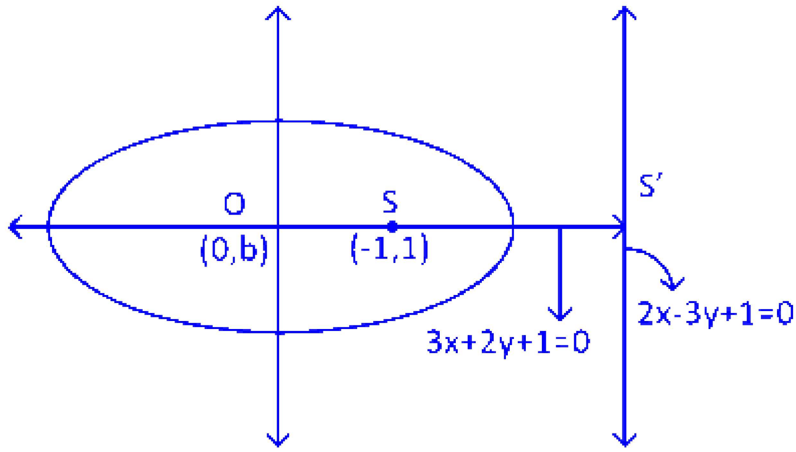
B. $\frac{4}{13}$

C. -1

D. 0

Answer: C

Solution:



Slope of line $SS' = \frac{-3}{2}$

Equation of $SS' \Rightarrow (y-1) = \frac{-3}{2}(x+1)$

$\Rightarrow \quad 3x + 2y + 1 = 0$

Now, point of intersection of

$3x + 2y + 1 = 0$ and $2x - 3y + 1 = 0$ is

$$S' \equiv \left(\frac{-5}{13}, \frac{1}{13}\right)$$

Let length of major axis $= P$

Now, $OS' = \frac{P}{e}$ and $OS = Pe$

$$\Rightarrow OS' - OS = \sqrt{\left(-1 + \frac{5}{13}\right)^2 + \left(1 - \frac{1}{13}\right)^2}$$

$$\Rightarrow \frac{P}{e} - Pe = \sqrt{\left(\frac{-13+5}{13}\right)^2 + \left(\frac{13-1}{13}\right)^2}$$

$$= \sqrt{\frac{64}{169} + \frac{144}{169}} = \sqrt{\frac{208}{169}}$$

Question9

a and b are the semi-major and semi-minor axes of an ellipse whose axes are along the coordinate axes, If its latus rectum is of length 4 units and the distance between its foci is $4\sqrt{2}$, then $a^2+b^2=$

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Options:

A. 24

B. 18

C. 16

D. 12

Answer: A

Solution:

Given:

The equation for the latus rectum of an ellipse is $\frac{2b^2}{a}=4$.

The distance between the foci is given as $4\sqrt{2}$.

Let's solve step-by-step:

From the latus rectum condition, we have $b^2=2a$.

The distance between the foci of the ellipse is defined as $2ae=4\sqrt{2}$.

$2ae=4\sqrt{2}$

Squaring both sides, we get:

$4a^2e^2=32$

We know the relationship between the eccentricity and the axes is $e^2=1-\frac{b^2}{a^2}$.

$a^2\left(1-\frac{b^2}{a^2}\right)=8$

Substituting $b^2=2a$ in the above equation:

$a^2-2a=8$

Solving for a, we get:

$a^2 - 2a - 8 = 0$

Solving this quadratic equation gives $a = 4$.

Subsequently, $b^2 = 2a = 8$.

Finally, calculating $a^2 + b^2$:

$a^2 = 16$

$b^2 = 8$

Therefore, $a^2 + b^2 = 16 + 8 = 24$.

Question10

If the extremities of the latus recta having positive ordinate of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ lie on the parabola $x^2 + 2ay - 4 = 0$, then the points (a, b) lie on the curve

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Options:

A. $xy = 4$

B. $x^2 + y^2 = 4$

C. $\frac{x^2}{4} + \frac{y^2}{1} = 1$

D. $\frac{x^2}{4} - \frac{y^2}{1} = 1$

Answer: B

Solution:

To determine the curve on which the points (a, b) lie, we begin by considering the properties of the ellipse given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$.

The extremities of the latus rectum of an ellipse for the positive ordinate are $\left(ae, \frac{b^2}{a}\right)$, where e is the eccentricity of the ellipse. For an ellipse, the relationship between the semi-major axis a, semi-minor axis b, and eccentricity e is given by:

$e = \sqrt{1 - \frac{b^2}{a^2}}$

Since these extremities lie on the parabola described by the equation $x^2 + 2ay = 4$, we substitute the point $\left(ae, \frac{b^2}{a}\right)$ into this equation:

$(ae)^2 + 2a\left(\frac{b^2}{a}\right) = 4$

This simplifies to:

$a^2e^2 + 2b^2 = 4$

Using the relationship for eccentricity, we substitute $a^2e^2 = a^2 - b^2$ into the equation:

$a^2 - b^2 + 2b^2 = 4$

This further simplifies to:

$a^2 + b^2 = 4$

Thus, the points (a, b) lie on the curve:

$x^2 + y^2 = 4$

Replacing (a, b) with (x, y), we confirm that the correct description for this relationship is given by:

$x^2 + y^2 = 4$

Question11

The length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ is $\frac{8}{3}$. If the distance from the centre of the ellipse to its focus is $\sqrt{5}$, then $\sqrt{a^2 + 6ab + b^2} =$

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Options:

A. 7

B. $12\sqrt{2}$

C. $3\sqrt{5}$

D. 11

Answer: A

Solution:

Length of latus rectum $= \frac{2b^2}{a} = \frac{8}{3}$

$\Rightarrow \quad b^2 = \frac{4a}{3}$

Also, given, $ae = \sqrt{5}$

On squaring both sides, we get

$$a^2e^2 = 5$$

$$a^2\left(1 - \frac{b^2}{a^2}\right) = 5 \Rightarrow a^2 - \frac{4a}{3} = 15$$

$$3a^2 - 4a = 15$$

$$3a^2 - 4a - 15 = 0$$

$$a = 3 \quad a = \frac{-5}{3}$$

If $a = 3$,

then, $b = 2$

then, $\sqrt{9 + 4 + 36} = \sqrt{49} = 7$

Question12

S is the focus of the ellips $\frac{x^2}{25} + \frac{y^2}{b^2} = 1, (b < 5)$ lying on the negative X-axis and $P(\theta)$ is a point on this ellipes. If the distance between the foci of this ellipse is 8 and $S'P = 7$, then $\theta =$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: B

Solution:

$\frac{x^2}{25}+\frac{y^2}{b^2}=1, s'=(-5e,0)$

$2\times5\times e=8$

$e=\frac{4}{5}\Rightarrow e^2=1-\frac{b^2}{a^2}\Rightarrow\frac{16}{25}=\frac{25-b^2}{25}$

$\Rightarrow \quad b=3$

$\because \quad S'\equiv(-4,0)$

$P=(5\cos\theta,3\sin\theta)$

$\mathrm{S'P}=7$

$=\sqrt{(5\cos\theta+4)^2+9\sin^2\theta}=49$

On squaring both side, we get

$\Rightarrow \quad 49=25\cos^2\theta+40\cos\theta$

$+16+9\sin^2\theta$

$\Rightarrow \quad 49=25\cos^2\theta+40\cos\theta$

$+16+9\left(1-\cos^2\theta\right)$

$\Rightarrow \quad 16\cos^2\theta+40\cos\theta-24=0$

$\Rightarrow \quad \cos\theta=\frac{1}{2}\text{ or }\cos\theta=-3$

$\therefore \quad \theta=\frac{\pi}{3}$

Question13

The equations of the directrices of the elmpse $9x^2+4y^2-18x-16y-11=0$ are

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Options:

A. $y=2\pm\frac{9}{\sqrt{5}}$

B. $x = 1 \pm \frac{6}{\sqrt{5}}$

C. $x = 2 \pm \frac{9}{\sqrt{5}}$

D. $y = 1 \pm \frac{6}{\sqrt{5}}$

Answer: A

Solution:

Given, ellipse :

$9x^2 + 4y^2 - 18x - 16y - 11 = 0$

The given equation of ellipse can be written as

$\frac{(x-1)^2}{2^2} + \frac{(y-2)^2}{3^2} = 1$

Here, $a = 2$ and $b = 3$, $h = 1$ and $k = 2$

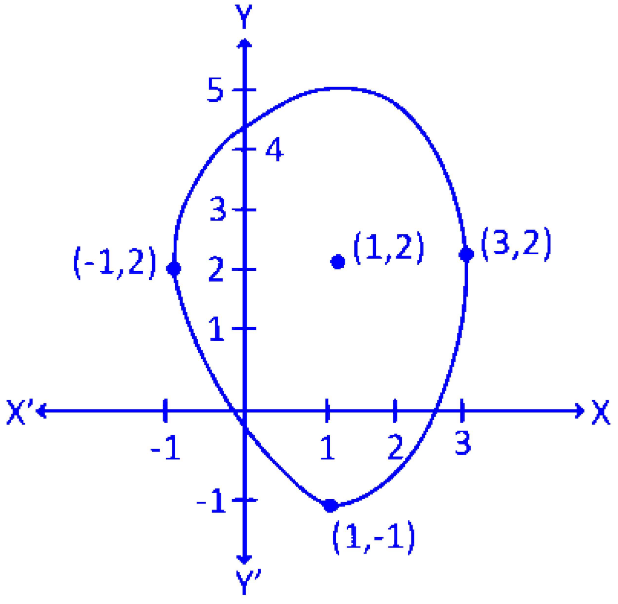
So, the given ellipse is a vertical ellipse.

$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

$\therefore$ Equations of directrices will be

$y = k \pm \frac{b}{e}$ i.e. $y = 2 \pm \frac{9}{\sqrt{5}}$

Ellipse: $\frac{(x-1)^2}{2^2} + \frac{(y-2)^2}{3^2} = 1$



Question14

L_1' is the end of a latus rectum of the ellipse $3x = 2 \pm \frac{\sqrt{5}}{\sqrt{5}}$ $3x^2 + 4y^2 = 12$ which is lying in the third quadrant. If the normal drawn at L_1' to this ellipse intersects the ellipse again at the point $P(a, b)$, then $a =$

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Options:

A. $\frac{63}{38}$

B. $\frac{11}{19}$

C. $-\frac{11}{19}$

D. $-\frac{63}{38}$

Answer: B

Solution:

Given, ellipse $3x^2 + 4y^2 = 12$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$

$$e = \sqrt{1 - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Latus rectums $x = \pm ae = \pm(2)\left(\frac{1}{2}\right) = \pm 1$

$\because L_1'$ is the end of a latus rectum and it is lying in the third quadrant

$\because \quad L_1' = \left(-1, -\sqrt{3\left(1 - \frac{(-1)^2}{4}\right)}\right) = \left(-1, -\frac{3}{2}\right)$

Equation tangent at $\left(-1, \frac{-3}{2}\right)$ is given by

$\frac{x(-1)}{4} + \frac{y\left(-\frac{3}{2}\right)}{3} = 1 \Rightarrow x + 2y + 4 = 0$

Slope of tangent $= -\frac{1}{2}$

$\therefore$ Slope of normal $= -\left(\frac{1}{-\frac{1}{2}}\right) = 2$. Equation of normal from $\left(-1, -\frac{3}{2}\right)$ is given by

$y + \frac{3}{2} = 2(x+1) \Rightarrow 4x - 2y + 1 = 0$

The points of intersection of the normal

$4x - 2y + 1 = 0$

and the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ are

$L_1'\left(-1, -\frac{3}{2}\right)$ and $P\left(\frac{11}{19}, \frac{63}{38}\right)$

Hence, $a = \frac{11}{19}$

Question15

If $6x - 5y - 20 = 0$ is a normal to the ellipse $x^2 + 3y^2 = K$, then $K =$

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Options:

A. 9

B. 17

C. 25

D. 37

Answer: D

Solution:

We have, $x^2 + 3y^2 = K$

$\Rightarrow \frac{x^2}{(\sqrt{K})^2} + \frac{y^2}{\left(\sqrt{\frac{K}{3}}\right)^2} = 1$

So, equation of normal is

$\sqrt{K} x \sec\theta - \sqrt{\frac{K}{3}} y \operatorname{cosec}\theta = K - \frac{K}{3} \quad \ldots$ (i)

Given equation of normal is

$6x-5y-20=0 \quad \ldots \text{(ii)}$

$\because$ Eqs, (i) and (ii) represent the same line

$$\therefore \frac{\sqrt{K}\sec\theta}{6}=\frac{\sqrt{\frac{K}{3}}\operatorname{cosec}\theta}{5}=\frac{K-\frac{K}{3}}{20}$$

$$\Rightarrow \frac{\sqrt{K}\sec\theta}{6}=\frac{\sqrt{K}\operatorname{cosec}\theta}{5\sqrt{3}}=\frac{K}{30}$$

$$\text{So, } \cos\theta=\frac{5}{\sqrt{K}}, \sin\theta=\frac{2\sqrt{3}}{\sqrt{K}}$$

$$\because \cos^2\theta+\sin^2\theta=1$$

$$\frac{25}{K}+\frac{12}{K}=1$$

$$\Rightarrow \quad K=37$$

Question16

The locus of the mid-points of the intercepted portion of the tangents by the coordinate axes, which are drawn to the ellipse $x^2+2y^2=2$ is

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Options:

A. $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

B. $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

C. $\frac{x^2}{2}+\frac{y^2}{4}=1$

D. $\frac{x^2}{4}+\frac{y^2}{2}=1$

Answer: A

Solution:

Given equation of ellipse is

$$x^2 + 2y^2 = 2$$
$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{1} = 1$$

Now, tangent at $(a\cos\theta, b\sin\theta)$ to the ellipse is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

This meets the axis at the points $A(a\sec\theta, 0)$ and $B(0, b\operatorname{cosec}\theta)$.

Now, Let $(x_1, y_1)x$ mid-point of AB.

Therefore, $2x_1 = a\sec\theta$ and

$2y_1 = b\operatorname{cosec}\theta$

$$\Rightarrow 2\cos\theta = \frac{a}{x_1} \text{ and } 2\sin\theta = \frac{b}{y_1}$$
$$\Rightarrow \quad 4\left(\cos^2\theta + \sin^2\theta\right) = \frac{a^2}{x_1^2} + \frac{b^2}{y_1^2}$$
$$\Rightarrow \quad 4 = \frac{a^2}{x_1^2} + \frac{b^2}{y_1^2} \Rightarrow 4 = \frac{2}{x_1^2} + \frac{1}{y_1^2}$$
$$\left(\because a^2 = 2, b^2 = 1\right)$$

Therefore, locus of (x_1, x_2) is

$$4 = \frac{2}{x^2} + \frac{1}{y^2} \Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

Question17

The product of the lengths of the perpendiculars drawn from the two foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ to the tangent at any point on the ellipse is

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Options:

A. 6

B. 7

C. 8

D. 9

Answer: D

Solution:

The equation of ellipse is

$\frac{x^2}{9}+\frac{y^2}{25}=1$

Now, equation of tangent at any point ($3 \cos \theta, 5 \sin \theta$) on the ellipse is

$\frac{x}{3} \cos \theta+\frac{y}{5} \sin \theta=1$

i.e. $5 x \cos \theta+3 y \sin \theta-15=0$

The product of perpendicular from $S(3 e, 0)$ and $S(-3 e, 0)$ on this tangent is

$$\begin{aligned} & \frac{(15 e \cos \theta-15)(-15 e \cos \theta-15)}{25 \cos ^2 \theta+9 \sin ^2 \theta} \\ & =\frac{225\left(1-e^2 \cos ^2 \theta\right)}{25 \cos ^2 \theta+25\left(1-e^2\right) \cos ^2 \theta} \\ & =\frac{225\left(1-e^2 \cos ^2 \theta\right)}{25\left(1-e^2 \cos ^2 \theta\right)}=9\end{aligned}$$

Question18

Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at all the ends of its latus recta. The area of the quadrilateral, so formed (in sq units) is

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Options:

A. 27

B. 36

C. 42

D. 45

Answer: A

Solution:

The equation of ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$

So, $e = \sqrt{1 - \frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $ae = 2$

Now, equation of tangent at $\left(2, \frac{5}{3}\right)$ is

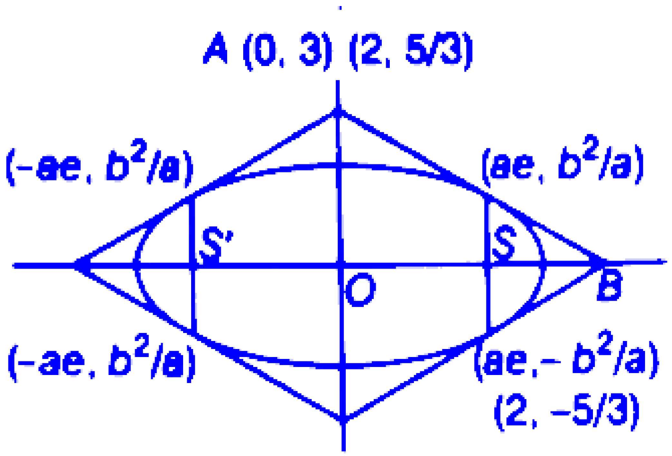
given as

$T : \left(\frac{2}{9}\right)x + \frac{y}{3} = 1 \qquad \ldots(i)$

and equation of tangent at $\left(2, -\frac{5}{3}\right)$ is

given as

$T^1 : \left(\frac{2}{9}\right)x + \left(-\frac{y}{3}\right) = 1 \qquad \ldots(ii)$



From Eq. (i),

x intercept, $y = 0 \Rightarrow x = \frac{9}{2}$

y intercept, $x = 0 \Rightarrow y = 3$

Now, area of $\triangle AOB$ is

$= \frac{1}{2} \times OB \times OA = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$

Thus, area of quadrilateral

$$= 4 \times (\text{ area of } \triangle AOB)$$
$$= 4 \times \frac{27}{4} = 27$$

Question19

A particle is travelling in clockwise direction on the ellipse $\frac{x^2}{100}+\frac{y^2}{25}=1$. If the particle leaves the ellipse the point $(-8,3)$ on it and travels along the tangent to the ellipse at that point, then the point where the particle crosses the Y-axis is

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Options:

A. $\left(0, \frac{7}{3}\right)$

B. $\left(0, \frac{25}{3}\right)$

C. $(0,9)$

D. $\left(0, \frac{-25}{3}\right)$

Answer: B

Solution:

The particle is moving clockwise along the ellipse described by the equation:

$\frac{x^2}{100}+\frac{y^2}{25}=1$

When the particle reaches the point $(-8,3)$ on the ellipse, it moves along the tangent line at this location. We need to determine where this tangent line intersects the Y-axis.

Equation of the Tangent Line:

For an ellipse with the general form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the equation of the tangent line at any point (x_1, y_1) on the ellipse is given by:

$\frac{x x_1}{a^2}+\frac{y y_1}{b^2}=1$

Substituting $a^2=100, b^2=25$, and the point $(-8,3)$, the equation of the tangent becomes:

$\frac{x(-8)}{100}+\frac{y(3)}{25}=1$

Simplifying:

$-2x+3y=25$

Finding the Intersection with the Y-Axis:

To find where the tangent line crosses the Y-axis, we set $x=0$:

$-2(0)+3y=25 \Rightarrow 3y=25 \Rightarrow y=\frac{25}{3}$

Thus, the particle crosses the Y-axis at the point:

$(0, \frac{25}{3})$

Question20

If an ellipse with foci at $(3,3)$ and $(-4,4)$ is passing through the origin, then the eccentricity of that ellipse is

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Options:

A. $5/7$

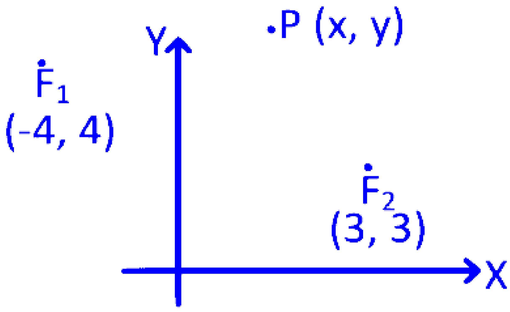
B. $3/7$

C. $1/7$

D. $4/7$

Answer: A

Solution:



$PF_1 + PF_2 = 2a$, where a is length of semi-major axis.

$\because$ Ellipse passes through $(0,0)$

$\Rightarrow \sqrt{(-4)^2 + 4^2} + \sqrt{3^2 + 3^2} = 2a$

$\Rightarrow 4\sqrt{2} + 3\sqrt{2} = 2a \Rightarrow a = \frac{7\sqrt{2}}{2} = \frac{7}{\sqrt{2}}$

$F_1F_2 = 2ae$

$$\Rightarrow \sqrt{(-4-3)^2 + (4-3)^2} = 2 \times \frac{7}{\sqrt{2}} \times e$$

$$\Rightarrow \sqrt{50} = \sqrt{2} \times 7e \Rightarrow e = \frac{5}{7}$$
