Hyperbola

Question1

If l is the maximum value of $-3x^2 + 4x + 1$ and m is the minimum value of $3x^2 + 4x + 1$, then the equation of the hyperbola having foci at $(l, 0), (7m, 0)$ and eccentricity as 2 is

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Options:

A.

$36x^2 - 12y^2 = 49$

B.

$49x^2 - 36y^2 = 12$

C.

$2x^2 - 5y^2 = 1$

D.

$36x^2 - 12y^2 = 1$

Answer: A

Solution:

Given, l is the maximum value of $-3x^2 + 4x + 1$ and m is the minimum value of $3x^2 + 4x + 1$

For $-3x^2 + 4x + 1$, $\frac{-b}{2a} = \frac{-4}{2(-3)} = \frac{2}{3}$

$$\therefore \text{ Maximum value } = -3\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) + 1$$

$$= -3 \times \frac{4}{9} + \frac{8}{3} + 1$$

$$\Rightarrow \quad l = \frac{-4}{3} + \frac{8}{3} + 1 = \frac{7}{3}$$

For $3x^2 + 4x + 1$, $\frac{-b}{2a} = \frac{-4}{2\times3} = \frac{-2}{3}$

$\therefore$ Minimum value

$$= 3\left(\frac{-2}{3}\right)^2 + 4\left(\frac{-2}{3}\right) + 1$$

$$= 3 \times \frac{4}{9} - \frac{8}{3} + 1$$

$$\Rightarrow \quad m = \frac{4}{3} - \frac{8}{3} + 1 = \frac{-1}{3}$$

Now, foci of hyperbola $= (l, 0)$

$= \left(\frac{7}{3}, 0\right)$

And $(7m, 0) = \left(\frac{-7}{3}, 0\right)$

$\therefore$ Distance between foci $=$

$\Rightarrow 2a \times 2 = 2 \times \frac{7}{3} \Rightarrow a = \frac{7}{6}$

Also, $b^2 = a^2\left(e^2 - 1\right)$

$$\Rightarrow \quad b^2 = \frac{49}{36}(4 - 1)$$

$$\Rightarrow \quad b^2 = \frac{49}{36} \times 3 = \frac{49}{12}$$

$\therefore$ Equation of hyperbola, $\frac{x^2}{\frac{49}{36}} - \frac{y^2}{\frac{49}{12}} = 1$

$$\Rightarrow \quad 36x^2 - 12y^2 = 49$$

Question2

The curve represented by $\frac{x^2}{12-\alpha} + \frac{y^2}{\alpha-10} = 1$ is

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Options:

A.

a hyperbola for some values of α in $(10, 12)$

B.

an ellipse for all values of α in $(10, 12)$

C.

a circle for some value of α in $(10, 12)$

D.

a hyperbola for all values of α in $(10, 12)$

Answer: C

Solution:

Given, equation of curve

$\frac{x^2}{12-\alpha} + \frac{y^2}{\alpha-10} = 1$

for circle, $12 - \alpha = \alpha - 10$

$\Rightarrow \quad 22 = 2\alpha \Rightarrow \alpha = 11$

$$\begin{aligned} \textit{and } & 12 - \alpha > 0 \Rightarrow \alpha < 12 \\ & \alpha - 10 > 0 \Rightarrow \alpha > 10 \end{aligned}$$

$\Rightarrow$ Curve represents a circle for some value of α in (10,12).

Option (c) is correct.

Similarly, for ellipse $12 - \alpha \neq \alpha - 10$

$\Rightarrow \quad 22 \neq 2\alpha \Rightarrow \alpha \neq 11$

Option (a) is wrong. for hyperbola one denominator must be positive and other negative

$\therefore \quad 12 - \alpha > 0$ and $\alpha - 10 < 0$

$\Rightarrow \quad \alpha < 12$ and $\alpha < 10$

$\Rightarrow \quad$ Means $\alpha < 10$

or $\quad 12 - \alpha < 0$ and $\alpha - 10 > 0$

$\Rightarrow \quad \alpha > 12$ and $\alpha > 10$

$\Rightarrow \quad$ Means $\alpha > 12$

Options (b) and (d) is wrong.

Question3

Let x be the eccentricity of a hyperbola whose transverse axis is twice its conjugate axis. Let y be the eccentricity of another hyperbola for which the distance between the focii is 3 times the distance between its directrices. Then $y^2 - x^2 =$

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Options:

A.

$\frac{23}{16}$

B.

$\frac{7}{4}$

C.

$\frac{4}{7}$

D.

$\frac{16}{23}$

Answer: B

Solution:

Let equation of hyperbola

$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$

Given, $2a = 2(2b)$

$$\Rightarrow \quad 2a = 4b$$
$$\Rightarrow \quad a = 2b$$

And $x^2 = 1 + \frac{b^2}{a^2}$

$$= 1 + \frac{b^2}{4b^2}$$
$$= 1 + \frac{1}{4} = \frac{5}{4}$$

Also, $2ay = 3\left(\frac{2a}{y}\right)$

(for another hyperbola)

$$\Rightarrow \qquad y^2 = 3$$
$$\therefore \qquad y^2 - x^2 = 3 - \frac{5}{4} = \frac{12-5}{4} = \frac{7}{4}$$

Question4

If the product of the perpendicular distances from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is $\frac{36}{13}$ and its eccentricity is $\frac{\sqrt{13}}{3}$, then $a - b =$

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Options:

A.

4

B.

3

C.

2

D.

1

Answer: D

Solution:

Since, the product of perpendicular distance from any point on the hyperbola to its asymptotes $= \frac{36}{13}$

So, $\frac{a^2b^2}{a^2+b^2} = \frac{36}{13}$ $\quad\quad\quad \ldots(i)$

Also, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow \frac{\sqrt{13}}{3} = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow \quad \frac{13}{9} = 1 + \frac{b^2}{a^2}$

(Squaring on both sides)

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{13}{9} - 1 = \frac{4}{9}$$
$$\Rightarrow \quad b^2 = \frac{4}{9}a^2$$

$\Rightarrow \quad b = \frac{2}{3}a \quad (\because a, b > 0)$

Substitute this into Eq. (i), we get

$$\frac{a^2 \cdot \left(\frac{2}{3}a\right)^2}{a^2 + \frac{4}{9}a^2} = \frac{36}{13}$$
$$\Rightarrow \frac{\frac{4}{9}a^4}{\frac{13}{9}a^2} = \frac{36}{13} \Rightarrow \frac{4}{13}a^2 = \frac{36}{13}$$
$$\Rightarrow 4a^2 = 36$$
$$\Rightarrow a^2 = 9$$

$\Rightarrow \quad a = 3 \quad (\because a > 0)$

And $b = \frac{2}{3}a = \frac{2}{3} \times 3 = 2$

Now, $a - b = 3 - 2 = 1$

Question5

$P(\theta)$ is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{9} = 1, S$ is its $\mathrm{fOO}_{4/}$ lying on the positive X-axis and $Q = (0, 1)$. If $SQ = \sqrt{26}$ and $SP = 6$, then $\theta =$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\cos^{-1}\left(\frac{?}{3}\right)$

Answer: C

Solution:

We begin with the hyperbola defined by the equation:

$\frac{x^2}{a^2} - \frac{y^2}{9} = 1$

We can determine the eccentricity e of the hyperbola using the equation $e^2 - 1 = \frac{9}{a^2}$. Solving for e:

$e^2 = \frac{a^2+9}{a^2}$

$e = \frac{\sqrt{9+a^2}}{a}$

The focus S is on the positive x-axis, given by the coordinates $(c, 0)$ where $c = \sqrt{9+a^2}$. We also have $Q = (0, 1)$.

Given that $SQ = \sqrt{26}$, substitute the values:

$\sqrt{(c-0)^2 + (0-1)^2} = \sqrt{26}$

$\sqrt{c^2+1} = \sqrt{26}$

$c^2 + 1 = 26$

$c^2 = 25$

$\Rightarrow c = 5$

Thus, $S = (5, 0)$.

Now, concerning the point P on the hyperbola, we represent it parametrically as $P(a\sec\theta, 3\tan\theta) = (4\sec\theta, 3\tan\theta)$, since $a = 4$.

We use the equation for the distance $SP = 6$:

$SP^2 = (4\sec\theta - 5)^2 + (3\tan\theta)^2 = 36$

Expanding and substituting $\tan^2\theta = \sec^2\theta - 1$:

$(4\sec\theta - 5)^2 + 9\tan^2\theta = 36$

$(4\sec\theta - 5)^2 + 9(\sec^2\theta - 1) = 36$

$16\sec^2\theta - 40\sec\theta + 25 + 9\sec^2\theta - 9 = 36$

$25\sec^2\theta - 40\sec\theta - 20 = 0$

Factoring:

$5\sec^2\theta - 8\sec\theta - 4 = 0$

$5\sec\theta(\sec\theta-2)+2(\sec\theta-2)=0$

$(\sec\theta-2)(5\sec\theta+2)=0$

Solving gives $\sec\theta=2$, leading to:

$\theta=\frac{\pi}{3}$

Question6

If the tangent drawn at a point $P(t)$ on the hyperbola $x^2-y^2=c^2$ cuts X-axis at T and the normal drawn at the same point P cuts the Y-axis at N, then the equation of the locus of the mid-point of TN is

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Options:

A. $\frac{c^2}{4x^2}-\frac{y^2}{c^2}=1$

B. $\frac{x^2}{c^2}-\frac{y^2}{4c^2}=1$

C. $\frac{x^2}{4c^2}+\frac{y^2}{c^2}=1$

D. $x^2+y^2=4c^2$

Answer: A

Solution:

Given the hyperbola $x^2-y^2=c^2$, we are examining the tangent at any point $P(t)=(c\sec\theta,c\tan\theta)$.

For the equation of the tangent at $P(t)$:

$c\sec\theta\cdot x-c\tan\theta\cdot y=c^2$

Simplifying gives:

$\sec\theta\cdot x-\tan\theta\cdot y=c$

This tangent line intersects the X-axis when $y=0$:

$T\equiv\left(\frac{c}{\sec\theta},0\right)$

Next, consider the normal's equation. The slope of the normal is:

$\frac{-1}{\frac{\sec\theta}{\tan\theta}}=\frac{-\tan\theta}{\sec\theta}$

The equation of the normal through $P(t)$ is:

$c \sec \theta \tan \theta + c \tan \theta \sec \theta = 2c \tan \theta \sec \theta$

Thus, the normal cuts the Y-axis (where $x = 0$) at:

$N \equiv (0, 2c \tan \theta)$

Consider the midpoint $M(x, y)$ of segment TN. The coordinates (x, y) satisfy:

$2x = \frac{c}{\sec \theta}$

$2y = 2c \tan \theta$

Therefore:

$\sec \theta = \frac{c}{2x}, \quad \tan \theta = \frac{y}{c}$

Using the identity $\sec^2 \theta - \tan^2 \theta = 1$, we have:

$\left(\frac{c}{2x}\right)^2 - \left(\frac{y}{c}\right)^2 = 1$

This simplifies to the equation of the locus of the midpoint:

$\frac{c^2}{4x^2} - \frac{y^2}{c^2} = 1$

Question7

The slope of the tangent drawn from the point $(1, 1)$ to the hyperbola $2x^2 - y^2 = 4$ is

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Options:

A. 2

B. $\frac{-2 \pm \sqrt{6}}{2}$

C. $-1 \pm \sqrt{6}$

D. $\frac{-2 \pm \sqrt{3}}{2}$

Answer: C

Solution:

Given,

$\frac{x^2}{2} - \frac{y^2}{4} = 1$

Let $y = mx \pm \sqrt{a^2m^2 - b^2}$ be the tangent

put $x = 1, y = 1$

$1 = m \pm \sqrt{2m^2 - 4}$

$1 - m = \pm\sqrt{2m^2 - 4}$

$m^2 + 1 - 2m = 2m^2 - 4$

$m^2 + 2m - 5 = 0$

$m = \frac{-2 \pm \sqrt{4+20}}{2} \Rightarrow m = -1 \pm \sqrt{6}$

Question8

(p, q) is the point of intersection of a latus rectum and an asymptote of the hyperbola $9x^2 - 16y^2 = 144$. If $p > 0$ and $q > 0$, then $q =$

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Options:

A. $\frac{9}{4}$

B. $\frac{7}{4}$

C. $\frac{15}{4}$

D. $\frac{13}{4}$

Answer: C

Solution:

Given, hyperbola, $9x^2 - 16y^2 = 144$

$\Rightarrow \quad \left(\frac{x}{4}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$

Here, $a = 4$ and $b = 3$

$\because \quad a > b$

$\therefore \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Asymptotes $y = \pm \frac{b}{a} x = \pm \frac{3}{4} x$

Latus rectums $x = \pm ae = \pm 4 \left(\frac{5}{4}\right) = \pm 5$

$\because \quad p > 0$

$\therefore$ Consider the latus rectum $x = 5$

The points of intersection of latus rectum $x = 5$ with asymptotes $y = \pm \frac{3}{4} x$ are

$$\left(5, \frac{3}{4}(5)\right) \text{ and } \left(5, -\frac{3}{4}(5)\right)$$

$\because \quad q > 0$

$\therefore \quad (p, q) = \left(5, \frac{15}{4}\right)$

Therefore, $q = \frac{15}{4}$

Question9

The point of intersection of two tangents drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{4} = 1$ lie on the circle $x^2 + y^2 = 5$. If these tangents are perpendicular to each other, then $a =$

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Options:

A. 25

B. 5

C. 9

D. 3

Answer: D

Solution:

We have,

$H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \ldots \text{(i)}$

$C : x^2 + y^2 = 5 \quad \ldots \text{(ii)}$

The equations of tangents drawn from an external point (x_1, y_1) to hyperbola are $(y - y_1) = m_1 (x - x_1)$ and $(y - y_1) = m_2 (x - x_2)$

where m_1, m_2 are the roots of the

equation $(x_1^2 - a^2)m^2 - 2x_1y_1m$

$+ y_1^2 + b^2 = 0 \quad \ldots \text{(iii)}$

$\Rightarrow m_1m_2 = \frac{y_1^2+b^2}{x_1^2-a^2} \quad \ldots \text{(iv)}$

$\because m_1m_2 = -1$ (tangents are perpendicular) and (x_1, y_1) lies on circle

$\Rightarrow \quad x_1^2 + y_1^2 = 5$

From Eq. (iv), $y_1^2 + b^2 = -x_1^2 + a^2$

$\Rightarrow \quad x_1^2 + y_1^2 = a^2 - b^2 \Rightarrow a^2 - 4 = 5$

$\Rightarrow \quad a^2 = 9 \Rightarrow a = 3$

Question10

$P(a \sec \theta, b \tan \theta)$ and $Q(a \sec \phi, b \tan \phi)$ are two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where, $\phi + \theta = \frac{\pi}{2}$. If (h, k) is the point of intersection of the normals drawn at P and Q, then $k =$

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Options:

A. $\frac{a^2-b^2}{b}$

B. $\frac{a^2+b^2}{b}$

C. $-\left(\frac{a^2-b^2}{b}\right)$

D. $-\left(\frac{a^2+b^2}{b}\right)$

Answer: D

Solution:

The given equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Now, normal equation at $P(a \sec \theta, b \tan \theta)$ and $Q(a \sec \phi, b \tan \phi)$ are

$ax \cos \theta + by \cot \theta = a^2 + b^2$

and $ax \cos \phi + by \cot \phi = a^2 + b^2$

Where, $\phi = \frac{\pi}{2} - \theta$ and (h, k) is the point of intersection of the normals drawn at P and Q.

So, $ah \cos \theta + bk \cot \theta = a^2 + b^2$

and $ah \sin \theta + bk \tan \theta = a^2 + b^2$

Now, multiply Eq. (i) by $\sin \theta$ and Eq. (ii) by $\cos \theta$ and subtract them, we get

$$bk(\cos \theta - \sin \theta) = \left(a^2 + b^2\right)(\sin \theta - \cos \theta)$$
$$\Rightarrow \left(a^2 + b^2 + bk\right)(\sin \theta - \cos \theta) = 0$$
$$\Rightarrow bk + a^2 + b^2 = 0 \quad (\because \sin \theta - \cos \theta \neq 0)$$
$$\therefore \quad k = -\frac{\left(a^2 + b^2\right)}{b} \quad \forall \theta$$

Hence, the value of $k = -\frac{(a^2+b^2)}{b}$

Question11

If the equation of a hyperbola is $9x^2 - 16y^2 + 72x - 32y - 16 = 0$, then the equation of conjugate hyperbola is

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Options:

A. $9x^2 - 16y^2 + 72x - 32y + 272 = 0$

B. $9x^2 - 16y^2 + 72x - 32y + 288 = 0$

C. $9x^2 - 16y^2 + 72x - 32y - 38 = 0$

D. $9x^2 - 16y^2 + 72x - 32y + 16 = 0$

Answer: A

Solution:

To find the equation of the conjugate hyperbola, we are given the equation of a hyperbola as:

$9x^2 - 16y^2 + 72x - 32y - 16 = 0$

First, we'll convert this into the standard form of a hyperbola. Starting with:

$9(x^2 + 8x) - 16(y^2 + 2y) = 16$

Complete the square for both x and y:

For x:

$x^2 + 8x \quad \Rightarrow \quad (x+4)^2 - 16$

For y:

$y^2 + 2y \quad \Rightarrow \quad (y+1)^2 - 1$

Substituting these back:

$9((x+4)^2 - 16) - 16((y+1)^2 - 1) = 16$

Simplify:

$9(x+4)^2 - 144 - 16(y+1)^2 + 16 = 16$

$9(x+4)^2 - 16(y+1)^2 = 144$

This yields the standard form of the hyperbola:

$\frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = 1$

Now, for the conjugate hyperbola, the equation is:

$\frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = -1$

This can be rewritten as:

$9(x+4)^2 - 16(y+1)^2 = -144$

Expanding this:

$9(x^2 + 8x + 16) - 16(y^2 + 2y + 1) = -144$

Which simplifies to:

$9x^2 + 72x + 144 - 16y^2 - 32y - 16 = -144$

Combining the constants gives:

$9x^2 + 72x - 16y^2 - 32y + 288 - 16 = 0$

Thus, the equation of the conjugate hyperbola is:

$$9x^2 - 16y^2 + 72x - 32y + 272 = 0$$
