Logarithms

Question1

$\sinh(\log(3+\sqrt{8})) =$

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $3^{\frac{3}{2}}$

B. $2^{\frac{3}{2}}$

C. $8^{\frac{2}{3}}$

D. $3^{\frac{1}{2}}$

Answer: B

Solution:

Given, $\sinh\ \log(3+\sqrt{8})\ )$

$$= \frac{e^{\log(3+\sqrt{8})} - e^{-\log(3+\sqrt{8})}}{2}$$

$$= \frac{(3+\sqrt{8}) - \left(\frac{1}{3+\sqrt{8}}\right)}{2}$$

$$= \frac{(3+\sqrt{8}) - (3-\sqrt{8})}{2} = \frac{2\sqrt{8}}{2} = \sqrt{8} = 2^{\frac{3}{2}}$$

Question2

The range of the function $f(x) = \log_{0.5}\left(x^4 - 2x^2 + 3\right)$ **is**

Options:

A. $(-\infty, \infty)$

B. $(-\infty, -1]$

C. $[-1, \infty)$

D. $[-1, 1]$

Answer: B

Solution:

To determine the range of the function $f(x) = \log_{0.5}(x^4 - 2x^2 + 3)$, we start by setting $x^2 = t$. The expression then becomes:

$t^2 - 2t + 3 = y$

To analyze the quadratic $t^2 - 2t + 3$, we calculate the discriminant D:

$D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times 3 = -8$

Since $D < 0$, the quadratic equation has no real roots; hence, $y = t^2 - 2t + 3$ is always positive.

Next, we find the value of t that minimizes the quadratic:

$t_{\min} = -\frac{b}{2a} = \frac{-(-2)}{2 \times 1} = 1$

Substituting back to find the minimum value of y:

$y_{\min} = -\frac{D}{4a} = \frac{-(-8)}{4 \times 1} = 2$

Given that the base of the logarithm, 0.5, is less than 1, the logarithmic function will be decreasing. Therefore, we can solve for when $f(x) = \log_{0.5}(2)$:

$$\begin{aligned} f(x) &= \log_{0.5}(2) \\ &= \log_{\left(\frac{1}{2}\right)}(2) \\ &= \log_{2^{-1}}(2) = -1 \cdot \log_2(2) = -1 \end{aligned}$$

Thus, the range of $f(x)$ is $(-\infty, -1]$.

Question3

If $\sinh(\log x) = -2$, then $x =$

Options:

A. $\sqrt{5}-2$

B. $2+\sqrt{5}$

C. $-(2+\sqrt{5})$

D. $2-\sqrt{5}$

Answer: A

Solution:

To solve for x given that $\sinh(\log x)=-2$, we start with the definition of the hyperbolic sine function:

$\sinh y=\frac{e^y-e^{-y}}{2}$

Substituting $\log x$ into the equation:

$\sinh(\log x)=\frac{e^{\log x}-e^{-\log x}}{2}$

Using the property $e^{\log x}=x$, this becomes:

$-2=\frac{x-\frac{1}{x}}{2}$

Multiplying through by 2 to clear the fraction:

$-4=x-\frac{1}{x}$

Rearranging terms and multiplying by x to eliminate the fraction:

$x^2+4x-1=0$

This is a quadratic equation, which we solve using the quadratic formula:

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

For our equation, $a=1$, $b=4$, and $c=-1$. Plugging in these values gives:

$x=\frac{-4\pm\sqrt{16+4}}{2}$

Simplifying further:

$x=\frac{-4\pm\sqrt{20}}{2}$

$x=\frac{-4\pm2\sqrt{5}}{2}$

$x=-2\pm\sqrt{5}$

This results in two possible solutions:

$x = \sqrt{5} - 2$

$x = -(2 + \sqrt{5})$

However, since $\log x$ implies $x > 0$, the valid solution is:

$x = \sqrt{5} - 2$
