Circle

Question1

The slope of a common tangent to the circles $x^2+y^2=16$ and $(x-9)^2+y^2=16$ is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$\frac{8}{\sqrt{13}}$

B.

$\frac{4}{\sqrt{13}}$

C.

$\frac{\sqrt{17}}{8}$

D.

$\frac{8}{\sqrt{17}}$

Answer: D

Solution:

Given, equation of circles

$x^2+y^2=16 \quad \ldots(i)$

And $(x-9)^2+y^2=16 \quad \ldots(ii)$

$$C_1=(0,0), r_1=4$$
$$C_2=(9,0), r_2=4$$
$$\therefore \quad C_1C_2=\sqrt{(9-0)^2+(0-0)^2}$$
$$=\sqrt{(9)^2}=9$$

And $r_1+r_2=4+4=8$

Since $C_1C_2>r_1+r_2$

Circles are separate to each other, any tangent to circle (i) with slope (m) is

$$y=mx\pm r\sqrt{1+m^2}$$
$$\Rightarrow \quad y=mx+4\sqrt{1+m^2}$$
$$\Rightarrow \quad mx-y\pm 4\sqrt{1+m^2}=0 \quad \ldots(ii)$$

Equation (iii) to be tangent to circle (ii)

$\therefore \quad \left|\frac{m(9)-0 \pm 4 \sqrt{1+m^2}}{\sqrt{m^2+1}}\right|=4$

$\Rightarrow \quad 16\left(m^2+1\right)=\left(9 m \pm 4 \sqrt{1+m^2}\right)^2$

$\Rightarrow \quad 16 m^2+16=81 m^2+16\left(1+m^2\right)$

$\pm 72 m \sqrt{1+m^2}$

$\Rightarrow \quad 81 m^2 \pm 72 m \sqrt{1+m^2}=0$

$\Rightarrow \quad 9 m\left(9 m \pm 8 \sqrt{1+m^2}\right)=0$

When $9 m=0 \Rightarrow m=0$

Then Eq. (iii) $y= \pm 4$ (horizontal lines)

If, $9 m \pm 8 \sqrt{1+m^2}=0$

$\Rightarrow \quad (9 m)^2=\left(\pm 8 \sqrt{1+m^2}\right)^2$

$\Rightarrow \quad 81 m^2=64\left(1+m^2\right)$

$\Rightarrow \quad 81 m^2-64 m^2=64$

$\Rightarrow \quad 17 m^2=64$

$\Rightarrow \quad m= \pm \sqrt{\frac{64}{17}}$

$\Rightarrow \quad m= \pm \frac{8}{\sqrt{17}}$

Question2

The equation of the circle whose radius is 3 and which touches the circle $x^2+y^2-4 x-6 y-12=0$ internally at $(-1,-1)$ is

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Options:

A.

$5 x^2+5 y^2-8 x-14 y-32=0$

B.

$x^2+y^2-12 x-14 y-28=0$

C.

$3 x^2+3 y^2-8 x-14 y-31=0$

D.

$x^2+y^2-5 x-7 y-14=0$

Answer: A

Solution:

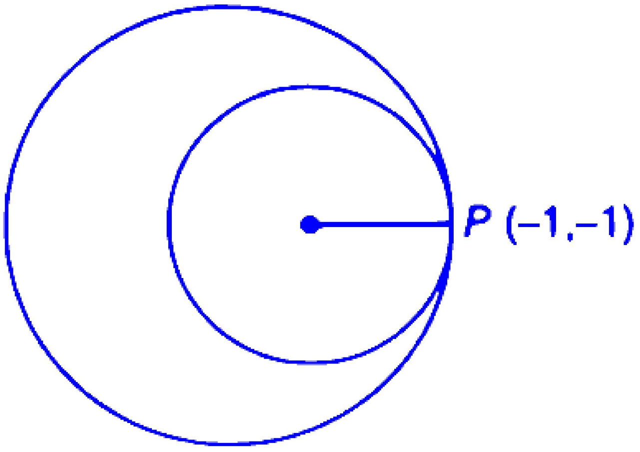
Given, equation of circle

$x^2+y^2-4x-6y-12=0 \quad \ldots(i)$

centre $(-g,-f)=(2,3)$

And radius $=\sqrt{g^2+f^2-c}$

$=\sqrt{4+9+12}=\sqrt{25}=5$



Equation of new circle whose radius is 3

$(x-h)^2+(y-k)^2=(3)^2$

$\Rightarrow \quad (x-h)^2+(y-k)^2=9 \quad \ldots(ii)$

Since circle (i) and (ii) touches each other internally at $(-1,-1)$

$\therefore \quad C_1C_2=|r_1-r_2|$

where $c_2=(h,k)$

$\therefore \quad C_1C_2=|5-3|=2$

P divides the line segment C_1C_2 in the ratio $r_1:r_2$

$\therefore C_1P=r_1$ and $C_2P=r_2$

$C_1P=\sqrt{(-1-2)^2+(-1-3)^2}$

$=\sqrt{(-3)^2+(-4)^2}$

$=\sqrt{9+16}=\sqrt{25}=5$

and $C_2P=r_2=3$

$\therefore$ The point of tangency P divides the line segment C_1C_2 externally in the ratio $r_1:-r$

$\therefore \quad P=\frac{r_1C_2-rC_1}{r_1-r}$

$=\frac{5C_2-3C_1}{5-3}=\frac{5C_2-3C_1}{2}$

$\Rightarrow \quad 2P=5C_2-3C_1$

$\Rightarrow \quad 5C_2=2P+3C_1$

$\Rightarrow \quad 5(h,k)=2(-1,-1)+3(2,3)$

$\Rightarrow \quad 5(h, k)=(-2,-2)+(6,9)$

$\Rightarrow \quad 5(h, k)=(4,7) \Rightarrow(h, k)=\left(\frac{4}{5}, \frac{7}{5}\right)$

$\therefore$ Equation of required circle

$\left(x-\frac{4}{5}\right)^2+\left(y-\frac{7}{5}\right)^2=(3)^2$

$\Rightarrow \quad\left(x-\frac{4}{5}\right)^2+\left(y-\frac{7}{5}\right)^2=9$

$\Rightarrow \quad x^2+\frac{16}{25}-\frac{8}{5} x+y^2+\frac{49}{25}-\frac{14}{5} y=9$

$x^2+y^2-\frac{8}{5} x-\frac{14}{5} y+\frac{13}{5}=9$

$\Rightarrow \quad 5 x^2+5 y^2-8 x-14 y-32=0$

Question3

Suppose C_1 and C_2 are two circles having no common points, then

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Options:

A.

There will be 3 common tangents to C_1 to C_2

B.

There will be exactly two common tangents to C_1 and C_2

C.

There will be no common tangent or there will be exactly two common tangents to C_1 and C_2

D.

There will be no common tangents or there will be four common tangents to C_1 and C_2

Answer: D

Solution:

If C_1 and C_2 are two circles having no common points, it means they do not intersect and do not touch each other then there will be no common tangents or there will be four common tangents C_1 and C_2

Question4

The locus of the centre of the circle touching the X-axis and passing through the point $(-1,1)$ is

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Options:

A.

a circle with centre at $\left(-1, \frac{1}{2}\right)$

B.

a pair of lines intersecting at $(-1, 1)$

C.

a parabola with focus at $(-1, 1)$

D.

a hyperbola with centre at $(-1, 1)$

Answer: C

Solution:

Equation of the circle touching the X-axis and passing through the point $(-1, 1)$

$$
\begin{aligned}
& (-1-h)^2+(1-k)^2=k^2 \\
\Rightarrow \quad & 1+h^2+2h+1+k^2-2k=k^2 \\
\Rightarrow \quad & 1+2h+h^2+1-2k+k^2=k^2 \\
\Rightarrow \quad & 1+2h+h^2+1-2k=0 \\
\Rightarrow \quad & h^2+2h-2k+2=0 \\
\Rightarrow \quad & x^2+2x-2y+2=0 \\
\Rightarrow \quad & 2y=x^2+2x+2 \\
\Rightarrow \quad & y=\frac{1}{2}\left(x^2+2x\right)+1 \\
\Rightarrow \quad & y=\frac{1}{2}(x+1)^2-\frac{1}{2}+1 \\
\Rightarrow \quad & y=\frac{1}{2}(x+1)^2+\frac{1}{2} \\
\Rightarrow \quad & (x+1)^2=2\left(y-\frac{1}{2}\right)
\end{aligned}
$$

Which represents parabola with focus $(-1, 1)$

Question5

The centres of all circles passing through the points of intersection of the circles $x^2+y^2+2x-2y+1=0$ and $x^2+y^2-2x+2y-2=0$ and having radius $\sqrt{14}$ lie on the curve

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$x+y=0$

B.

$y^2=4x-2$

C.

$3x^2+5x=y$

D.

$2x^2+3y^2=7$

Answer: A

Solution:

Given, equation of circles

$x^2+y^2+2x-2y+1=0$ and
$x^2+y^2-2x+2y-2=0$

$\therefore$ Equation of circle passing through the point of intersection of two given circle is

$$\begin{aligned} & \left(x^2+y^2+2x-2y+1\right)+\lambda\left(x^2+y^2-2x+2y-2\right)=0 \\ \Rightarrow \quad & (1+\lambda)x^2+(1+\lambda)y^2+(2-2\lambda)x+(-2+2\lambda)y+(1-2\lambda)=0 \\ & \Rightarrow \quad x^2+y^2+\tfrac{(2-2\lambda)}{1+\lambda}x+\tfrac{(-2+2\lambda)}{1+\lambda}y+\tfrac{1-2\lambda}{1+\lambda}=0 \end{aligned}$$

Coordinate of centre of circle

$=(-g,-f)=\left(-\frac{1-\lambda}{1+\lambda},-\frac{-1+\lambda}{1+\lambda}\right)$

$\therefore$ Radius of

$$\left(-\frac{1-\lambda}{1+\lambda}\right)^2+\left(-\frac{-1+\lambda}{1+\lambda}\right)^2-\left(\frac{1-2\lambda}{1+\lambda}\right)=14$$
$$\Rightarrow \quad \frac{(1-\lambda)^2}{(1+\lambda)^2}+\frac{(\lambda-1)^2}{(1+\lambda)^2}-\frac{1-2\lambda}{1+\lambda}=14$$
$$\Rightarrow \quad (1-\lambda)^2+(\lambda-1)^2-(1+\lambda)(1-2\lambda)$$
$$=14(1+\lambda)^2$$
$$\Rightarrow \quad 2(\lambda-1)^2-\left(1-2\lambda+\lambda-2\lambda^2\right)$$
$$=14\left(1+\lambda^2+2\lambda\right)$$
$$\Rightarrow \quad 2\left(\lambda^2+1-2\lambda-\left(1-\lambda-2\lambda^2\right)\right.$$
$$=14\left(1+\lambda^2+2\lambda\right)$$
$$\Rightarrow \quad 4\lambda^2+1-3\lambda=14+14\lambda^2+28\lambda$$
$$\Rightarrow \quad 10\lambda^2+31\lambda+13=0$$

$\Rightarrow$ This is a quadrate equation is λ the existence of real values for λ conforms that such circle exists.

Coordinates of centre

$$\begin{aligned} & h=\tfrac{\lambda-1}{1+\lambda}, k=-\tfrac{\lambda-1}{1+\lambda} \\ \Rightarrow \quad & h=-k \Rightarrow h+k=0 \\ \Rightarrow \quad & x+y=0 \end{aligned}$$

Question6

A circle S given by $x^2+y^2-14x+6y+33=0$ cuts the X-axis at A and $B(OB>OA)$. C is mid-point of AB. L is a line through C and having slope (-1). If L is the diameter of a circle S' and also the radical axis of the circles S and S', then the equation of the circle S' is

TG EAPCET 2025 (Online) 2nd May Evening Shift

Options:

A.

$x^2+y^2-17x+3y+54=0$

B.

$x^2+y^2+17x-3y-54=0$

C.

$x^2+y^2-17x+3y+51=0$

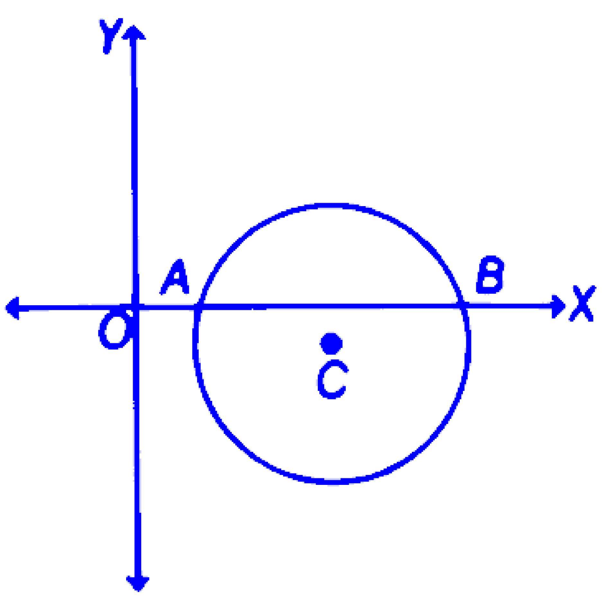
D.

$x^2+y^2-3x+17y-51=0$

Answer: A

Solution:

Given equation of circle
$x^2+y^2-14x+6y+33=0 \ldots$ (i)



$$\begin{aligned} \text{Centre of circle } &= (-g,-f) \\ &= (7,-3) \\ \text{and radius of circle } &= \sqrt{g^2+f^2-c} \\ &= \sqrt{49+9-33} \\ &= \sqrt{25}=5 \end{aligned}$$

Since, circle cuts the X-axis, $y=0$

$$\begin{aligned}
& x^2-14x+33=0 \\
\Rightarrow\ & x^2-11x-3x+33=0 \\
\Rightarrow\ & x(x-11)-3(x-11)=0 \\
\Rightarrow\ & (x-3)(x-11)=0 \\
\Rightarrow\ & x=3,11 \\
\therefore\ & A=(3,0) \text{ and } B=(11,0)
\end{aligned}$$

Since, C is the mid-point of AB

$$\begin{aligned}
\therefore\quad C &= \left(\frac{3+11}{2}, \frac{0+0}{2}\right) \\
&= (7,0)
\end{aligned}$$

$\therefore$ Equation of line L, which passes through C and having slope (-1) is

$$\begin{aligned}
& y-0=(-1)(x-7) \\
\Rightarrow\ & y=-x+7 \\
\Rightarrow\ & x+y-7=0 \qquad \ldots(ii)
\end{aligned}$$

Since, L is the radical axis of the circles s and S' $(S-S'=0)$

$$\begin{aligned}
\therefore\ & \left(x^2+y^2-14x+6y+33\right)-\left(x^2+y^2+2g'x+2f'y+c'\right)=0 \\
\Rightarrow\ & \left(-14-2g'\right)x+\left(6-2f'\right)y+\left(33-c'\right)=0
\end{aligned}$$

Which is identical with line (ii)

$$\begin{aligned}
\therefore\ & \tfrac{-14-2g'}{1}=\tfrac{6-2f'}{1}=\tfrac{33-c'}{-7}=k \\
\Rightarrow\ & -14-2g'=k \text{ and } 6-2f'=k \\
\therefore\ & -14-2g'=6-2f' \\
\Rightarrow\ & g'-f'=-10 \qquad \ldots(iii)
\end{aligned}$$

Since, centre of S' lies on (ii)

$$\begin{aligned}
\therefore\ & -g'-f'=7 \\
\Rightarrow\ & g'+f'=-7 \qquad \ldots(iv)
\end{aligned}$$

Solving Eqs. (iii) and (iv), we get

$$\begin{aligned}
& g'=\tfrac{-17}{2}, f'=\tfrac{3}{2} \\
\therefore\ & -14-2\left(-\tfrac{17}{2}\right)=k \\
\Rightarrow\ & k=3
\end{aligned}$$

Also, $\frac{33-c'}{-7}=k=3$

$$\begin{aligned}
\Rightarrow\ & 33-c'=-21 \\
\Rightarrow\ & c'=54
\end{aligned}$$

$\therefore$ Equation of required circle is

$$\begin{aligned}
& x^2+y^2+2\left(\frac{-17}{2}\right)x+2\left(\frac{3}{2}\right)y+54=0 \\
\Rightarrow\ & x^2+y^2-17x+3y+54=0
\end{aligned}$$

Question7

If the equation of the circle passing through the points $(-1,0),(-1,1),(1,1)$ is $ax^2+ay^2+2gx+2fy-2=0$, then $a=$

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Options:

A.

1

B.

-1

C.

2

D.

-2

Answer: C

Solution:

Given equation is

$ax^2 + ay^2 + 2gx + 2fy - 2 = 0$

Put $(-1, 0)$ into this equation, we get

$$a(-1)^2 + a(0)^2 + 2g(-1) + 2f(0) - 2 = 0$$
$$\Rightarrow \quad a - 2g - 2 = 0 \qquad \ldots(i)$$

Put $(-1, 1)$, we get

$$a(-1)^2 + a(1)^2 + 2g(-1) + 2f(1) - 2 = 0$$
$$\Rightarrow \quad a + a - 2g + 2f - 2 = 0$$

$$\Rightarrow \quad 2a - 2g + 2f - 2 = 0$$
$$\Rightarrow \quad a - g + f - 1 = 0 \qquad \ldots(ii)$$

Put (1, 1), we get

$$a(1)^2 + a(1)^2 + 2g(1) + 2f(1) - 2 = 0$$
$$\Rightarrow \quad a + g + f - 1 = 0 \qquad \ldots(iii)$$

Subtract Eq. (ii) from Eq. (iii),

$$(a + g + f - 1) - (a - g + f - 1) = 0$$
$$\Rightarrow \quad 2g = 0$$
$$\Rightarrow \quad g = 0$$

Put $g = 0$ into Eq. (i), we get

$$a - 2(0) - 2 = 0$$
$$\Rightarrow \quad a = 2$$

Question8

For the circle $x-2=5\cos\theta, y+1=5\sin\theta$, where θ is the perimeter, the line $x=1+\frac{r}{2}, y=-2+\frac{\sqrt{3}}{2}r$ where r is the perimeter, is a

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

Chord of the circle other than diameter

B.

Tangent of the circle

C.

Diameter of the circle

D.

Line that does not meet the circle

Answer: A

Solution:

Given circle, $x-2=5\cos\theta, y+1=5\sin\theta$, where θ is the perimeter.

$$\Rightarrow (x-2)^2+(y+1)^2=(5\cos\theta)^2+(5\sin\theta)^2$$
$$=25\left(\cos^2\theta+\sin^2\theta\right)=25$$

$\therefore$ The circle has a centre at $(h,k)=(2,-1)$ and a radius, $r_c=5$

Now, the line equation is $x=1+\frac{r}{2}$ and $y=-2+\frac{\sqrt{3}}{2}r$. This is parametric form.

Let $x_1=1, y_1=-2$

The direction vector of the line is $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

So, the slope of the line, $m=\frac{\Delta y}{\Delta x}$

$$\Rightarrow \quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$

Now, the equation of line is

$$y-y_1=m\left(x-x_1\right)$$
$$\Rightarrow \quad y-(-2)=\sqrt{3}(x-1)$$
$$\Rightarrow \quad y+2=\sqrt{3}x-\sqrt{3}$$
$$\Rightarrow \quad \sqrt{3}x-y-(2+\sqrt{3})=0$$

Now, calculate the distance from the center of the circle to line,

$$d=\frac{\left|Ax_0+By_0+C\right|}{\sqrt{A^2+B^2}}$$

Here, $(x_0,y_0)=(2,-1)$ and the line is $\sqrt{3}x-y-(2+\sqrt{3})=0$

So, $A=\sqrt{3}, B=-1, C=-(2+\sqrt{3})$

$$\begin{aligned} d &= \frac{|\sqrt{3}(2)+(-1)(-1)-(2+\sqrt{3})|}{\sqrt{(\sqrt{3})^2+(-1)^2}} \\ &= \frac{|2\sqrt{3}+1-2-\sqrt{3}|}{\sqrt{3+1}} \\ &= \frac{\sqrt{3}-1}{2} \approx 0.366 < 5 \end{aligned}$$

So, the line intersects the circle at two distinct points, meaning it is a secant to the circle.

So, the line is a chord of circle other than diameter.

Question9

If $x-2y=0$ is a tangent drawn at a point P on the circle $x^2+y^2-6x+2y+c=0$, then the distance of the point $(6,3)$ from P is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\sqrt{5}$

B.

$2\sqrt{5}$

C.

$4\sqrt{5}$

D.

$5\sqrt{2}$

Answer: B

Solution:

The equation of the circle is

$$\begin{aligned} & x^2+y^2-6x+2y+c=0 \\ \Rightarrow\ & (x-3)^2+(y+1)^2=10-c \end{aligned}$$

So, the center of the circle is $C(3,-1)$ and the radius is $r^2=10-C$

Now, since the equation of tangent is

$x-2y=0$

Slop, $m_t=1/2$

Slope of the radius, connecting the centre $(3,-1)$ to the point of tangency

$P(x_p, y_p)$ is $m_r = \frac{y_p-(-1)}{x_p-3} = \frac{y_p+1}{x_p-3}$

Now, since radius is perpendicular to the tangent, so

$$\begin{aligned} & m_t \cdot m_r = -1 \\ \Rightarrow \quad & \frac{1}{2} \cdot \frac{y_p+1}{x_p-3} = -1 \Rightarrow y_p + 1 = -2x_p + 6 \\ \Rightarrow \quad & y_p = -2x_p + 5 \end{aligned}$$

But, P lies on the tangent line, its coordinates satisfy $x_p - 2y_p = 0$

$$\begin{aligned} \Rightarrow \quad & x_p = 2y_p \\ \text{So,} \quad & y_p = -2x_p + 5 \\ \Rightarrow \quad & -2(2y_p) + 5 = -4y_p + 5 \\ \Rightarrow \quad & 5y_p = 5 \Rightarrow y_p = 1 \end{aligned}$$

So, $x_p = 2y_p = 2 \times 1 = 2$

So, the point P is $(2, 1)$.

Now, the distance between $(6, 3)$ and $(2, 1)$ is

$$\begin{aligned} & d = \sqrt{(2-6)^2 + (1-3)^2} \\ & \Rightarrow \sqrt{(-4)^2 + (-2)^2} \Rightarrow \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$\therefore$ The distance of the point $(6, 3)$ from P is $2\sqrt{5}$.

Question10

If A, B are the points of contact of the tangents drawn from the point $(-3, 1)$ to the circle $x^2 + y^2 - 4x + 2y - 4 = 0$, then the equation of the circumcircle of the $\triangle PAB$ is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$x^2 + y^2 - 6x + 2y - 6 = 0$

B.

$x^2 + y^2 - x + 7 = 0$

C.

$x^2 + y^2 + x - 7 = 0$

D.

$x^2 + y^2 + 6x - 2y - 6 = 0$

Answer: C

Solution:

The equation of the circle is

$x^2+y^2-4x+2y-4=0$

Comparing this with the general form

$x^2+y^2+2gx+2fy+c=0$

We have

$$
\begin{aligned}
& 2g=-4 \\
\Rightarrow \quad & g=-2 \text{ and } 2f=2 \\
\Rightarrow \quad & f=1
\end{aligned}
$$

So, the center of the circle C is

$(-g,-f)=(2,-1)$

And, the radius is $r=\sqrt{g^2+f^2-c}$

$$
\begin{aligned}
& =\sqrt{(-2)^2+(1)^2-(-4)} \\
& =\sqrt{4+1+4}=\sqrt{9}=3
\end{aligned}
$$

The point from which tangents are drawn is $P=(-3,1)$

Now, the circumcircle of $\triangle PAB$ is the circle with PC as its diameter. The center of this circumcircle is the mid-point of PC.

Mid-point $M=\left(\frac{-3+2}{2}, \frac{1+(-1)}{2}\right)$

$\Rightarrow\left(\frac{-1}{2}, \frac{0}{2}\right)=\left(\frac{-1}{2}, 0\right)$

Distance $PC=\sqrt{(2-(-3))^2+(-1-1)^2}$

$$
\begin{aligned}
\Rightarrow \quad & \sqrt{5^2+(-2)^2} \\
\Rightarrow \quad & \sqrt{25+4}=\sqrt{29}
\end{aligned}
$$

$\therefore$ The radius of the circumcircle

$=\frac{1}{2} \cdot$ distance $PC=\frac{\sqrt{29}}{2}$

Now, the equation of a circle with center (h, k) and radius R is

$(x-h)^2+(y-k)^2=R^2$

Here, $(h, k)=\left(\frac{-1}{2}, 0\right)$ and $R=\frac{\sqrt{29}}{2}$

So, $\left(x-\left(-\frac{1}{2}\right)\right)^2+(y-0)^2=\left(\frac{\sqrt{29}}{2}\right)^2$

$$
\begin{aligned}
\Rightarrow \quad & \left(x+\frac{1}{2}\right)^2+y^2=\frac{29}{4} \\
\Rightarrow \quad & x^2+x+\frac{1}{4}+y^2=\frac{29}{4} \\
\Rightarrow \quad & x^2+y^2+x-\frac{28}{4}=0 \\
\Rightarrow \quad & x^2+y^2+x-7=0
\end{aligned}
$$

Question11

A circle C passing through the point $(1,1)$ bisects the circumference of the circle $x^2+y^2-2x=0$. If C is orthogonal to the circle $x^2+y^2+2y-3=0$, then the centre of the circle C is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$\left(-\frac{1}{2}, 0\right)$

B.

$\left(\frac{5}{2}, 0\right)$

C.

$\left(0, \frac{5}{2}\right)$

D.

$\left(0, -\frac{1}{2}\right)$

Answer: B

Solution:

Let the equation of circle C be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since, it passes through $(1, 1)$ then

$$\begin{aligned} & 1^2 + 1^2 + 2g + 2f + c = 0 \\ \Rightarrow \quad & 2 + 2g + 2f + c = 0 \ \ldots(i) \end{aligned}$$

Now, $S_1 : x^2 + y^2 + 2gx + 2fy + c = 0$

and $S_2 = x^2 + y^2 - 2x = 0$

So, $S_1 - S_2 = 0$ is the common chord.

$\Rightarrow (2g + 2x + 2fy + c = 0$

Now, center of $s_2 = x^2 + y^2 - 2x = 0$ is $(1, 0)$

Since, the common chord is a diameter of S_2, then centre of S_2 must lies on the common chord.

$$\begin{aligned} \therefore \quad & (2g + 2(l) + 2f(0) + c = 0 \\ \Rightarrow \quad & 2g + 2 + c = 0 \qquad\qquad \ldots(ii) \end{aligned}$$

Also, circle C is orthogonal to

$x^2 + y^2 + 2y - 3 = 0$

Here, $g_1 = g, f_1 = f, c_1 = c$ and $g_2 = 0$,

$f_2 = 1, c_2 = -3 \qquad\qquad \ldots(iii)$

Using the orthogonality condition

$$\begin{aligned} & 2g_1g_2 + 2f_1f_2 = c_1 + c_2 \\ \Rightarrow \quad & 2g(0) + 2f(1) = c + (-3) \\ \Rightarrow \quad & 2f = c - 3 \end{aligned}$$

From Eq. (ii), $c = -2g - 2$

Put this in Eq. (iii), we get

$$2f = c - 3 = -2g - 2 - 3$$
$$\Rightarrow \quad 2f = -2g - 5 \qquad \ldots (iv)$$

Substitute $c = -2g - 2$ into Eq. (i), we get

$$\Rightarrow \quad 2 + 2g + 2f - 2g - 2 = 0$$
$$\Rightarrow \quad 2f = 0$$
$$\Rightarrow \quad f = 0$$

Put $f = 0$ in Eq. (iv), we get

$$2(0) = -2g - 5$$
$$\Rightarrow \quad 2g = -5$$
$$\Rightarrow \quad g = \frac{-5}{2}$$

The center of circle C is

$$(-g, -f) = \left(-\left(-\frac{5}{2}\right), 0\right)$$
$$\Rightarrow \left(\frac{5}{2}, 0\right)$$

Question12

P and Q are the points of trisection of the line segment joining the points $(3, -7)$ and $(-5, 3)$. If PQ subtends right angle at a variable point R, then the locus of R is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. a circle with radius $\frac{\sqrt{41}}{3}$

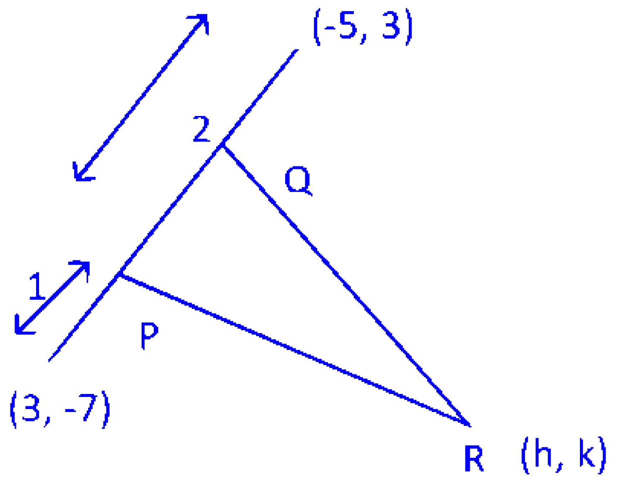
B. a circle with radius $\sqrt{409}$

C. a pair of straight lines passing through $(-1, -2)$

D. a pair of straight lines passing through $(1, 2)$

Answer: A

Solution:



$P\left(\frac{-5+6}{3}, \frac{3-14}{3}\right)=\left(\frac{1}{3}, \frac{-11}{3}\right)$

$Q\left(\frac{-10+3}{3}, \frac{6-7}{3}\right)=\left(-\frac{7}{3},-\frac{1}{3}\right)$

Slope of $PR=\frac{K+\frac{11}{3}}{h-\frac{1}{3}}=\frac{3K+11}{3h-1}$

Slope of $QR=\frac{K+\frac{1}{3}}{h+\frac{7}{3}}=\frac{3K+1}{3h+7}$

$PR \perp QR$

$\frac{3K+11}{3h-1} \times \frac{3K+1}{3h+7}=-1$

$(3K+11)(3K+1)=-(3h-1)(3h+7)$

$9K^2+36K+11=-\left[9h^2+18h-7\right]$

$9h^2+18h-7+9K^2+36K+11=0$

$9h^2+9K^2+18h+36K+4=0$

$h^2+K^2+2h+4K+\frac{4}{9}=0$

Taking locus, we get

$\Rightarrow x^2+y^2+2x+4y+\frac{4}{9}=0$

$\therefore$ radius $=\sqrt{1+4-\frac{4}{9}}=\sqrt{\frac{41}{9}}=\frac{\sqrt{41}}{3}$

Question13

If $A(1,2), B(2,1)$ are two vertices of an acute angled triangle and $S(0,0)$ is its circumcenter, then the angle subtended by AB at the third vertex is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\tan^{-1}\left(\frac{1}{3}\right)$

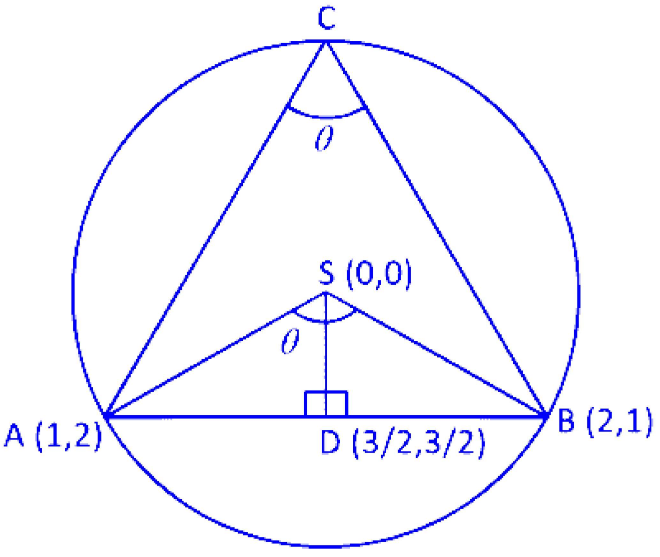
B. $\tan^{-1}\left(\frac{1}{2}\right)$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: A

Solution:



$AB=\sqrt{(1)^1+(l)^2}=\sqrt{2}$ units

$\because D$ is the mid point of AB

So, $AD=1/\sqrt{2}$ units

$\therefore \quad D\equiv\left(\frac{3}{2},\frac{3}{2}\right)$

Now, shortest distance $=\sqrt{\frac{9}{4}+\frac{9}{4}}$

$=\frac{3\sqrt{2}}{2}$ units

$$\tan\theta=\frac{AD}{SD}\Rightarrow\tan\theta=\frac{1}{\sqrt{2}}\times\frac{2}{3\sqrt{2}}=\frac{1}{3}$$

$\therefore \quad \theta=\angle ACB=\angle ASD=\tan^{-1}\left(\frac{1}{3}\right)$

Hence, the angle substances by AB at the third vertex is $\tan^{-1}\left(\frac{1}{3}\right)$.

Question14

A circle passing through the points $(1,1)$ and $(2,0)$ touches the line $3x-y-1=0$. If the equation of this circle is $x^2+y^2+2gx+2fy+c=0$, then a possible value of g is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $-\frac{5}{2}$

B. $-\frac{3}{2}$

C. 6

D. -5

Answer: A

Solution:

To find a possible value of g for a circle described by the equation $x^2+y^2+2gx+2fy+c=0$, which passes through the points $(1,1)$ and $(2,0)$ and touches the line $3x-y-1=0$, we begin with the following steps:

Pass through Points:

For point $(1,1)$:

$1^2+1^2+2g\cdot 1+2f\cdot 1+c=02+2g+2f+c=0 \quad \ldots$(i)

For point $(2,0)$:

$2^2+0^2+2g\cdot 2+2f\cdot 0+c=04+4g+c=0 \quad \ldots$(ii)

Solve the System of Equations:

Subtract equation (ii) from (i):

$(2+2g+2f+c)-(4+4g+c)=0-2-2g+2f=0\Rightarrow 2f=2g+2f=g+1$

From equation (ii), solve for c:

$4+4g+c=0c=-4g-4$

Condition for Tangency:

Given the line $3x-y-1=0$ touches the circle, use the tangency condition:

$\frac{-3g+f-1}{\sqrt{3^2+(-1)^2}}=\sqrt{g^2+f^2-c}$

$\frac{-3g+f-1}{\sqrt{10}}=\sqrt{g^2+f^2-c}$

Substitute $f=g+1$ and $c=-4g-4$:

$(-3g+(g+1)-1)^2=10(g^2+(g+1)^2-(-4g-4))$

Simplifying:

$(-3g+g)^2=10((g+1)^2-(-4g-4))4g^2=10(g^2+2g+1+4g+4)4g^2=10(2g^2+6g+5)4g^2=20g^2+60g+50$

Simplifying further:

$16g^2+60g+50=08g^2+30g+25=0$

Solving this quadratic equation gives:

$g=-\frac{5}{2}$

Question15

A circle passes through the points $(2,0)$ and $(1,2)$. If the power of the point $(0,2)$ with respect to this circle is 4 , then the radius of the circle is

Options:

A. 2

B. $\sqrt{\frac{5}{2}}$

C. $\sqrt{5}$

D. 4

Answer: B

Solution:

Let the equation of the circle be:

$x^2 + y^2 + 2gx + 2fy + c = 0$

The circle passes through the points $(2, 0)$ and $(1, 2)$.

From the point $(2, 0)$:

$4 + 4g + c = 0 \quad \ldots \text{(i)}$

From the point $(1, 2)$:

$5 + 2g + 4f + c = 0 \quad \ldots \text{(ii)}$

Subtracting equation (i) from equation (ii):

$(-1) + 2g - 4f = 0$

$2g - 4f = 1 \quad \ldots \text{(iii)}$

Denoting the power of the point $(0, 2)$ with respect to this circle as 4:

$0 + 4 + 4f + c = 4$

$4f + c = 0$

$c = -4f \quad \ldots \text{(iv)}$

From equation (iii):

$2g - 4f = 1$

$4g - 4f = -4$

Solving for f:

$-2f = -3 \Rightarrow f = -\frac{3}{2}$

Substituting the value of f into equation (iv):

$c = -4\left(-\frac{3}{2}\right) = 6$

Solving for g:

$g = f - 1 = -\frac{3}{2} - 1 = -\frac{5}{2}$

Now, calculate the radius:

$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{25}{4} + \frac{9}{4} - 6}$

$= \sqrt{\frac{17}{2} - 6} = \sqrt{\frac{5}{2}}$

Question16

$x - 2y - 6 = 0$ is a normal to the circle $x^2 + y^2 + 2gx + 2fy - 8 = 0$. If the line $y = 2$ touches this circle, then the radius of the circle can be

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\sqrt{32}$

B. 6

C. 4

D. $\sqrt{18}$

Answer: C

Solution:

The given circle equation is:

$x^2 + y^2 + 2gx + 2fy - 8 = 0$

The center of this circle is at $(-g, -f)$.

We are told that the line $x - 2y - 6 = 0$ is a normal to the circle, which means it passes through the center $(-g, -f)$ of the circle. Therefore, we have:

$-g + 2f = 6 \quad \ldots \text{(i)}$

Additionally, the line $y = 2$ is tangent to the circle. The general equation for the distance from a point to a line is given by:

$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

The distance from the center $(-g, -f)$ to the line $y = 2$ is:

$\frac{|f-2|}{1} = \sqrt{g^2 + f^2 + 8}$

Squaring both sides yields:

$(f - 2)^2 = g^2 + f^2 + 8$

Simplifying further:

$f^2 - 4f + 4 = g^2 + f^2 + 8$

$-4f + 4 = g^2 + 8$

Rearranging, we get:

$4f = g^2 + 4 \quad \ldots \text{(ii)}$

Substitute $g = 4$ and $g = -2$ into the equation obtained above:

For $g = 4$,

$4f = 16 + 4 = 20$

$f = 5$

For $g = -2$,

$4f = 4 + 4 = 8$

$f = 2$

Calculating the radius:

For $f = 5$,

Radius $= \sqrt{16 + 25 + 8} = \sqrt{49} = 7$

For $f = 2$,

Radius $= \sqrt{4 + 4 + 8} = \sqrt{16} = 4$

Thus, the radius of the circle can be 4.

Question17

The line $x + y + 1 = 0$ intersects the circle $x^2 + y^2 - 4x + 2y - 4 = 0$ at the points A and B. If $M(a, b)$ is the mid-point of AB, then $a - b =$

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. 0

B. 1

C. 2

D. 3

Answer: D

Solution:

Given:

$y = -x - 1$

Substitute $y = -x - 1$ into the circle's equation:

$x^2 + (-x - 1)^2 - 4x + 2(-x - 1) - 4 = 0$

Simplify:

$x^2 + (x^2 + 2x + 1) - 4x - 2x - 2 - 4 = 0$

$2x^2 - 4x - 5 = 0$

Let x_1 and x_2 be the roots of the quadratic equation above. By Vieta's formulas:

$x_1 + x_2 = 4/2 = 2$

The midpoint $M(a, b)$ of AB on the x-axis is:

$\frac{x_1 + x_2}{2} = 1$

Now substitute $x = -y - 1$ into the circle equation:

$(-y-1)^2 + y^2 - 4(-y-1) + 2y - 4 = 0$

Simplify:

$(y^2 + 2y + 1) + y^2 + 4y + 4 + 2y - 4 = 0$

$2y^2 + 8y + 1 = 0$

Let y_1 and y_2 be the roots of this quadratic equation. By Vieta's formulas:

$y_1 + y_2 = -8/2 = -4$

The midpoint $M(a, b)$ on the y-axis is:

$\frac{y_1+y_2}{2} = -2$

Thus, the midpoint coordinates are $(1, -2)$.

Therefore, the values are:

$a = 1, \quad b = -2$

Calculating $a - b$:

$a - b = 1 - (-2) = 3$

Question18

A circle S passes through the points of intersection of the circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 2y = 0$. If $x + y + 1 = 0$ is a tangent to the circle S, then equation of S is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $2x^2 + 2y^2 + 2x + 2y + 3 = 0$

B. $2x^2 + 2y^2 - 2x - 2y + 3 = 0$

C. $x^2 + y^2 - 2x - 2y + 3 = 0$

D. $2x^2 + 2y^2 - 2x - 2y - 3 = 0$

Answer: D

Solution:

Equation of circle passes through the point of intersection of

$x^2 + y^2 - 2x - 3 = 0$

and $x^2 + y^2 - 2y = 0$ is

$x^2 + y^2 - 2x - 3 + \lambda\left(x^2 + y^2 - 2y\right) = 0$

$x^2(1+\lambda) + y^2(1+\lambda) - 2x - 2\lambda y - 3 = 0$

$x^2 + y^2 - \frac{2x}{1+\lambda} - \frac{\lambda}{1+\lambda}2y - \frac{3}{1+\lambda} = 0$

Centre $\left(\frac{1}{1+\lambda}, \frac{\lambda}{1+\lambda}\right)$

Radius $= \sqrt{\frac{1}{(1+\lambda)^2} + \frac{\lambda^2}{1+\lambda^2} + \frac{3}{1+\lambda}}$

$$= \sqrt{\frac{1+\lambda^2+3+3\lambda}{(1+\lambda)^2}} = \sqrt{\frac{\lambda^2+3\lambda+4}{(\lambda+1)^2}}$$

$x+y+1=0$ touches the circle

$$\frac{\frac{1}{1+\lambda} + \frac{\lambda}{1+\lambda} + 1}{\sqrt{2}} = \frac{\sqrt{\lambda^2+3\lambda+4}}{\lambda+1}$$

$$\frac{1+\lambda}{1+\lambda} + 1 = \frac{\sqrt{\lambda^2+3\lambda+4}}{\lambda+1}(\sqrt{2})$$

$$\sqrt{2}(\lambda+1) = \sqrt{\lambda^2+3\lambda+4}$$

$$2\lambda^2 + 2 + 4\lambda = \lambda^2 + 3\lambda + 4$$

$$\lambda^2 + \lambda - 2 = 0$$

$$(\lambda+2)(\lambda-1) = 0$$

$\lambda = -2$ or 1

Take $\lambda = 1$, we get

$2x^2 + 2y^2 - 2x - 2y - 3 = 0$

Question19

If the common chord of the circles $x^2+y^2-2x+2y+1=0$ and $x^2+y^2-2x-2y-2=0$ is the diameter of a circle S, then the center of the circles is

TG EAPCET 2024 (Online) 11th May Morning Shift

Options:

A. $\left(\frac{1}{2}, -\frac{3}{4}\right)$

B. $\left(1, -\frac{3}{4}\right)$

C. $\left(1, \frac{3}{4}\right)$

D. $\left(-\frac{1}{2}, -\frac{3}{4}\right)$

Answer: B

Solution:

To find the center of circle S, where the common chord of the given circles serves as its diameter, we first determine the equation of the common chord.

The equations of the two circles are:

$x^2 + y^2 - 2x + 2y + 1 = 0$

$x^2 + y^2 - 2x - 2y - 2 = 0$

To find the common chord, subtract the second circle's equation from the first:

$$\begin{aligned} &(x^2 + y^2 - 2x + 2y + 1) - (x^2 + y^2 - 2x - 2y - 2) = 0 \\ \Rightarrow &4y + 3 = 0 \\ \Rightarrow &y = -\frac{3}{4} \end{aligned}$$

Next, find the centers of the circles:

First circle, center $C_1 = (1, -1)$

Second circle, center $C_2 = (1, 1)$

The midpoint of C_1 and C_2 is calculated as follows:

$\left(\frac{1+1}{2}, \frac{-1+1}{2}\right) = (1, 0)$

Thus, the center of circle S is at the intersection of the common chord $y = -\frac{3}{4}$ and the line connecting C_1 and C_2 which is the diameter of S. Hence, the center of circle S is:

$(1, -\frac{3}{4})$

Question20

A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$. If the line joining the centres of these circles and the common chord of them are the diagonals of this rhombus, then the area (in sq units) of the rhombus is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $16\sqrt{3}$

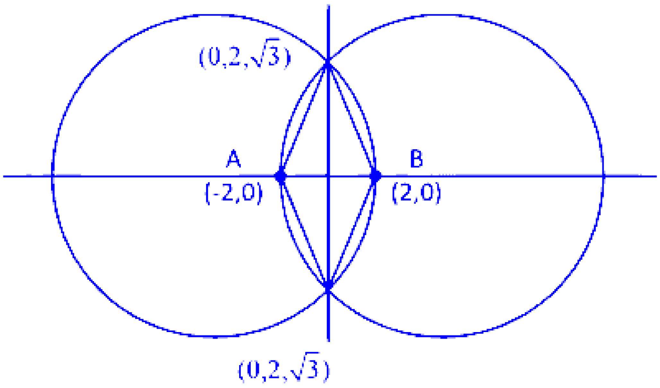
B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. $8\sqrt{3}$

Answer: D

Solution:



Points of intersection of circles is

$x^2 + y^2 + 4x - 12 = x^2 + y^2 + 4x - 12$

$\therefore \quad x = 0$

Now, $y = \pm 2\sqrt{3}$

Now, area of rhombus

$= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 4\sqrt{3} \times 4$

$= 8\sqrt{3}$ sq units

Question21

If m is the slope and $P(8, \beta)$ is the mid-point of a chord of contact of the circle $x^2 + y^2 = 125$, then the number of values of β such that β and m are integers is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. 2

B. 4

C. 6

D. 8

Answer: D

Solution:

To find the equation of a chord of contact with a given mid-point for the circle $x^2 + y^2 = 125$, we use the formula:

$T = S_1$

Substituting the mid-point $P(8, \beta)$ into this formula gives us:

$8x + \beta y = 64 + \beta^2$

From the equation, the slope m of the line is:

$m = \frac{-8}{\beta}$

For both β and m to be integers, β must be a divisor of -8, because the slope simplifies in this way. The integer divisors of -8 are:

$\beta = -1, -2, -4, -8, 1, 2, 4, 8$

Therefore, there are 8 possible integer values for β.

Question22

A rectangle is formed by the lines $x = 4, x = -2, y = 5, y = -2$ and a circle is drawn through the vertices of this rectangle. The pole of the line $y + 2 = 0$ with respect to this circle is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $\left(1, \frac{-85}{14}\right)$

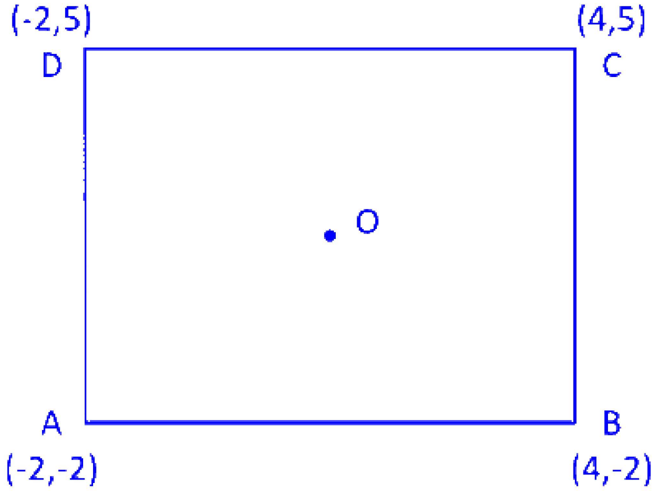
B. $\left(1, \frac{-32}{7}\right)$

C. $(-2, -2)$

D. $(1, -4)$

Answer: B

Solution:



Centre $(O) \equiv \left(\frac{4-2}{2}, \frac{5-2}{2}\right) = \left(1, \frac{3}{2}\right)$

$\Rightarrow$ Equation of circle

$= x^2 + y^2 - 2x - 3y + k = 0$

Circle passes through $= (-2, -2)$

$4 + 4 + 4 + 6 + K = 0$

$k = -18$

$x^2+y^2-2x-3y-18=0$

Let pole (h, k)

Equation $T=0$

$hx+ky-x-h-\frac{3y}{2}-\frac{3k}{2}-18=0$

$(h-1)x+\left(k-\frac{3}{2}\right)y-h-\frac{3k}{2}-18=0$

On comparing with $y+2=0$, we get

$\Rightarrow \quad h=1$

and $\frac{k-\frac{3}{2}}{1}=\frac{-1-\frac{3k}{2}-18}{2}$

$\Rightarrow \quad 2k-3=\frac{-3k-38}{2}$

$\Rightarrow \quad 7k=-38+6$

$\therefore \quad k=\frac{-32}{7}$

Pole $\left(1, \frac{-32}{7}\right)$

Question23

The equation of a circle which passes through the points of intersection of the circles $2x^2+2y^2-2x+6y-3=0, x^2+y^2+4x+2y+1=0$ and whose centre lies on the common chord of these circles is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $2x^2+2y^2-3x+4y-2=0$

B. $x^2+y^2+2x+5y-2=0$

C. $3x^2+3y^2-2x+4y-3=0$

D. $4x^2+4y^2+6x+10y-1=0$

Answer: D

Solution:

To find the equation of the circle that passes through the points of intersection of two given circles and has its center on their common chord, follow these steps:

Common Chord of the Two Circles

The common chord is found by subtracting the equations of the two circles:

First Circle:

$2x^2+2y^2-2x+6y-3=0$

Second Circle:

$x^2+y^2+4x+2y+1=0$

Subtract the second circle from the first:

$$(x^2 + y^2 + 4x + 2y + 1) - (2x^2 + 2y^2 - 2x + 6y - 3) = 0$$

Simplifying, we have:

$$x^2 + y^2 + 4x + 2y + 1 - x^2 - y^2 + x - 3y + \frac{3}{2} = 0$$

This simplifies to:

$$5x - y + \frac{5}{2} = 0$$

To eliminate the fraction, multiply the entire equation by 2:

$$10x - 2y + 5 = 0$$

Equation of the New Circle

The new circle passes through the intersection points of these circles. Its equation is of the form:

$$(x^2 + y^2 - x + 3y - \frac{3}{2}) + \mu(x^2 + y^2 + 4x + 2y + 1) = 0$$

Let's simplify this:

$$x^2 + y^2 + \frac{(4\mu-1)}{(1+\mu)}x + \frac{(3+2\mu)}{(1+\mu)}y + \mu - \frac{3}{2} = 0$$

Center of the Circle

The center (h, k) of the circle is given by:

$$\left(\frac{1-4\mu}{2(1+\mu)}, \frac{-(3+2\mu)}{2(1+\mu)}\right)$$

Substitute into the chord equation (Equation (i)):

$$5\left(\frac{1-4\mu}{1+\mu}\right) + \left(\frac{3+2\mu}{1+\mu}\right) + 5 = 0$$

Simplifying, we get:

$$5 - 20\mu + 3 + 2\mu + 5 + 5\mu = 0$$

This results in:

$$13 - 13\mu = 0$$

Thus:

$$\mu = 1$$

Final Equation

Substitute $\mu = 1$ back into Equation (ii):

$$2x^2 + 2y^2 + 3x + 5y - \frac{1}{2} = 0$$

Multiply the entire equation by 2 to clear the fraction:

$$4x^2 + 4y^2 + 6x + 10y - 1 = 0$$

This is the equation of the required circle.

Question24

If the equation of the circle which cuts each of the circles $x^2 + y^2 = 4, x^2 + y^2 - 6x - 8y + 10 = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the

extremities of a diameter of these circles is $x^2 + y^2 + 2gx + 2fy + c = 0$, then $g + f + c =$

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. 9

B. -9

C. 12

D. -12

Answer: B

Solution:

We have the circle $C : x^2 + y^2 + 2gx + 2fy + c = 0$, which intersects the following circles at the extremities of their diameters:

$C_1 : x^2 + y^2 = 4$

$C_2 : x^2 + y^2 - 6x - 8y + 10 = 0$

$C_3 : x^2 + y^2 + 2x - 4y - 2 = 0$

Finding c

The radical axis of C and C_1 passes through the origin $(0, 0)$. Therefore:

$2gx + 2fy + c + 4 = 0$

By substituting $x = 0$ and $y = 0$, we find:

$c + 4 = 0 \implies c = -4$

Finding g, f

The radical axis of C and C_2 passes through $(3, 4)$. The radical axis equation is:

$(2g + 6)x + (2f + 8)y - 14 = 0$

By substituting $x = 3$ and $y = 4$, we get:

$3(2g + 6) + 4(2f + 8) - 14 = 0$

Simplifying:

$6g + 18 + 8f + 32 - 14 = 0 \implies 3g + 4f + 18 = 0$

The radical axis of C and C_3 passes through $(-1, 2)$. The equation is:

$(2g - 2)x + (2f + 4)y - 2 = 0$

Substituting $x = -1$ and $y = 2$:

$(-1)(2g - 2) + 2(2f + 4) - 2 = 0$

Simplifying:

$-2g + 2 + 4f + 8 - 2 = 0 \implies -g + 2f + 4 = 0$

Solving the Equations

From the equations:

$3g + 4f + 18 = 0$

$-g + 2f + 4 = 0$

From equation 2:

$-g = 2f + 4 \implies g = -2f - 4$

Substitute $g = -2f - 4$ in equation 1:

$3(-2f - 4) + 4f + 18 = 0$

$-6f - 12 + 4f + 18 = 0$

$-2f + 6 = 0 \implies f = 3$

Now substitute back to find g:

$g = -2(3) - 4 = -10$

Finally, calculate $g + f + c$:

$g + f + c = -10 + 3 - 4 = -11$

Correcting for f:

$f = 3, \quad g = -2 \quad \Rightarrow \quad g + f + c = -2 - 3 - 4 = -9$

Question25

The equation of the circle passing through the origin and cutting the circles $x^2 + y^2 + 6x - 15 = 0$ and $x^2 + y^2 - 8y - 10 = 0$ orthogonally is

TG EAPCET 2024 (Online) 10th May Evening Shift

Options:

A. $2x^2 + 2y^2 - 5x + 10y = 0$

B. $x^2 + y^2 - 2x + 5y = 0$

C. $2x^2 + 2y^2 - 10x + 5y = 0$

D. $x^2 + y^2 - 5x + 2y = 0$

Answer: B

Solution:

To find the equation of the circle passing through the origin and cutting the given circles orthogonally, we use the condition for orthogonality between circles.

The condition for two circles to be orthogonal is expressed as:

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Applying this condition to the circles in question, we have:

For the circle $x^2 + y^2 + 6x - 15 = 0$:

Center: $(-3, 0)$

Radius term: -15

For the circle $x^2 + y^2 - 8y - 10 = 0$:

Center: $(0, 4)$

Radius term: -10

Let's determine g and f for the required circle $x^2 + y^2 + 2gx + 2fy = 0$.

For the first condition of orthogonality with $x^2 + y^2 + 6x - 15 = 0$, we get:

$2g(-3) + 2f(0) = 0 - 15$

Solving for g:

$-6g = -15 \quad \Rightarrow \quad g = \frac{15}{6} = \frac{5}{2}$

For the second condition of orthogonality with $x^2 + y^2 - 8y - 10 = 0$, we have:

$2g(0) + 2f(4) = -10$

Solving for f:

$8f = -10 \quad \Rightarrow \quad f = -\frac{5}{4}$

The equation of the required circle is then:

$x^2 + y^2 - 5x + \frac{5}{2}y = 0$

Multiplying the entire equation by 2 to eliminate fractions, we obtain:

$2x^2 + 2y^2 - 10x + 5y = 0$

This is the equation of the required circle.

Question26

$(1, k)$ is a point on the circle passing through the points $(-1, 1), (0, -1)$ and $(1, 0)$. If $k \neq 0$, then $k =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. $-\frac{1}{2}$

Answer: B

Solution:

Equation of circle passing through 3 point is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1+k^2 & 1 & k & 1 \\ 2 & -1 & 1 & 1 \\ 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & 1 \end{vmatrix} = 0$$

$C_1 = C_1 - C_4, C_4 = C_4 - C_2$

$$\begin{vmatrix} K^2 & 1 & K & 0 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} = 0$$

$R_2 = R_2 + R_3$, we get

$$\begin{vmatrix} K^2 & 1 & K & 0 \\ 1 & -1 & 0 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow K^2(3) - 1(K) = 0$

$\Rightarrow 3K^2 - K = 0; K = 0, K = \frac{1}{3}$

Question27

If the tangents $x + y + k = 0$ and $x + ay + b = 0$ drawn to the circle $S = x^2 + y^2 + 2x - 2y + 1 = 0$ are perpendicular to each other and k, b are both greater than 1 , then $b - k =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\sqrt{2}$

B. 0

C. 2

D. $2\sqrt{2}$

Answer: C

Solution:

If lines are perpendicular

then, $m_1 m_2 = -1 = -1 \times \frac{-1}{a} = -1$

$a = -1$

If this lines are tangent mean $\perp$ distance from centre $=$ Radius of circle

Centre $\equiv (-1, 1)$ radius $= \sqrt{1+1-1} = 1$

Perpendicular distance $= r$

$\Rightarrow \left|\frac{-1+1+K}{\sqrt{2}}\right| = 1$

$|K| = \sqrt{2}, K = +\sqrt{2} \quad \{\because K > 1\}$

Perpendicular distance $= r$

$\Rightarrow \quad \left|\frac{-1+a+b}{\sqrt{1+a^2}}\right| = 1$

$\left|\frac{-1-1+b}{\sqrt{2}}\right| = 1 \ (\because a = -1)$

$\Rightarrow \quad |2-b| = \sqrt{2}$

$\Rightarrow \quad 2-b = -\sqrt{2}$ and $2-b = +\sqrt{2}$

$\Rightarrow \quad b = 2+\sqrt{2}$ and $b = 2-\sqrt{2}$

$b - K = 2+\sqrt{2}-\sqrt{2} = 2$

Question28

If (h, k) is the internal centre of similitude of the circles $x^2+y^2+2x-6y+1=0$ and $x^2+y^2-4x+2y+4=0$, then $4h =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 0

B. 3

C. 1

D. 5

Answer: D

Solution:

Given, $c_1 \equiv x^2+y^2+2x-6y+1=0$

$c_2 \equiv x^2+y^2-4x+2y+4=0$

$c_1 \equiv (-1,3), r_1=\sqrt{1+9-1}=3$

$c_2 \equiv (2,-1), r_2=\sqrt{4+1-4}=1$

$\therefore$ Internal similitude (P) centre divide c_1 and c_2 in ratio $r_1 : r_2$

$\Rightarrow \quad P \equiv (h,k)$

$$h=\frac{-1+6}{3+1} \Rightarrow h=\frac{5}{4} \Rightarrow 4h=5$$

Question29

The slope of a common tangent to the circles $x^2+y^2-4x-8y+16=0$ and $x^2+y^2-6x-16y+64=0$ is

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 0

B. $\frac{15}{8}$

C. 1

D. $\frac{17}{4}$

Answer: B

Solution:

Given equation

$x^2+y^2-4x-8y+16=0$ and

$x^2+y^2-6x-16y+64=0$ can be

written as

$S_1 \equiv (x-2)^2+(y-4)^2=2^2$

$\Rightarrow \quad$ Centre $\equiv (2,4)$ and Radius $=2$

$S_2 \equiv (x-3)^2+(y-8)^2=3^2$

$\Rightarrow \quad$ Centre $\equiv (3,8)$ and Radius $=3$

Equation of tangent to S_1

$\Rightarrow \quad (y-4)=m(x-2)+2\sqrt{1+m^2}$

$\Rightarrow \quad y-mx=4-2m+2\sqrt{1+m^2}..$

Equation of tangent to S_2

$\Rightarrow \quad (y-8)=m(x-3)+3\sqrt{1+m^2}$

$\Rightarrow \quad y-mx=8-3m+3\sqrt{1+m^2}\ldots$

If tangent is common, then Eq. (i) and

(ii) will be same

$4-2m+2\sqrt{1+m^2}=8-3m+3\sqrt{1+m^2}$

$\Rightarrow \quad m-4=\sqrt{1+m^2}$

On squaring both sides, we get

$\Rightarrow \quad m^2+16-8m=1+m^2$

$\Rightarrow \quad 8m=15 \Rightarrow m=\frac{15}{8}$

Question30

$x^2+y^2+2x-6y-6=0$ and $x^2+y^2-6x-2y+k=0$ are two intersecting circles and k is not an integer. If θ is the angle between the two circles and $\cos\theta=\frac{-5}{24}$, then $k=$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. $\frac{6}{5}$

B. $\frac{74}{9}$

C. $\frac{37}{3}$

D. $\frac{53}{7}$

Answer: B

Solution:

Angle between 2 circle is $\cos\theta=\frac{r_1^2+r_2^2-d^2}{2r_1r_2}$

$c_1 \equiv(-1,3), r_1=\sqrt{1+9+6}=4$

$c_2 \equiv(3,1), r_2=\sqrt{9+1-K}=\sqrt{10-K}$

$c_1c_2=d=\sqrt{4^2+2^2}=\sqrt{20}$

$\frac{-5}{24}=\frac{16+10-K-20}{2\cdot4\cdot\sqrt{10-K}}$

$\frac{-5}{3}=\frac{6-K}{\sqrt{10-K}}$

$5\sqrt{10-K}=3K-18$

$250-25K=9K^2+324-108K$

$9K^2-83K+74=0$

$9K^2 - 74K - 9K + 74 = 0$

$K = \frac{74}{9}, K = 1$

Question31

If (p, q) is the centre of the circle which cuts the three circles $x^2 + y^2 - 2x - 4y + 4 = 0, x^2 + y^2 + 2x - 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y - 11 = 0$ orthogonally, then $p + q =$

TG EAPCET 2024 (Online) 10th May Morning Shift

Options:

A. 9

B. $\frac{35}{4}$

C. $\frac{15}{2}$

D. 7

Answer: A

Solution:

$c = (p, q)$, constant tan $= c$

$c_1 = (1, 2), r_1 = \sqrt{1 + 4 - 4} = 1$

$c_2 = (-1, 2), r_2 = \sqrt{1 + 4 - 1} = 2$

$c_3 = (2, 1), r_3 = \sqrt{4 + 1 + 11} = 4$

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

[$\because$ Condition for orthogonal]

$2p + 4q = c + 4$

$-2p + 4q = c + 1$

$4p + 2q = c - 11$

btracting Eq: (i) fro

$4p = 3 \Rightarrow p = \frac{3}{4}$

On subtracting Eq. (i) from (ii), we get

On subtracting Eqs. (iii) from (ii), we get

$-6p + 2q = 12 \Rightarrow -6 \times \frac{3}{4} + 2q = 12$

$2q = 12 + \frac{9}{2} \Rightarrow 2q = \frac{33}{2} \Rightarrow q = \frac{33}{4}$

$\therefore \quad p + q = \frac{33}{4} + \frac{3}{4} = \frac{36}{4} = 9$

Question32

If $P\left(\frac{\pi}{4}\right), Q\left(\frac{\pi}{3}\right)$ are two points on the circle $x^2+y^2-2x-2y-1=0$, then the slope of the tangent to this circle which is parallel to the chord PQ is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $2+\sqrt{2}-\sqrt{3}-\sqrt{6}$

B. $2+\sqrt{2}+\sqrt{3}+\sqrt{6}$

C. $\sqrt{2}+\sqrt{3}$

D. $2+\sqrt{2}$

Answer: A

Solution:

Given, circle

$x^2+y^2-2x-2y-1=0, P\left(\frac{\pi}{4}\right)$ and $Q\left(\frac{\pi}{3}\right)$

$$(x-1)^2+(y-1)^2=1^2$$

Center, $(h, k)=(1,1)$ and radius, $r=1$

$$\therefore \quad P\left(\frac{\pi}{4}\right)=\left(h+r\cos\frac{\pi}{4}, y+r\sin\frac{\pi}{4}\right)$$

$$=\left(1+\frac{1}{\sqrt{2}}, 1+\frac{1}{\sqrt{2}}\right)$$

and $\quad Q\left(\frac{\pi}{3}\right)=\left(h+r\cos\frac{\pi}{3}, y+r\sin\frac{\pi}{3}\right)$

$$=\left(1+\frac{1}{2}, 1+\frac{\sqrt{3}}{2}\right)$$

Slope of described tangent $=$ Slope of chord PQ

$$=\frac{\left(1+\frac{\sqrt{3}}{2}\right)-\left(1+\frac{1}{\sqrt{2}}\right)}{\left(1+\frac{1}{2}\right)-\left(1+\frac{1}{\sqrt{2}}\right)}=\frac{\sqrt{6}-2}{\sqrt{2}-2}\times\frac{\sqrt{2}+2}{\sqrt{2}+2}$$

$$=\frac{\sqrt{12}-2\sqrt{2}+2\sqrt{6}-4}{2-4}$$

$$=-\frac{1}{2}(2\sqrt{3}-2\sqrt{2}+2\sqrt{6}-4)$$

$$=2+\sqrt{2}-\sqrt{3}-\sqrt{6}$$

Question33

The power of a point $(2,0)$ with respect to a circle S is -4 and the length of the tangent drawn from the point $(1,1)$ to S is 2 . If the circle S passes through the point $(-1,-1)$, then the radius of the circle S is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. 2

B. $\sqrt{13}$

C. 3

D. $\sqrt{10}$

Answer: B

Solution:

Let $S : x^2 + y^2 + 2gx + 2fy + c = 0$

$\because$ The power of the point $(2, 0)$ with respect to circle S is -4

$\therefore (2)^2 + (0)^2 + 2g(2) + 2f(0) + c = -4$

$\Rightarrow \quad 4g + c + 8 = 0 \quad \ldots$ (i)

$\because$ The length of tangent drawn from the point $(1, 1)$ to S is 2

$\therefore \quad \sqrt{(1)^2 + (l)^2 + 2g(l) + 2f(l) + c} = 2$

$\Rightarrow \quad 2g + 2f + c + 2 = 4$

$\Rightarrow \quad 2g + 2f + c - 2 = 0 \quad \ldots$ (ii)

$\because$ The circle S is passing through $(-1, -1)$.

$\therefore (-1)^2 + (-1)^2 + 2g(-1) + 2f(-1) + c = 0$

$\Rightarrow \quad -2g - 2f + c + 2 = 0$

$\Rightarrow \quad 2g + 2f - c - 2 = 0 \ldots$ (iii)

On solving Eqs. (i), (ii) and (iii), we get

$g = -2f = 3$ and $c = 0$

$\therefore \quad S = x^2 + y^2 - 4x + 6y = 0$

Radius of $S = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-2)^2 + (3)^2 - 0}$

$= \sqrt{4 + 9} = \sqrt{13}$

Question34

The pole of the line $x - 5y - 7 = 0$ with respect to the circle $S \equiv x^2 + y^2 - 2x + 4y + 1 = 0$ is $P(a, b)$. If C is the centre of the circle $S = 0$, then $PC =$

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $\sqrt{a+b-1}$

B. $\sqrt{a^2+b^2-1}$

C. $\sqrt{a^3+b^3-1}$

D. $3ab$

Answer: C

Solution:

Given, the pole of the line $x - 5y - 7 = 0$ with respect to the circle

$S \equiv x^2 + y^2 - 2x + 4y + 1 = 0$ is $P(a, b)$

$\therefore$ Equation of polar $T = 0$

$ax + by - (a + x) + 2(b + y) + 1 = 0$

$(a - 1)x + (b + 2)y + (2b - a + 1) = 0$

This equation will be same as

$x - 5y - 7 = 0$

$\therefore \quad \frac{a-1}{1} = \frac{b+2}{-5} = \frac{2b-a+1}{-7}$

$\Rightarrow \quad a = 0$ and $b = 3$

$\therefore \quad P(a, b) = P(0, 3)$

Center of circle $S = 0$ is $C(1, -2)$

$PC = \sqrt{(1-0)^2 + (-2-3)^2} = \sqrt{26}$

$\sqrt{a^3+b^3-1} = \sqrt{0+27-1} = \sqrt{26}$

Hence, $\quad PC = \sqrt{a^3+b^3-1}$

Question35

The equation of the pair of transverse common tangents drawn to the circles $x^2 + y^2 + 2x + 2y + 1 = 0$ and $x^2 + y^2 - 2x - 2y + 1 = 0$ is

Options:

A. $x^2 - y^2 = 0$

B. $xy = 0$

C. $x^2 - y^2 + 2x + 1 = 0$

D. $x^2 - y^2 - 2y - 1 = 0$

Answer: B

Solution:

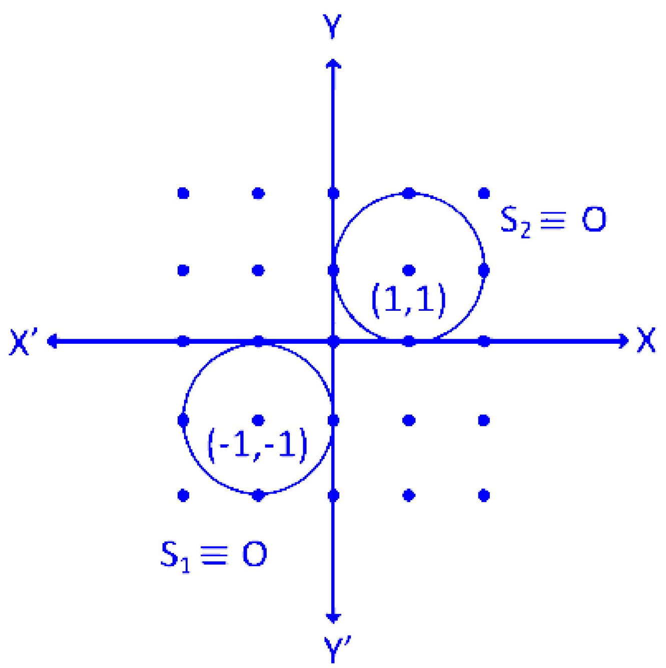
We have,

$S_1 \equiv x^2 + y^2 + 2x + 2y + 1 = 0$

$\therefore S_1 \equiv (x+1)^2 + (y+1)^2 = 1$

and $S_2 \equiv x^2 + y^2 - 2x - 2y + 1 = 0$

$\therefore S_2 \equiv (x-1)^2 + (y-1)^2 = 1$



From the diagram it is clear that transverse common tangents are $x = 0$ and $y = 0$ (i.e. Y-axis and X-axis)

The combined equation of lines $x = 0$ and $y = 0$ is $xy = 0$

Question36

If a circle passing through the point $(1, 1)$ cuts the circles $x^2 + y^2 + 4x - 5 = 0$ and $x^2 + y^2 - 4y + 3 = 0$ orthogonally, then the centre of that circle is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $\left(\frac{3}{4}, \frac{5}{4}\right)$

B. $\left(\frac{3}{2}, \frac{5}{2}\right)$

C. $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

D. $\left(-\frac{3}{4}, -\frac{5}{2}\right)$

Answer: A

Solution:

Let required circle

$S : x^2 + y^2 + 2gx + 2fy + c = 0$

$S_1 : x^2 + y^2 + 4x - 5 = 0$

$S_2 : x^2 + y^2 - 4y + 3 = 0$

Here, $g_1 = 2, f_1 = 0, g_2 = 0, f_2 = -2, c_1 = -5$ and $c_2 = 3$

$\because$ Circle S is passing throught the point (1, 1)

$\therefore 1^2 + 1^2 + 2g(1) + 2f(1) + c = 0$

$\Rightarrow 2g + 2f + c + 2 = 0 \quad \ldots$ (i)

$\because$ S and S_1 are orthogonal circles

$\therefore \quad 2gg_1 + 2ff_1 = c + c_1$

$\Rightarrow \quad 2g(2) + 2f(0) = c + (-5)$

$\Rightarrow \quad 4g - c + 5 = 0 \quad \ldots$ (ii)

$\because$ S and S_2 are orthogonal circles.

$\therefore \quad 2gg_2 + 2ff_2 = c + c_2$

$\Rightarrow \quad 2g(0) + 2f(-2) = c + 3$

$\Rightarrow \quad -4f - c - 3 = 0 \quad \ldots$ (iii)

On solving Eqs. (i), (ii) and(iii), we get

$g = -\frac{3}{4}, f = -\frac{5}{4}$

and $\quad c = 2$

$\therefore$ Center of $S = (-g, -f) = \left(\frac{3}{4}, \frac{5}{4}\right)$

Question37

Length of the common chord of the circles $x^2 + y^2 - 6x + 5 = 0$ and $x^2 + y^2 + 4y - 5 = 0$ is

TG EAPCET 2024 (Online) 9th May Evening Shift

Options:

A. $\sqrt{13}$

B. $\frac{12}{\sqrt{13}}$

C. $\frac{6}{\sqrt{13}}$

D. $2\sqrt{13}$

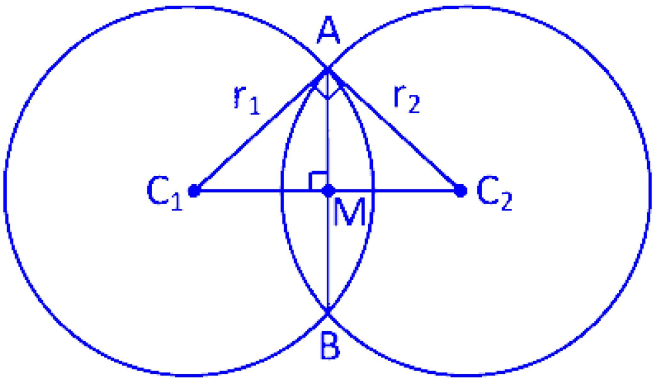
Answer: B

Solution:

Given,

$S_1 \equiv x^2 + y^2 - 6x + 5 = 0$

$S_2 \equiv x^2 + y^2 + 4y - 5 = 0$



Here, $C_1 = (3, 0), r_1 = \sqrt{(-3)^2 + 0^2 - 5} = 2$

$C_2 = (0, -2), r_2 = \sqrt{0^2 + 2^2 - (-5)} = 3$

$C_1C_2 = \sqrt{(0-3)^2 + (-2-0)^2} = \sqrt{13}$

Area of $\Delta AC_1C_2 = \frac{1}{2}r_1r_2 = \frac{1}{2}AM \times C_1C_2$

$\Rightarrow \frac{1}{2}(2)(3) = \frac{1}{2}(AM)(\sqrt{13}) \Rightarrow AM = \frac{6}{\sqrt{13}}$

$\therefore$ Required length of common tangent

$= AB = 2(AM) = 2\left(\frac{6}{\sqrt{13}}\right) = \frac{12}{\sqrt{13}}$

Question38

The centroid of a variable $\triangle ABC$ is at the distance of 5 units from the origin. If $A=(2,3)$ and $B=(3,2)$, then the locus of C is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. a circle of radius 225 units

B. a rectangular hyperbola

C. a circle of diameter 30 units

D. an ellipse with eccentricity $\frac{4}{5}$

Answer: C

Solution:

Let coordinates of C be (h, k).

$A=(2,3), B=(3,2)$

Coordinates of centroid

$$=\left(\frac{2+h+3}{3}, \frac{k+3+2}{3}\right)=\left(\frac{h+5}{3}, \frac{k+5}{3}\right)$$

$$\therefore \quad \left(\frac{h+5}{3}\right)^2+\left(\frac{k+5}{3}\right)^2=25 \quad \text{[given]}$$

$$(h+5)^2+(k+5)^2=(15)^2$$

$$(x+5)^2+(Y+5)^2=(15)^2$$

which is the equation of circle with radius 15 units.

$\Rightarrow$ Diameter $=30$ units

Question39

If $(1,1),(-2,2)$ and $(2,-2)$ are 3 points on a circle S, then the perpendicular distance from the centre of the circle S to the line $3x-4y+1=0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $\frac{1}{2}$

B. 1

C. $\frac{23}{10}$

D. 2

Answer: A

Solution:

Three points on circle are

$(1,1),(-2,2),(2,-2)$

Let equation of circle be

$x^2+y^2+2gx+2fy+c=0$

$\because$ Above points lie on the circle

$$\therefore \quad 1+1+2g+2f+c=0$$

$$\Rightarrow \quad 2g+2f+c+2=0 \quad \ldots \text{(i)}$$

$$(-2)^2+(2)^2+2(-2)g+2\times 2\times f+c=0$$

$$8-4g+4f+c=0$$

$$4g-4f-c-8=0 \quad \ldots \text{(ii)}$$

and

$$(2)^2+(-2)^2+2\times 2\times g+2(-2)f+c=0$$

$$4g-4f+c+8=0 \quad \ldots \text{(iii)}$$

On solving Eqs. (i), (ii) and (iii), we get

$c=-8, g=\frac{3}{2}, f=\frac{3}{2}$

$\therefore$ Centre of circle $\equiv\left(\frac{-3}{2},\frac{-3}{2}\right)$

Now, perpendicular distance from centre of the circle to the line

$$=\left|\frac{3\left(-\frac{3}{2}\right)-4\left(-\frac{3}{2}\right)+1}{\sqrt{3^2+(-4)^2}}\right|$$

$$=\frac{\left|-\frac{9}{2}+\frac{12}{2}+1\right|}{5}=\frac{1}{2}$$

Question40

If the line $4x-3y+p=0(p+3>0)$ touches the circle $x^2+y^2-4x+6y+4=0$ at the point (h,k), then $h-2k=$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $-\frac{8}{5}$

B. 2

C. $\frac{6}{5}$

D. 3

Answer: B

Solution:

Given circle is

$x^2 + y^2 - 4x + 6y + 4 = 0$

$(x-2)^2 + (y+3)^2 = (3)^2$

Centre $= (2, -3)$ and radius $= r = 3$

$\because$ Perpendicular distance between centre and line $= r$

$$\therefore \left|\frac{4 \times 2 - 3(-3) + p}{\sqrt{4^2 + (3)^2}}\right| = 3$$

$\Rightarrow \quad 17 + p = \pm 15$

$p = -2 \quad (\because p + 3 > 0)$

So, equation of line is

$4x - 3y - 2 = 0 \quad \ldots$ (i)

Let line touches the circle at (h, k).

Then, equation of tangent at (h, k) is

$hx + ky - 2(x + h) + 3(y + k) + 4 = 0$

$(h-2)x + (k+3)y - 2h + 3k + 4 = 0 \quad \ldots$ (ii)

On comparing Eq. (ii) with Eq. (i), we get

$$\frac{4}{h-2} = \frac{-3}{k+3}$$

$4k + 3h = -6 \quad \ldots$ (iii)

and $\frac{-3}{k+3} = \frac{-2}{-2h+3k+4}$

$\Rightarrow \quad 7k - 6h = -6 \quad \ldots$ (iv)

On solving Eqs. (iii) and (iv), we get

$h = \frac{-2}{5}, k = \frac{-6}{5}$

Hence, $h - 2k = \frac{-2}{5} + \frac{12}{5} = 2$

Question41

If the inverse point of the point $P(3, 3)$ with respect to the circle $x^2 + y^2 - 4x + 4y + 4 = 0$ is $Q(a, b)$, then $a + 5b =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. 4

B. 0

C. -4

D. 1

Answer: C

Solution:

We know that inverse of any point $P(x, y)$ with respect to circle centred at (h, k) is $Q\left(x', y'\right)$, where

$x' = \alpha(x - h) + h \Rightarrow y' = \alpha(y - k) + k$

and $\alpha = \frac{r^2}{(x-h)^2+(y-k)^2}$

$r =$ radius of circle

Here, $h = 2, k = -2$ and $r = 2$,

$x = 3, y = 3$

$$\Rightarrow \alpha = \frac{2^2}{(3-2)^2+(3+2)^2} = \frac{4}{26} = \frac{2}{13}$$

Now,

$$x' = a = \frac{2}{13}(3-2) + (2) = \frac{28}{13}$$

$$y' = b = \frac{2}{13}(3+2) - 2 = -\frac{16}{13}$$

Hence, $a + 5b = \frac{28}{13} + 5\left(-\frac{16}{13}\right)$

$= \frac{-}{-52}\frac{52}{13} = -4$

Question42

If the equation of the transverse common tangent of the circles $x^2 + y^2 - 4x + 6y + 4 = 0$ and $x^2 + y^2 + 2x - 2y - 2 = 0$ is $ax + by + c = 0$, then $\frac{a}{c} =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $-\frac{3}{4}$

B. $\frac{4}{3}$

C. 1

D. -1

Answer: D

Solution:

We have,

$S_1 : x^2 + y^2 - 4x + 6y + 4 = 0$

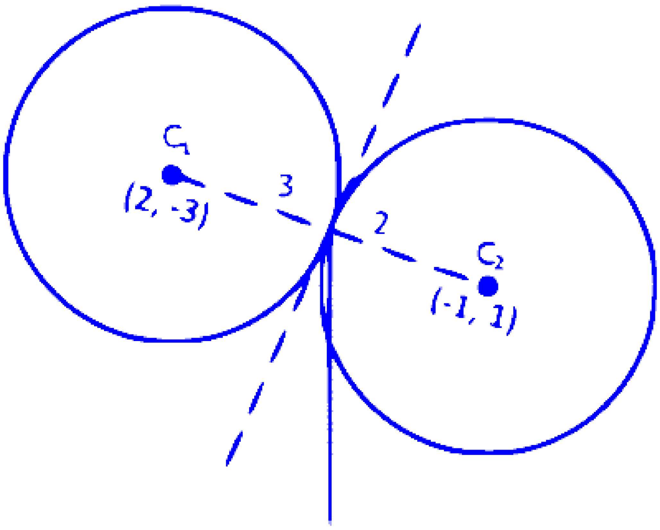
$C_1 : (2 - 3), r_1 = 3$

$S_2 : x^2 + y^2 + 2x - 2y - 2 = 0$

$C_2 : (-1, 1), r_2 = 2$

Here, $C_1C_2 = r_1 + r_2$

So, the point P divides C_1C_2 in the ratio of 3 : 2



$P\left(\frac{1}{5}, \frac{-3}{5}\right)$

Slope of line passing through C_1 and C_2 is $\frac{1+3}{-1-2} = \frac{4}{-3} \Rightarrow P : \left(\frac{1}{5}, -\frac{3}{5}\right)$ and here only one transverse common tangent which is perpendicular to C_1C_2

$\Rightarrow$ slope of tangent $= -\frac{1}{-\frac{4}{3}} = \frac{3}{4}$

$\because$ Tangent is also passes through $\left(\frac{1}{5}, -\frac{3}{5}\right)$

So, equation of tangent is

$y + \frac{3}{5} = \frac{3}{4}\left(x - \frac{1}{5}\right) \Rightarrow 3x - 4y - 3 = 0$

On comparing with

$ax + by + c = 0, \frac{a}{c} = -1$

Question43

A circle $S \equiv x^2 + y^2 + 2gx + 2fy + 6 = 0$ cuts another circle $x^2 + y^2 - 6x - 6y - 6 = 0$ orthogonally. If the angle between the circles $S = 0$ and $x^2 + y^2 + 6x + 6y + 2 = 0$ is 60°, then the radius of the circle $S = 0$ is

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. 2

B. 1

C. 4

D. 5

Answer: A

Solution:

We have,

$S \equiv x^2 + y^2 + 2gx + 2fy + 6 = 0$

cuts another circle

$x^2 + y^2 - 6x - 6y - 6 = 0$ orthogonally.

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$2g(-3) + 2f(-3) = 6 - 6 \Rightarrow g + f = 0$

and angle between $S = 0$ and

$x^2 + y^2 + 6x + 6y + 2 = 0$ is 60°.

The condition for angle θ between two circles is given by

$$\cos\theta = \frac{|2(g_1g_2 + f_1f_2) - (c_1 + c_2)|}{2\sqrt{r_1^2 r_2^2}}$$

$$\cos 60^\circ = \frac{2(3g + 3f) - (6 + 2)}{2\sqrt{4^2 \cdot r^2}}$$

(Let radius of $\dot{s} = 0$ is r)

$$\frac{1}{2} = \frac{|2 \times 3 \times 0 - 8|}{2 \times 4 \times r} (\because g + f = 0)$$

$4r = g \Rightarrow r = 2$

Question44

If m_1 and m_2 are the slopes of the direct common tangents drawn to the circles $x^2 + y^2 - 2x - 8y + 8 = 0$ and $x^2 + y^2 - 8x + 15 = 0$, then $m_1 + m_2 =$

TG EAPCET 2024 (Online) 9th May Morning Shift

Options:

A. $-\frac{24}{5}$

B. $\frac{12}{5}$

C. $\frac{24}{5}$

D. $-\frac{12}{5}$

Answer: A

Solution:

We have,

$C_1 : x^2 + y^2 - 2x - 8y + 8 = 0 \quad \ldots\ldots.(i)$

Centre $(1, 4), r_1 = 3$

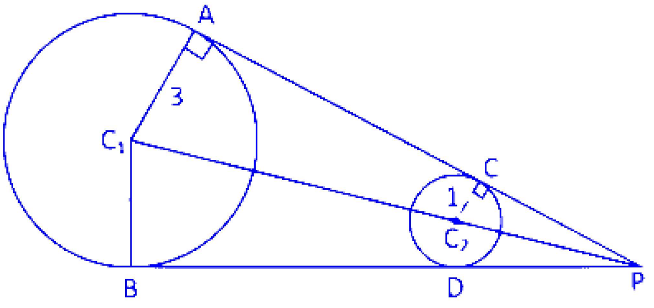
$C_2 : x^2 + y^2 - 8x + 15 = 0 \quad \ldots\ldots.(ii)$

Centre $(4, 0), r_2 = 1$

As $\frac{C_1P}{C_2P} = \frac{C_1A}{C_2C} = \frac{3}{1}$

$\Rightarrow P_x = \frac{12-1}{2} = \frac{11}{2} \quad$ (external division)

$P_y = \frac{0-4}{2} = -2$



Let slope of tangent AP be m.

Then, equation of AP is :

$y + 2 = m\left(x - \frac{11}{2}\right)$

$2mx - 2y - (4 + 11m) = 0$

$\because$ Perpendicular distance of C_2 from AP is 1 .

$\therefore \left|\frac{2m\times4-2\times0-(4+11m)}{\sqrt{(2m)^2+(-2)^2}}\right| = 1$

$(3m + 4)^2 = 4m^2 + 4$

$5m^2 + 24m + 12 = 0$

So, $m_1 + m_2 = -\frac{24}{5}$ where m_1 and m_2 are slope of AP and BP.

Question45

If a circle passing through $(1, -2)$ has $x - y = 2$ and $2x + 3y = 14$ as its diameters, then the radius of the circle is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. 2

B. 3

C. 4

D. 5

Answer: D

Solution:

The given two diameters are,

$x - y = 2$
$2x + 3y = 14$

Now, solving these equations, we get

$x = 4$ and $y = 2$

So, centre of circle is $(4, 2)$.

Now, equation of circle is

$(x-4)^2 + (y-2)^2 = r^2$

where, ' r ' is radius.

Since, it also passes through $(1, -2)$, then

$$\begin{aligned} (1-4)^2 + (-2-2)^2 &= r^2 \\ \Rightarrow \quad 9 + 16 &= r^2 \\ \Rightarrow \quad r^2 &= 25 \\ \Rightarrow \quad r &= 5 \end{aligned}$$

Question46

The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $2x^2 + 2y^2 + 2x + 6y + 1 = 0$

B. $x^2 + y^2 - 2x + 3y - 1 = 0$

C. $x^2 + y^2 + 2x + 3y - 4 = 0$

D. $2x^2 + 2y^2 - x + 2y + 1 = 0$

Answer: A

Solution:

To find the equation of the circle whose diameter is the common chord of two given circles, we start by considering the equations of the circles:

$S_1 = x^2 + y^2 + 2x + 3y + 1 = 0 \quad \text{(Equation 1)}$

$S_2 = x^2 + y^2 + 4x + 3y + 2 = 0 \quad \text{(Equation 2)}$

The common chord of these two circles is also known as the radical axis, which can be found by subtracting the second equation from the first:

$S_1 - S_2 = 0$

This results in:

$(x^2 + y^2 + 2x + 3y + 1) - (x^2 + y^2 + 4x + 3y + 2) = 0$

Simplifying this, we obtain:

$2x + 1 = 0$

$\Rightarrow x = -\frac{1}{2}$

Now, considering this line as $S_3 = 2x + 1 = 0$.

To find the circle that passes through the points of intersection of S_1 and S_3, its general equation is:

$S_1 + \lambda S_3 = 0$

Substituting the values, we have:

$(x^2 + y^2 + 2x + 3y + 1) + \lambda(2x + 1) = 0$

Simplifying further:

$x^2 + y^2 + (2 + 2\lambda)x + 3y + (1 + \lambda) = 0$

The center of this circle is $(-1 - \lambda, \frac{3}{2})$.

The line $2x + 1 = 0$ will be a diameter if the center of our circle lies on this line. So, we solve:

$2(-1 - \lambda) + 1 = 0$

$-2 - 2\lambda + 1 = 0$

$-2\lambda - 1 = 0$

$\lambda = -\frac{1}{2}$

Substituting $\lambda = -\frac{1}{2}$ back into our circle equation gives:

$x^2 + y^2 + (2 - 1)x + 3y + \frac{1}{2} = 0$

$\Rightarrow 2(x^2 + y^2) + 2x + 6y + 1 = 0$

Therefore, the equation of the circle whose diameter is the common chord is:

$2x^2 + 2y^2 + 2x + 6y + 1 = 0$

Question47

The number of common tangents to the circles $x^2+y^2-2x-6y+9=0$ and $x^2+y^2+6x-2y+1=0$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

The equation of circle are

$x^2+y^2-2x-6y+9=0$

and $x^2+y^2+6x-2y+1=0$

Centre and radii of the circles are $c_1(1,3)$ and $r_1=1$

and $c_2=(-3,1)$ and $r_2=3$

Clearly,

$$c_1c_2=\sqrt{(1+3)^2+(3-1)^2}=\sqrt{16+4}=\sqrt{20}$$
$$r_1+r_2=4=\sqrt{16}$$

Thus, $c_1c_2>r_1+r_2$

Hence, the circles are non-intersecting. Thus, there will be four common tangents.

Transverse common tangents are tangents drawn from the point P which divides c_1, c_2 internally in the ratio of radii 1: 3 coordinate of P are

$$\left(\frac{1\cdot(-3)+3\cdot1}{1+3}, \frac{1\cdot1+3\cdot3}{1+3}\right)=\left(0,\frac{5}{2}\right)$$

Equation of tangent through the point $P\left(0,\frac{5}{2}\right)$.

Any line through $P\left(0,\frac{5}{2}\right)$ is

$y-\frac{5}{2}=m(x-0)\Rightarrow mx-y+\frac{5}{2}=0$

Now, $\frac{m\cdot1-3+\frac{5}{2}}{\sqrt{m^2+1}}=1$

$\Rightarrow\quad\left(m-\frac{1}{2}\right)=\sqrt{m^2+1}$

$\Rightarrow m^2+\frac{1}{4}-m=m^2+1\Rightarrow m=\frac{-3}{4}$ and '∞'

Thus, $x=0$ is a tangent and $y-\frac{5}{2}=-\frac{3x}{4}$

$\Rightarrow 3x+4y-10=0$

is another tangent.

Direct common tangents are tangents drawn from the point Q which divides c_1c_2 externally in the ratio 1 : 3.

Direct tangent, are tangent drawn from the point $Q(3,4)$.

Now, proceeding as for transverse tangents their equations are $y=4$,

$4x-3y=0$

Thus, the number of common tangent are ' 4 '.

Question48

The pole of the straight line $9x+y-28=0$ with respect to the circle $2x^2+2y^2-3x+5y-7=0$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $(3,1)$

B. $(-3,1)$

C. $(-2,1)$

D. $(3,-1)$

Answer: D

Solution:

The given straight line,

$$9x+y-28=0 \qquad \ldots(i)$$

and the given circle is

$$\begin{aligned} & 2x^2+2y^2-3x+5y-7=0 \\ \text{i.e.} \quad & x^2+y^2-\frac{3}{2}x+\frac{5}{2}y-\frac{7}{2}=0 \qquad \ldots(ii) \end{aligned}$$

Let, (h,k) be the pole of the given line Eq. (i) w.r.t. the circle Eq. (ii).

The equation of the polar

$$\begin{aligned} & hx+ky-\frac{3}{4}(x+h)+\frac{5}{4}(y+k)-\frac{7}{2}=0 \\ & \Rightarrow x\left(h-\frac{3}{4}\right)+y\left(k+\frac{5}{4}\right)-\frac{3}{4}h+\frac{5}{4}k-\frac{7}{2}=0 \\ & \Rightarrow x(4h-3)+y(4k+5)-3h+5k-14=0 \end{aligned}$$

This equation and $9x+y-28=0$ represent same line so,

$$\begin{aligned} & \frac{4h-3}{9}=\frac{4k+5}{1}=\frac{-3h+5k-14}{-28}=\lambda(\text{ say }) \\ \Rightarrow \quad & h=\frac{9\lambda+3}{4}, k=\frac{\lambda-5}{4}, -3h+5k-14 \\ & =-28\lambda \end{aligned}$$

Now,

$$-3\left(\frac{9\lambda+3}{4}\right)+5\left(\frac{\lambda-5}{4}\right)-14=-28\lambda$$

$\Rightarrow \quad -9-27\lambda+5\lambda-25-56=-112\lambda$

$\Rightarrow -22\lambda-90=-112\lambda \Rightarrow 90\lambda=90$

$\Rightarrow \quad \lambda=1$

Thus, $h=\frac{3+9.1}{4}=\frac{12}{4}=3$

$$k=\frac{1-5}{4}=\frac{-4}{4}=-1$$

$\therefore (h,k)=(3,-1)$

Question49

The equation of the line perpendicular to the radical axis of two circles $x^2+y^2-5x+6y+12=0$, $x^2+y^2+6x-4y-14=0$ and passing through $(1,1)$ is

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $2x+3y-5=0$

B. $x+y-2=0$

C. $10x+11y-21=0$

D. $11x+10y-21=0$

Answer: C

Solution:

The given equation of circles are

$S_1=x^2+y^2-5x+6y+12=0$

and $S_2=x^2+y^2+6x-4y-14=0$

Now, equation of radical axis of two circle is $S_1-S_2=0$

$$\Rightarrow \quad \left(x^2+y^2-5x+6y+12\right) - \left(x^2+y^2+6x-4y-14\right)=0$$

$\Rightarrow \quad -11x+10y+26=0$

$\Rightarrow \quad 11x-10y-26=0 \qquad \ldots(i)$

So, slope of line (i) is $\frac{11}{10}$ (say m_1).

Let slope of perpendicular line be m_2. Thus, $m_1m_2=-1 \Rightarrow m_2=-\frac{10}{11}$ Now, equation of line is $y=-\frac{10}{11}x+c$ which passes through $(1,1)$

$\Rightarrow \quad c=\frac{21}{11}$

Thus, the required equation of line is

$$y=-\frac{10}{11}x+\frac{21}{11}$$
$\Rightarrow 11y+10x-21=0$
$\Rightarrow 10x+11y-21=0$

Question50

If the angle between the circles

$x^2+y^2-2x-4y+c=0$ and $x^2+y^2-4x-2y+4=0$

is 60°**, then** $c=$

TS EAMCET 2023 (Online) 12th May Evening Shift

Options:

A. $\frac{3\pm\sqrt{5}}{2}$

B. $\frac{6\pm\sqrt{5}}{2}$

C. $\frac{7\pm\sqrt{5}}{2}$

D. $\frac{9\pm\sqrt{5}}{2}$

Answer: C

Solution:

The given equations of circles are,

$x^2+y^2-2x-4y+c=0 \qquad \ldots(i)$

and $x^2+y^2-4x-2y+4=0 \qquad \ldots(ii)$

From Eq. (i), center $c_1=(1,2)$ and radius $=\sqrt{5-c}=r_1$ (say)

From Eq. (ii), centre $c_2=(2,1)$ and radius $=1=r_2$ (say)

Now, $d=c_1c_2=\sqrt{(1-2)^2+(2-1)^2}=\sqrt{2}$

$\therefore \quad d^2=2$

Now, $\theta=60^\circ(given)$

$$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

$$\Rightarrow \cos 60^\circ = \frac{(5-c)+1-2}{2\sqrt{(5-c)}\times 1} \Rightarrow \frac{1}{2} = \frac{4-c}{2\sqrt{5-c}}$$

$$\Rightarrow \quad 5 - c = (4-c)^2$$

$$\Rightarrow \quad 5 - c = 16 - 8c + c^2$$

$$\Rightarrow \quad c^2 - 7c + 11 = 0$$

$$\because \quad c = \frac{7 \pm \sqrt{49-44}}{2} \Rightarrow c = \frac{7 \pm \sqrt{5}}{2}$$

Question51

If a diameter of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is a chord of a circle S whose centre is at $(-3, 2)$, then the radius of S is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $5\sqrt{3}$

B. $4\sqrt{3}$

C. $2\sqrt{3}$

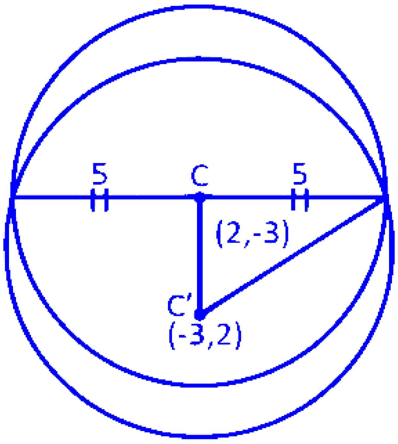
D. 5

Answer: A

Solution:

Given, equation of circle

$$x^2 + y^2 - 4x + 6y - 12 = 0$$
$$C \equiv (2, -3)$$
$$l(CC') = \sqrt{(-3-2)^2 + (2-(-3))^2}$$



$$= \sqrt{25+25} = 5\sqrt{2}$$
$$R^2 = (5\sqrt{2})^2 + 5^2 = 50 + 25 = 75 = 25 \times 3$$
$$R = 5\sqrt{3}$$

Question52

If a circle passing through $A(1,1)$ touches the X-axis, then the locus of the other end of the diameter through A is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $(x+1)^2 = 4y$

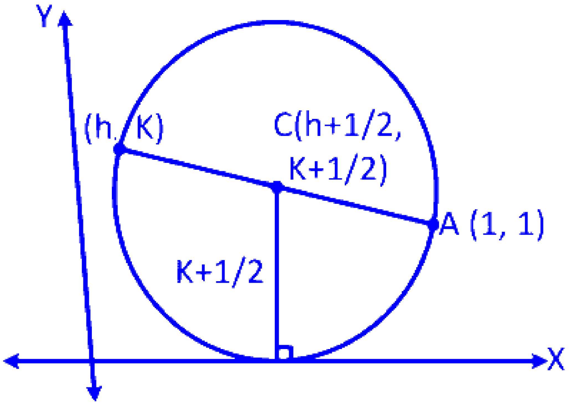
B. $(y-1)^2 = 4x$

C. $(x-1)^2 = 4y$

D. $(y+1)^2 = 4x$

Answer: C

Solution:



$$\therefore \quad \frac{k+1}{2} = \frac{1}{2}\sqrt{(h-1)^2 + (k-1)^2}$$
$$(k+1)^2 = (h-1)^2 + (k-1)^2$$
$$k^2 + 2k + 1 = (h-1)^2 + k^2 - 2k + 1$$
$$(h-1)^2 = 4k$$
$$\therefore \quad (x-1)^2 = 4y$$

Question53

If $C(\alpha, \beta)(a < 0)$ is the centre of the circle that touches the Y-axis at $(0,3)$ and makes an intercept of length 2 units on positive X-axis, then $(\alpha, \beta) =$

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $(-3, \sqrt{10})$

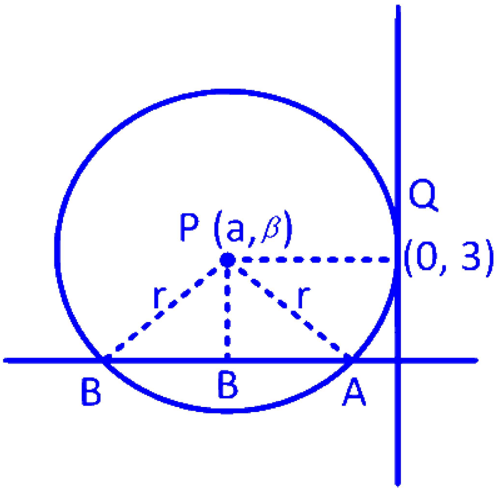
B. $(-3, -\sqrt{10})$

C. $(-\sqrt{10}, 3)$

D. $(-\sqrt{10}, -3)$

Answer: C

Solution:



$\because \alpha < 0 \Rightarrow$ centre of circle lies on 2nd quadrant

circle is touching Y-axis

$\Rightarrow \quad r = |\alpha| = -\alpha$
and $\quad \beta = 3$

Also, $AB = 2$

In $\triangle PQA$,
$PQ = 3, PA = r, AQ = 1$
$PQ^2 + AQ^2 = PA^2$
$9 + 1 = r^2 \Rightarrow r = \sqrt{10}$

(given)

From Eq. (i), we get $\alpha = -\sqrt{10}$

Centre $\equiv (-\sqrt{10}, 3)$

Question54

The equations of the tangents to the circle $x^2 + y^2 = 4$ drawn from the point $(4, 0)$ are

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $\sqrt{3}y = \pm(x - 4)$

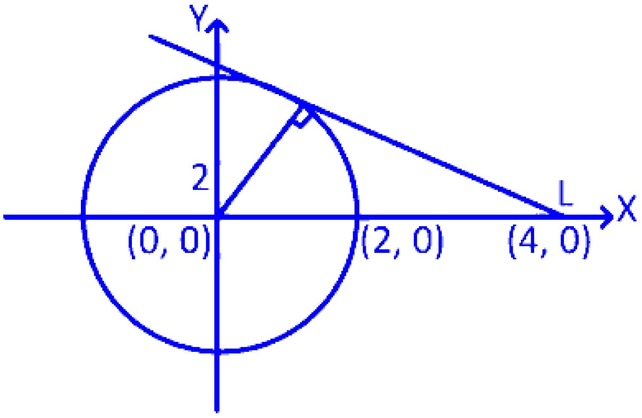
B. $\sqrt{3}y = \pm 2(x - 4)$

C. $\sqrt{3}x = \pm(y - 4)$

D. $\sqrt{3}x = \pm 2(y - 4)$

Answer: A

Solution:



Given equation of circle

$x^2 + y^2 = 4$

Let the slope of tangent be m

$$y = m(x - 4)$$
$$\Rightarrow y - mx + 4m = 0$$

Distance of L from origin will be 2 .

$$\frac{|4m|}{\sqrt{1+m^2}} \doteq 2$$

$$\Rightarrow \quad 16m^2 = 4m^2 + 4$$

$$\Rightarrow \quad 12m^2 = 4$$

$$\Rightarrow \quad m^2 = \frac{1}{3}$$

$$\Rightarrow \quad m = \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$$

$$\sqrt{3}y = \pm(x-4)$$

Question55

The image of every point lying on the curve $x^2+y^2=1$ in the line $x+y=1$ satisfies the equation

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $x^2+y^2+2x+2y+1=0$

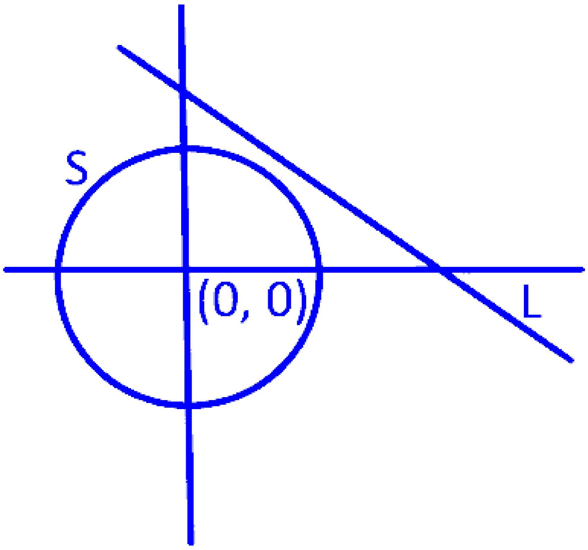
B. $x^2+y^2-2x+2y+1=0$

C. $x^2+y^2+2x-2y+1=0$

D. $x^2+y^2-2x-2y+1=0$

Answer: D

Solution:



To find image of circle in the line find the image of its centre and radius will be equal to radius of given circle.

$$\begin{aligned} \frac{x-x_1}{a} &= \frac{y-y_1}{b} \\ &= -\frac{2\left(a x_1+b y_1\right)+c}{a^2+b^2} \\ \frac{x}{1} &= \frac{y}{1}=-2 \frac{(-1)}{1+1} \\ x &= 1, y=1 \end{aligned}$$

Centre of image circle is $(1,1)$ and radius $=1$

$$\begin{aligned} & (x-1)^2+(y-1)^2=1^2 \\ \Rightarrow & x^2+y^2-2 x-2 y+1=0 \end{aligned}$$

Question56

If the inverse of $P(-3,5)$ with respect to a circle is $(1,3)$ then polar of P with respect to that circle is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. $x+2 y=7$

B. $2 x-2 y+4=0$

C. $2 x-y+1=0$

D. $2 x+y-5=0$

Answer: C

Solution:

Given that the point $Q(1,3)$ is the inverse of $P(-3,5)$ with respect to a circle, we need to determine the polar of point P with respect to the circle.

Since Q is the inverse of P, the line PQ is related to the polar we are looking for. To find the polar line, we first determine the slope of the line segment joining the points $P(-3, 5)$ and $Q(1, 3)$.

The slope m_1 of line PQ is calculated as:

$m_1 = \frac{5-3}{-3-1} = -\frac{1}{2}$

The polar of P will be perpendicular to this line, so its slope m_2 must satisfy the condition:

$m_1 \cdot m_2 = -1 \quad \Rightarrow \quad -\frac{1}{2} \cdot m_2 = -1 \quad \Rightarrow \quad m_2 = 2$

Using the slope $m_2 = 2$, we can write the equation of the line passing through $Q(1, 3)$ with this slope:

$y - 3 = 2(x - 1)$

Rearrange this equation:

$y - 3 = 2x - 2 \quad \Rightarrow \quad 2x - y + 1 = 0$

Thus, the equation of the polar of the point P with respect to the circle is:

$2x - y + 1 = 0$

Question57

If the tangent drawn at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the Y-axis, then the length of PQ is

TS EAMCET 2023 (Online) 12th May Morning Shift

Options:

A. 5

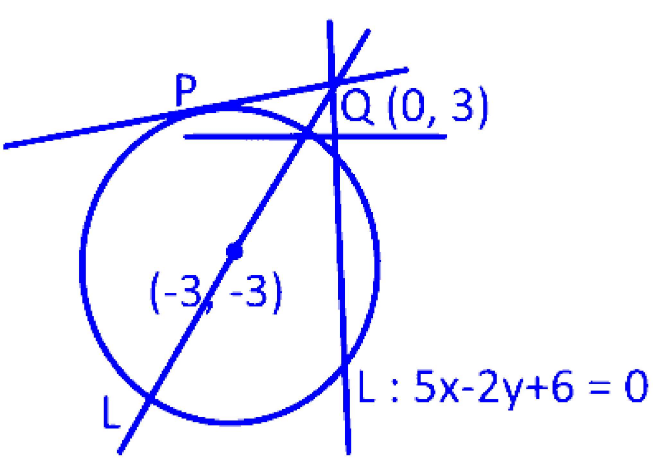
B. 4

C. 2

D. 1

Answer: A

Solution:



$L : 5x - 2y + 6 = 0$

Length of tangent from an external point is $\sqrt{S_1}$

$$\Rightarrow \quad PQ = \sqrt{0+9+0+18-2}$$
$$= \sqrt{25} = 5$$

Question58

If the angle between the circles $x^2 + y^2 - 2x + ky + 1 = 0$ and $x^2 + y^2 - kx - 2y + 1 = 0$ is $\cos^{-1}\left(\frac{1}{4}\right)$ and $k < 0$, then the point which lies on the radical axis of the given circle is

TG EAPCET 2025 (Online) 2nd May Morning Shift

Options:

A.

$(1, -3)$

B.

$(-1, 3)$

C.

$(-1, -3)$

D.

$(1, 3)$

Answer: A

Solution:

For $S_1 : x^2 + y^2 - 2x + ky + 1 = 0$,

Center $C_1 = \left(1, -\frac{k}{2}\right)$

Radius, $r_1 = \sqrt{1^2 + \left(-\frac{k}{2}\right)^2 - 1} = \sqrt{\frac{k^2}{4}} = \frac{k}{2}$

For $S_2 : x^2 + y^2 - kx - 2y + 1 = 0$

Center $C_2 = \left(\frac{k}{2}, 1\right)$

Radius, $r_2 = \sqrt{\left(\frac{k}{2}\right)^2 + 1^2 - 1}$

$\sqrt{\frac{k^2}{4}} = \frac{k}{2}$

Since, angle θ between two circles is given by $\cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}$, where d is the distance between the centers.

Given, $\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$$\Rightarrow \quad \cos\theta = \frac{1}{4}$$

$$\begin{aligned} d^2 &= \left(1-\frac{k}{2}\right)^2 + \left(\frac{-k}{2}-1\right)^2 \\ &= 1-k+\frac{k^2}{4}+1+k+\frac{k^2}{4} \\ &= 2+\frac{k^2}{2} \end{aligned}$$

Since $k < 0, r_1 = \frac{-k}{2}$ and $r_2 = \frac{-k}{2}$

Substituting into the formula

$$\frac{1}{4} = \frac{\left(2+\frac{k^2}{2}\right)-\left(\frac{-k}{2}\right)^2-\left(\frac{-k}{2}\right)^2}{2\left(\frac{-k}{2}\right)\left(\frac{-k}{2}\right)}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{2+\frac{k^2}{2}-\frac{k^2}{4}-\frac{k^2}{4}}{2\left(\frac{k^2}{4}\right)}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{4}{k^2}$$

$$\Rightarrow \quad k^2 = 16$$

$$\Rightarrow \quad k = \pm 4$$

But since $k < 0, k = -4$

Now, the equation of the radical axis is

$$S_1 - S_2 = 0$$
$$\left(x^2+y^2-2x+ky+1\right) - \left(x^2+y^2-kx\right.$$
$$\left.-2y+1\right) = 0$$
$$\Rightarrow -2x+ky+kx+2y = 0$$
$$\Rightarrow -6x-2y = 0 \quad (\because k = -4)$$
$$\Rightarrow 3x+y = 0$$

The points satisfying this equation are $(-1, 3)$ and $(1, -3)$ lies on the radical axis.
