Elasticity

Question1

A spring has length L and force constant K . It is cut into two springs of length L_1 and L_2 such that $\mathrm{L}_1=\mathrm{NL}_2$ (N is an integer). The force constant of spring of length L_1 is

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Options:

A. $(\mathrm{N}+1)\mathrm{K}$

B. $\frac{\mathrm{K}}{\mathrm{N}}(1+\mathrm{N})$

C. K

D. $\frac{\mathrm{K}}{\mathrm{N}+1}$

Answer: B

Solution:

Given:

$\mathrm{L}_1=\mathrm{NL}_2 \quad$ (Equation i)

The relationship between spring constant (k) and length (L) is:

$\mathrm{k} \propto \frac{1}{\mathrm{L}}$

From Equation (i), we can derive:

$\mathrm{k}_2=\mathrm{Nk}_1$

Before being cut, the springs are effectively in series, which implies:

$\frac{1}{\mathrm{K}}=\frac{1}{\mathrm{k}_1}+\frac{1}{\mathrm{k}_2}$

Substitute $\mathrm{k}_2=\mathrm{Nk}_1$ into the equation:

$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{Nk_1}$

$\frac{1}{K} = \frac{N+1}{Nk_1}$

Solving for k_1, we get:

$k_1 = \frac{K}{N}(1 + N)$

Question2

A metal rod has length, cross-sectional area and Young's modulus as L, A and Y, respectively. If the elongation in the rod produced is I, then work done is proportional to

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Options:

A. ℓ^4

B. ℓ

C. ℓ^3

D. ℓ^2

Answer: D

Solution:

The work done to elongate a metal rod can be calculated using the formula:

$W = \frac{1}{2} \frac{AY}{L} (\Delta L)^2$

Where:

W is the work done,

A is the cross-sectional area,

Y is Young's modulus,

L is the original length of the rod,

ΔL is the change in length, or elongation.

Given that $\Delta L = \ell$, we can rewrite the expression for work done as:

$W \propto \ell^2$

This shows that the work done is proportional to the square of the elongation, ℓ.

Question3

The compressibility of water is $5 \times 10^{-10}\ \mathrm{m}^2/\mathrm{N}$. Pressure of $15 \times 10^6\ \mathrm{Pa}$ is applied on 100 mL volume of water. The change in the volume of water is

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Options:

A. zero

B. 0.75 mL decrease

C. 0.75 mL increase

D. 1.50 mL increase

Answer: B

Solution:

The compressibility of a fluid, such as water, can be quantified using the formula:

$K = \frac{-\Delta V}{\Delta p\, V}$

Where:

K is the compressibility factor

ΔV is the change in volume

Δp is the change in pressure

V is the original volume

Given:

Compressibility of water, $K = 5 \times 10^{-10}\ \mathrm{m}^2/\mathrm{N}$

Pressure change, $\Delta p = 15 \times 10^6$ Pa

Initial volume, $V = 100\,\text{mL}$

To find the change in volume (ΔV) when the pressure is applied, we rearrange the formula as follows:

$\Delta V = -K \Delta p V$

Substituting the given values into the equation:

$\Delta V = -(5 \times 10^{-10} \times 15 \times 10^6 \times 100)$

Calculating this yields:

$\Delta V = -0.75\,\text{mL}$

Thus, the volume of water decreases by 0.75 mL when the specified pressure is applied.

Question4

Two wires of different materials have same length L and same diameter d. The second wire is connected at the end of the first wire and forms one single wire of double the length. This wire is subjected to stretching force F to produce the elongation I. The two wires have

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Options:

A. different stress but same strain

B. different stress and different strain

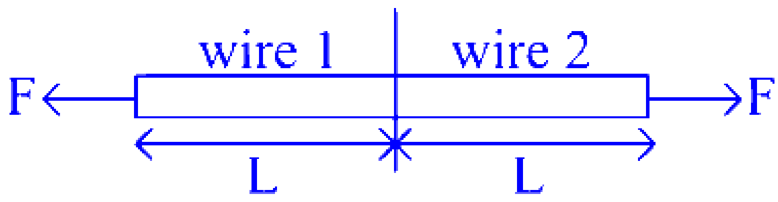
C. same stress but different strain

D. same stress and same strain

Answer: C

Solution:

Consider the figure shown below



$\text{Stress} = \frac{\text{Force}}{\text{Area}}$

As, force on both wire is same and their areas are also same.

Hence, the stress on both wire will be same.

The strain is given by

$\text{Strain} = \frac{\text{Stress}}{Y}$

where, Y is the Young's modulus.

For different material the value of Young's modulus is different.

Hence, strain for both wires are different.

Question5

Two wires A and B are stretched by the same load. The radius of wire A is double the radius of wire B. The stress on the wire B as compared to the stress on the wire A is

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Options:

A. half

B. equal

C. twice

D. four times

Answer: D

Solution:

Stress $= \frac{\text{Force}}{\text{Area}}$

The stress on wire A,

$(\text{Stress})_A = \frac{F}{\pi r_A^2} = \frac{F}{\pi(2r_B)^2}$ (given, $r_A = 2r_B$)

The stress on wire B,

$$(\text{ Stress })_B = \frac{F}{\pi r_B^2}$$

$$\therefore \quad \frac{(\text{ Stress })_B}{(\text{ Stress })_A} = \frac{\frac{F}{\pi r_B^2}}{\frac{F}{\pi(2r_B)^2}} = 4$$

$$\Rightarrow \quad (\text{ Stress })_B = 4(\text{Stress})_A$$

Question6

The density of a metal at normal pressure p is ρ. When it is subjected to an excess pressure, the density becomes ρ'. If K is the bulk modulus of the metal, then the ratio $\frac{\rho'}{\rho}$ is

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Options:

A. $\frac{1}{1-\frac{K}{p}}$

B. $1 + \frac{K}{p}$

C. $\frac{1}{1-\frac{p}{K}}$

D. $1 + \frac{p}{K}$

Answer: C

Solution:

The bulk modulus of metal is

$K = -\frac{\Delta p}{\frac{\Delta V}{V}}$ or $\Delta V = -\frac{\Delta p V}{K}$

New volume of metal, $V' = V + \Delta V$

$= V - \frac{\Delta p V}{K} = V\left(1 - \frac{\Delta p}{K}\right)$

Since, mass of metal remains same,

$$\rho V = p' V'$$
$$\Rightarrow \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V\left(1 - \frac{\Delta p}{K}\right)} = \frac{1}{\left(1 - \frac{\Delta p}{K}\right)}$$

When $\Delta p = p$, then $\frac{\rho'}{\rho} = \frac{1}{\left(1 - \frac{p}{K}\right)}$

Question7

Two rods of same material and volume having circular cross-section are subjected to tension T. Within the elastic limit, same force is applied to both the rods. Diameter of the first rod is half of the second rod, then the extensions of first rod to second rod will be in the ratio

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Options:

A. 4 : 1

B. 32 : 1

C. 16 : 1

D. 2 : 1

Answer: C

Solution:

Let's start by understanding the basic relationships in play when a force is applied to rods within the elastic limit. The extension (ΔL) seen in the rod when a tensile force is applied can be described by Hooke's law for elastic deformation, which is given by:

$\Delta L = \frac{FL}{AE}$

where:

- F is the force applied,
- L is the original length of the rod,
- A is the cross-sectional area of the rod,
- E is the Young's modulus of the material (a property of the material).

Given that both rods are of the same material, the value of E is constant for both. Additionally, since both rods are subjected to the same tensile force (T), F is constant as well. The question tells us that the volume of both rods is the same, and the diameter of the first rod is half that of the second rod. Let's denote the diameter of the first rod as D and that of the second rod as $2D$.

The cross-sectional area of a rod with a circular cross-section is given by $A = \pi\left(\frac{D}{2}\right)^2$. So, for the first rod, the area A_1 is:

$$A_1 = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$

And for the second rod, the area A_2 is:

$$A_2 = \pi\left(\frac{2D}{2}\right)^2 = \pi D^2$$

Since the volume of both rods is the same and they are made up of the same material, their volume is $V = AL$, indicating that their length and area are inversely proportional. If the diameter of the second rod is twice that of the first, its cross-sectional area is quadruple the first rod's area, which implies that the length of the first rod is four times that of the second to maintain the same volume for both rods.

Therefore, the length of the first rod $L_1 = 4L_2$, where L_2 is the length of the second rod.

Substituting these values back into the extension formula and comparing the extensions of the first and second rods, we get:

$$\Delta L_1 = \frac{T \times 4L_2}{\frac{\pi D^2}{4} E} = \frac{16TL_2}{\pi D^2 E}$$

$$\Delta L_2 = \frac{T \times L_2}{\pi D^2 E}$$

Comparing ΔL_1 to ΔL_2, we find:

$$\frac{\Delta L_1}{\Delta L_2} = \frac{\frac{16TL_2}{\pi D^2 E}}{\frac{TL_2}{\pi D^2 E}} = 16$$

Therefore, the correct ratio of extension of the first rod to the second rod is $16 : 1$, which corresponds to Option C.

Question8

For homogeneous isotropic material, which one of the following cannot be the value of Poisson's ratio?

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Options:

A. 0.1

B. -1

C. 0.5

D. 0.8

Answer: D

Solution:

The Poisson's ratio for homogeneous isotropic material must be between -1.0 to +0.5 because of the requirement. For Young's modulus, the shear modulus and bulk modulus to have positive values.

Hence, 0.8 cannot be the value of Poisson's ratio for homogeneous isotropic material.

Question9

A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in length is I. If another wire of the same material but double the length and radius is stretched with a force $2F$, then increase in length is

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Options:

A. $\frac{l}{4}$

B. $2l$

C. $\frac{l}{2}$

D. l

Answer: D

Solution:

Let’s analyze the given situation with the two wires made from the same material:

Initial Wire:

Length, L

Radius, r

Force applied, F

Increase in length (elongation), I

Using Young’s modulus (Y), we know:

$$Y = \frac{\frac{F}{\pi r^2}}{\frac{I}{L}} = \frac{FL}{\pi r^2 I} \quad \text{(Equation 1)}$$

Second Wire:

Length, $L' = 2L$

Radius, $r' = 2r$

Force applied, $F' = 2F$

The elongation for the second wire is denoted as I'.

The Young’s modulus for the second wire is:

$$Y = \frac{\frac{F'}{\pi (r')^2}}{\frac{I'}{L'}} = \frac{F'L'}{\pi (r')^2 I'}$$

Substituting the given dimensions and force:

$$Y = \frac{2F \cdot 2L}{\pi (2r)^2 \cdot I'} = \frac{4FL}{4\pi r^2 \cdot I'}$$

Simplifying gives:

$$Y = \frac{FL}{\pi r^2 I'} \quad \text{(Equation 2)}$$

Comparing Equation 1 and Equation 2:

Both equations are equal since the material's Young's modulus Y remains constant, giving us:

$$\frac{FL}{\pi r^2 I} = \frac{FL}{\pi r^2 I'}$$

Hence, $I = I'$.

Therefore, the increase in length of the second wire is the same as the first, making the final elongation $I' = I$.

Question10

A wire of length ' L ' and area of cross section ' A ' is made of material of Young's modulus ' r. It is stretched by an amount ' x '.

The work done in stretching the wire is

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Options:

A. $\frac{Y x^2 A}{2L}$

B. $\frac{2 Y x^2 A}{L}$

C. $\frac{Y \times A}{2L}$

D. $\frac{Y x^2 A}{2}$

Answer: A

Solution:

If a force F is applied along the length L of wire for stretching by an amount x, then Young's modulus is given by

$Y = \frac{\text{stress}}{\text{strain}} = \frac{FL}{Ax}$

where, $A =$ area of cross-sectional $\Rightarrow F = \frac{YA}{L} x$

The work done in stretching the wire is given by

$W = \int_0^x F \cdot dx = \int_0^x \frac{YA}{L} x = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^x = \frac{YAx^2}{2L}$

Question11

A lift is tied with thick iron ropes having mass ' M '. The maximum acceleration of the lift is ' a ' $\mathrm{m/s^2}$ and maximum safe stress is ' S ' $\mathrm{N/m^2}$. The minimum diameter of the rope is

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Options:

A. $\left[\frac{6M(g+a)}{\pi S}\right]^{\frac{1}{2}}$

B. $\left[\frac{4M(g+a)}{\pi S}\right]^{\frac{1}{2}}$

C. $\left[\frac{M(g+a)}{\pi S}\right]^{\frac{1}{2}}$

D. $\left[\frac{M(g-a)}{\pi S}\right]^{\frac{1}{2}}$

Answer: B

Solution:

The maximum stress produced in a rope is given by

$\sigma_{\max}=\frac{\text{Force}}{\text{Area}}=\frac{Mg}{\pi r^2}$

As the lift is accelerating with acceleration a, then

$$\sigma_{\max}=\frac{M(g\pm a)}{\pi r^2}$$
$$\Rightarrow \quad r^2=\frac{M(g\pm a)}{\pi S} \quad [\text{Given}, \sigma_{\max}=S\,]$$
$$\frac{d^2}{4}=\frac{M(g\pm a)}{\pi S} \quad \left[\because r=\frac{d}{2}\right]$$
$$\Rightarrow d=\sqrt{\frac{4M(g\pm a)}{\pi S}}$$

As acceleration is maximum i.e., $g'=g+a$, so

$$d=\sqrt{\frac{4M(g+a)}{\pi S}}$$

Question12

Two identical wires of substances ' P ' and ' Q ' are subjected to equal stretching force along the length. If the elongation of ' Q ' is more than that of ' P ', then

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Options:

A. both P and Q are equally elastic

B. P is more elastic than Q

C. P is plastic and Q is elastic

D. Q is more elastic than P

Answer: B

Solution:

To analyze the problem, we can use the relation that describes the extension of a wire when a force is applied. For a wire of initial length L_0, cross-sectional area A, and Young's modulus Y, the extension ΔL is given by:

$\Delta L = \frac{FL_0}{AY}$

Here's a step-by-step breakdown:

Since both wires are identical in dimensions and are subjected to the same force F, the other factors (L_0 and A) remain the same for both.

Therefore, the extension ΔL depends only on the modulus of elasticity Y. The relationship shows that:

$\Delta L \propto \frac{1}{Y}$

Given that the elongation of substance Q is more than that of P, it means:

$\frac{1}{Y_Q} > \frac{1}{Y_P}$

or equivalently,

$Y_Q < Y_P$

A higher modulus of elasticity indicates that the material is stiffer, or in other words, more elastic in the sense that it is less easily deformed.

Thus, since wire P has a higher Young's modulus than wire Q, it is more elastic (stiffer) compared to wire Q.

Therefore, the correct choice is:

Option B: P is more elastic than Q.

Question13

Work done in stretching a wire through 1 mm is 2 J . What amount of work will be done for elongating another wire of same material, with half the length and double the radius of cross section, by 1 mm ?

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Options:

A. 2 J

B. 4 J

C. 8 J

D. 16 J

Answer: D

Solution:

Given that, work done, $W = 2$ J

As, we have two wires of same material and they are elongated to the same length.

So, this means, their Young's modulus (Y) is same and $\Delta I_1 = \Delta I_2 = \Delta I$ (say)

The work done in stretching a wire,

$W = \frac{1}{2} F \Delta I$

Now, for both the wires we have

$$\frac{W_1}{W_2} = \frac{\frac{1}{2} F_1 \Delta l}{\frac{1}{2} F_2 \Delta l}$$
$$\Rightarrow \quad W_2 = 2 \times \frac{F_2}{F_1} \quad .... \text{(i)}$$

The forces by which the wires are stretched can be expressed as the ratio,

$$\frac{F_1}{F_2} = \frac{Y A_1 \frac{\Delta I}{L_1}}{Y A_2 \frac{\Delta I}{L_2}}$$
$$= \frac{r_1^2 L_2}{r_2^2 L^2} \quad ... \text{(ii)}$$

Given, $r_2 = 2r_1$ and $L_2 = \frac{L_4}{2}$

Now, substituting the given values in Eq. (ii), we get

$\frac{F_1}{F_2} = \frac{r_1^2 L_1}{4 r_1^2 2 L_2} = \frac{1}{8} \quad .... \text{(iii)}$

From Eqs. (i) and (iii), we get

$$W_2 = 2 \times 8$$
$$= 16 \text{ J}$$
