Gravitation

Question1

A uniform sphere has radius ' R ' and mass ' M '. The magnitude of gravitational field at distances ' r_1 ' and ' r_2 ' from the centre of the sphere are ' E_1 ' and ' E_2 ' respectively. The ratio $E_1 : E_2$ is $(r_1 > R$ and $\left.r_2

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Options:

A.

$\frac{R^2}{r_1^2 r_2}$

B.

$\frac{R^3}{r_1 r_2}$

C.

$\frac{R^3}{r_1^2 r_2}$

D.

$\frac{R^3}{r_1 r_2^2}$

Answer: C

Solution:

Step 1: Gravitational field outside the sphere

For $r_1 > R$, the gravitational field is as if all mass is concentrated at the center:

$E_1 = \frac{GM}{r_1^2}$

Step 2: Gravitational field inside the sphere

For $r_2 < R$, the gravitational field is due only to the mass inside radius r_2.

The enclosed mass is

$$M_{enclosed} = M \cdot \frac{r_2^3}{R^3}$$

So the field is

$$E_2 = \frac{GM_{enclosed}}{r_2^2} = \frac{GM}{R^3} \cdot r_2$$

Step 3: Ratio

$$\frac{E_1}{E_2} = \frac{\frac{GM}{r_1^2}}{\frac{GM}{R^3} r_2} = \frac{GM}{r_1^2} \cdot \frac{R^3}{GM r_2} = \frac{R^3}{r_1^2 r_2}$$

Final Answer:

$$E_1 : E_2 = \frac{R^3}{r_1^2 r_2}$$

✅ Correct Option: **C**

Question2

The depth at which acceleration due to gravity becomes $\frac{g'}{n}$ is ($R =$ radius of earth, $\mathrm{g} =$ acceleration due to gravity) ($\mathrm{n} =$ integer)

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Options:

A.

$\frac{R(n-1)}{n}$

B.

$\frac{R(n+1)}{n}$

C.

$\frac{R(n-1)^2}{n}$

D.

$\frac{R(n+1)^2}{n}$

Answer: A

Solution:

Step 1: Relation between gravity and depth

At depth d below Earth's surface (assuming uniform density of Earth):

$g' = g\left(1 - \frac{d}{R}\right)$

where R = Earth's radius, g = acceleration due to gravity at surface.

Step 2: Condition given

We want:

$g' = \frac{g}{n}$

Thus:

$g\left(1 - \frac{d}{R}\right) = \frac{g}{n}$

Cancel g (non-zero):

$1 - \frac{d}{R} = \frac{1}{n}$

Step 3: Solve for depth d

$\frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n}$

So:

$d = \frac{R(n-1)}{n}$

Final Answer:

$$\boxed{\frac{R(n-1)}{n}}$$

Correct Option: A ✅

Question3

A body weighs 45 N on the surface of the earth. The gravitational force on a body due to earth at a height equal to half the radius of earth will be

Options:

A. 20 N

B. 22.5 N

C. 30 N

D. 36 N

Answer: A

Solution:

The gravitational force on a body is given by this formula:

$F = \frac{GMm}{r^2}$

On the Earth's surface, the distance from the center of Earth is R. So, the weight (force) is:

$W = \frac{GMm}{R^2} = 45 \text{ N}$

If the body is lifted to a height that equals half the Earth's radius ($h = \frac{R}{2}$), then the total distance from the center becomes:

Distance from center = $R + \frac{R}{2} = \frac{3R}{2}$

The new force of gravity at this distance is:

$F' = \frac{GMm}{(\frac{3R}{2})^2}$

This simplifies to:

$F' = \frac{GMm}{\frac{9R^2}{4}} = \frac{4}{9} \cdot \frac{GMm}{R^2}$

We already know that $\frac{GMm}{R^2} = 45$ N, so substitute this value:

$F' = \frac{4}{9} \times 45 = 20 \text{ N}$

Question4

The depth at which the value of acceleration due to gravity becomes $\left(\frac{1}{n}\right)$ times the value at the surface of the earth is

(R = radius of the earth)

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Options:

A. $\frac{R(n-1)}{n}$

B. $\frac{R(n+1)}{n}$

C. $\frac{\mathrm{Rn}}{(\mathrm{n}-1)}$

D. $\frac{R}{n}$

Answer: A

Solution:

Step 1: Gravity inside the Earth

For a uniform Earth of radius R,

$g(h)=g\left(1-\frac{h}{R}\right)$

where:

- g is surface gravity,
- h is depth below the surface.

Equivalently, at radial distance $r=R-h$:

$g(r)=g \cdot \frac{r}{R}$

Step 2: Set condition

We want:

$g(h)=\frac{1}{n} g$

So:

$g\left(1-\frac{h}{R}\right)=\frac{1}{n} g$

Cancel g:

$1-\frac{h}{R}=\frac{1}{n}$

Step 3: Solve for depth

$\frac{h}{R} = 1 - \frac{1}{n} = \frac{n-1}{n}$

$h = \frac{R(n-1)}{n}$

✅ **Final Answer:**

$\boxed{\frac{R(n-1)}{n}}$

Correct Option: A

Question5

Two planets A and B have densities ' ρ_1 ', ' ρ_2 ' and have radii ' r_1 ', ' r_2 ', respectively. The ratio of acceleration due to gravity on A to that of B is

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Options:

A. $r_1 : r_2$

B. $r_1\rho_1 : r_2\rho_2$

C. $r_1^2\rho_1 : r_2^2\rho_2$

D. $r_1\rho_2 : r_2\rho_1$

Answer: B

Solution:

We know,

$g = \frac{GM}{R^2}$

If the density is ρ, then

$\Rightarrow \mathrm{M} = \frac{4}{3}\pi\mathrm{R}^3\rho \quad \ldots(\because \mathrm{M} = \mathrm{V} \times \mathrm{d})$

$\therefore \quad \mathrm{g} = \frac{4}{3}\pi\mathrm{GR}\rho$

From given,

$$g_A = \frac{4}{3}\pi G r_1 \rho_1$$
$$g_B = \frac{4}{3}\pi G r_2 \rho_2$$

Therefore,

$$\frac{g_A}{g_B} = \frac{r_1}{r_2} \times \frac{\rho_1}{\rho_2} \Rightarrow \frac{g_A}{g_B} = \frac{r_1\rho_1}{r_2\rho_2}$$

Question6

The gravitational pull of the moon is $\left(\frac{1}{6}\right)^{th}$ of the earth and mass of moon is $\left(\frac{1}{8}\right)^{\text{th}}$ of the earth. This implies that the

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Options:

A. radius of moon is $(1/4)^{\text{th}}$ of the earth's radius.

B. radius of the earth is $(\sqrt{4/3})^{\text{th}}$ of the moon's radius.

C. moon's radius is half that of the earth.

D. radius of the earth is $(4/3)^{\text{th}}$ of the moon's radius.

Answer: B

Solution:

Gravitational pull depends upon the acceleration due to gravity on that planet.

$$M_m = \frac{1}{6}M_e, g_m = \frac{1}{8}g_e$$
$$\therefore \quad \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(6 \times \frac{1}{8}\right)^{1/2}$$
$$\therefore \quad R_m = \sqrt{\frac{4}{3}}R_e$$

Question7

A uniform solid sphere of mass ' m ' and radius ' r ' is surrounded by a uniform thin spherical shell of radius ' $2r$ ' and mass ' m ' then the gravitational field

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Options:

A.

at a distance of 15 r from the centre is

$\frac{2}{9} \frac{\mathrm{Gm}}{\mathrm{r}^2}$

B.

at a distance of (2.5)r from the centre is

$\frac{8}{25} \frac{\mathrm{Gm}}{\mathrm{r}^2}$

C. at a distance of (1.5)r from the centre is zero.

D. between the sphere and spherical shell is uniform.

Answer: B

Solution:

Gravitational field for $\mathrm{x} < \mathrm{r}$,

$g = \frac{Gm}{r^3} \times x$

Gravitational field for $\mathrm{r} < \mathrm{x} < 2\mathrm{r}$,

$\mathrm{g} = \frac{\mathrm{Gm}}{\mathrm{x}^2} \quad \ldots\ldots \left(g_{\text{shell}} = 0\right)$

Gravitational field for $x > 2r$,

$\mathrm{g} = \frac{2\mathrm{Gm}}{\mathrm{x}^2} \quad \ldots\ldots \left(\mathrm{m}_{\text{net}} = \mathrm{m}_{\text{sphere}} + \mathrm{m}_{\text{shell}} = 2\ \mathrm{m}\right)$

At a distance of 15 r ,

$g = \frac{2\mathrm{Gm}}{(15r)^2} = \frac{2}{225} \frac{\mathrm{Gm}}{\mathrm{r}^2}$

At a distance of 2.5 r ,

$g=\frac{2Gm}{(2.5r)^2}=\frac{2}{6.25}\frac{Gm}{r^2}=\frac{8}{25}\frac{Gm}{r^2}$

At a distance of 1.5r,

$g=\frac{Gm}{(1.5r)^2}=\frac{1}{2.25}\frac{Gm}{r^2}=\frac{4}{9}\frac{Gm}{r^2}$

$\therefore$ The correct option is (B).

Question8

The magnitude of gravitational potential energy of a body at a distance ' R ' from the centre of the earth is ' E '. Its weight at a distance ' 1.5 R ' from the centre of the earth is

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Options:

A. $\frac{2E}{9R}$

B. $\frac{4E}{5R}$

C. $\frac{4E}{9R}$

D. $\frac{2E}{7R}$

Answer: C

Solution:

Step 1: Recall expressions

- Gravitational potential energy of a body of mass m at distance r from Earth's centre:

$U=-\frac{GMm}{r}$.

So, the magnitude is:

$|U|=\frac{GMm}{r}$.

- Weight (force of gravitation) at distance r:

$F=\frac{GMm}{r^2}$.

Step 2: Relating to the given E

At $r = R$, the magnitude of potential energy:

$E = \frac{GMm}{R}$.

So:

$GMm = ER$.

Step 3: Force at $r = 1.5R$

At $1.5R$:

$F = \frac{GMm}{(1.5R)^2} = \frac{ER}{(1.5R)^2} = \frac{E}{(1.5^2 R)}$.

Since $1.5^2 = 2.25 = \frac{9}{4}$,

$F = \frac{E}{\frac{9}{4}R} = \frac{4E}{9R}$.

Step 4: Match with options

That corresponds to **Option C**:

$\boxed{\frac{4E}{9R}}$

Question9

Time period of a simple pendulum on earth's surface is ' T '. It time period becomes ' xT ' when taken to a height ' $2R$ ' above earth's surface. The value of x will be $(R =$ radius of earth)

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Options:

A. 2

B. 4

C. 1

D. 3

Answer: D

Solution:

Step 1: Formula for simple pendulum

On Earth's surface:

$T = 2\pi\sqrt{\frac{l}{g}}$

At height h:

$g' = g\left(\frac{R}{R+h}\right)^2$

So:

$T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g\left(\frac{R}{R+h}\right)^2}} = 2\pi\sqrt{\frac{l}{g} \cdot \left(\frac{R+h}{R}\right)^2}$

$T' = T \cdot \left(\frac{R+h}{R}\right)$

Step 2: Substitute $h = 2R$

$T' = T \cdot \left(\frac{R+2R}{R}\right) = T \cdot \frac{3R}{R} = 3T$

So

$x = 3$

✅ **Final Answer:**

Option D: 3

Question10

Two satellites P and Q go round a planet in circular orbits having radii ' $3R$ ' and ' R ' respectively. If the speed of satellite P is ' $2V$ ', the speed of the satellite Q will be

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Options:

A. $2\sqrt{3}$ V

B. $\frac{2 \text{ V}}{\sqrt{3}}$

C. $\frac{V}{2}$

D. $\frac{V}{\sqrt{3}}$

Answer: A

Solution:

Step 1: Expression for orbital velocity

The orbital velocity for a circular orbit at radius r:

$$v = \sqrt{\frac{GM}{r}}$$

Here GM is the planet's gravitational parameter.

Step 2: Relation between velocities

Let v_P = orbital speed of satellite P at radius $r_P = 3R$.

$$v_P = \sqrt{\frac{GM}{3R}}$$

Given $v_P = 2V$.

For satellite Q, orbit radius $r_Q = R$:

$$v_Q = \sqrt{\frac{GM}{R}}.$$

Step 3: Ratio of velocities

$$\frac{v_Q}{v_P} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{GM}{3R}}} = \sqrt{\frac{1}{1/3}} = \sqrt{3}.$$

So

$$v_Q = \sqrt{3}\, v_P.$$

Step 4: Substitute $v_P = 2V$

$$v_Q = \sqrt{3} \cdot 2V = 2\sqrt{3}\, V.$$

Final Answer:

The speed of satellite Q is:

$$\boxed{2\sqrt{3}\, V}$$

Correct option: A.

Question11

The radii of circular orbits of two satellites A and B of the earth are ' $4R$ ' and ' R ' respectively, where R is the radius of earth. If the speed of satellite B is 6 V , then the speed of satellite A will be

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Options:

A. 3 V

B. 4 V

C. 12 V

D. $\frac{3}{4}$ V

Answer: A

Solution:

We have two satellites A and B orbiting Earth in circular orbits.

- Radius of orbit of A: $r_A = 4R$
- Radius of orbit of B: $r_B = R$
- Speed of B: $v_B = 6V$

We need the speed of A.

Step 1: Orbital velocity formula

For a satellite in circular orbit,

$$v = \sqrt{\frac{GM}{r}}$$

where M is mass of Earth and r is the orbital radius.

So,

$$\frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}}$$

Step 2: Plug in given values

$\frac{v_A}{v_B} = \sqrt{\frac{R}{4R}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Thus,

$v_A = \frac{1}{2} v_B$

Step 3: Evaluate

$v_A = \frac{1}{2} \times 6V = 3V$

✅ **Final Answer:**

$\boxed{3V}$

Option A: **3V**

Question12

Water rises up to height ' x ' in a capillary tube immersed vertically in water. When the whole arrangement is taken to a depth ' d ' in a mine, the water level rises up to height Y . If R is the radius of the earth then the ratio $\mathrm{Y} : \mathrm{x}$ is

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Options:

A. $\mathrm{R} : (\mathrm{R} + \mathrm{d})$

B. $\mathrm{R} : (\mathrm{R} - \mathrm{d})$

C. $\mathrm{R} : (\mathrm{R} - \mathrm{d})^2$

D. $\mathrm{R} : (\mathrm{R} + \mathrm{d})^2$

Answer: B

Solution:

Step 1: Capillary Rise Formula

The height h to which a liquid rises in a capillary tube is given by:

$h = \frac{2T\cos\theta}{r\rho g}$

Where:

- T = Surface tension
- θ = Angle of contact
- r = Radius of capillary
- ρ = Density of liquid
- g = Acceleration due to gravity

Step 2: Effect of Gravitational Acceleration

At earth's surface:

- Let capillary rise be x.
- The value of gravity at the surface is g.

At depth d below earth's surface:

- The value of gravity g' reduces according to:

  $g' = g\left(1 - \frac{d}{R}\right)$

  Where R is the radius of the Earth.

Let the new height of water in the tube be Y.

Step 3: Write Heights at Surface and Depth

At surface:

$x = \frac{2T\cos\theta}{r\rho g}$

At depth:

$Y = \frac{2T\cos\theta}{r\rho g'}$

So,

$Y = \frac{2T\cos\theta}{r\rho\left[g\left(1-\frac{d}{R}\right)\right]} = \frac{2T\cos\theta}{r\rho g} \cdot \frac{1}{1-\frac{d}{R}}$

So,

$Y = x \cdot \frac{1}{1-\frac{d}{R}}$

Step 4: Find the Ratio $\frac{Y}{x}$

$\frac{Y}{x} = \frac{1}{1-\frac{d}{R}}$

Rewrite denominator:

$\frac{Y}{x} = \frac{1}{\frac{R-d}{R}} = \frac{R}{R-d}$

Step 5: Compare With Options

Thus, the correct ratio is:

$Y : x = R : (R - d)$

Final Answer:

Option B

$\boxed{\mathrm{R : (R - d)}}$

Question13

The total energy of a circularly orbiting satellite is

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Options:

A. half the kinetic energy of the satellite.

B. half the potential energy of the satellite.

C. twice the kinetic energy of the satellite.

D. twice the potential energy of the satellite.

Answer: B

Solution:

First, recall the NCERT results for a satellite of mass m orbiting a planet of mass M at radius r:

- Gravitational potential energy:

  $U = -\frac{GMm}{r}$

- Kinetic energy:

  $K = \frac{GMm}{2r}$

The total energy E is the sum of kinetic and potential energy:

$E = K + U$

Substitute the values:

$E = \frac{GMm}{2r} + (-\frac{GMm}{r})$

$E = \frac{GMm}{2r} - \frac{GMm}{r}$

$E = -\frac{GMm}{2r}$

Now compare E with K and U:

- $E = -\frac{GMm}{2r} = -K$
- $U = -\frac{GMm}{r} = 2E$

Therefore, the total energy is **half the potential energy** of the satellite.

Correct answer:

Option B: half the potential energy of the satellite.

Question14

The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value then its new time period will become

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Options:

A. 3 hours

B. 6 hours

C. 24 hours

D. 12 hours

Answer: A

Solution:

Let initial separation between Earth and satellite be r and its time period is $T = 24$ hours.

If the separation is decreased to one fourth, new separation $r' = \frac{r}{4}$.

From **NCERT**, the time period of a satellite in orbit at distance r from the center of Earth is

$T = 2\pi\sqrt{\frac{r^3}{GM}}$

where G is gravitational constant and M is mass of the Earth.

Let new time period be T':

$T' = 2\pi\sqrt{\frac{(r')^3}{GM}}$

Substitute $r' = \frac{r}{4}$:

$T' = 2\pi\sqrt{\frac{\left(\frac{r}{4}\right)^3}{GM}} = 2\pi\sqrt{\frac{r^3}{GM} \times \frac{1}{4^3}}$

$= 2\pi\sqrt{\frac{r^3}{GM} \times \frac{1}{64}}$

$= 2\pi\sqrt{\frac{r^3}{GM}} \times \frac{1}{8}$

But $2\pi\sqrt{\frac{r^3}{GM}} = T = 24$ hours

So,

$T' = T \times \frac{1}{8} = 24 \times \frac{1}{8} = 3$ hours

Correct option: A) 3 hours

Question15

A body is projected vertically from earth's surface with $\left(\frac{1}{3}\right)^{\text{rd}}$ of escape velocity. The maximum height reached by the body is ($R =$ radius of earth)

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Options:

A. $\frac{\mathrm{R}}{4}$

B. $\frac{\mathrm{R}}{8}$

C. $\frac{R}{9}$

D. $\frac{R}{6}$

Answer: B

Solution:

Given:

- The body is projected vertically upward with velocity $u = \frac{1}{3}v_{esc}$.
- R = radius of Earth.

Step 1: Escape velocity from Earth

Escape velocity from Earth is given by:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Step 2: Initial Kinetic Energy and Potential Energy

At Earth's surface:

- Potential Energy: $U_1 = -\frac{GMm}{R}$
- Kinetic Energy: $K_1 = \frac{1}{2}mu^2$

Total initial energy:

$$E_1 = K_1 + U_1 = \frac{1}{2}mu^2 - \frac{GMm}{R}$$

Step 3: At Maximum Height h

At maximum height h:

- Final velocity = 0 (momentarily stops)
- Distance from centre = $R + h$

Total energy at top:

$$E_2 = 0 - \frac{GMm}{R+h} = -\frac{GMm}{R+h}$$

Step 4: Conservation of Mechanical Energy

By conservation of energy,

$$E_1 = E_2$$

Substitute values:

$$\frac{1}{2}mu^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

Step 5: Substitute $u = \frac{1}{3}v_{esc}$

Recall $v_{esc}^2 = \dfrac{2GM}{R}$

So:

$$u^2 = \left(\frac{1}{3}v_{esc}\right)^2 = \frac{1}{9}v_{esc}^2 = \frac{1}{9}\left(\frac{2GM}{R}\right) = \frac{2GM}{9R}$$

So,

$$\frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{2GM}{9R}\right) = \frac{GMm}{9R}$$

Plug this into the energy equation:

$$\frac{GMm}{9R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$-\frac{GMm}{R} + \frac{GMm}{9R} = -\frac{GMm}{R+h}$$

Combine:

$$\frac{GMm}{9R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{GMm}{9R} - \frac{9GMm}{9R} = -\frac{GMm}{R+h}$$

$$\frac{GMm - 9GMm}{9R} = -\frac{GMm}{R+h}$$

$$\frac{-8GMm}{9R} = -\frac{GMm}{R+h}$$

Step 6: Simplify and Solve for h

Remove minus signs and GMm:

$$\frac{8}{9R} = \frac{1}{R+h}$$

Take reciprocal:

$$R + h = \frac{9R}{8}$$

$$h = \frac{9R}{8} - R = \frac{9R - 8R}{8} = \frac{R}{8}$$

Final Answer:

$$\boxed{\frac{R}{8}}$$

So, the correct option is **Option B**.

Question16

The escape velocity of a satellite from the surface of earth does NOT depend on

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Options:

A. mass of the earth.

B. mass of the object to be projected.

C. radius of the earth.

D. gravitational constant.

Answer: B

Solution:

The escape velocity v_{esc} from the surface of Earth is given by:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Where:

G = gravitational constant

M = mass of the Earth

R = radius of the Earth

- It **depends** on G (gravitational constant), M (mass of Earth) and R (radius of Earth).
- It **does not depend** on m (the mass of the object to be projected).

Correct answer:

Option B: mass of the object to be projected.

Question17

Two particles of equal mass ' m ' move in a circle of radius ' r ' under the action of their mutual gravitational attraction. The speed of each particle will be ($\mathrm{G}=$ Universal gravitational constant)

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Options:

A. $\sqrt{\frac{\mathrm{Gm}}{4 \mathrm{r}}}$

B. $\sqrt{\frac{\mathrm{Gm}}{\mathrm{r}}}$

C. $\sqrt{\frac{\mathrm{Gm}}{2 \mathrm{r}}}$

D. $\sqrt{\frac{4 \mathrm{Gm}}{\mathrm{r}}}$

Answer: A

Solution:

Let the two particles, each of mass m, move in a circle of radius r due to their mutual gravitational attraction.

Step 1: Gravitational Force

The gravitational force between the particles separated by distance $2r$ is:

$F=\frac{G m^2}{(2 r)^2}=\frac{G m^2}{4 r^2}$

Step 2: Required Centripetal Force

Each particle moves in a circle of radius r about the center of mass. The centripetal force for each is:

$F_{\text{centripetal}}=\frac{m v^2}{r}$

Step 3: Equate Gravitational and Centripetal Force

The only force providing this centripetal acceleration is the gravitational force:

$\frac{m v^2}{r}=\frac{G m^2}{4 r^2}$

Step 4: Solve for v

Divide both sides by m:

$\frac{v^2}{r}=\frac{G m}{4 r^2}$

Multiply both sides by r:

$v^2 = \frac{Gm}{4r}$

Take the square root:

$v = \sqrt{\frac{Gm}{4r}}$

So, the correct answer is:

$$\boxed{\sqrt{\frac{\mathrm{Gm}}{4\mathrm{r}}}}$$

Option A is correct.

Question18

A boy weighs 72 N on the surface of earth. The gravitational force on a body due to earth at a height equal to half the radius of earth will be

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Options:

A. 32 N

B. 48 N

C. 96 N

D. 162 N

Answer: A

Solution:

When calculating the gravitational force on a body at a height above Earth's surface, we use the formula for acceleration due to gravity at a height h equal to half the radius of Earth, $\left(h = \frac{R}{2}\right)$. The modified acceleration due to gravity, g_h, is expressed as:

$g_h = \left(\frac{n}{n+1}\right)^2 g$

Here, $n = 2$, because the height is half the Earth's radius. Substituting the value of n:

$g_h = \left(\frac{2}{2+1}\right)^2 g = \left(\frac{2}{3}\right)^2 g = \frac{4}{9} g$

The weight of the boy at this height, W_h, is calculated by multiplying his weight at Earth's surface by this modified gravitational acceleration:

$W_h = m \cdot g_h = \frac{4}{9} \times mg$

Given that $mg = 72\,\text{N}$, we substitute this into the equation:

$W_h = \frac{4}{9} \times 72 = 32\,\text{N}$

Thus, the gravitational force on the boy at this height is 32 N.

Question19

A satellite is orbiting just above the surface of the planet of density 'ρ' with periodic time 'T'. The quantity $\mathrm{T}^2\rho$ is equal to ($\mathrm{G} =$ universal gravitational constant)

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Options:

A. $\frac{4\pi^2}{G}$

B. $\frac{3\pi^2}{G}$

C. $\frac{3\pi}{G}$

D. $\frac{\pi}{G}$

Answer: C

Solution:

Time period of a nearby satellite is given by,

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$$\therefore \quad T^2 = 4\pi^2 \times \frac{R}{g}$$

But, $\mathrm{g} = \frac{4}{3}\pi\rho\mathrm{GR}$

$\therefore \quad T^2 = \frac{4\pi^2 R}{\frac{4}{3}\pi\rho G R}$

$\therefore \quad T^2\rho = \frac{3\pi}{G}$

Question20

The speed with which the earth would have to rotate about its axis so that a person on the equator would weigh $\frac{3}{5}$ th as much as at present weight is (g = gravitational acceleration, R = equatorial radius of the earth)

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Options:

A. $\sqrt{\frac{2g}{5R}}$

B. $\sqrt{\frac{3\mathrm{~g}}{5\mathrm{R}}}$

C. $\sqrt{\frac{5R}{3g}}$

D. $\sqrt{\frac{3}{5}\mathrm{gR}}$

Answer: A

Solution:

$$\begin{aligned} & \mathrm{g}' = \mathrm{g} - \mathrm{R}\omega^2\cos^2\theta \\ \therefore \quad & \mathrm{g}' = \mathrm{g} - \mathrm{R}\omega^2 \quad \ldots.(\because \theta = 0) \end{aligned}$$

$$\text{Given } \mathrm{g}' = \frac{3\mathrm{~g}}{5}$$

$$\therefore \quad \omega^2 = \frac{2g}{5R} \text{ or } \omega = \sqrt{\frac{2g}{5R}}$$

Question21

A simple pendulum has a periodic time ' T_1 ' when it is on the surface of earth of radius ' R '. Its periodic time is ' T_2 ' when it is taken to a height ' R ' above the earth's surface. The value of $\frac{T_2}{T_1}$ is

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Options:

A. $\sqrt{2}$

B. 1

C. 2

D. $\frac{1}{2}$

Answer: C

Solution:

$\mathrm{T}_1=2 \pi \sqrt{\frac{l}{\mathrm{g}}}$

At a height ' $h=2 R$ ' from earth's surface,

$$\mathrm{T}_2=2 \pi \sqrt{\frac{l}{\mathrm{g}_{\mathrm{h}}}}$$

$$\therefore \quad \frac{\mathrm{T}_2}{\mathrm{T}_1}=\sqrt{\frac{\mathrm{g}}{\mathrm{g}_{\mathrm{h}}}} \quad \ldots . \text{(i)}$$

Now, $\mathrm{g}_{\mathrm{h}}=\frac{\mathrm{GM}}{(\mathrm{R}+\mathrm{h})^2}$

$\therefore \quad \mathrm{g}_{\mathrm{h}}=\frac{\mathrm{GM}}{4 \mathrm{R}^2} \ldots .(\because \mathrm{R}+\mathrm{h}=2 \mathrm{R})$

$\therefore \quad g_h=\frac{g}{4}$

$\therefore \quad$ From equations (i) and (ii),

$$\frac{T_2}{T_1}=\sqrt{\frac{\mathrm{g}}{\mathrm{g} / 4}}=2$$

Question22

The minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$ is

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Options:

A. $\frac{5GMm}{6R}$

B. $\frac{2GMm}{3R}$

C. $\frac{GMm}{2R}$

D. $\frac{GMm}{3R}$

Answer: A

Solution:

Orbital energy, $E_0 = \frac{-GMm}{2(R+h)}$

$\therefore \quad E_0 = \frac{-GMm}{2(R+2R)} = \frac{-GMm}{6R} \quad \ldots(\because\ h = 2R)$

Energy at surface $E = \frac{-GMm}{R}$

$$\begin{aligned} \therefore \quad \text{Min. energy required } &= E_0 - E \\ &= \frac{-GMm}{6R} - \left(\frac{-GMm}{R}\right) \\ &= \frac{5GMm}{6R} \end{aligned}$$

Question23

The density of a new planet is twice that of earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of earth. If R is the radius of earth, then radius of the planet would be

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Options:

A. 4 R

B. R/2

C. $\frac{\mathrm{R}}{4}$

D. $2R$

Answer: B

Solution:

Given, $\rho_{\mathrm{p}}=2\rho_{\mathrm{e}}, \mathrm{g}_{\mathrm{p}}=\mathrm{g}_{\mathrm{e}}$

$g=\frac{4}{3}\pi\rho GR$

$$\therefore \quad \frac{R_p}{R_e}=\left(\frac{g_p}{g_e}\right)\left(\frac{\rho_e}{\rho_p}\right)=(1)\times\left(\frac{1}{2}\right)$$

$$\therefore \quad R_p=\frac{R_e}{2}=\frac{R}{2}$$

Question24

The weights of an object are measured in a coal mine of depth ' h_1 ', then at sea level of height ' h_2 ' and lastly at the top of a mountain of height ' h_3 ' as W_1, W_2 and W_3 respectively. Which one of the following relation is correct?[h $h_1 \ll R, h_3 \gg h_2=R, R=$ radius of the earth]

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Options:

A. $\mathrm{W}_1=\mathrm{W}_2=\mathrm{W}_3$

B. $\mathrm{W}_1<\mathrm{W}_2<\mathrm{W}_3$

C. $W_1 > W_2 < W_3$

D. $W_1 < W_2 > W_3$

Answer: D

Solution:

Weight $= mg$
Value of ' g ' is maximum on the surface of earth. As we go above or below the surface of earth, value of ' g ' decreases
$\therefore \quad W_2 > W_1$ and $W_2 > W_3$
or $W_1 < W_2 > W_3$

Question25

A satellite of mass ' m ' is revolving around the earth of mass ' M ' in an orbit of radius ' r ' with constant angular velocity ' ω '. The angular momentum of satellite is($G =$ Universal constant of gravitation)

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Options:

A. $m(GMr)^{3/2}$

B. $m(GMr)$

C. $m(GMr)^{1/2}$

D. $m(GMr)^{-1/2}$

Answer: C

Solution:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\therefore \quad V = \sqrt{\frac{GM}{r}}$$

$$L = mvr = m\sqrt{\frac{GM}{r}}r$$

$$= m(GMr)^{1/2}$$

Question26

For a satellite moving in an orbit around the earth at height ' h ' the ratio of kinetic energy to potential energy is

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Options:

A. 2 : 1

B. 1 : 2

C. 1 : $\sqrt{2}$

D. $\sqrt{2}$: 1

Answer: B

Solution:

Let M_e = Mass of earth

m = Mass of satellite

R_e = Radius of earth

G = Gravitational constant

$\therefore$ Potential Energy,

$U = \frac{GM_e m}{R_e}$

Kinetic Energy,

$$K = \frac{1}{2}\frac{GM_e\, m}{R_e}$$

$$\frac{K}{U} = \frac{\frac{1}{2}\frac{GMM_e}{R_e}}{\frac{GM_e\, m}{R_e}}$$

$$\frac{K}{U} = \frac{1}{2}$$

Question27

A body is projected in vertically upward direction from the surface of the earth of radius ' R ' into space with velocity ' nV_e ' $(n < 1)$. The maximum height from the surface of earth to which a body can reach is

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Options:

A. $\frac{n^2R}{(1-n^2)}$

B. $\frac{n^2R^2}{(1-n)}$

C. $\frac{nR^2}{(1+n^2)}$

D. $\frac{n^2R^2}{(1+n)}$

Answer: A

Solution:

As per the law of conservation of energy,

$$(K.E + P.E)_{\text{surface}} = (K.E + P.E)_{\text{max height}}$$

$$\therefore \quad \frac{-GMm}{R} + \frac{1}{2}m(nV_e)^2 = -\frac{GMm}{R+h} + \frac{1}{2}m(0)^2$$

$$\therefore \quad \frac{1}{2} m(nV_e)^2 = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\therefore \quad (nV_e)^2 = 2GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$\therefore \quad (nV_e)^2 = V_e^2\left(\frac{h}{(R+h)}\right)$$

$$\therefore \quad \frac{1}{n^2} = \frac{R}{h} + 1$$

$$\therefore \quad \frac{R}{h} = \frac{1-n^2}{n^2}$$

$$\therefore \quad h = \frac{n^2 R}{(1-n^2)}$$

Question28

The acceleration due to gravity at the surface of the planet is same as that at the surface of the earth, but the density of planet is thrice that of the earth. If 'R' is the radius of the earth, the radius of the planet will be

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Options:

A. $\frac{R}{9}$

B. $\frac{R}{3}$

C. 3R

D. 9R

Answer: B

Solution:

$g = \frac{GM}{R^2} = \frac{4}{3}\pi R \rho G$

As, g is same on earth and the planet, $R \propto \frac{1}{\rho}$

$\therefore \quad \frac{R_2}{R} = \frac{\rho}{3\rho}$

$\therefore \quad R_2 = \frac{R}{3}$

Question29

The depth 'd' at which the value of acceleration due to gravity becomes $\frac{1}{n-1}$ times the value at the earth's surface is ($R =$ radius of the earth)

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Options:

A. $R\left(\frac{n}{n-1}\right)$

B. $R\left(\frac{n-2}{n-1}\right)$

C. $R\left(\frac{2n-1}{n}\right)$

D. $R\left(\frac{n-1}{2n-1}\right)$

Answer: B

Solution:

$$g_d = g\left(1 - \frac{d}{R}\right)$$

$$g\left(\frac{1}{n-1}\right) = g\left(1 - \frac{d}{R}\right) \quad \ldots\left(\text{Given: } g_d = g\left(\frac{1}{n-1}\right)\right.$$

$$\therefore \quad \frac{d}{R} = 1 - \left(\frac{1}{n-1}\right)$$

$$\therefore \quad d = R\left(\frac{n-2}{n-1}\right)$$

Question30

Assuming that the earth is revolving around the sun in circular orbit of radius R , the angular momentum is directly proportional to R^n.

The value of ' n ' is

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Options:

A. 2

B. 1.5

C. 1

D. 0.5

Answer: D

Solution:

The angular momentum of the Earth revolving around the Sun can be derived using the formula for angular momentum in circular motion:

$L = mvr$

where:

L is the angular momentum,

m is the mass of the Earth,

v is the tangential velocity,

r is the radius of the circular orbit, given as R.

For circular orbits under the gravitational influence of the Sun, the centripetal force is provided by gravitational force:

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

where:

M is the mass of the Sun,

G is the gravitational constant.

From this, solving for the velocity v, we have:

$v^2 = \frac{GM}{r}$

Taking the square root:

$v = \sqrt{\frac{GM}{r}}$

Substituting this back into the angular momentum expression:

$L = m \times \sqrt{\frac{GM}{r}} \times r$

Simplifying the equation:

$L = m \times r \times \sqrt{\frac{GM}{r}}$

The term $\sqrt{\frac{GM}{r}}$ simplifies to:

$L = m \times \sqrt{GMr}$

Since the problem states that angular momentum L is directly proportional to R^n, we can compare this with

$L \propto \sqrt{r}$

or:

$L \propto r^{0.5}$

Thus, $n = 0.5$. Therefore, the value of n is 0.5.

Correct Option: D. 0.5

Question31

The height ' h ' from the surface of the earth at which the value of ' g ' will be reduced by 64% than the value at surface of the earth is ($\mathrm{R} =$ radius of the earth)

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Options:

A. $\frac{1}{3} R$

B. $\frac{2}{3} R$

C. $\frac{3}{2} R$

D. 2R

Answer: B

Solution:

Since, $g = \frac{GM}{R^2}$

$g_h = \frac{GM}{(R+h)^2}$

When value of g is reduced by 64%, $g_h = 36\%$ of g

$$\therefore \quad \frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{36}{100}$$
$$\therefore \quad \frac{R}{(R+h)} = \frac{6}{10}$$
$$\therefore \quad 4R = 6h$$
$$\therefore \quad h = \frac{4}{6}R = \frac{2}{3}R$$

Question32

A body starts from rest from a distance R_0 from the centre of the earth. The velocity acquired by the body when it reaches the surface of the earth will be ($R =$ radius of earth, $M =$ mass of earth)

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Options:

A. $2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$

B. $\sqrt{2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)}$

C. $GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$

D. $2GM\sqrt{\left(\frac{1}{R} - \frac{1}{R_0}\right)}$

Answer: B

Solution:

According to law of conservation of energy,

$$\frac{1}{2}mv^2 = -\frac{GMm}{R_0} - \left(\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R_0}\right)$$

$$\therefore \quad V^2 = 2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$$

$\therefore$ The velocity acquired by the body when it reaches the surface of earth is:

$$v=\sqrt{2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)}$$

Question33

The radius and mean density of the planet are four times as that of the earth. The ratio of escape velocity at the earth to the escape velocity at a planet is

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Options:

A. $1:\sqrt{8}$

B. $1:8$

C. $1:\sqrt{3}$

D. $1:3$

Answer: B

Solution:

$$v_c=\sqrt{\frac{2GM}{R}}$$

$$v_c=R\sqrt{\frac{8}{3}\pi G\rho}\ldots.\left(\because M=\frac{4}{3}\pi R^3\rho\right)$$

Also, $\rho_p=4\rho_E$ and $R_p=4R_E$

$$\frac{v_c}{v_p}=\frac{R_E\sqrt{\rho_E}}{4R_E\sqrt{4\rho_E}}=\frac{1}{8}=1:8$$

Question34

A small planet is revolving around a very massive star in a circular orbit of radius ' R ' with a period of revolution ' T '. If the

gravitational force between the planet and the star were proportional to '$R^{-5/2}$', then 'T' would be proportional to

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Options:

A. $R^{3/2}$

B. $R^{3/5}$

C. $R^{7/2}$

D. $R^{7/4}$

Answer: D

Solution:

In this scenario, we are given that the gravitational force between the planet and the star is proportional to $R^{-5/2}$. The gravitational force F provides the necessary centripetal force for the planet's circular motion. Therefore, we can write:

$$F = \frac{k}{R^{5/2}}$$

where k is a proportionality constant.

The centripetal force required to keep the planet in a circular orbit of radius R with a period T is given by:

$$F_{\text{centripetal}} = \frac{mv^2}{R} = m\left(\frac{2\pi R}{T}\right)^2 \frac{1}{R}$$

Simplifying the expression:

$$F_{\text{centripetal}} = \frac{4\pi^2 mR}{T^2}$$

Equating the two expressions for the force, we have:

$$\frac{k}{R^{5/2}} = \frac{4\pi^2 mR}{T^2}$$

Solving for T^2:

$$T^2 = \frac{4\pi^2 mR^{7/2}}{k}$$

Taking the square root to solve for T:

$$T = \sqrt{\frac{4\pi^2 m}{k}} \cdot R^{7/4}$$

Thus, the period T is proportional to $R^{7/4}$.

Therefore, the correct answer is: **Option D:** $R^{7/4}$.

Question35

A satellite is revolving around a planet in a circular orbit close to its surface. Let ' ρ ' be the mean density and ' R ' be the radius of the planet. Then the period of the satellite is ($\mathrm{G}=$ universal constant of gravitation)

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Options:

A. $\sqrt{\frac{4\pi}{\rho G}}$

B. $\sqrt{\frac{\pi}{\rho G}}$

C. $\sqrt{\frac{3\pi}{\rho G}}$

D. $\sqrt{\frac{2\pi}{\rho G}}$

Answer: C

Solution:

From Kepler's third law,

$$\mathrm{T}^2=\frac{4\pi^2\mathrm{r}^3}{\mathrm{GM}}$$

$$\therefore \quad \mathrm{T}=2\pi\sqrt{\frac{\mathrm{r}^3}{\mathrm{GM}}}$$

As the satellite is very close to the planet, $r=R$.

$\therefore \quad \mathrm{T}=2\pi\sqrt{\frac{\mathrm{R}^3}{\mathrm{GM}}}$ .... (i)

We know, Mass $=$ Volume $\times$ Density (ρ)

$=\frac{4}{3}\pi\mathrm{R}^3\times\rho$ .... (ii)

Putting (ii) into (i)

$T = 2\pi\sqrt{\frac{R^3}{G\times\frac{4}{3}\pi R^3\rho}} = \sqrt{\frac{3\pi}{\rho G}}$

Question36

The radius of the planet is double that of the earth, but their average densities are same. V_p and V_E are the escape velocities of planet and earth respectively. If $\frac{V_P}{V_E} = x$, the value of ' x ' is

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Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 2

D. 4

Answer: C

Solution:

Escape velocity is given by,

$$\begin{aligned} v_c &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho} = \sqrt{\frac{8G}{3}\pi R^2\rho} = \sqrt{\frac{8G\pi\rho}{3}}\times R \end{aligned}$$

As the planets have the same density,

$$\begin{aligned} & V_e \propto R \\ & \frac{V_P}{V_E} = \frac{R_P}{R_E} = \frac{2R}{R} = 2 \\ \therefore \quad & \frac{V_P}{V_E} = 2 \\ \therefore \quad & X = 2 \end{aligned}$$

Question37

Two satellites A and B having ratio of masses $3:1$ are revolving in circular orbits of radii ' r ' and ' 4 r '. The ratio of total energy of satellites A to that of B is

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Options:

A. 1 : 3

B. 3 : 1

C. 3 : 4

D. 12 : 1

Answer: D

Solution:

$$\begin{aligned} \frac{E_A}{E_B} &= \frac{M_A}{M_B} \times \frac{r_B}{r_A} \quad \ldots\left(E \propto \frac{M}{r}\right) \\ &= \frac{3}{1} \times \frac{4r}{r} \\ \frac{E_A}{E_B} &= \frac{12}{1} \end{aligned}$$

Question38

The period of a planet around the sun is 8 times that of earth. The ratio of radius of planet's orbit to the radius of the earth's orbit is

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Options:

A. 4

B. 8

C. 16

D. 64

Answer: A

Solution:

Kepler's Third Law of Planetary Motion states that the square of the period of orbit (T) of a planet is directly proportional to the cube of the semi-major axis of its orbit (r). Mathematically, this is given by:

$T^2 \propto r^3$

For the Earth, we have:

$T_{\text{earth}}^2 = k \cdot r_{\text{earth}}^3$

For the planet with a period 8 times that of the Earth, we have:

$(8T_{\text{earth}})^2 = k \cdot r_{\text{planet}}^3$

Simplifying this expression:

$64T_{\text{earth}}^2 = k \cdot r_{\text{planet}}^3$

Using the equation for Earth, $T_{\text{earth}}^2 = k \cdot r_{\text{earth}}^3$, we substitute:

$64(k \cdot r_{\text{earth}}^3) = k \cdot r_{\text{planet}}^3$

The equation simplifies to:

$64r_{\text{earth}}^3 = r_{\text{planet}}^3$

Taking the cube root on both sides:

$4r_{\text{earth}} = r_{\text{planet}}$

Therefore, the ratio of the radius of the planet's orbit to the radius of the Earth's orbit is 4.

Option A: 4

Question39

A pendulum is oscillating with frequency ' n ' on the surface of earth. If it is taken to a depth $\frac{R}{4}$ below the surface of earth, new frequency of oscillation of depth $\frac{\mathrm{R}}{4}$ is ($\mathrm{R} =$ radius of earth)

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Options:

A. $\frac{2}{\sqrt{3}n}$

B. $\frac{\sqrt{3}n}{2}$

C. $\frac{2\mathrm{n}}{\sqrt{3}}$

D. $\frac{\mathrm{n}}{4}$

Answer: B

Solution:

The frequency of the pendulum at the surface is given as $\mathrm{f}=\frac{1}{2\pi}\sqrt{\frac{\mathrm{g}}{l}}$

At depth the formula for gravitational acceleration is $\mathrm{g}_{\mathrm{cff}}=\mathrm{g}\left(1-\frac{\mathrm{d}}{\mathrm{R}}\right)$

For $\mathrm{d}=\frac{\mathrm{R}}{4}$, $\mathrm{g}_{\mathrm{eff}}=\mathrm{g}\left(1-\frac{1}{4}\right)=\frac{3}{4}g$

The frequency at depth $\mathrm{d}=\frac{\mathrm{R}}{4}$

$\mathrm{f}_{\mathrm{d}}=\frac{1}{2\pi}\sqrt{\frac{\frac{3}{4}\mathrm{g}}{l}}=\frac{1}{2\pi}\sqrt{\frac{3\mathrm{g}}{4l}}$

Take the ratio of both frequencies

$$\frac{f_d}{f}=\frac{\sqrt{3}}{2}$$

$$f_d=\frac{\sqrt{3}}{2}f=\frac{\sqrt{3}n}{2} \quad \ldots(\because f=n)$$

Question40

The escape velocity from earth surface is $11\ \mathrm{km/s}$. The escape velocity from a planet having twice the radius and same mean density as earth is

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Options:

A. 22 km/s

B. 11 km/s

C. 5.5 km/s

D. 15.5 km/s

Answer: A

Solution:

Escape velocity is given by,

$$\begin{aligned} v_e &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8G}{3}\pi R^2 \rho} = \sqrt{\frac{8G\pi\rho}{3}} \times R \end{aligned}$$

As the planets have the same density,

$$\begin{aligned} & \mathrm{v_e} \propto \mathrm{R} \\ & \frac{\mathrm{v_e'}}{\mathrm{v_e}} = \frac{\mathrm{R'}}{\mathrm{R}} = \frac{2\mathrm{R}}{\mathrm{R}} = 2 \\ \therefore \quad & \mathrm{v_e'} = 2\mathrm{v_e} = 2 \times 11 = 22 \mathrm{~km/s} \end{aligned}$$

Question41

If ' R ' is the radius of earth \& ' g ' is acceleration due to gravity on earth's surface, then mean density of earth is

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Options:

A. $\frac{4\pi G}{3gR}$

B. $\frac{3\pi R}{4gG}$

C. $\frac{3 \mathrm{~g}}{4 \pi \mathrm{RG}}$

D. $\frac{\pi R G}{12 \mathrm{~g}}$

Answer: C

Solution:

Mass of the Earth is given by,

$\mathrm{M}=\mathrm{V} \rho=\frac{4}{3} \pi \mathrm{R}^3 \rho \quad \ldots .(\mathrm{i})$

Acceleration due to gravity is given by,

$\mathrm{g}=\frac{\mathrm{GM}}{\mathrm{R}^2} \quad \ldots .(\mathrm{ii})$

Substituting equation (i) in equation (ii),

$g=\frac{G}{R^2} \times \frac{4}{3} \pi R^3 \rho \Rightarrow \rho=\frac{3 g}{4 \pi R G}$

Question42

The height ' h ' above the earth's surface at which the value of acceleration due to gravity (g) becomes $\left(\frac{\mathrm{g}}{3}\right)$ is ($\mathrm{R}=$ radius of the earth)

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Options:

A. $(\sqrt{3}+1) R$

B. $(\sqrt{3}-1) R$

C. $\sqrt{3} \mathrm{R}$

D. $3 \sqrt{R}$

Answer: B

Solution:

We know that,

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

For $g_h = \frac{g}{3}$

$$\frac{g}{3} = g\left(\frac{R}{R+h}\right)^2$$

$$\frac{1}{\sqrt{3}} = \frac{R}{R+h}$$

$$\sqrt{3}R = R+h$$

$$\therefore \quad h = (\sqrt{3}-1)R$$

Question43

The height at which the weight of the body becomes $\frac{1^{\text{th}}}{16}$ of its weight on the surface of the earth of radius ' R ' is

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Options:

A. 2 R

B. 3 R

C. 4 R

D. 5 R

Answer: B

Solution:

To determine the height at which the weight of a body becomes $\frac{1}{16}$ of its weight on the surface of the Earth, we start by using the formula for gravitational force:

$F = \frac{GMm}{r^2}$

Where:

F is the gravitational force (weight),

G is the gravitational constant,

M is the mass of the Earth,

m is the mass of the body,

r is the distance from the center of the Earth.

On the surface of the Earth, the weight W_0 is given by:

$W_0 = \frac{GMm}{R^2}$

At a height h above the Earth's surface, the new distance from the center of the Earth becomes $R + h$. The weight at this height, W_h, is:

$W_h = \frac{GMm}{(R+h)^2}$

According to the problem, $W_h = \frac{1}{16} W_0$. Plugging in the expressions for W_0 and W_h:

$\frac{GMm}{(R+h)^2} = \frac{1}{16} \frac{GMm}{R^2}$

Canceling GMm from both sides and solving for h:

$\frac{1}{(R+h)^2} = \frac{1}{16R^2}$

Taking the reciprocal gives:

$(R + h)^2 = 16R^2$

Taking the square root of both sides:

$R + h = 4R$

Solving for h:

$h = 4R - R = 3R$

Thus, the height at which the weight becomes $\frac{1}{16}$ of its weight on the surface is:

Option B: 3R

Question44

Two identical metal spheres are kept in contact with each other, each having radius ' R ' cm and ' ρ ' is the density of material of metal spheres. The gravitational force ' F ' of attraction between them is proportional to

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Options:

A. $\mathrm{R}^3\rho$

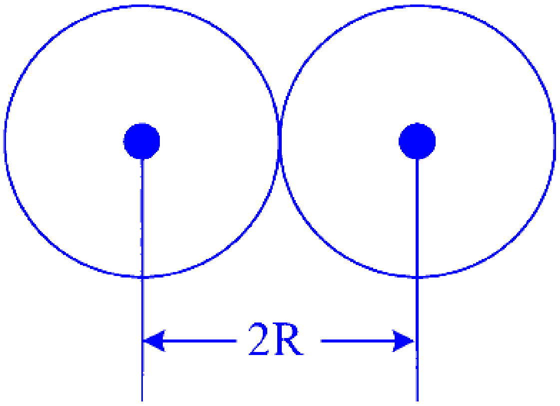
B. $R^4\rho^2$

C. $R^4\rho$

D. R^3p^2

Answer: B

Solution:



Force due to gravity,

$$\mathrm{F}=\frac{\mathrm{GMM}}{(2\mathrm{R})^2}$$

$$\mathrm{F}=\frac{\mathrm{GM}^2}{4\mathrm{R}^2} \quad \text{.... (i)}$$

Density, $\rho=\frac{M}{V}$

$M=\rho\times\frac{4}{3}\pi R^3$

Substituting in (i),

$F=\frac{G\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2}$

$\therefore \quad F\propto R^4\rho^2$

Question45

The distance of the two planets A and B from the sun are r_A and r_B respectively. Also r_B is equal to $100r_A$. If the orbital speed of the planet A is ' v ' then the orbital speed of the planet B is

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Options:

A. $\frac{\text{v}}{10}$

B. $\frac{v}{2}$

C. $\sqrt{2}v$

D. 10 v

Answer: A

Solution:

The orbital speed of a planet is determined by the formula for circular orbital velocity:

$$v = \sqrt{\frac{GM}{r}}$$

where:

v is the orbital speed,

G is the gravitational constant,

M is the mass of the central object (in this case, the Sun),

r is the distance from the center of the planet to the center of the central object (orbital radius).

For two planets A and B with radii r_A and r_B (where $r_B = 100r_A$), the orbital speed for each planet can be set as follows:

For planet A:

$$v_A = \sqrt{\frac{GM}{r_A}}$$

For planet B:

$$v_B = \sqrt{\frac{GM}{r_B}}$$

Since $r_B = 100r_A$, substitute into the equation for v_B:

$v_B = \sqrt{\frac{GM}{100r_A}}$

This can be further simplified to:

$v_B = \frac{1}{\sqrt{100}}\sqrt{\frac{GM}{r_A}}$

$v_B = \frac{1}{10}\sqrt{\frac{GM}{r_A}}$

Thus, since $v_A = \sqrt{\frac{GM}{r_A}}$, it follows that:

$v_B = \frac{1}{10}v_A$

Therefore, the orbital speed of planet B is $\frac{v}{10}$.

So, the correct answer is **Option A**:

$\frac{v}{10}$

Question46

Earth has mass ' M_1 ' radius ' R_1 ' and for moon mass ' M_2 ' and radius ' R_2 '. Distance between their centres is ' r '. A body of mass ' M ' is placed on the line joining them at a distance $\frac{\mathrm{r}}{3}$ from the centre of the earth. To project a mass ' M ' to escape to infinity, the minimum speed required is

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Options:

A. $\left[\frac{2G}{r}\left(M_2+\frac{M_1}{2}\right)\right]^{1/2}$

B. $\left[\frac{4\mathrm{G}}{\mathrm{r}}\left(\mathrm{M}_1+\frac{\mathrm{M}_2}{2}\right)\right]^{1/2}$

C. $\left[\frac{3G}{r}(M_1+M_2)\right]^{1/2}$

D. $\left[\frac{6G}{r}\left(M_1+\frac{M_2}{2}\right)\right]^{1/2}$

Answer: D

Solution:

The binding energy of the body is given by

$$\begin{aligned} \text{B.E.} &= \frac{GM_1M}{\frac{r}{3}} + \frac{\mathrm{GM_2M}}{\frac{2r}{3}} = \frac{3GM_1M}{r} + \frac{3\mathrm{GM_2}M}{2\mathrm{r}} \\ &= \frac{3GM}{r}\left[M_1 + \frac{M_2}{2}\right] \end{aligned}$$

If v is the velocity given to the body, then

$$\frac{1}{2}Mv^2 = \frac{3GM}{r}\left[M_1 + \frac{M_2}{2}\right]$$

$$\therefore \quad v = \left[\frac{6G}{r}\left(M_1 + \frac{M_2}{2}\right)\right]^{\frac{1}{2}}$$

Question47

The gravitational potential energy required to raise a satellite of mass ' m ' to height ' h ' above the earth's surface is ' $\mathrm{E_1}$ '. Let the energy required to put this satellite into the orbit at the same height be ' E_2 '. If M and R are the mass and radius of the earth respectively then $E_1 : E_2$ is

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Options:

A. h : R

B. h : 2R

C. R : h

D. 2 h : R

Answer: D

Solution:

Energy required to raise a satellite upto a height h,

$E_1 = \Delta U = \frac{mgh}{1+\frac{h}{R}}$ .... (i)

Energy required to put satellite into orbit,

$E_2 = \frac{1}{2} m v_0^2 = \frac{1}{2} m \left(\frac{GM}{r}\right)$ as v_0 is orbital speed

$= \frac{1}{2} m \left(\frac{GM}{R+h}\right)$

Dividing numerator and denominator by R^2,

$$E_2 = \frac{1}{2} m \left(\frac{\frac{GM}{R^2}}{\frac{R+h}{R} \times \frac{1}{R}}\right)$$

$$= \frac{1}{2} m \left(\frac{g}{1+\frac{h}{R}}\right) R \quad \ldots \left(\because g = \frac{GM}{R^2}\right)$$

$$E_2 = \frac{mgR}{2\left(1+\frac{h}{R}\right)} \quad \text{... (ii)}$$

$$\frac{E_1}{E_2} = \frac{2h}{R} \quad \text{.. [From (i) and (ii)]}$$

Question48

The height above the earth's surface at which the acceleration due to gravity becomes $\left(\frac{1}{n}\right)$ times the value at the surface is ($R =$ radius of earth)

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Options:

A. $\frac{R}{\sqrt{n}}$

B. $R \cdot \sqrt{n}$

C. $(\sqrt{n}+1) R$

D. $(\sqrt{n}-1) R$

Answer: D

Solution:

To find the height above the Earth's surface where the acceleration due to gravity (g') becomes $\frac{1}{n}$ times the value at the surface (g), we can use the formula for gravitational acceleration at a distance:

$g' = \frac{GM}{(R+h)^2}$

where:

G is the gravitational constant,

M is the mass of the Earth,

R is the radius of the Earth,

h is the height above the Earth's surface,

$g = \frac{GM}{R^2}$ is the acceleration due to gravity at the surface of the Earth.

Given that $g' = \frac{1}{n} \cdot g$, we have:

$\frac{GM}{(R+h)^2} = \frac{1}{n} \cdot \frac{GM}{R^2}$

The gravitational constant G and the Earth's mass M cancel out, leading to:

$\frac{1}{(R+h)^2} = \frac{1}{n \cdot R^2}$

Taking the reciprocal of both sides and then taking the square root:

$(R+h)^2 = n \cdot R^2$

$R + h = \sqrt{n} \cdot R$

Isolating h, we find:

$h = \sqrt{n} \cdot R - R$

Therefore, the height h is:

$h = (\sqrt{n} - 1) \cdot R$

The correct answer is Option D: $(\sqrt{n} - 1) \cdot R$.

Question49

The magnitude of gravitational field at distance ' r_1 ' and ' r_2 ' from the centre of a uniform sphere of radius ' R ' and mass ' M ' are ' F_1 ' and ' F_2 ' respectively. The ratio ' (F_1/F_2) ' will be (if $r_1 > R$ and $r_2

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Options:

A. $\frac{R^2}{r_1 r_2}$

B. $\frac{R^3}{r_1 r_2^2}$

C. $\frac{R^3}{r_1^2 r_2}$

D. $\frac{R^4}{r_1^2 r_2^2}$

Answer: C

Solution:

To find the ratio $\left(\frac{F_1}{F_2}\right)$, we first calculate the gravitational field at r_1 and r_2.

For a point outside the uniform sphere ($r_1 > R$), the gravitational field F_1 is given by:

$$F_1 = \frac{GM}{r_1^2}$$

For a point inside the sphere ($r_2 < R$), the gravitational field F_2 is given by:

$$F_2 = \frac{GM_{\text{enclosed}}}{r_2^2}$$

where the enclosed mass M_{enclosed} for a solid sphere of uniform density is given by

$$M_{\text{enclosed}} = M\left(\frac{r_2^3}{R^3}\right)$$

because the mass is proportional to the volume. Therefore:

$$F_2 = \frac{GM\left(\frac{r_2^3}{R^3}\right)}{r_2^2} = \frac{GMr_2}{R^3}$$

Now, we find the ratio $\left(\frac{F_1}{F_2}\right)$:

$$\frac{F_1}{F_2} = \frac{\frac{GM}{r_1^2}}{\frac{GMr_2}{R^3}}$$

Simplifying this expression, we have:

$$\frac{F_1}{F_2} = \frac{R^3}{r_1^2 r_2}$$

Hence, the correct option is **Option C:**

$\frac{R^3}{r_1^2 r_2}$

Question50

Earth is assumed to be a sphere of radius R. If 'g_ϕ' is value of effective acceleration due to gravity at latitude 30° and 'g' is the value at equator, then the value of $|g - g_\phi|$ is (ω is angular velocity of rotation of earth, $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

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Options:

A. $\frac{1}{4}\omega^2\mathrm{R}$

B. $\frac{3}{4}\omega^2\mathrm{R}$

C. $\omega^2\mathrm{R}$

D. $\frac{1}{2}\omega^2\mathrm{R}$

Answer: A

Solution:

The acceleration due to gravity on the surface of the Earth varies with latitude due to the centrifugal force resulting from the Earth's rotation. The effective acceleration due to gravity at a latitude ϕ, denoted as g_ϕ, takes into account the centrifugal force and is less than the acceleration due to gravity that would be experienced if the Earth were not rotating (denoted as g_0). At the equator ($\phi = 0^\circ$), this effect is maximal because the velocity due to Earth's rotation is maximal. As we go to the poles, the effect of rotation becomes minimal.

The effective acceleration due to gravity at latitude ϕ is given by:

$$g_\phi = g_0 - \omega^2\mathrm{R}\cos^2\phi$$

Where:

- g_0 is the acceleration due to gravity without the Earth's rotation, at the equator
- ω is the angular velocity of the Earth's rotation
- R is the radius of the Earth
- ϕ is the latitude

At the equator, the effective acceleration due to gravity, g, is:

$$g = g_0 - \omega^2\mathrm{R}$$

because $\cos 0^\circ = 1$.

At latitude 30°, the effective acceleration due to gravity, g_ϕ, using $\cos 30^\circ = \frac{\sqrt{3}}{2}$, is:

$g_\phi = g_0 - \omega^2 R \left(\frac{\sqrt{3}}{2}\right)^2$

$g_\phi = g_0 - \omega^2 R \times \frac{3}{4}$

To find $|g - g_\phi|$, we subtract the above two equations:

$|g - g_\phi| = \left|(g_0 - \omega^2 R) - (g_0 - \omega^2 R \times \frac{3}{4})\right|$

$|g - g_\phi| = \omega^2 R - \omega^2 R \times \frac{3}{4}$

$|g - g_\phi| = \omega^2 R \left(1 - \frac{3}{4}\right)$

$|g - g_\phi| = \omega^2 R \times \frac{1}{4}$

So the value of $|g - g_\phi|$ is given by the option A, which is:

$\frac{1}{4}\omega^2 R$

Question51

A body (mass m) starts its motion from rest from a point distant R_0 $(R_0 > R)$ from the centre of the earth. The velocity acquired by the body when it reaches the surface of earth will be ($\mathrm{G} =$ universal constant of gravitation, $\mathrm{M} =$ mass of earth, R = radius of earth)

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Options:

A. $2\mathrm{GM}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{R}_0}\right)$

B. $\left[2\mathrm{GM}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{R}_0}\right)\right]^{\frac{1}{2}}$

C. $\mathrm{GM}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{R}_0}\right)$

D. $2\mathrm{GM}\left[\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{R}_0}\right)\right]^{\frac{1}{2}}$

Answer: B

Solution:

According to law of conservation of energy,

$$\frac{1}{2} m v^2=-\frac{G M m}{R_0}-\left(\frac{G M m}{R}\right)=G M m\left(\frac{1}{R}-\frac{1}{R_0}\right)$$

$$\therefore \quad v^2=2 G M\left(\frac{1}{R}-\frac{1}{R_0}\right)$$

$\therefore$ The velocity acquired by the body when it reaches the surface of earth is:

$$\mathrm{V}=\left[2 \mathrm{GM}\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{R}_0}\right)\right]^{\frac{1}{2}}$$

Amongst the given options, only option (B) dimensionally equates to velocity.

Question52

Considering earth to be a sphere of radius 'R' having uniform density 'ρ', then value of acceleration due to gravity 'g' in terms of R, ρ and G is

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Options:

A. $g=\sqrt{\frac{3 \pi R}{\rho G}}$

B. $\mathrm{g}=\sqrt{\frac{4}{3} \pi \rho \mathrm{GR}}$

C. $\mathrm{g}=\frac{4}{3} \pi \rho \mathrm{GR}$

D. $g=\frac{G M}{\rho R^2}$

Answer: C

Solution:

To derive an expression for the acceleration due to gravity 'g' at the surface of the Earth, in terms of its radius 'R', its uniform density 'ρ', and the gravitational constant 'G', we can start by calculating the mass 'M' of the Earth in terms of its density and volume.

The volume 'V' of a sphere is given by

$$V=\frac{4}{3} \pi R^3$$

For a sphere with uniform density 'ρ', the mass 'M' can be calculated by multiplying the volume by the density:

$M = \rho V = \rho \left(\frac{4}{3}\pi R^3\right)$

Now that we have the mass, we can use Newton's law of universal gravitation to find the force 'F' exerted on a mass 'm' at the surface of the Earth:

$F = \frac{GMm}{R^2}$

The acceleration 'g' due to gravity at the Earth's surface is simply the force per unit mass 'm':

$g = \frac{F}{m} = \frac{GM}{R^2}$

Substituting the expression for 'M' in terms of 'ρ' and 'R', we get:

$g = \frac{G(\rho(\frac{4}{3}\pi R^3))}{R^2}$

When we simplify this expression, we have:

$g = \frac{G\rho\frac{4}{3}\pi R^3}{R^2} g = \frac{4}{3}\pi G\rho R$

Therefore, the correct answer is:

Option C

$g = \frac{4}{3}\pi\rho GR$

Question53

The value of acceleration due to gravity at a depth 'd' from the surface of earth and at an altitude 'h' from the surface of earth are in the ratio

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Options:

A. 1 : 1

B. $\frac{R-2h}{R-d}$

C. $\frac{R-d}{R-2h}$

D. $\frac{\mathrm{R-d}}{\mathrm{R-h}}$

Answer: C

Solution:

$$\frac{g_d}{g_h} = \frac{g\left[1 - \frac{d}{R}\right]}{g\left[1 - \frac{2h}{R}\right]}$$
$$\frac{g_d}{g_h} = \frac{R - d}{R - 2h}$$

Question54

If two planets have their radii in the ratio $x : y$ and densities in the ratio $m : n$, then the acceleration due to gravity on them are in the ratio

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Options:

A. $\frac{ny}{mx}$

B. $\frac{my}{nx}$

C. $\frac{nx}{my}$

D. $\frac{mx}{ny}$

Answer: D

Solution:

Acceleration due to gravity of planet A is given below.

$g_A = \frac{GM_A}{R_A^2}$ ..... (i)

$\because$ We know that, (Mass $=$ Density $\times$ Volume) $(M = D \times V)$

So, from Eq. (i)

$$g_A = \frac{GD_A \times \frac{4}{3}\pi R_A^3}{R_A^2} = GD_A \times \frac{4}{3}\pi R_A$$

$$g_A = \frac{4}{3}\pi G R_A D_A \quad .... \text{(ii)}$$

Similarly, Acceleration due to gravity of planet B is

$g_B = \frac{GM_B}{R_B^2}$ .... (iii)

and $\quad g_B = \frac{4}{3}\pi G D_B R_B$ .... (iv)

We have to find the ratio of $g_A : g_B$

So, according to question, we have

$$\frac{R_A}{R_B} = \frac{x}{y} \quad .... \text{(v)}$$

and $$\frac{D_A}{D_B} = \frac{m}{n} \quad .... \text{(vi)}$$

Now, taking the ratio of Eqs. (ii) and (iv), we get

$\frac{g_A}{g_B} = \frac{R_A D_A}{R_B D_B}$

Put the value from Eqs. (v) and (vi) in this equation, we get

$\frac{g_A}{g_B} = \frac{x \cdot m}{y \cdot n}$

or $\quad g_A / g_B = mx/ny$

Question55

A mine is located at depth $R/3$ below earth's surface. The acceleration due to gravity at that depth in mine is ($R =$ radius of earth, $g =$ acceleration due to gravity)

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Options:

A. g

B. $3g$

C. $\frac{2g}{3}$

D. $\frac{g}{3}$

Answer: C

Solution:

Here, g' be the acceleration due to gravity in mine.

Then, $g' = g\left(1 - \frac{d}{R}\right)$ ...... (i)

Here,

$$d(\text{ depth }) = \frac{R}{3}$$
$$R = \text{ Radius of earth}$$

Now, from Eq. (i), we get

$$\Rightarrow \quad g' = g\left(1 - \frac{R}{3 \times R}\right)$$
$$g' = \frac{2g}{3}$$

Question56

A body of mass 'm' is raised through a height above the earth's surface so that the increase in potential energy is $\frac{\mathrm{mgR}}{5}$. The height to which the body is raised is (R = radius of earth, g = acceleration due to gravity)

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Options:

A. R

B. $\frac{\mathrm{R}}{2}$

C. $\frac{\mathrm{R}}{4}$

D. $\frac{\mathrm{R}}{8}$

Answer: C

Solution:

When a particle of mass m is taken from the Earth's surface to a height $h = nR$, then the change in P.E. can be calculated as,

$$\Delta U = mgR\left(\frac{n}{n+1}\right)$$
$$\therefore \quad \frac{mgR}{5} = mgR\left(\frac{n}{n+1}\right)$$
$$\therefore \quad n + 1 = 5n$$
$$\therefore \quad n = \frac{1}{4}$$
$$\therefore \quad h = \frac{R}{4}$$

Question57

If two identical spherical bodies of same material and dimensions are kept in contact, the gravitational force between them is proportional to R^X, where x is non zero integer [Given : R is radius of each spherical body]

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Options:

A. -4

B. 4

C. 2

D. -2

Answer: B

Solution:

$$F = \frac{G \times m \times m}{(2R)^2} = \frac{G \times \left(\frac{4}{3}\pi R^3 \rho\right)^2}{4R^2}$$
$$= \frac{4}{9}\pi^2 \rho^2 R^4$$
$$\therefore \quad F \propto R^4$$

Question58

A body is projected vertically upwards from earth's surface of radius 'R' with velocity equal to $\frac{1^{\text{rd}}}{3}$ of escape velocity. The maximum height reached by the body is

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Options:

A. $\frac{R}{8}$

B. $\frac{\mathrm{R}}{6}$

C. $\frac{\mathrm{R}}{4}$

D. $\frac{\mathrm{R}}{9}$

Answer: A

Solution:

Δ K.E. $= \Delta$U

Let mass of the particle be M and that of the Earth be M_e

$\therefore \quad \frac{1}{2}\mathrm{Mv}^2 = \mathrm{GM_eM}\left(\frac{1}{\mathrm{R}} - \frac{1}{\mathrm{R+h}}\right)$ ..... (i)

Also, $\mathrm{g} = \frac{\mathrm{GM_e}}{\mathrm{R}^2}$ ..... (ii)

Equation (i) can be written as,

$$\frac{1}{2}v^2 = G_e\left[\frac{R+h-R}{R(R+h)}\right] = \frac{G_e}{R^2}\left[\frac{Rh}{(R+h)}\right]$$

$$\therefore \quad \frac{1}{2}\left(\frac{1}{3}v_e\right)^2 = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{1}{2}\left(\frac{1}{3}\sqrt{2gR}\right)^2 = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{1}{2} \times \frac{1}{9}(2gR) = \frac{gRh}{R+h}$$

$$\therefore \quad \frac{h}{R+h} = \frac{1}{9}$$

$\therefore \quad 9h = R + h$

$\therefore \quad 8h = R$

$\therefore \quad h = \frac{R}{8}$

Given $V = \frac{Ve}{3}$

$\therefore \quad h = \frac{R}{\left[\frac{V_e}{V_e/3}\right]^2 - 1} = \frac{R}{9-1} = \frac{R}{8}$

Question59

A simple pendulum is oscillating with frequency 'F' on the surface of the earth. It is taken to a depth $\frac{R}{3}$ below the surface of earth. ($R =$ radius of earth). The frequency of oscillation at depth $R/3$ is

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Options:

A. $\frac{2F}{3}$

B. $\frac{F}{\sqrt{1.5}}$

C. F

D. $\frac{F}{3}$

Answer: B

Solution:

The frequency of the pendulum at the surface is given as

$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

At depth the formula for gravitational acceleration is $g_{eff} = g\left(1 - \frac{d}{R}\right)$

For $d = \frac{R}{3}$, $g_{eff} = g\left(1 - \frac{1}{3}\right)$

The frequency at depth $d = \frac{R}{3}$

$f_d = \frac{1}{2\pi}\sqrt{\frac{g(1-\frac{1}{3})}{l}} = \frac{1}{2\pi}\sqrt{\frac{2g}{3l}}$

Take the ratio of both frequencies

$$\frac{f_d}{f} = \sqrt{\frac{2}{3}}$$
$$\therefore \quad f_d = \frac{F}{\sqrt{1.5}} \quad \ldots.(\because f = F)$$

Question60

The depth at which acceleration due to gravity becomes $\frac{\mathrm{g}}{2\mathrm{n}}$ is $(\mathrm{R} =$ radius of earth, $\mathrm{g} =$ acceleration due to gravity on earth's surface, n is integer)

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Options:

A. $\frac{\mathrm{R}(1-2\mathrm{n})}{\mathrm{n}}$

B. $\frac{\mathrm{R}(1-\mathrm{n})}{2\mathrm{n}}$

C. $\frac{\mathrm{R}(\mathrm{n}-1)}{\mathrm{n}}$

D. $\frac{\mathrm{R}(2\mathrm{n}-1)}{2\mathrm{n}}$

Answer: D

Solution:

The gravitational acceleration at depth is given as $g_d = g\left[1 - \frac{d}{R}\right]$

Given $\mathrm{g_d} = \frac{\mathrm{g}}{2\mathrm{n}}$

$$\therefore \quad \frac{g}{2n} = g\left[1 - \frac{d}{R}\right]$$
$$\frac{d}{R} = 1 - \frac{1}{2n}$$
$$d = \left[\frac{2n-1}{2n}\right]R$$

Question61

Time period of simple pendulum on earth's surface is 'T'. Its time period becomes 'xT' when taken to a height R (equal to earth's radius) above the earth's surface. Then the value of 'x' will be

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Options:

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

$\mathrm{T}=2 \pi \sqrt{\frac{l}{\mathrm{~g}}}$

At a height 'h' from earth's surface,

$$\mathrm{xT}=2 \pi \sqrt{\frac{l}{\mathrm{~g}_{\mathrm{h}}}}$$

$$\therefore \quad \mathrm{x}=\sqrt{\frac{\mathrm{g}}{\mathrm{g}_{\mathrm{h}}}} \quad \ldots . .(\mathrm{i})$$

Now, $\mathrm{g}_{\mathrm{h}}=\frac{\mathrm{GM}}{(\mathrm{R}+\mathrm{h})^2}$

$\therefore \quad \mathrm{g}_{\mathrm{h}}=\frac{\mathrm{GM}}{4 \mathrm{R}^2} \quad \ldots . .(\because \mathrm{h}=\mathrm{R})$

$\therefore \quad \mathrm{g}_{\mathrm{h}}=\frac{\mathrm{g}}{4} \quad \ldots .(\mathrm{ii})$

$\therefore$ From equations (i) and (ii),

$x=\sqrt{\frac{g}{g / 4}}=\sqrt{4}=2$

Question62

The height at which the weight of the body becomes $\left(\frac{1}{9}\right)^{\text{th}}$ its weight on the surface of earth is $(\mathrm{R}=$ radius of earth)

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Options:

A. 8R

B. 4R

C. 3R

D. 2R

Answer: D

Solution:

$$
\begin{aligned}
& \mathrm{W_h}=\frac{\mathrm{W}}{9} \\
& \mathrm{mg_h}=\frac{\mathrm{mg}}{9} \\
& \mathrm{g_h}=\frac{\mathrm{g}}{9} \\
\therefore \quad & \frac{\mathrm{GM}}{(\mathrm{R}+\mathrm{h})^2}=\frac{\mathrm{GM}}{9 \mathrm{R}^2} \\
\therefore \quad & \mathrm{R}+\mathrm{h}=3 \mathrm{R} \\
\therefore \quad & \mathrm{h}=2 \mathrm{R}
\end{aligned}
$$

Question63

Consider a light planet revolving around a massive star in a circular orbit of radius 'r' with time period 'T'. If the gravitational force of attraction between the planet and the star is proportional to $\mathrm{r}^{\frac{7}{2}}$, then T^2 is proportional to

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Options:

A. $r^{9/2}$

B. $r^{7/2}$

C. $r^{5/2}$

D. $r^{3/2}$

Answer: A

Solution:

For the planet to orbit around the star, the centripetal force must be provided by gravitational force. Hence, $\mathrm{F_G = F_a}$.

$\mathrm{F_a \propto -r^{-7/2}}$ .... (Given)

(−ve sign indicates force is towards the centre of orbit)

Hence, $\mathrm{a \propto -r^{-7/2}}$

$\therefore \quad -\omega^2 \mathrm{r} \propto -\mathrm{r}^{-7/2}$

$\therefore \quad \omega^2 \propto \mathrm{r}^{-9/2}$

$\therefore \quad \frac{4\pi^2}{\mathrm{T}^2} \propto \mathrm{r}^{-9/2}$

$\Rightarrow \mathrm{T}^2 \propto \mathrm{r}^{9/2}$

Question64

The radius of the orbit of a geostationary satellite is (mean radius of earth is 'R', angular velocity about own axis is 'ω' and acceleration due to gravity on earth's surface is 'g')

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Options:

A. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{1}{3}}$

B. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{2}{3}}$

C. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{1}{2}}$

D. $\frac{\mathrm{gR}^2}{\omega^2}$

Answer: A

Solution:

$$\mathrm{mr}\,\omega^2=\frac{GMm}{r^2}$$

$$\omega^2=\frac{GM}{r^3}=\frac{gR^2}{r^3}\quad\ldots.\left(\because g=\frac{GM}{R^2}\right)$$

$\therefore$ Radius of the orbit of the satellite is:

$$r=\left(\frac{gR^2}{\omega^2}\right)^{\frac{1}{3}}$$

Question65

The ratio of energy required to raise a satellite to a height 'h' above the earth's surface to that required to put it into the orbit at the same height is ($\mathrm{R}=$ radius of earth)

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Options:

A. $\frac{2\mathrm{~h}}{\mathrm{R}}$

B. $\frac{h}{R}$

C. $\frac{\mathrm{R}}{\mathrm{h}}$

D. $\frac{\mathrm{R}}{2\mathrm{~h}}$

Answer: A

Solution:

The formula for the energy required to raise a satellite to height h is

$E_1 = \Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mghR}{R+h}$

The formula for the energy required to set the satellite in orbit is

$$\begin{aligned} E_2 &= \frac{-GMm}{2(R+h)} + \frac{GMm}{R} \\ &= mgR\left[1 - \frac{1}{2\left(1+\frac{h}{R}\right)}\right] \left(\because GM = gR^2\right) \end{aligned}$$

$$\therefore \quad E_2 = \frac{mgR\left(\frac{2h}{R}+1\right)}{2\left(1+\frac{h}{R}\right)}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{mgh}{1+\frac{h}{R}} \times \frac{2\left(1+\frac{h}{R}\right)}{mgR}$$

$= \frac{2h}{R} \quad \left(\because h < R \Rightarrow 1 + \frac{2h}{R} \approx 0\right)$

Question66

The radius of earth is $6400\ \mathrm{km}$ and acceleration due to gravity $g = 10\ \mathrm{ms}^{-2}$. For the weight of body of mass $5\ \mathrm{kg}$ to be zero on equator, rotational velocity of the earth must be (in $\mathrm{rad/s}$)

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Options:

A. $\frac{1}{80}$

B. $\frac{1}{400}$

C. $\frac{1}{800}$

D. $\frac{1}{1600}$

Answer: C

Solution:

At equator, for the weight to be zero, the gravitational force must be equal to centrifugal force.

$$mR\omega^2 = mg$$

$$\omega^2 = \frac{g}{R}$$

$$\omega = \sqrt{\frac{g}{R}}$$

$$\omega = \sqrt{\frac{10}{6.4 \times 10^6}}$$

$$\omega = \frac{1}{800} \frac{rad}{s}.$$

Question67

A body of mass 'm' kg starts falling from a distance 3R above earth's surface. When it reaches a distance 'R' above the surface of the earth of radius 'R' and Mass 'M', then its kinetic energy is

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Options:

A. $\frac{2}{3} \frac{GMm}{R}$

B. $\frac{1}{3} \frac{GMm}{R}$

C. $\frac{1}{2} \frac{GMm}{R}$

D. $\frac{1}{4} \frac{GMm}{R}$

Answer: D

Solution:

Initial height:

$h = 3R + R = 4R$

The potential energy of the body initially will be:

$U_1 = -\frac{1}{4} \frac{GMm}{R}$

$\therefore$ At the height R,

$h = R + R = 2R$

Potential energy:

$U_2 = -\frac{1}{2}\frac{GMm}{R}$

Gain in kinetic energy is equal to loss in potential energy.

$$\therefore \quad KE = U_1 - U_2$$
$$= -\frac{1}{4}\frac{GMm}{R} - \left(-\frac{1}{2}\frac{GMm}{R}\right) = \frac{1}{4}\frac{GMm}{R}$$

Question68

A body is projected vertically from earth's surface with velocity equal to half the escape velocity. The maximum height reached by the satellite is (R = radius of earth)

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Options:

A. R

B. $\frac{R}{2}$

C. $\frac{R}{3}$

D. $\frac{R}{4}$

Answer: C

Solution:

Given: $v = \frac{v_e}{2}$

If body is projected with velocity v $(v < v_e)$ then height up to which it will rise, $h = \frac{R}{\left(\frac{v_e^2}{v^2} - 1\right)}$

$$\therefore \quad h = \frac{R}{\left(\frac{v_e}{v_e/2}\right)^2 - 1} = \frac{R}{4-1} = \frac{R}{3}$$

Question69

A system consists of three particles each of mass 'm_1' placed at the corners of an equilateral triangle of side '$\frac{L}{3}$', A particle of mass 'm_2' is placed at the mid point of any one side of the triangle. Due to the system of particles, the force acting on m_2 is

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Options:

A. $\frac{3Gm_1 m_2}{L^2}$

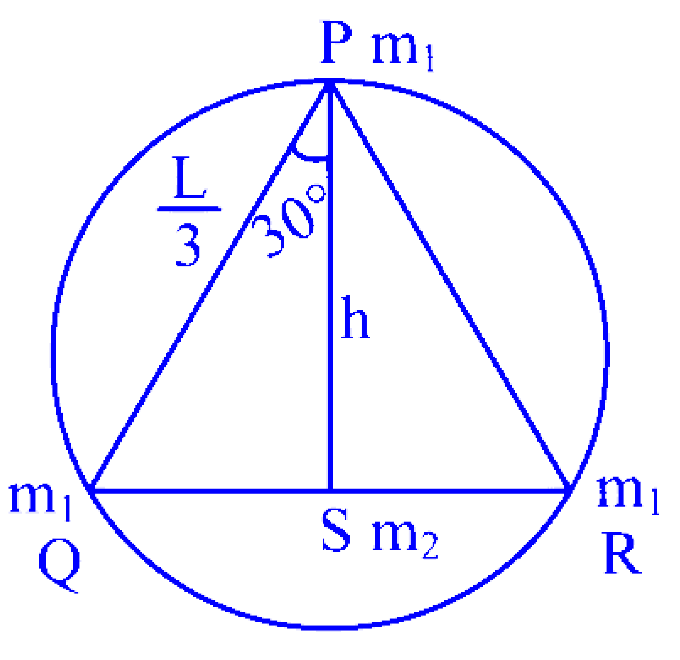
B. $\frac{6Gm_1 m_2}{L^2}$

C. $\frac{9Gm_1 m_2}{L^2}$

D. $\frac{12Gm_1 m_2}{L^2}$

Answer: D

Solution:



From the fig., we can see that the forces due to masses at Q and R cancel each other as they are equal and opposite. The force at P is only due to m_1.

In $\triangle$PQS,

$h = \frac{L}{3}\cos 30^\circ = \frac{L\sqrt{3}}{6}$

$\therefore$ Force on m_2 due to m_1 at P is

$$F = \frac{Gm_1m_2}{\left(\frac{L\sqrt{3}}{6}\right)^2} = \frac{Gm_1m_2 \cdot 12}{L^2} = \frac{12Gm_1 m_2}{L^2}$$

Question70

A satellite moves in a stable circular orbit round the earth if (where V_H, V_c and V_e are the horizontal velocity, critical velocity and escape velocity respectively)

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Options:

A. $V_H < V_c$

B. $V_H = V_e$

C. $V_H = V_c$

D. $V_H > V_e$

Answer: C

Solution:

To understand the conditions for a stable circular orbit around Earth, we need to know the meanings of the critical velocity (V_c) and escape velocity (V_e).

Critical Velocity (V_c) is the necessary orbital velocity a body must have to be in a stable circular orbit around Earth without the need for propulsion. It depends on the gravitational force providing the necessary centripetal force to keep the satellite in orbit.

Escape Velocity (V_e) is the minimum velocity an object must have to break free from the gravitational attraction of a celestial body, like earth, without further propulsion.

For a satellite to maintain a stable orbit, its horizontal velocity (V_H) must be equal to the critical velocity (V_c). If the horizontal velocity is less than critical velocity, the gravitational pull will cause the satellite to spiral downwards towards Earth. If the horizontal velocity is greater than critical velocity but less than escape velocity, the satellite will enter an elliptical orbit. And if the horizontal velocity equals or exceeds escape velocity, the satellite will leave Earth's orbit entirely.

In simple terms, for a stable, circular orbit, the option is:

Option C :$V_H = V_c$

Options A and D would not result in a stable circular orbit. Option A would lead to a decaying orbit, and Option D would cause the satellite to leave orbit entirely. Option B implies that the satellite would be on an escape path and thus not in a stable orbit around the Earth.

Question71

There is a second's pendulum on the surface of earth. It is taken to the surface of planet whose mass and radius are twice that of earth. The period of oscillation of second's pendulum on the planet will be

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Options:

A. $2\sqrt{2}$ s

B. 2 s

C. $\frac{1}{\sqrt{2}}$ s

D. $\frac{1}{2}$ s

Answer: A

Solution:

$$\text{As } g = \frac{GM}{R^2}$$

$$\therefore \quad \frac{g_{\text{Earth}}}{g_{\text{plunet}}} = \frac{M_{\text{Earth}}}{M_{\text{planet}}} \times \frac{R^2_{\text{planet}}}{R_{\text{Earth}}} = \frac{M}{2M} \times \frac{(2R)^2}{R} = \frac{2}{1}$$

$$\text{Also } T \propto \frac{1}{\sqrt{\mathrm{g}}}$$

$$\therefore \quad \frac{\mathrm{T}_{\text{Earth}}}{\mathrm{T}_{\text{planet}}} = \sqrt{\frac{\mathrm{g}_{\text{planet}}}{\mathrm{g}_{\text{Earrth}}}}$$

$$\frac{2}{\mathrm{T}_{\text{plunat}}} = \sqrt{\frac{1}{2}}$$

$$\therefore \quad \mathrm{T}_{\text{planet}} = 2\sqrt{2}\ \mathrm{s}$$

Question72

For a satellite orbiting around the earth in a circular orbit, the ratio of potential energy to kinetic energy at same height is

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Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. 2

Answer: D

Solution:

$$\text{K.E} = \frac{GMm}{2r}$$

$$P.E = -\frac{GMm}{r}$$

$$\therefore \quad \frac{K.E}{|P.E|} = \frac{GMm}{2r} \times \frac{r}{GMm}$$

$$\therefore \quad \frac{P.E}{K.E} = \frac{2}{1} = 2$$

Question73

Periodic time of a satellite revolving above the earth's surface at a height equal to radius of the earth 'R' is [$g =$ acceleration due to gravity]

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Options:

A. $2\pi\sqrt{\frac{2R}{g}}$

B. $4\pi\sqrt{\frac{2R}{g}}$

C. $2\pi\sqrt{\frac{R}{g}}$

D. $8\pi\sqrt{\frac{R}{g}}$

Answer: B

Solution:

$$\begin{aligned} T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} &= 2\pi\sqrt{\frac{(2R)^3}{gR^2}} \quad \ldots.(\because h = R) \\ &= 4\pi\sqrt{\frac{2R}{g}} \end{aligned}$$

Question74

Consider a planet whose density is same as that of the earth but whose radius is three times the radius 'R' of the earth. The acceleration due to gravity 'g_n' on the surface of planet is $g_n = x.\, g$ where g is acceleration due to gravity on surface of earth. The value of 'x' is

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Options:

A. 9

B. 3

C. $\frac{1}{3}$

D. $\frac{1}{9}$

Answer: B

Solution:

As, $g = \frac{GM}{R^2}$ and $M = \rho V$

$\therefore \quad g = \frac{G\rho V}{R^2} = \frac{G\rho\frac{4}{3}\pi R^3}{R^2}$

$\therefore \quad g \propto R$

For the planet: Radius $R = 3R$

$\therefore \quad g_{\text{planet}} = \frac{G\rho V_{\text{planet}}}{(3R)^2}$

where $V_{\text{planet}} = \frac{4}{3}\pi(3R)^3$

$\therefore \quad g_{\text{planet}} = \frac{G\rho\frac{4}{3}\pi(3R)^3}{(3R)^2}$

$\therefore \quad g_{\text{planet}} \propto 3R$

$\therefore \quad \frac{g}{g_{\text{planet}}} = \frac{R}{3R}$ ....(since ρ is constant)

$\therefore \quad g_{\text{planet}} = 3g$

$\therefore \quad x = 3$

Question75

A thin rod of length 'L' is bent in the form of a circle. Its mass is 'M'. What force will act on mass 'm' placed at the centre of this circle?

($\mathrm{G} =$ constant of gravitation)

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Options:

A. zero

B. $\frac{\mathrm{GMm}}{4\ \mathrm{L}^2 \pi^2}$

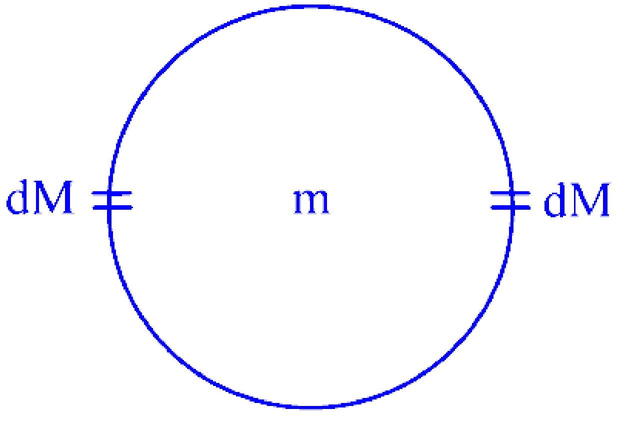
C. $\frac{4\pi^2 \mathrm{GMm}}{\mathrm{L}}$

D. $\frac{2\mathrm{GMm}}{\mathrm{L}^2}$

Answer: A

Solution:

Consider two diametrically and equal small and equal mass segments dM_1 and dM_2



$\therefore$ Force at the centre due to dM_1 is

$\mathrm{F}_1 = \frac{\mathrm{GmdM}_1}{\mathrm{r}^2}$

Similarly,

Force at the centre due to dM_2 is

$F_2 = \frac{GmdM_2}{r^2}$

But $\mathrm{F}_1 = -\mathrm{F}_2$

$\Rightarrow \mathrm{F}_1 + \mathrm{F}_2 = 0$ ($\because$ the forces cancel each other out as they are equal and opposite)

If this process is done for all such dM segments, we find the net force at the centre of the circle to be zero.

Question76

A body weighs 300 N on the surface of the earth. How much will it weigh at a distance $\frac{R}{2}$ below the surface of earth? ($R \rightarrow$ Radius of earth)

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Options:

A. 300 N

B. 250 N

C. 200 N

D. 150 N

Answer: D

Solution:

Acceleration due to gravity at depth d,

$$\begin{aligned} g_d &= g\left(1-\frac{d}{R}\right) \\ &= g\left(1-\frac{1}{2}\right) \quad \because d=\frac{R}{2} \\ &= \frac{g}{2} \end{aligned}$$

$\therefore$ Weight of the body at depth d_1

$W_d = mg_d = 300 \times \frac{1}{2} = 150$ N

Question77

A seconds pendulum is placed in a space laboratory orbiting round the earth at a height '3R' from the earth's surface. The time period of the pendulum will be ($R=$ radius of earth)

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Options:

A. zero

B. $\frac{2}{3}$ s

C. 4 s

D. infinite

Answer: D

Solution:

In outer space, g = 0. Therefore, T = ∞.

Question78

A body weighs $500 \mathrm{~N}$ on the surface of the earth. At what distance below the surface of the earth it weighs $250 \mathrm{~N}$? (Radius of earth, $\mathrm{R}=6400 \mathrm{~km}$)

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Options:

A. $6400 \mathrm{~km}$

B. $800 \mathrm{~km}$

C. $1600 \mathrm{~km}$

D. $3200 \mathrm{~km}$

Answer: D

Solution:

The value of g at a depth h below the surface of the earth of radius R is

$$\mathrm{g}^{\prime}=\mathrm{g}\left[1-\frac{\mathrm{d}}{\mathrm{R}}\right]$$
$$\therefore \frac{\mathrm{g}^{\prime}}{\mathrm{g}}=1-\frac{\mathrm{d}}{\mathrm{R}} \quad \ldots . .(1)$$

It is given that $\mathrm{mg}=500 \mathrm{~N}$ and $\mathrm{mg}^{\prime}=250 \mathrm{~N}$

$\therefore \frac{g'}{g} = \frac{250}{500} = \frac{1}{2}$ ..... (2)

$\therefore$ From (1) and (2), $\frac{1}{2} = 1 - \frac{d}{R}$

$\therefore \frac{d}{R} = \frac{1}{2}$

$\therefore d = \frac{R}{2} = \frac{6400}{2} = 3200$ km

Question79

The masses and radii of the moon and the earth are M_1, R_1 and M_2, R_2 respectively. Their centres are at a distance d apart. What should be the minimum speed with which a body of mass 'm' should be projected from a point midway between their centres, so as to escape to infinity?

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Options:

A. $\frac{G(M_1+M_2)}{d}$

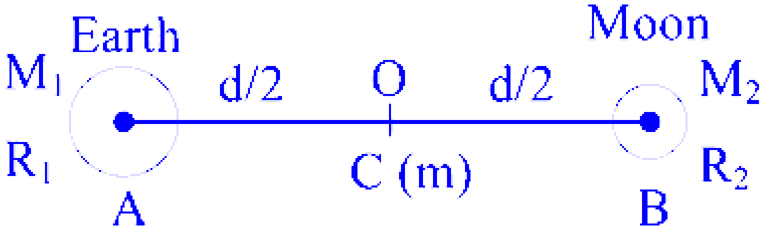
B. $\sqrt[2]{\frac{G(M_1+M_2)}{d}}$

C. $\sqrt{\frac{Gd}{M_1+M_2}}$

D. $\sqrt{\frac{M_1+M_2}{Gd}}$

Answer: B

Solution:



O is the midpoint of the line joining the centres of A and B. and a body (C) of mass 'm' is kept at O

The P.E. of C is

$$U = -\frac{GM_1 m}{d/2} - \frac{GM_2 m}{d/2} = -\frac{2Gm}{d}(M_1 + M_2)$$

Initially, C is at rest, its K.E. $= 0$

$\therefore$ Total energy of C $= -\frac{2Gm}{d}(M_1 + M_2)$

$\therefore$ Its binding energy $= \frac{2GM}{d}(M_1 + M_2)$ ..... (1)

Let v_e be the velocity that should be given to the body to escape to infinity.

For this its K.E. = Binding energy

$$\therefore \frac{1}{2}mv_e^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\therefore v_e^2 = \frac{4G(M_1 + M_2)}{d}$$

$$\therefore v_e = 2\sqrt{\frac{G(M_1 + M_2)}{d}}$$

Question80

The average density of the earth is [g is acceleration due to gravity]

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Options:

A. inversely proportional to g^2.

B. directly proportional to g.

C. inversely proportional to g.

D. directly proportional to g^2.

Answer: B

Solution:

To determine the relationship between the average density of the Earth and the acceleration due to gravity (g), we start with the formula for gravitational force:

$F = G\frac{m_1 m_2}{r^2}$

Here, G is the gravitational constant, m_1 and m_2 are the masses of two objects, and r is the distance between their centers of mass. For an object near the Earth's surface, this force is also equal to the weight of the object, which is given by:

$F = mg$

Setting these two expressions for the force equal to each other, we get:

$mg = G\frac{mM_{\text{earth}}}{r^2}$

Here, M_{earth} is the mass of the Earth, and r (or R) is the radius of the Earth. Simplifying, we obtain the equation:

$g = G\frac{M_{\text{earth}}}{R^2}$

Now, the mass of the Earth can be expressed in terms of its volume and average density (ρ_{earth}) as follows:

$M_{\text{earth}} = \rho_{\text{earth}} \cdot V_{\text{earth}}$

Since the volume of a sphere is given by

$V_{\text{earth}} = \frac{4}{3}\pi R^3$

we can substitute this into our expression for M_{earth} to get:

$M_{\text{earth}} = \rho_{\text{earth}} \cdot \frac{4}{3}\pi R^3$

Substituting this back into our equation for g, we have:

$g = G\frac{\rho_{\text{earth}} \cdot \frac{4}{3}\pi R^3}{R^2}$

Simplifying the equation, we get:

$g = \frac{4\pi G}{3}\rho_{\text{earth}} R$

From this equation, we can see that the average density of the Earth, ρ_{earth}, is directly proportional to g, i.e.,

$\rho_{\text{earth}} \propto g$

Hence, the correct answer is:

Option B

directly proportional to g.

Question81

The depth from the surface of the earth of radius R, at which acceleration due to gravity will be 60% of the value on the earth surface is

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Options:

A. $\frac{2R}{3}$

B. $\frac{2R}{5}$

C. $\frac{3R}{5}$

D. $\frac{5R}{3}$

Answer: B

Solution:

To determine the depth from the surface of the Earth at which the acceleration due to gravity is 60% of its value on the surface, we can use the concept of the gravitational force inside the Earth.

Let g_0 be the acceleration due to gravity at the surface of the Earth, and let the radius of the Earth be R. When at a depth d from the surface, the gravity is determined by the distance from the center of the Earth, which is $R - d$.

The gravity inside the Earth at a depth d is given by the formula:

$g_d = g_0 \left(1 - \frac{d}{R}\right)$

Given that at depth d, the gravity is 60% of g_0, we get:

$g_d = 0.60 \cdot g_0$

Substitute this into the formula:

$0.60 \cdot g_0 = g_0 \left(1 - \frac{d}{R}\right)$

Divide both sides by g_0:

$0.60 = 1 - \frac{d}{R}$

Rearrange the equation to solve for d:

$\frac{d}{R} = 1 - 0.60 = 0.40$

Hence:

$d = 0.40R$

Thus, the depth d is:

$d = 0.40R$

To find the depth from the surface, use:

$d = R - \frac{d}{R}$

Rearranging for d we get:

$d = 0.40R$

Therefore, the depth from the surface is:

$d = 0.40R$

Which corresponds to the option:

Option B

$\frac{2R}{5}$

Question82

Three point masses, each of mass 'm' are kept at the corners of an equilateral triangle of side 'L'. The system rotates about the centre of the triangle without any change in the separation of masses during rotation. The period of rotation is directly proportional to $\left(\cos 30^{\circ} = \frac{\sqrt{3}}{2}\right)$

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Options:

A. L

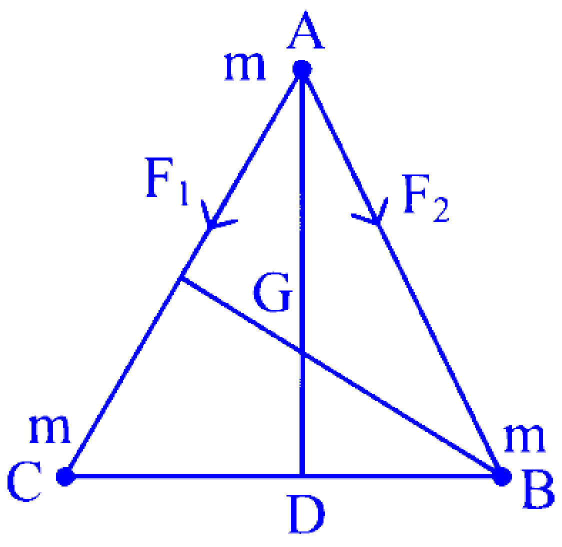
B. $L^{1/2}$

C. $L^{3/2}$

D. L^{-2}

Answer: C

Solution:



Consider mass m at A.

The forces exerted on it by the other two masses are given by

$F_1 = G\frac{m^2}{L^2} = F_2$

The angle between the two forces is 60°.

Hence the resultant force

$$F = \sqrt{F_1^2 + F_1^2 + 2\,F_1^2 \cos 60^\circ} = \sqrt{3}\,F_1$$

$$\therefore \quad F = \sqrt{3} \cdot G\frac{m^2}{L^2}$$

Mass m rotates around the centre G. The radius of the circular motion is AG.

$$AG = \frac{2}{3}AD$$

$$AD = AC \sin 60^\circ = L \sin 60^\circ = \frac{\sqrt{3}}{2}\,L$$

$$\therefore \quad AG = \frac{2}{3} \cdot \frac{\sqrt{3}}{2}\,L = \frac{L}{\sqrt{3}}$$

$$\therefore \text{ radius } r = \frac{L}{\sqrt{3}}$$

For uniform circular motion, the gravitational force provides the centripetal force.

$$\therefore mr\frac{L}{\sqrt{3}} \cdot \omega^2 = \sqrt{3}G\frac{m^2}{L^2}$$

$$\therefore \omega^2 = 3G\frac{m}{L^3}$$

$$\therefore \quad \omega=\left(3G\frac{m}{L^3}\right)^{1/2}$$

$$\therefore \frac{2\pi}{T}=\left(\frac{3G\,m}{L^3}\right)^{1/2}$$

$$\therefore T=2\pi\left(\frac{L^3}{3Gm}\right)^{1/2}$$

$$\therefore T\propto L^{3/2}$$

Question83

The length of the seconds pendulum is lm on earth. If the mass and diameter of the planet is 1.5 times that of the earth, the length of the seconds pendulum on the planet will be nearly

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Options:

A. 0.67 m

B. 0.45 m

C. 0.60 m

D. 0.76 m

Answer: A

Solution:

If g is the acceleration due to gravity on earth's surface and g' on the planet, then

$$g=\frac{GM}{r^2} \text{ and } g'=\frac{G\times 1.5M}{(1.5)^2r^2}=\frac{1}{1.5}\frac{GM}{r^2}$$

$$\therefore \quad g'=\frac{g}{1.5}.$$

For second's pendulum $T=2$ s

$$\therefore \mathrm{T} = 2\pi\sqrt{\frac{\ell}{\mathrm{g}}} = 2\pi\sqrt{\frac{\ell'}{\mathrm{g}'}}$$

$$\therefore \frac{\ell}{\mathrm{g}} = \frac{\ell'}{\mathrm{g}'} \quad \therefore \ell' = \ell \cdot \frac{\mathrm{g}'}{\mathrm{g}} = 1 \times \frac{1}{1.5} = \frac{2}{3} = 0.67 \mathrm{~m}$$

Question84

If the horizontal velocity given to a satellite is greater than critical velocity but less than the escape velocity at the height, then the satellite will

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Options:

A. be lost in space

B. falls on the earth along parabolic path

C. revolve in circular orbit

D. revolve in elliptical orbit

Answer: D

Solution:

- **Critical velocity (also called orbital velocity)** is the minimum horizontal velocity required for a satellite to go into a **circular orbit** around Earth.
- If horizontal velocity is **less than critical**, the body falls back to Earth.
- If horizontal velocity is **equal to critical**, it goes in a **circular orbit**.
- If horizontal velocity is **more than critical but less than escape velocity**, the satellite is bound to Earth but not in a circular path — it follows a **closed elliptical orbit**.
- If horizontal velocity is **equal to or greater than escape velocity**, it leaves Earth's gravitational field.

✔ So in this case:

$$V_c < V < V_e \quad \Rightarrow \quad \text{Elliptical orbit}$$

Therefore, option D: **revolve in elliptical orbit** is correct.

Question85

The period of revolution of planet A around the sun is 8 times that of planet B. How many times the distance of A from the sun is greater than that of B from the sun?

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Options:

A. 5 times

B. 2 times

C. 3 times

D. 6 times

Answer: C

Solution:

$$\left(\frac{R_1}{R_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2 = (8)^2 = 64$$

$$\therefore \frac{R_1}{R_2} = (64)^{\frac{1}{3}} = 4$$

$$\therefore R_1 = 4R_2$$

$$\therefore R_1 - R_2 = 4R_2 - R_2 = 3R_2$$

Question86

The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite will satellite is increased to four times the previous value, the new time period of the satellite will be

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Options:

A. 20 hours

B. 40 hours

C. 80 hours

D. 10 hours

Answer: B

Solution:

$T^2 \alpha r^3$

$\therefore \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = (4)^3 = 64$

$\therefore \frac{T_2}{T_1} = 8$

or $T_2 = 8\ T_1 = 8 \times 5 = 40$ hours.

Question87

A body of mass 'M' and radius 'R', situated on the surface of the earth becomes weightless at its equator when the rotational kinetic energy of the earth reaches a critical value 'K'. The value of 'K' is given by [Assume the earth as a solid sphere, g = gravitational acceleration on the earth's surfacde]

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Options:

A. $\frac{1}{2}MgR$

B. $\frac{1}{3}MgR$

C. $\frac{1}{4}MgR$

D. $\frac{1}{5}$MgR

Answer: D

Solution:

To determine the rotational kinetic energy at which a body becomes weightless at the equator, we need to consider the balance between the centrifugal force and the gravitational force acting on the body.

Let's start by analyzing the forces acting on the body at the equator. For a body to become weightless, the centrifugal force must equal the gravitational force:

Centrifugal Force:

$F_{\text{centrifugal}} = M\omega^2 R$

Gravitational Force:

$F_{\text{gravitational}} = Mg$

For weightlessness:

$M\omega^2 R = Mg$

Solving for ω^2:

$\omega^2 R = g \implies \omega^2 = \frac{g}{R}$

Next, the rotational kinetic energy of the Earth (considering it a solid sphere) is:

$K = \frac{1}{2} I\omega^2$

where I is the moment of inertia of the Earth. For a solid sphere, the moment of inertia is given by:

$I = \frac{2}{5} MR^2$

Substituting the value of I and ω^2:

$K = \frac{1}{2}\left(\frac{2}{5} MR^2\right)\left(\frac{g}{R}\right)$

Simplifying:

$K = \frac{1}{2} \cdot \frac{2}{5} MR^2 \cdot \frac{g}{R}$

$K = \frac{1}{2} \cdot \frac{2}{5} \cdot M \cdot R \cdot g$

$K = \frac{1}{5} MgR$

Thus, the value of the critical rotational kinetic energy K is:

$\frac{1}{5}$MgR

The correct option is:

Option D:

$\frac{1}{5}$MgR

Question88

The mass of a spherical planet is 4 times the mass of the earth, but its radius (R) is same as that of the earth. How much work is done is lifting a body of mass 5 kg through a distance of 2 m on the planet ? (g = 10 ms^{-2})

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Options:

A. 400 J

B. 200 J

C. 800 J

D. 300 J

Answer: A

Solution:

To determine the work done in lifting a body of mass 5 kg through a distance of 2 m on the spherical planet with the given conditions, we'll proceed step-by-step.

First, let's establish the gravitational force acting on the body on the planet, specifically the relationship between the gravitational forces on Earth and the planet.

Given:

- The mass of the planet is 4 times the mass of Earth.
- The radius (R) of the planet is the same as that of Earth.
- The gravitational acceleration on Earth, $g = 10\,\text{m/s}^2$.

The gravitational force at the surface of a planet is given by Newton's law of gravitation:

$F = \frac{GMm}{R^2}$

where:

- G is the gravitational constant,
- M is the mass of the planet,

- m is the mass of the object, and
- R is the radius of the planet.

Since the planet has 4 times the mass of Earth and the same radius, the gravitational acceleration g' on the planet can be written as:

$$g' = \frac{G \cdot (4M_{\text{Earth}})}{R^2}$$

Comparing with Earth's gravity:

$$g_{\text{Earth}} = \frac{GM_{\text{Earth}}}{R^2}$$

Thus,

$$g' = 4g_{\text{Earth}}$$

Given $g_{\text{Earth}} = 10\,\text{m/s}^2$, we have:

$$g' = 4 \times 10\,\text{m/s}^2 = 40\,\text{m/s}^2$$

Now, the work done in lifting a body through a vertical distance h is given by:

$$W = mg'h$$

Substituting the given values:

- $m = 5\,\text{kg}$
- $g' = 40\,\text{m/s}^2$
- $h = 2\,\text{m}$

$$W = 5\,\text{kg} \times 40\,\text{m/s}^2 \times 2\,\text{m} = 400\,\text{J}$$

Therefore, the work done is:

400 J

The correct answer is Option A: 400 J.

Question89

The radius of a planet is twice the radius of the earth. Both have almost equal average mass densities. If 'V_P' and 'V_E' are escape velocities of the planet and the earth respectively, then

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Options:

A. $\mathrm{V_E = 1.5\ V_P}$

B. $\mathrm{V_P = 1.5\ V_E}$

C. $\mathrm{V_P = 2V_E}$

D. $\mathrm{V_E = 3V_P}$

Answer: C

Solution:

Escape velocity $\mathrm{v} = \sqrt{\frac{2GM}{R}}$

$M = \frac{4}{3}\pi R^3 \cdot \rho$

$$\therefore v = \sqrt{\frac{2G \times \frac{4}{3}R^3 \cdot \rho}{R}} = \sqrt{\frac{8G}{3}R^2\rho} = R\sqrt{\frac{8G}{3}\rho}$$
$$\therefore v \propto R\sqrt{\rho}$$

$\therefore$ If ρ is constant, the $\mathrm{v} \propto \mathrm{R}$

$$\therefore \frac{v_P}{v_E} = \frac{R_P}{R_E} = 2$$
$$\therefore v_p = 2v_E$$

Question90

Two satellites of same mass are launched in circular orbits at heights 'R' and '$2R$' above the surface of the earth. The ratio of their kinetic energies is ($R =$ radius of the earth)

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Options:

A. 1 : 3

B. 3 : 2

C. 4 : 9

D. 9 : 4

Answer: B

Solution:

To find the ratio of the kinetic energies of two satellites orbiting at different heights above the Earth, we use the formula for kinetic energy in terms of gravitational force and orbital radius. The formula for the kinetic energy (KE) of a satellite in orbit around the Earth is given by the expression: $KE = \frac{GM_Em}{2r}$ where:

- KE is the kinetic energy of the satellite,
- G is the gravitational constant,
- M_E is the mass of the Earth,
- m is the mass of the satellite, and
- r is the distance from the center of the Earth to the satellite (i.e., the Earth's radius plus the height of the satellite above the Earth's surface).

For the first satellite at a height R above the surface, the total distance from the center of the Earth is $r_1 = R + R = 2R$. Thus, its kinetic energy (KE_1) would be:

$$KE_1 = \frac{GM_Em}{2(2R)} = \frac{GM_Em}{4R}.$$

For the second satellite at a height $2R$ above the surface, the total distance from the center of the Earth is $r_2 = R + 2R = 3R$. Thus, its kinetic energy (KE_2) would be:

$$KE_2 = \frac{GM_Em}{2(3R)} = \frac{GM_Em}{6R}.$$

To find the ratio of their kinetic energies, $\frac{KE_1}{KE_2}$, we divide KE_1 by KE_2:

$$\frac{KE_1}{KE_2} = \frac{\frac{GM_Em}{4R}}{\frac{GM_Em}{6R}} = \frac{GM_Em}{4R} \cdot \frac{6R}{GM_Em} = \frac{6}{4} = \frac{3}{2}.$$

Therefore, the correct answer is Option B, which shows the ratio of their kinetic energies as 3 : 2.

Question91

At a height 'R' above the earth's surface the gravitational acceleration is (R = radius of earth, g = acceleration due to gravity on earth's surface)

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Options:

A. g

B. $\frac{g}{8}$

C. $\frac{g}{4}$

D. $\frac{g}{2}$

Answer: C

Solution:

The gravitational acceleration at a distance 'r' from the center of the Earth, where 'r' is greater than the radius of the Earth (R), is given by the formula:

$g' = \frac{GM}{r^2}$

Here,

- g' is the gravitational acceleration at distance 'r' from the center of the Earth,
- G is the universal gravitational constant,
- M is the mass of the Earth,
- r is the distance from the center of the Earth to the point where the gravitational acceleration is being calculated.

In this case, we're looking at a height 'R' above the Earth's surface, so the distance 'r' from the center of the Earth becomes $r = R + R = 2R$. Substituting this into our formula gives us:

$g' = \frac{GM}{(2R)^2}$

$g' = \frac{GM}{4R^2}$

Now, the acceleration due to gravity on the surface of the Earth (g) is given by:

$g = \frac{GM}{R^2}$

Comparing the two expressions, we can now express g' in terms of g:

$g' = \frac{g}{4}$

Thus, the correct answer is Option C, which states that the gravitational acceleration at a height 'R' above the Earth's surface is $\frac{g}{4}$. This result demonstrates how gravitational acceleration decreases as one moves away from the surface of the Earth.

Question92

The mass of a planet is six times that of the earth. The radius of the planet is twice that of the earth. If the escape velocity from the earth is 'V_e', then the escape velocity from the planet is

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Options:

A. $\sqrt{3}\ \mathrm{V_e}$

B. $\sqrt{2}\ \mathrm{V_e}$

C. $\mathrm{V_e}$

D. $\sqrt{5}\ \mathrm{V_e}$

Answer: A

Solution:

To find the escape velocity from the planet, we need to use the formula for escape velocity. The escape velocity V from a celestial body is given by:

$$V = \sqrt{\frac{2GM}{R}}$$

where:

- G is the gravitational constant,
- M is the mass of the celestial body, and
- R is the radius of the celestial body.

For Earth, the escape velocity is:

$$V_e = \sqrt{\frac{2GM_e}{R_e}}$$

where M_e is the mass of the Earth and R_e is the radius of the Earth.

Now, for the planet in question, the mass of the planet M_p is 6 times that of the Earth, i.e.,

$$M_p = 6M_e$$

The radius of the planet R_p is twice that of the Earth, i.e.,

$$R_p = 2R_e$$

The escape velocity V_p from the planet is then given by:

$$V_p = \sqrt{\frac{2GM_p}{R_p}}$$

Substituting the values of M_p and R_p in the escape velocity formula, we get:

$$V_p = \sqrt{\frac{2G(6M_e)}{2R_e}}$$

Simplifying the expression inside the square root:

$V_p = \sqrt{\frac{12GM_e}{2R_e}}$

$V_p = \sqrt{6\frac{GM_e}{R_e}}$

Since we know that:

$V_e = \sqrt{\frac{2GM_e}{R_e}}$

we can substitute V_e^2 in place of $\frac{2GM_e}{R_e}$:

$V_e^2 = \frac{2GM_e}{R_e}$

Therefore:

$V_p = \sqrt{3 \cdot V_e^2}$

$V_p = \sqrt{3} \cdot V_e$

Thus, the escape velocity from the planet is $\sqrt{3} \cdot V_e$.

The correct answer is Option A: $\sqrt{3}$ V_e.

Question93

For a body of mass 'm', the acceleration due to gravity at a distance 'R' from the surface of the earth is $\left(\frac{g}{4}\right)$. Its value at a distance $\left(\frac{R}{2}\right)$ from the surface of the earth is ($R =$ radius of the earth, $g =$ acceleration due to gravity)

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Options:

A. $\left(\frac{g}{8}\right)$

B. $\left(\frac{9g}{4}\right)$

C. $\left(\frac{4g}{9}\right)$

D. $\left(\frac{g}{2}\right)$

Answer: C

Solution:

$g=\frac{GM}{r^2}$ where r is the distance from the centre of the earth

In the first case, $r_1=R+R=2R$

In the second case, $r_2=R+\frac{R}{2}=\frac{3}{2}R$

$$\therefore \frac{g_2}{g_1}=\left(\frac{r_1}{r_2}\right)^2=\frac{4\times4}{9}=\frac{16}{9}$$
$$g_2=\frac{16}{9}g_1=\frac{16}{9}\times\frac{9}{4}=\frac{4}{9}g$$

Question94

The ratio of energy required to raise a satellite of mass 'm' to height 'h' above the earth's surface to that required to put it into the orbit at same height is [$\mathrm{R}=$ radius of earth]

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Options:

A. $\frac{h}{R}$

B. $\frac{2h}{\mathrm{R}^2}$

C. $\frac{3\mathrm{~h}}{\mathrm{R}^2}$

D. $\frac{2\mathrm{~h}}{\mathrm{R}}$

Answer: D

Solution:

Energy required to raise to satellite of m to a height h is equal to change in its potential energy.

$\therefore W=-\frac{GMm}{R+h}+\frac{GMm}{R}=\frac{GMmh}{(R+h)R}$ .... (1)

The energy of a satellite moving in a circular orbit is given by

$$E=\frac{GMm}{2(R+h)} \quad ......(2)$$

$$\therefore \quad \frac{W}{E}=\frac{2h}{R}$$

Question95

A pendulum is oscillating with frequency 'n' on the surface of the earth. It is taken to a depth $\frac{R}{2}$ below the surface of earth. New frequency of oscillation at depth $\frac{R}{2}$ is

[R is the radius of earth]

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Options:

A. $\frac{n}{3}$

B. $\frac{n}{\sqrt{2}}$

C. 2n

D. $\frac{n}{2}$

Answer: B

Solution:

Frequency of a simple pendulum is given by

$$n=\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$\therefore \frac{n'}{n}=\sqrt{\frac{g'}{g}}$$

$$g'=g\left(1-\frac{d}{R}\right)=g\left(1-\frac{1}{2}\right)=\frac{g}{2}$$

$$\therefore \quad n'=\frac{n}{\sqrt{2}}$$

Question96

When the value of acceleration due to gravity 'g' becomes $\frac{g}{3}$ above surface of height 'h' then relation between 'h' and 'R' is ($\mathrm{R}=$ radius of earth)

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Options:

A. $\mathrm{h}=\frac{\mathrm{R}}{\sqrt{3}-1}$

B. $\mathrm{h}=\frac{\sqrt{3}}{\mathrm{R}}$

C. $h=(\sqrt{2}-1) R$

D. $\mathrm{h}=(\sqrt{3}-1) \mathrm{R}$

Answer: D

Solution:

$$\mathrm{g}^{\prime}=\mathrm{g} \frac{\mathrm{R}^2}{(\mathrm{R}+\mathrm{h})^2} \quad \mathrm{g}^{\prime}=\frac{\mathrm{g}}{3}$$

$$\therefore \frac{1}{3}=\frac{\mathrm{R}^2}{(\mathrm{R}+\mathrm{h})^2} \quad \therefore \frac{1}{\sqrt{3}}=\frac{\mathrm{R}}{\mathrm{R}+\mathrm{h}}$$

$$\therefore \sqrt{3} \mathrm{R}=\mathrm{R}+\mathrm{h}$$

$$\therefore \mathrm{h}=(\sqrt{3}-1) \mathrm{R}$$

Question97

A particle of mass 'm' is kept at rest at a height $3R$ from the surface of earth, where 'R' is radius of earth and 'M' is the mass of earth. The minimum speed with which it should be projected, so that it does not return back is ($g=$ acceleration due to gravity on the earth's surface)

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Options:

A. $\left[\frac{GM}{2R}\right]^{1/2}$

B. $\left[\frac{gR}{4}\right]^{1/2}$

C. $\left[\frac{2g}{R}\right]^{1/2}$

D. $\left[\frac{GM}{R}\right]^{1/2}$

Answer: A

Solution:

For the particle to escape from earth's gravitational field, it should be given kinetic energy equal to its binding energy.

$$\therefore \quad \frac{1}{2}mv^2 = \frac{GMm}{4R}$$

$$\therefore V = \left[\frac{GM}{2R}\right]^{1/2}$$

Question98

A body is projected from earth's surface with thrice the escape velocity from the surface of the earth. What will be its velocity when it will escape the gravitational pull?

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Options:

A. $2\ V_e$

B. $4\ V_e$

C. $2\sqrt{2}\ V_e$

D. $\frac{V_e}{2}$

Answer: C

Solution:

Energy required to escape the earth's gravitational field is $\frac{1}{2} mV_e^2$

Energy given to the body is $= \frac{1}{2}\ m(3\ V_e)^2$

$= \frac{9}{2} mV_e^2$

$\therefore$ If V is the velocity of the body when it has escaped from earth's gravitational field then

$$\frac{1}{2} mV^2 = \frac{9}{2} mV_e^2 - \frac{1}{2} mV_e^2$$
$$\therefore \quad \frac{1}{2} mV^2 = 4mV_e^2$$
$$\therefore \quad V^2 = 8\ V_e^2$$
$$\therefore \quad V = 2\sqrt{2}\ V_e$$

Question99

The depth at which acceleration due to gravity becomes $\frac{g}{n}$ is [$R =$ radius of earth, $g =$ acceleration due to gravity, $n =$ integer]

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Options:

A. $\frac{R(n-1)}{n}$

B. $\frac{(n-1)}{nR}$

C. $\frac{Rn}{(n-1)}$

D. $\frac{n}{R(n-1)}$

Answer: A

Solution:

$$g' = g\left(1 - \frac{d}{R}\right)$$
$$\therefore \frac{g}{n} = g\left(1 - \frac{d}{R}\right)$$
$$\therefore \frac{1}{n} = \left(1 - \frac{d}{R}\right)$$
$$\therefore \frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n}$$
$$\therefore d = \frac{R(n-1)}{n}$$

Question100

The depth 'd' below the surface of the earth where the value of acceleration due to gravity becomes $\left(\frac{1}{n}\right)$ times the value at the surface of the earth is $(R =$ radius of the earth)

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Options:

A. $\mathrm{R}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)$

B. $R\left(\frac{n}{n+1}\right)$

C. $\frac{R}{n}$

D. $\frac{\mathrm{R}}{\mathrm{n}^2}$

Answer: A

Solution:

$\mathrm{g}' = \mathrm{g}\left(1 - \frac{\mathrm{d}}{\mathrm{R}}\right)$

If $g' = \frac{g}{n}$ then $\frac{g}{n} = g\left(1 - \frac{d}{R}\right)$

$$\therefore \frac{1}{n} = 1 - \frac{d}{R}$$
$$\therefore \frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n} \quad \therefore \quad d = R\left(\frac{n-1}{n}\right)$$

Question101

The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is 'V'. For the satellite orbiting at an altitude of half the earth's radius, the orbital velocity is

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Options:

A. $\frac{3}{2}V$

B. $\sqrt{\frac{3}{2}}V$

C. $\sqrt{\frac{2}{3}}V$

D. $\frac{2}{3}V$

Answer: C

Solution:

The orbital velocity near the surface of the earth

$V = \sqrt{\frac{GM}{R}}$

At an altitude $\frac{R}{2}$, the orbital velocity

$V' = \sqrt{\frac{GM}{R+\frac{R}{2}}} = \sqrt{\frac{2GM}{3R}} \quad \therefore \frac{V'}{V} = \sqrt{\frac{2}{3}} \quad$ or $\quad V' = \sqrt{\frac{2}{3}}\,V$

Question102

Earth has mass M_1 and radius R_1. Moon has mass M_2 and radius R_2. Distance between their centre is r. A body of mass M is placed on the line joining them at a distance $\frac{r}{3}$ from centre of the earth. To project the mass M to escape to infinity, the minimum speed required is

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Options:

A. $\left[\frac{3G}{r}\left(M_1+\frac{M_2}{2}\right)\right]^{\frac{1}{2}}$

B. $\left[\frac{6G}{r}\left(M_1+\frac{M_2}{2}\right)\right]^{\frac{1}{2}}$

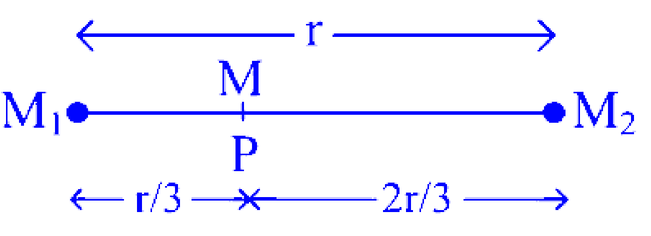
C. $\left[\frac{6G}{r}\left(M_1-\frac{M_2}{2}\right)\right]^{\frac{1}{2}}$

D. $\left[\frac{3G}{r}\left(M_1-\frac{M_2}{2}\right)\right]^{\frac{1}{2}}$

Answer: B

Solution:

The given situation can be drawn as



The gravitational potential at P is

$$V_p=-\left(\frac{GM_1}{\frac{r}{3}}+\frac{GM_2}{\frac{2r}{3}}\right)$$
$$=\frac{-3G\left(2M_1+M_2\right)}{2r}$$

The work done to escape the mass M to infinity is

$$W=M\left(V_\infty-V_p\right)=\frac{3GM(2M_1+M_2)}{2r}$$

As, work done is equal to kinetic energy of mass M.

$$\Rightarrow \quad \frac{1}{2} M v_e^2 = \frac{3GM(2M_1+M_2)}{2r}$$

$$v_e = \left[\frac{3G}{r}(2M_1 + M_2)\right]^{1/2}$$

$$\text{or} \quad v_e = \left[\frac{6G}{r}\left(M_1 + \frac{M_2}{2}\right)\right]^{1/2}$$

Question103

The escape velocity of a body from any planet, whose mass is six times the mass of earth and radius is twice the radius of earth will (v_e = escape velocity of a body from the earth's surface)

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Options:

A. $2\sqrt{2}v_e$

B. $\frac{3}{2}v_e$

C. $2v_e$

D. $\sqrt{3}v_e$

Answer: D

Solution:

The escape velocity of body from earth's surface is

$$v_e = \sqrt{\frac{2GM}{R}}$$

where, G is universal gravitational constant, M^{is} mass of earth and R is the radius of earth.

For the planet, $M_1 = 6M$ and $R_1 = 2R$

$\therefore$ The escape velocity from the planet will be

$$v_p = \sqrt{\frac{2G(6M)}{(2R)}} = \sqrt{3}v_e$$

Question104

The ratio of energy required to raise a satellite of mass m to a height h above the earth's surface of that required to put it into the orbit at same height is [$R =$ radius of the earth]

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Options:

A. $\frac{h}{R}$

B. $\frac{3h}{R}$

C. $\frac{4h}{R}$

D. $\frac{2h}{R}$

Answer: D

Solution:

The energy required to raise a satellite of mass m to a height h,

$E_1 = \frac{mgh}{\left(1+\frac{h}{R}\right)}$ ... (i)

The energy required to put it into orbit,

$$E_2 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)$$

($\because$ orbital velocity, $v_0 = \frac{GM}{r}$)

$$= \frac{1}{2}m\left(\frac{GM}{R+h}\right) = \frac{1}{2}\frac{mgR^2}{R\left(1+\frac{h}{R}\right)} (\because GM = gR^2)$$

$$= \frac{mgR}{2\left(1+\frac{h}{R}\right)}$$

The ratio of $E_1 : E_2$ becomes,

$$\frac{E_1}{E_2} = \frac{mgh}{\left(1+\frac{h}{R}\right)} \times \frac{2\left(1+\frac{h}{R}\right)}{mgR} = \frac{2h}{R}$$

Question105

As we go from the equator of the earth to pole of the earth, the value of acceleration due to gravity

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Options:

A. decreases

B. decreases up to latitude of 45° and increases thereafter

C. remains same

D. increases

Answer: D

Solution:

The acceleration due to gravity increases as we go from equator of earth to pole of earth because the equatorial radius is more than the polar radius of earth and $g \propto \frac{1}{R}$.

Question106

The mass of earth is 81 times the mass of the moon and the distance between their centres is R. The distance from the centre of the earth, where gravitational force will be zero is

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Options:

A. $\frac{R}{4}$

B. $\frac{R}{2}$

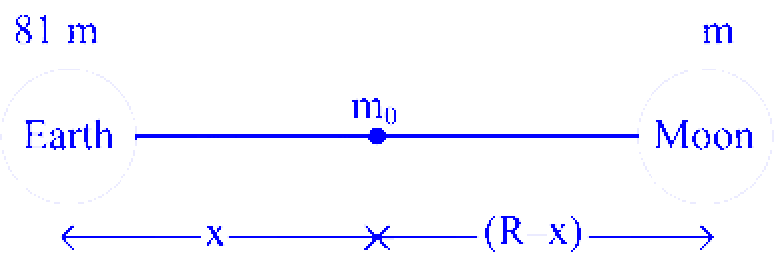
C. $\frac{9R}{10}$

D. $\frac{R}{81}$

Answer: C

Solution:

Let x be the distance from the centre of earth, where gravitational force is zero and m_0 be the point mass as shown below



Using Newton's law of gravitation,

Force between earth and point mass,

$F_1 = \frac{G \times 81m \times m_0}{x^2}$ ..... (i)

and force between moon and point mass,

$F_2 = \frac{G \times m \times m_0}{(R-x)^2}$ .... (ii)

According to the question,

$$F_1 = F_2$$

$$\Rightarrow \quad \frac{G \times 81m \times m_0}{x^2} = \frac{F_2}{(R-x)^2}$$

$$\Rightarrow \quad \frac{81}{x^2} = \frac{1}{(R-x)^2}$$

$$\Rightarrow \quad \frac{9}{x} = \frac{1}{R-x}$$

$$\Rightarrow \quad x = \frac{9}{10}R$$

Question107

A body is thrown from the surface of the earth velocity v/s. The maximum height above the earth's surface upto which it will reach is ($R =$ radius of earth, $g =$ acceleration due to gravity)

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Options:

A. $\frac{vR}{2gR-v}$

B. $\frac{2gA}{v^2(R-1)}$

C. $\frac{vR^2}{gR-v}$

D. $\frac{v^2R}{2gR-v^2}$

Answer: D

Solution:

At maximum height, the velocity of body becomes zero. Let maximum height be h.

Applying principle of conservation of energy, we get

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(0)^2 - \frac{GMm}{(R+h)}$$

[where, $m =$ mass of body
and $M =$ mass of earth.]

$$\Rightarrow \frac{1}{2}mv^2 = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow \quad \frac{h}{R(R+h)} = \frac{v^2}{2GM}$$

$$\Rightarrow \quad \frac{h}{R+h} = \frac{v^2R}{2GM} = \frac{v^2R}{2gR^2} \quad (\because GM = gR^2)$$

$$\Rightarrow \quad \frac{R+h}{h} = \frac{2gR}{v^2}$$

$$\Rightarrow \quad \frac{R}{h} + 1 = \frac{2gR}{v^2}$$

$$\Rightarrow \quad \frac{R}{h} = \frac{2gR}{v^2} - 1 = \frac{2gR - v^2}{v^2}$$

or $\quad h = \frac{v^2R}{2gR - v^2}$

Question108

Consider a particle of mass m suspended by a string at the equator. Let R and M denote radius and mass of the earth. If ω is the angular velocity of rotation of the earth about its own axis, then the tension on the string will be $(\cos 0^\circ = 1)$

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Options:

A. $\frac{GMm}{R^2}$

B. $\frac{GMm}{2R^2}$

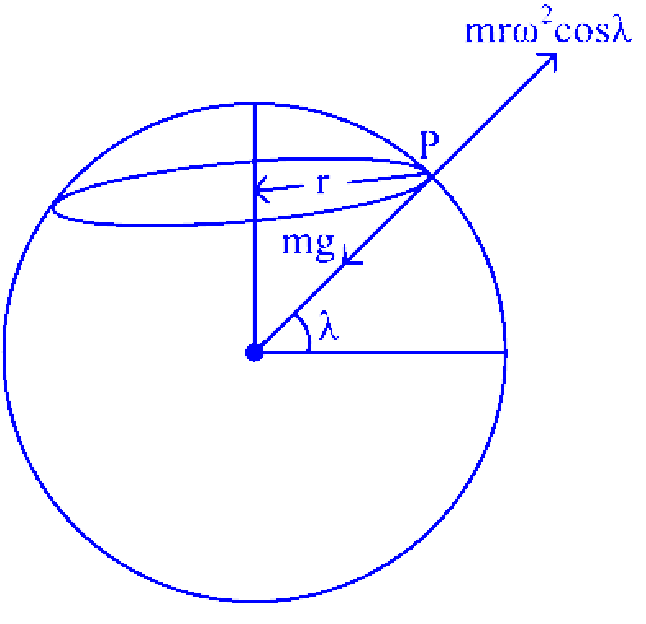
C. $\frac{GMm}{2R^2} + m\omega^2 R$

D. $\frac{GMm}{R^2} - m\omega^2 R$

Answer: D

Solution:

When a body suspended by the string situated at position P as shown in the figure, where latitude is λ, then body is also rotated with angular frequency (ω) of earth, hence tension on the string is given by



$$T = mg - mr\omega^2 \cos\lambda$$

$$T = m \cdot \frac{GM}{R^2} - mr\omega^2 \cos\lambda \left[\therefore g = \frac{GM}{R^2}\right]$$

$$T = \frac{GMm}{R^2} - mr\omega^2 \cos\lambda \quad .... \text{(i)}$$

When body is suspended at equator, then

$\lambda = 0$ and $r = R$

$\therefore$ From Eq. (i), we have,

$$T=\frac{GMm}{R^2}-mR\omega^2\cos 0^{\circ}$$
$$T=\frac{GMm}{R^2}-mR\omega^2$$

Question109

A hole is drilled half way to the centre of the earth. A body weighs 300 N on the surface of the earth. How much will, it weigh at the bottom of the hole?

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Options:

A. 200 N

B. 250 N

C. 120 N

D. 150 N

Answer: D

Solution:

Given, distance of bottom of hole from the surface of earth $(d)=$ half the radius of earth $=\frac{R_e}{2}$

If g be the value of gravitational acceleration on the surface of earth, then weight of body

$mg=300$ N

If g' be the gravitational acceleration at the bottom of hole, then

$$g'=g\left(1-\frac{d}{R_\theta}\right)=g\left(1-\frac{\frac{R_\theta}{2}}{R_e}\right)\Rightarrow g'=\frac{g}{2}$$

$\therefore$ Weight of the body on the bottom of hole,

$mg'=\frac{mg}{2}=\frac{300}{2}=150$ N

Question110

What is the minimum energy required to launch a satellite of mass ' m ' from the surface of the earth of mass ' M ' and radius ' R ' at an altitude $2R$?

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Options:

A. $\frac{GMm}{2R}$

B. $\frac{2GMm}{3R}$

C. $\frac{GMm}{3R}$

D. $\frac{5GMm}{6R}$

Answer: B

Solution:

The minimum energy required to launch a satellite from surface of earth is equal to the amount of work done against gravity to move it from the surface of earth to a height h and is given by

$$U = W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$= GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

Here, $h = 2R$

$\Rightarrow \quad U = GMm\left(\frac{1}{R} - \frac{1}{3R}\right) = \frac{2GMm}{3R}$

Question111

The radius of the earth and the radius of orbit around the sun are 6371 km and 149×10^6 km respectively. The order of magnitude of the diameter of the orbit is greater than that of earth by

Options:

A. 10^3

B. 10^2

C. 10^4

D. 10^5

Answer: C

Solution:

Given, radius of earth, $R_e = 6371$ km and radius of orbit around the sun, $R_0 = 149 \times 10^6$ km

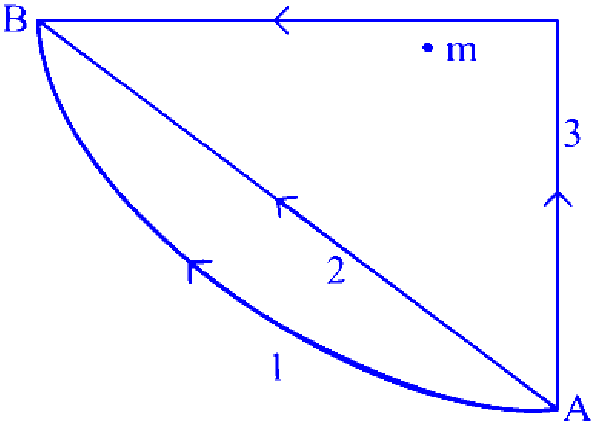
Now, the ratio of the diameters of the earth and orbit around the sun can be expressed as

$\frac{D_0}{D_\theta} = \frac{2R_0}{2R_\theta} = \frac{2\times14.9\times10^6}{2\times6371} = 2.33 \times 10^4$

From above ratio, it can be concluded that the order of magnitude of the diameter of the orbit is greater than that of earth by multiple of 10^4.

Question112

If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 (as shown in fig) in the gravitational field of the point mass ' m '. Find the correct relation between ' W_1 ', ' W_2 ' and ' W_3 '



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Options:

A. $W_1 < W_3 < W_2$

B. $W_1 < W_2 < W_3$

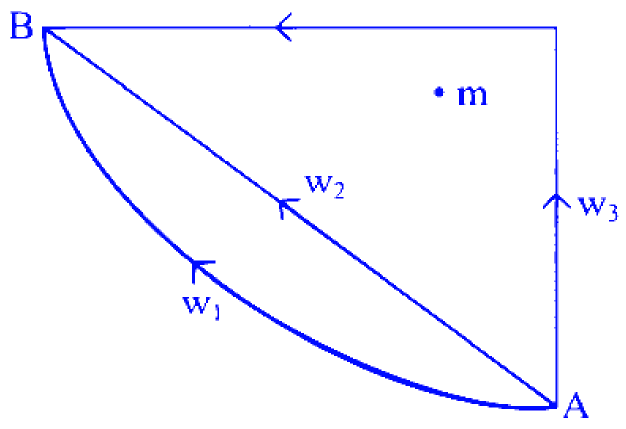
C. $W_1 = W_2 = W_3$

D. $W_1 > W_3 > W_2$

Answer: C

Solution:

Gravitational force is a conservative frorce. As we know that the work done by a conservative force is independent of the path followed and depends only on the end points of the motion. Since in the given pattern w_1, w_2 and w_3 have the same end point A and B as shown in the figure below,



So, the correct relation between is $w_1 = w_2 = w_3$

Question113

The kinetic energy of a revolving satellite (mass m) at a height equal to thrice the radius of the earth (R) is

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Options:

A. $\frac{mgR}{8}$

B. $\frac{mgR}{16}$

C. $\frac{mgR}{2}$

D. $\frac{mgR}{4}$

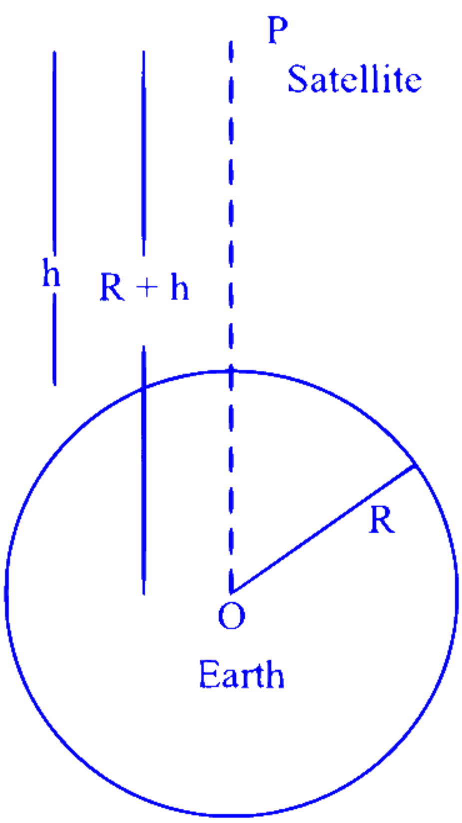
Answer: A

Solution:

As we know that the kinetic energy of a revolving satellite at a height h above the earth's surface is given as

$$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r}\left(\because v = \frac{\sqrt{GM}}{r} \text{ for circular orbit}\right)$$
$$= \frac{GMm}{2(R+h)}$$

where, R is the radius of the earth.



Given, height of a satellite from the earth surface $h=3R$ and mass of the satellite $=m$

So,

$$
\begin{aligned}
& KE=\frac{GMm}{2(R+3R)}=\frac{GMm}{2(4R)} \\
& KE=\frac{GMm}{8R}\times\frac{R}{R} \\
\therefore \quad & GMm\frac{R}{8R^2}=\frac{gmR}{8} \quad \left(\therefore g=\frac{GM}{R^2}\right)
\end{aligned}
$$

So, the kinetic energy, $KE=\frac{gmR}{8}$.

Question114

A body is projected vertically from the surface of the earth of radius ' R ' with velocity equal to half of the escape velocity. The maximum height reached by the body is

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Options:

A. $\frac{R}{5}$

B. $\frac{R}{3}$

C. $\frac{R}{2}$

D. $\frac{R}{4}$

Answer: B

Solution:

According to the law of conservation of energy,

$$
\begin{aligned}
& (\text{ Total energy) })_{\text{surtace}} = (\text{ Total energy })_{(\text{max height})} \\
\Rightarrow \quad & (KE+PE)_{\text{surtace}} = (KE+PE)_{\text{max height}} \\
\Rightarrow & \frac{1}{2}mv^2+\left(-\frac{GMm}{R}\right)=0+\left(-\frac{GMm}{R+h}\right)
\end{aligned}
$$

Given, $v = \frac{1}{2} v_e = \frac{1}{2} \sqrt{\frac{2GM}{R}}$

$\Rightarrow \frac{1}{2} m \left[\frac{1}{4} \cdot \frac{2GM}{R}\right] - \frac{GMm}{R} = -\frac{GMm}{R+h}$

$\Rightarrow \quad \frac{1}{4} \frac{GMm}{R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$

$\Rightarrow \quad \frac{1-4}{4R} = \frac{1}{R+h} \Rightarrow 3R + 3h = 4R$

$\Rightarrow \quad h = \frac{R}{3}$

So, option (b) is the correct answer.