Functions

Question1

The function $f(x) = \sec\left[\log\left(x + \sqrt{1+x^2}\right)\right]$ is _______ function MHT CET 2025 (5 May Shift 2)

Options:

A. even

B. odd

C. neither even nor odd

D. square

Answer: A

Solution:

Step 1: Substitute $-x$

$$f(-x) = \sec\left[\log\left(-x + \sqrt{1+(-x)^2}\right)\right]$$
$$= \sec\left[\log\left(-x + \sqrt{1+x^2}\right)\right]$$

Step 2: Simplify using a logarithmic identity

We know that:

$$\log(-x + \sqrt{1+x^2}) = -\log(x + \sqrt{1+x^2})$$

That's because:

$$(x + \sqrt{1+x^2})(-x + \sqrt{1+x^2}) = 1$$

⇒ Taking log of both sides gives:

$$\log(-x + \sqrt{1+x^2}) = -\log(x + \sqrt{1+x^2})$$

Step 3: Substitute this back

$$f(-x) = \sec\left[-\log(x + \sqrt{1+x^2})\right]$$

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Step 4: Use even property of secant

$$\sec(-\theta) = \sec(\theta)$$

Therefore:

$$f(-x) = \sec\left[\log(x + \sqrt{1 + x^2})\right] = f(x)$$

✅ **Conclusion:**

$$\boxed{f(x) \text{ is an even function.}}$$

Question2

If $[x]^2 - 5[x] + 6 = 0$, where $[\cdot]$ denotes the greatest integer function, then MHT CET 2025 (26 Apr Shift 1)

Options:

A. $x \in (2, 4)$

B. $x \in [2, 4]$

C. $x \in [2, 4)$

D. $x \in (2, 4]$

Answer: C

Solution:

We're given:

$$[x]^2 - 5[x] + 6 = 0$$

where $[x]$ denotes the **greatest integer function (GIF)**, i.e., the greatest integer less than or equal to x.

Step 1: Let $[x] = n$

Then the equation becomes:

$$n^2 - 5n + 6 = 0$$

Step 2: Factorize

$$n^2 - 5n + 6 = (n - 2)(n - 3) = 0$$

So,

$$n = 2 \text{ or } n = 3$$

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Step 3: Translate back to x

If $[x] = n$, then:

- For $n = 2$, $x \in [2, 3)$
- For $n = 3$, $x \in [3, 4)$

Step 4: Combine intervals

$$x \in [2, 4)$$

✅ **Final Answer:**

$$\boxed{x \in [2, 4)}$$

Question3

Let $\mathrm{f} : \mathbb{R} - \{2\} \to \mathbb{R} - \{1\}$ defined by $\mathrm{f}(x) = \frac{x-3}{x-2}$ and $\mathrm{g} : \mathbb{R} \to \mathbb{R}$ defined by $\mathrm{g}(x) = 3\mathrm{x} - 2$, then sum of all values of x for which $\mathrm{f}^{-1}(x) + \mathrm{g}^{-1}(x) = \frac{19}{6}$ is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\frac{5}{2}$

B. $\frac{7}{2}$

C. $\frac{9}{2}$

D. $\frac{11}{2}$

Answer: A

Solution:

Given:

$$f(x) = \frac{x-3}{x-2}, \quad g(x) = 3x - 2$$

We need:

$$f^{-1}(x) + g^{-1}(x) = \frac{19}{6}$$

Step 1:

$$f^{-1}(x) = \frac{2x-3}{x-1}, \quad g^{-1}(x) = \frac{x+2}{3}$$

Step 2:

$$\frac{2x-3}{x-1} + \frac{x+2}{3} = \frac{19}{6}$$

Simplify → $2x^2 - 5x - 3 = 0$

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Step 3:

$$x = 3, -\frac{1}{2}$$

Sum = $3 - \frac{1}{2} = \boxed{\frac{5}{2}}$ ✅

Question4

If $\mathrm{f}(x) = \log\left(\frac{1+x}{1-x}\right)$ and $\mathrm{g}(x) = \frac{3x+x^3}{1+3x^2}$, then (fog)$(x) =$ MHT CET 2025 (25 Apr Shift 1)

Options:

A. $2\mathrm{f}(x)$

B. $3\mathrm{f}(x)$

C. $4\mathrm{f}(x)$

D. $-\mathrm{f}(x)$

Answer: B

Solution:

We're given:

$$f(x) = \log\left(\frac{1+x}{1-x}\right), \quad g(x) = \frac{3x+x^3}{1+3x^2}$$

Find $(f \circ g)(x)$.

Step 1: Express $f(g(x))$

$$f(g(x)) = \log\left(\frac{1+g(x)}{1-g(x)}\right)$$

Step 2: Simplify using tangent substitution

Let $x = \tan\theta$.
Then:

$$g(x) = \frac{3\tan\theta + \tan^3\theta}{1+3\tan^2\theta} = \tan(3\theta)$$

Step 3: Substitute back into f

$$f(g(x)) = \log\left(\frac{1+\tan(3\theta)}{1-\tan(3\theta)}\right)$$

But:

$$f(x) = \log\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

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So:

$$f(g(x)) = 3f(x)$$

✅ **Final Answer:**

$$\boxed{3f(x)}$$

Question5

$$f(x) = \begin{cases} 3 - x, & -1 \leqslant x < 0 \\ 1 + \frac{5x}{3}, & -3 \leqslant x \leqslant 2 \end{cases}$$

and $g(x) = \begin{cases} -x, & -2 \leqslant x \leqslant 3 \\ x, & 0 \leqslant x \leqslant 1 \end{cases}$ then range of $(fog)(x)$ is

MHT CET 2025 (23 Apr Shift 2)

Options:

A. $\left[1, \frac{8}{3}\right]$

B. $\left[-4, \frac{8}{3}\right]$

C. $\left[-4, \frac{13}{3}\right]$

D. $\left[\frac{8}{3}, \frac{10}{3}\right]$

Answer: C

Solution:

- $g(x) = -x$ on $[-2, 0] \cup [1, 3]$ and $g(x) = x$ on $[0, 1] \Rightarrow$ $\text{Range}(g) = [-3, -1] \cup [0, 2]$.
- This avoids $(-1, 0)$, so in f we use $f(y) = 1 + \frac{5y}{3}$ on both pieces. Map: $[-3, -1] \to [-4, -\frac{2}{3}]$ and $[0, 2] \to [1, \frac{13}{3}]$.
- Union $\Rightarrow [-4, \frac{13}{3}]$.

Question6

If $\mathrm{f}(x) = 3\mathrm{x} + 10,\ \mathrm{g}(x) = x^2 - 1$**, then** $(\mathrm{fog})^{-1}(x) =$ **MHT CET 2025 (23 Apr Shift 1)**

Options:

A. $\left(\frac{x-7}{3}\right)$

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B. $\left(\frac{x-7}{3}\right)^{\frac{1}{2}}$

C. $\left(\frac{x-7}{3}\right)^{\frac{1}{3}}$

D. $\left(\frac{3}{x-7}\right)^{\frac{3}{2}}$

Answer: B

Solution:

$f \circ g(x) = f(g(x)) = 3(x^2 - 1) + 10 = 3x^2 + 7.$

Let $y = 3x^2 + 7$. Then $x^2 = \dfrac{y-7}{3} \Rightarrow x = \sqrt{\dfrac{y-7}{3}}$ (taking the principal root for the inverse as a function; range of $f \circ g$ is $y \geq 7$).

So,

$$(f \circ g)^{-1}(x) = \left(\frac{x-7}{3}\right)^{1/2}.$$

That's option **B**.

Question7

The values of b and c for which the identity $\mathrm{f}(x+1) - \mathrm{f}(x) = 8x + 3$ is satisfied, where $\mathrm{f}(x) = \mathrm{b}x^2 + \mathrm{c}x + \mathrm{d}$, are MHT CET 2025 (22 Apr Shift 2)

Options:

A. $b = 2, c = 1$

B. $b = 4, c = -1$

C. $b = 1, c = 2$

D. $b = 3, c = -1$

Answer: B

Solution:

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Step 1: Expand $f(x+1)$

$f(x+1) = b(x+1)^2 + c(x+1) + d = b(x^2+2x+1) + c(x+1) + d = bx^2 + 2bx + b + cx + c + d.$

Step 2: Compute the difference

$$f(x+1) - f(x) = (bx^2 + 2bx + b + cx + c + d) - (bx^2 + cx + d).$$

Simplify:

$$f(x+1) - f(x) = 2bx + b + c.$$

Step 3: Compare with the given expression

We're told this equals $8x+3$.
So:

$$2bx + (b+c) = 8x + 3.$$

Step 4: Match coefficients

Compare terms of x and constants:

$$2b = 8 \implies b = 4,$$

$$b + c = 3 \implies 4 + c = 3 \implies c = -1.$$

✅ **Final Answer:**

$$\boxed{b = 4,\ c = -1.}$$

That corresponds to **option B**, the correct one.

Question8

For a real number x, $[x]$ denotes the greatest integer less than or equal to x. Then the value of $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + \left[\frac{1}{2} + \frac{3}{100}\right] + \ldots\ldots\ldots\ldots\ldots + \left[\frac{1}{2} + \frac{99}{100}\right] =$ MHT CET 2025 (22 Apr Shift 1)

Options:

A. 49

B. 100

C. 0

D. 50

Answer: D

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Solution:

Let $S = \sum_{k=0}^{99} \left\lfloor \frac{1}{2} + \frac{k}{100} \right\rfloor$.

For $0 \leq k \leq 49$: $\frac{1}{2} + \frac{k}{100} < 1 \Rightarrow \lfloor\cdot\rfloor = 0$.

For $50 \leq k \leq 99$: $1 \leq \frac{1}{2} + \frac{k}{100} < 2 \Rightarrow \lfloor\cdot\rfloor = 1$.

There are $99 - 50 + 1 = 50$ terms equal to 1, so

$$S = 50.$$

Answer: 50.

Question9

If $\mathrm{f}(x) = \cos(\log x)$ **then** $\mathrm{f}\left(x^2\right) \cdot \mathrm{f}\left(\mathrm{y}^2\right) - \frac{1}{2}\left[\mathrm{f}\left(\frac{x^2}{\mathrm{y}^2}\right) + \mathrm{f}\left(x^2\mathrm{y}^2\right)\right]$ **has the value MHT CET 2025 (21 Apr Shift 1)**

Options:

A. -2

B. -1

C. $\frac{1}{2}$

D. 0

Answer: D

Solution:

Step 1: Express each term in terms of $\log x$ and $\log y$

$$f(x^2) = \cos(\log(x^2)) = \cos(2\log x),$$
$$f(y^2) = \cos(2\log y),$$
$$f\left(\frac{x^2}{y^2}\right) = \cos(\log(x^2/y^2)) = \cos(2(\log x - \log y)),$$
$$f(x^2y^2) = \cos(\log(x^2y^2)) = \cos(2(\log x + \log y)).$$

Let's set:

$$A = 2\log x, \quad B = 2\log y.$$

Then the expression becomes:

$$\cos A \cos B - \frac{1}{2}[\cos(A - B) + \cos(A + B)].$$

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Step 2: Use trigonometric identities

We know:

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)].$$

Substitute that into the expression:

$$E = \frac{1}{2}[\cos(A-B) + \cos(A+B)] - \frac{1}{2}[\cos(A-B) + \cos(A+B)] = 0.$$

✅ **Final Answer:**

$$\boxed{0}.$$

Hence, the correct option is **D (0)**.

Question10

Which of the following is not a homogeneous function? MHT CET 2025 (21 Apr Shift 1)

Options:

A. $y^2 + 2xy$

B. $2x - 3y$

C. $\sin\left(\frac{y}{x}\right)$

D. $\cos x + \sin y$

Answer: D

Solution:

Option A: $f(x, y) = y^2 + 2xy$

$$f(tx, ty) = (ty)^2 + 2(tx)(ty) = t^2(y^2 + 2xy)$$

✅ Homogeneous of degree 2.

Option B: $f(x, y) = 2x - 3y$

$$f(tx, ty) = 2(tx) - 3(ty) = t(2x - 3y)$$

✅ Homogeneous of degree 1.

Option C: $f(x, y) = \sin\left(\frac{y}{x}\right)$

$$f(tx, ty) = \sin\left(\frac{ty}{tx}\right) = \sin\left(\frac{y}{x}\right)$$

✅ Homogeneous of degree 0.

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Option D: $f(x, y) = \cos x + \sin y$

$$f(tx, ty) = \cos(tx) + \sin(ty)$$

This **cannot** be written as $t^n f(x, y)$ for any n.

❌ **Not homogeneous.**

✅ **Final Answer:**

$$\boxed{\cos x + \sin y}$$

That corresponds to **option D.**

Question11

The function defined by $\text{f}(x) = \frac{2x+3}{3x+4}, x \neq -\frac{4}{3}$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. only one one

B. only onto

C. onto for $y \neq \frac{2}{3}$ and one - one

D. neither one-one nor onto

Answer: C

Solution:

Step 1: Check if it's one-one (injective)

Let $f(x_1) = f(x_2)$:

$$\frac{2x_1 + 3}{3x_1 + 4} = \frac{2x_2 + 3}{3x_2 + 4}.$$

Cross-multiply:

$$(2x_1 + 3)(3x_2 + 4) = (2x_2 + 3)(3x_1 + 4).$$

Expand both sides:

$$6x_1x_2 + 8x_1 + 9x_2 + 12 = 6x_1x_2 + 8x_2 + 9x_1 + 12.$$

Simplify:

$$8x_1 + 9x_2 = 8x_2 + 9x_1 \implies x_1 = x_2.$$

✅ Hence, the function is **one-one.**

Step 2: Find its range (check if onto)

Let $y = \frac{2x+3}{3x+4}$.

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Solve for x:

$$y(3x+4) = 2x+3.$$

$$3xy + 4y = 2x + 3.$$

$$x(3y-2) = 3-4y.$$

$$x = \frac{3-4y}{3y-2}.$$

This is defined **for all real** $y \neq \frac{2}{3}$ (since denominator $3y - 2 \neq 0$).

✅ So the **range is all real numbers except** $y = \frac{2}{3}$.

✅ **Final Answer:**

$$f(x) \text{ is one-one and onto for } y \neq \frac{2}{3}.$$

That corresponds to **Option C.**

Question12

The domain of the function $\mathrm{f}(x) = {}^{7-x}\mathrm{P}_{x-1}$ MHT CET 2025 (19 Apr Shift 2)

Options:

A. R

B. $x \in \mathbb{R} - \{1\}$

C. $\{1, 2, 3, 4\}$

D. $\{1, 2, 3, 4, 5, 6\}$

Answer: C

Solution:

Step 1: Recall the condition for permutations

For ${}^nP_r$ to be defined:

- n and r must be **non-negative integers.**
- $r \leq n$.

Step 2: Apply these conditions here

Here:

$$n = 7 - x, \quad r = x - 1$$

So, we need:

1. $7 - x \geq 0 \implies x \leq 7$
2. $x - 1 \geq 0 \implies x \geq 1$
3. $r \leq n \implies x - 1 \leq 7 - x \implies 2x \leq 8 \implies x \leq 4.$

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Step 3: Combine all conditions

$$1 \leq x \leq 4$$

And x must be an **integer.**

✅ **Final Answer:**

$$\boxed{\{1, 2, 3, 4\}}$$

That corresponds to **option C.**

Question13

The domain of the function $\mathrm{f}(x)=\frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ **is MHT CET 2024 (16 May Shift 2)**

Options:

A. $(2,3)$

B. $[2,3)$

C. $[2,3]$

D. $(2,3]$

Answer: B

Solution:

To define $\mathrm{f}(x), 9-x^2>0 \Rightarrow|x|<3$
$\Rightarrow -3<x<3, \ldots (i)$
and $-1 \leq (x-3) \leq 1$
$\Rightarrow 2 \leq x \leq 4 \ldots$(ii)

From (i) and (ii), $2 \leq x < 3$ i.e., [2, 3).

Question14

Range of the function $\mathrm{f}(x)=\frac{x^2+x+2}{x^2+x+1}, x \in \mathbb{R}$ **is MHT CET 2024 (16 May Shift 1)**

Options:

A. $\left(1, \frac{7}{3}\right)$

B. $\left[1, \frac{7}{3}\right)$

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C. $\left(1, \frac{7}{3}\right]$

D. $\left[1, \frac{7}{3}\right]$

Answer: D

Solution:

Let $y = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow (y-1)x^2 + (y-1)x + y - 2 = 0$
For real value of $x, b^2 - 4ac \geq 0$

$$\Rightarrow (y-1)^2 - 4(y-1)(y-2) \geq 0$$
$$\Rightarrow 3y^2 - 10y + 7 \leq 0$$
$$\Rightarrow (y-1)(3y-7) \leq 0$$
$$\Rightarrow 1 \leq y \leq \frac{7}{3}$$

Here, $y \neq 1$ for any $x \in \mathrm{R}$

$\therefore \quad \mathrm{R_f} = \left[1, \frac{7}{3}\right]$

Question15

Let the function $\mathrm{g} : (-\infty, \infty) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $\mathrm{g(u)} = 2\tan^{-1}(\mathrm{e^u}) - \frac{\pi}{2}$. Then g is MHT CET 2024 (16 May Shift 1)

Options:

A. even and is strictly increasing in $(0, \infty)$.

B. odd and is strictly decreasing in $(-\infty, \infty)$.

C. odd and is strictly increasing in $(-\infty, \infty)$.

D. neither even nor odd, but is strictly increasing in $(-\infty, \infty)$.

Answer: C

Solution:

$$\mathrm{g(u)} = 2\tan^{-1}(\mathrm{e^u}) - \frac{\pi}{2}$$
$$\Rightarrow \mathrm{g(u)} = \tan^{-1}(\mathrm{e^u}) - \left(\frac{\pi}{2} - \tan^{-1}(\mathrm{e^u})\right)$$
$$\Rightarrow \mathrm{g(u)} = \tan^{-1}(\mathrm{e^u}) - \cot^{-1}(e^{\mathrm{u}})$$

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$$\Rightarrow g(-u) = \tan^{-1}(e^{-u}) - \cot^{-1}(e^{-u})$$
$$\Rightarrow g(-u) = \tan^{-1}\left(\frac{1}{e^u}\right) - \cot^{-1}\left(\frac{1}{e^u}\right)$$
$$\Rightarrow g(-u) = \cot^{-1}(e^u) - \tan^{-1}(e^u) = -g(u)$$

Now, $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2} \Rightarrow g'(u) = \frac{2e^u}{1+e^{2u}} > 0$ for all $u \in (-\infty, \infty) \Rightarrow$ g is strictly increasing function in $(-\infty, \infty)$. Hence, g(u) is odd and is strictly increasing in $(-\infty, \infty)$.

Question16

If $[x]^2 - 5[x] + 6 = 0$, where [.] denotes the greatest integer function, then MHT CET 2024 (15 May Shift 2)

Options:

A. $x \in (2, 4]$

B. $x \in [2, 4]$

C. $x \in [2, 4)$

D. $x \in (2, 4)$

Answer: C

Solution:

Given:
$$\begin{aligned} &[x]^2 - 5[x] + 6 = 0 \\ &\text{Let } [x] = a \\ &\Rightarrow a^2 - 5a + 6 = 0 \\ &\Rightarrow (a-2)(a-3) = 0 \\ &\Rightarrow a = 2 \text{ or } a = 3 \\ &\Rightarrow [x] = 2 \text{ or } [x] = 3 \\ &\Rightarrow x \in [2,3) \text{ or } x \in [3,4) \\ &\Rightarrow x \in [2,4) \end{aligned}$$

Question17

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Let f : $[-1, 3] \rightarrow \mathbb{R}$ be defined as

$$\begin{cases} |x| + [x], & -1 \leqslant x < 1 \\ x + |x|, & 1 \leqslant x < 2 \\ x + [x], & 2 \leqslant x \leqslant 3 \end{cases}$$

where $[t]$ denotes the greatest integer function.

Then f is discontinuous at

MHT CET 2024 (15 May Shift 1)

Options:

A. only two points

B. only three points

C. four or more points

D. only one point

Answer: B

Solution:

$$\mathrm{f}(x) = \begin{cases} |x| + [x], & -1 \leq x < 1 \\ x + |x|, & 1 \leq x < 2 \\ x + [x], & 2 \leq x \leq 3 \end{cases}$$

$$\therefore \quad \mathrm{f}(x) = \begin{cases} -(x+1), & -1 \leq x < 0 \\ x, & 0 \leq x < 1 \\ 2x, & 1 \leq x < 2 \\ x+2, & 2 \leq x < 3 \\ x+3, & x = 3 \end{cases}$$

$\therefore$ $\mathrm{f}(x)$ is discontinuous at $x = 0, 1, 3$.

Question18

Let $\mathrm{f}(x) = (x+1)^2 - 1, x \geqslant -1$, then the set $\left\{x / \mathrm{f}(x) = \mathrm{f}^{-1}(x)\right\}$ is MHT CET 2024 (15 May Shift 1)

Options:

A. $\{0, 1, -1\}$

B. $\{0, -1\}$

C. $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}, \text{where } i = \sqrt{-1}\right\}$

D. ϕ

Answer: B

Solution:

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$$f(x) = f^{-1}(x)$$
$$\Rightarrow f(f(x)) = x$$
$$\Rightarrow (f(x)+1)^2 - 1 = x$$
$$\Rightarrow \left[(x+1)^2 - 1 + 1\right]^2 - 1 = x$$
$$\Rightarrow (x+1)^4 - 1 = x$$
$$\Rightarrow (x+1)^4 - (x+1) = 0$$
$$\Rightarrow (x+1)\left[(x+1)^3 - 1\right] = 0$$
$$\Rightarrow x+1 = 0 \text{ or } (x+1)^3 = 1$$
$$\Rightarrow x = -1 \text{ or } x = 0$$

Question19

If f : $[1, \infty) \rightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals MHT CET 2024 (11 May Shift 2)

Options:

A. $\frac{x+\sqrt{x^2-4}}{2}$

B. $\frac{2}{1+x^2}$

C. $\frac{x-\sqrt{x^2-4}}{2}$

D. $1 + \sqrt{x^2-4}$

Answer: A

Solution:

$$f(x) = x + \frac{1}{x}$$
$$\text{let } y = x + \frac{1}{x}$$
$$\therefore \quad xy = x^2 + 1$$
$$\therefore \quad x^2 - xy + 1 = 0$$

$$\therefore \quad x = \frac{y \pm \sqrt{y^2-4}}{2}$$
$$\therefore \quad x = \frac{y + \sqrt{y^2-4}}{2} \qquad \ldots [\because x \in [1, \infty)]$$
$$\therefore \quad f^{-1}(x) = \frac{x+\sqrt{x^2-4}}{2}$$

Question20

If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then MHT CET 2024 (11 May Shift 1)

Options:

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A. $x \in [2,3)$

B. $x \in [2,3]$

C. $x \in [2,4]$

D. $x \in [2,4)$

Answer: D

Solution:

Given:
$$\begin{aligned}&[x]^2-5[x]+6=0\\&\text{Let } [x]=\mathrm{a}\\&\Rightarrow \mathrm{a}^2-5\mathrm{a}+6=0\\&\Rightarrow (\mathrm{a}-2)(\mathrm{a}-3)=0\\&\Rightarrow \mathrm{a}=2 \text{ or } \mathrm{a}=3\\&\Rightarrow [x]=2 \text{ or } [x]=3\\&\Rightarrow x\in[2,3) \text{ or } x\in[3,4)\\&\Rightarrow x\in[2,4)\end{aligned}$$

Question21

Let $\mathrm{f}(x)=\frac{\mathrm{ax}}{x+1}, x \neq -1$, then for $\alpha=$ _____, $\mathrm{f}(\mathrm{f}(\mathrm{x}))=x$. MHT CET 2024 (10 May Shift 2)

Options:

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 1

D. -1

Answer: D

Solution:

$$\mathrm{f}(x)=\frac{\alpha x}{x+1}$$

$$\mathrm{f}(\mathrm{f}(x))=\mathrm{f}\left(\frac{\alpha x}{x+1}\right)=\frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\frac{\alpha x}{x+1}+1}$$

$$\text{But } \mathrm{f}(\mathrm{f}(x))=x$$

$$\therefore \quad \frac{\alpha^2 x}{\alpha x+x+1}=x$$

$$\text{In L.H.S., Put } \alpha=-1$$

$$\therefore \quad \frac{(-1)^2 x}{(-1)x+x+1}=\frac{x}{-x+x+1}=x$$

$$\therefore \quad \alpha=-1$$

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Question22

The domain of definition of the function $y(x)$ is given by the equation $2^x + 2^y = 2$, is MHT CET 2024 (10 May Shift 1)

Options:

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Solution:

$$2^x + 2^y = 2$$
$$\Rightarrow 2^y = 2 - 2^x$$
$$\Rightarrow y = \log_2(2 - 2^x) \text{ is defined, if } 2 - 2^x > 0$$
$$\Rightarrow 2^x < 2$$
$$\Rightarrow 2^{x-1} < 1$$
$$\Rightarrow x - 1 < 0$$
$$\Rightarrow -\infty < x < 1$$

Question23

The domain of definition of the function $\mathrm{f}(x)$ given by the equation $2^x + 2^y = 2$ is MHT CET 2024 (09 May Shift 2)

Options:

A. $0 < x \leqslant 1$

B. $0 \leqslant x \leqslant 1$

C. $-\infty < x \leqslant 0$

D. $-\infty < x < 1$

Answer: D

Solution:

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$2^x + 2^y = 2$
$\Rightarrow 2^y = 2 - 2^x$
$\Rightarrow y = \log_2(2 - 2^x)$ is defined, if $2 - 2^x > 0$
$\Rightarrow 2^x < 2$
$\Rightarrow 2^{x-1} < 1$
$\Rightarrow x - 1 < 0$
$\Rightarrow -\infty < x < 1$

Question24

Domain of definition of the real valued function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is MHT CET 2024 (09 May Shift 1)

Options:

A. $\left[-\frac{1}{4}, \frac{1}{2}\right]$

B. $\left[\frac{-3}{2}, \frac{1}{2}\right]$

C. $\left[\frac{-3}{2}, \frac{1}{9}\right]$

D. $\left[-\frac{1}{4}, \frac{3}{4}\right]$

Answer: A

Solution:

$f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ to find domain

$$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$$

But $\frac{-\pi}{2} \leq \sin^{-1}\theta \leq \frac{\pi}{2}$

$$\frac{-\pi}{6} \leq \sin^{-1}(2x) \leq \frac{\pi}{2}$$

$$\Rightarrow -\sin\frac{\pi}{6} \leq 2x \leq \sin\frac{\pi}{2}$$

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$$\Rightarrow \frac{-1}{2} \leq 2x \leq 1$$

$$\Rightarrow \frac{-1}{4} \leq x \leq \frac{1}{2}$$

$\therefore$ Domain of $\sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is $\left[-\frac{1}{4}, \frac{1}{2}\right]$

Question25

If $\mathrm{f}(x) = \frac{x}{2-x}$, $\mathrm{g}(x) = \frac{x+1}{x+2}$, then (gogof) $(x) =$ MHT CET 2024 (04 May Shift 2)

Options:

A. $\frac{6+x}{10-2x}$

B. $\frac{6-x}{10+2x}$

C. $\frac{6+x}{10+2x}$

D. $\frac{6-x}{10-2x}$

Answer: D

Solution:

$f(x) = \frac{x}{2-x}, g(x) = \frac{x+1}{x+2}$

$$(\text{ gof })(x) = \frac{\frac{x}{2-x}+1}{\frac{x}{2-x}+2}$$

$$(\text{gogof})\ (x) = \frac{\frac{\frac{x}{2-x}+1}{x} + 1}{\frac{\frac{x}{2-x}+2}{\frac{x}{2-x}+2} + 2}$$

$$= \frac{\frac{x+2-x}{x+4-2x} + 1}{\frac{x+2-x}{x+4-2x} + 2}$$

$$= \frac{\frac{2}{4-x} + 1}{\frac{2}{4-x} + 2}$$

$$= \frac{2+4-x}{2+8-2x}$$

$$= \frac{6-x}{10-2x}$$

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Question26

If the function $f(x)=x^3+e^{\frac{x}{2}}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is MHT CET 2024 (04 May Shift 1)

Options:

A. 1

B. 0

C. 2

D. $\frac{1}{2}$

Answer: C

Solution:

$$
\begin{aligned}
& f(x)=x^3+e^{\frac{x}{2}} \\
& \Rightarrow f'(x)=3x^2+\frac{e^{\frac{x}{2}}}{2} \\
& \text{Given that } g(x)=f^{-1}(x) \\
\therefore \quad & gof(x)=x \\
\therefore \quad & f(x))=x \\
& \text{Differentiating w.r.t. } x, \text{ we get} \\
& g'(f(x))f'(x)=1 \\
& \text{for } x=0, \text{ we get} \\
& g'(f(0))\cdot f'(0)=1 \\
\therefore \quad & g'(1)=\frac{1}{f'(0)}=\frac{1}{0+\frac{1}{2}}=2
\end{aligned}
$$

Question27

If $f(x)=\log_c\left(\frac{1-x}{1+x}\right),|x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to MHT CET 2024 (04 May Shift 1)

Options:

A. $2f\left(x^2\right)$

B. $(f(x))^2$

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C. $-2f(x)$

D. $2f(x)$

Answer: D

Solution:

$$f\left(\frac{2x}{1+x^2}\right) = \log_e\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$
$$= \log_e\left(\frac{(1-x)^2}{(1+x)^2}\right)$$
$$= 2\log_e\left(\frac{1-x}{1+x}\right)$$
$$= 2f(x)$$

Question28

The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is MHT CET 2024 (03 May Shift 2)

Options:

A. $R - \{1, 2\}$

B. $(-2, \infty)$

C. $R - \{-1, -2, -3\}$

D. $(-3, \infty) - \{-1, -2\}$

Answer: D

Solution:

Given, $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$

Now, $f(x)$ is defined if $x^2 + 3x + 2 \neq 0$

$\Rightarrow (x+1)(x+2) \neq 0$

$\Rightarrow x \neq -1, -2$.

Also, $x + 3 > 0$.

$x > -3$

$\therefore \quad x \in (-3, \infty) - \{-1, -2\}$

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Question29

If $\mathrm{g}(x)=x^2+x-1$ and (gof) $(x)=4x^2-10x+5$, then $\mathrm{f}(2)$ is equal to MHT CET 2024 (03 May Shift 1)

Options:

A. 1

B. -1

C. 2

D. -2

Answer: D

Solution:

If $g(x)=x^2+x-1$ and $(g \circ f)(x)=4x^2-10x+5$, then $f(2)$?

1. Relationship:

$g(f(x))=4x^2-10x+5$

2. Substitute $g(x)$: Given $g(x)=x^2+x-1$, substitute $f(x)$:

$(f(x))^2+f(x)-1=4x^2-10x+5$

3. Solve for $f(x)$: Use quadratic identities to isolate $f(x)$. Substitute $x=2$. After simplifications: Answer: -2, Option 4.

Question30

For a suitable chosen real constant a, let a function $\mathrm{f}: \mathbb{R}-\{-\mathrm{a}\} \rightarrow \mathbb{R}$ be defined by $\mathrm{f}(x)=\frac{\mathrm{a}-x}{\mathrm{a}+x}$. Further suppose that for any real number $x \neq -\mathrm{a}$ and $\mathrm{f}(x) \neq -\mathrm{a}$, (fof) $(x)=x$. Then $\mathrm{f}\left(-\frac{1}{5}\right)$ is equal to MHT CET 2024 (02 May Shift 1)

Options:

A. 1.5

B. $2 \cdot 0$

C. $1 \cdot 0$

D. $3 \cdot 0$

Answer: A

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Solution:

Given: $f(x)=\frac{a-x}{a+x}$

$$\because \quad f(f(x))=x$$
$$\Rightarrow \frac{a-f(x)}{a+f(x)}=x$$
$$\Rightarrow \frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}=x$$
$$\Rightarrow \frac{a^2+ax-a+x}{a^2+ax+a-x}=x$$
$$\Rightarrow \left(a^2-a\right)+(a+1)x=\left(a^2+a\right)x+(a-1)x^2$$
$$\Rightarrow (a-1)x^2+\left(a^2-1\right)x-a^2+a=0$$
$$\Rightarrow (a-1)\left[x^2+(a+1)x-a\right]=0$$

This is possible when $a=1$

$$\therefore \quad f(x)=\frac{1-x}{1+x}$$
$$f\left(\frac{-1}{5}\right)=\frac{1-\left(\frac{-1}{5}\right)}{1+\left(\frac{-1}{5}\right)}$$
$$=\frac{1+\frac{1}{5}}{1-\frac{1}{5}}$$
$$=\frac{\frac{6}{5}}{\frac{4}{5}}=\frac{6}{4}=1.5$$

Question31

If $f(x)=\frac{2x-3}{3x-4}, x\neq\frac{4}{3}$, then the value of $f^{-1}(x)$ is MHT CET 2023 (14 May Shift 2)

Options:

A. $\frac{4x-3}{3x-2}$

B. $\frac{3x-2}{4x+3}$

C. $\frac{3x-4}{4x-2}$

D. $\frac{2x+3}{4x-3}$

Answer: A

Solution:

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Let $f(x)=y \Rightarrow x=f^{-1}(y)$

$$y=\frac{2x-3}{3x-4}$$
$$\Rightarrow 3xy-4y=2x-3$$
$$\Rightarrow x(3y-2)=4y-3$$
$$\Rightarrow x=\frac{4y-3}{3y-2}$$
$$\Rightarrow f^{-1}(y)=\frac{4y-3}{3y-2}$$
$$\Rightarrow f^{-1}(x)=\frac{4x-3}{3x-2}$$

Question32

The range of the function $f(x)=\frac{x^2}{x^2+1}$ is MHT CET 2023 (14 May Shift 1)

Options:

A. $(0,1)$

B. $[0,1)$

C. $(0,1]$

D. $[0,1]$

Answer: B

Solution:

$$\text{Let } y=\frac{x^2}{x^2+1}$$
$$\Rightarrow yx^2+y=x^2$$
$$\Rightarrow x^2(y-1)+y=0$$
$$\Rightarrow x^2=\frac{y}{1-y}$$

For x to be real,

$$y(1-y)\geq 0 \text{ and } 1-y\neq 0$$
$$\Rightarrow y(y-1)\leq 0 \text{ and } y\neq 1$$
$$\Rightarrow 0\leq y<1$$

Question33

The domain of the definition of the function $y(x)$ is given by the equation $2^x+2^y=2$ is MHT CET 2023 (13 May Shift 2)

Options:

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A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Solution:

$$
\begin{aligned}
& 2^x + 2^y = 2 \\
& \Rightarrow 2^y = 2 - 2^x \\
& \Rightarrow y = \log_2(2 - 2^x) \text{ is defined, if } 2 - 2^x > 0 \\
& \Rightarrow 2^x < 2 \\
& \Rightarrow 2^{x-1} < 1 \\
& \Rightarrow x - 1 < 0 \\
& \Rightarrow -\infty < x < 1
\end{aligned}
$$

Question34

If $3\mathrm{f}(x) - \mathrm{f}\left(\frac{1}{x}\right) = 8\log_2 x^3, x > 0$, then $\mathrm{f}(2), \mathrm{f}(4)\ \mathrm{f}(8)$ are in MHT CET 2023 (13 May Shift 1)

Options:

A. A.P.

B. G.P.

C. H.P.

D. Arithmetico Geometric Progression.

Answer: A

Solution:

$$
\begin{gathered}
3 f(x) - f\left(\frac{1}{x}\right) = 8\log_2 x^3 \\
\Rightarrow 3\mathrm{f}\left(\frac{1}{x}\right) - \mathrm{f}(x) = 8\log_2\left(\frac{1}{x}\right)^3.
\end{gathered}
$$

From (i) and (ii), we get

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$$8f(x) = 24\log_2 x^3 + 8\log_2\left(\frac{1}{x}\right)^3$$
$$\Rightarrow 8f(x) = 72\log_2 x - 24\log_2 x$$
$$\Rightarrow 8f(x) = 48\log_2 x$$
$$\Rightarrow f(x) = 6\log_2 x$$

$\therefore \quad f'(2) = 6\log_2 2 = 6$

$f'(4) = 6\log_2 4 = 12$

$f^{(}(8) = 6\log_2 8 = 18$

$\therefore \quad f(2), f(4), f(8)$ are in A.P.

Question35

If $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x}$, then the value of $f(g(g(f(x))))$ at $x = 1$ is MHT CET 2023 (12 May Shift 2)

Options:

A. 4

B. 1

C. 5

D. 3

Answer: C

Solution:

$$f(x) = x^2 + 1, g(x) = \frac{1}{x}$$
$$\therefore \quad f(g(g(f(x)))) = f\left(g\left(g\left(x^2+1\right)\right)\right)$$
$$= f\left(g\left(\frac{1}{x^2+1}\right)\right)$$
$$= f\left(x^2+1\right)$$
$$= \left(x^2+1\right)^2 + 1$$

$\therefore$ At $x = 1$, we get the value of above function

$$= \left[(1)^2 + 1\right]^2 + 1$$
$$= 5$$

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Question36

If $\mathrm{g}(x)=1+\sqrt{x}$ and $\mathrm{f}(\mathrm{g}(x))=3+2\sqrt{x}+x$, then $\mathrm{f}(\mathrm{f}(x))$ is MHT CET 2023 (12 May Shift 1)

Options:

A. x^2+4x+6

B. x^4+x^2+6

C. x^2+x+6

D. x^4+4x^2+6

Answer: D

Solution:

$$g(x)=1+\sqrt{x} \text{ and } \mathrm{f}(\mathrm{g}(x))=3+2\sqrt{x}+x$$
$$\therefore \quad \mathrm{f}(\mathrm{g}(x))=\left[(\sqrt{x})^2+2\sqrt{x}+1\right]+2$$
$$=(\sqrt{x}+1)^2+2$$
$$=[\mathrm{g}(x)]^2+2$$
$$\Rightarrow \mathrm{f}(x)=x^2+2$$
$$\Rightarrow \mathrm{f}(\mathrm{f}(x))=\left(x^2+2\right)^2+2=x^4+4x^2+6$$

Question37

For all real x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is MHT CET 2023 (11 May Shift 2)

Options:

A. 0

B. 1

C. $\frac{1}{3}$

D. 3

Answer: C

Solution:

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$$f(x)=\frac{1-x+x^2}{1+x+x^2}$$
$$\therefore \quad f'(x)=\frac{\left(1+x+x^2\right)(-1+2x)-\left(1-x+x^2\right)(1+2x)}{\left(1+x+x^2\right)^2}$$
$$=\frac{\left(-1+2x-x+2x^2-x^2+2x^3\right)}{\left(1+x+x^2\right)^2}$$
$$=\frac{-2+2x^2}{\left(1+x+x^2\right)^2}$$

If $f'(x)=0$, then $\frac{-2+2x^2}{\left(1+x+x^2\right)^2}=0 \Rightarrow x^2=1$

$\Rightarrow x=\pm 1$

$\therefore$ $f(x)$ at $x=1$ is $\frac{1}{3}$ and $f(x)$ at $x=-1$ is 1 ,

$\therefore$ Minimum value of $f(x)$ is $\frac{1}{3}$.

Question38

If $f(x)=\frac{3x+4}{5x-7}$ and $g(x)=\frac{7x+4}{5x-3}$, then $f(g(x))=$ MHT CET 2023 (11 May Shift 2)

Options:

A. $\frac{x^3+1}{x^2+2}$

B. $41x$

C. $g(f(x))$

D. $\frac{5x-7}{41}$

Answer: C

Solution:

$$f(g(x))=f\left(\frac{7x+4}{5x-3}\right)$$
$$=\frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7}$$
$$=\frac{21x+12+20x-12}{35x+20-35x+21}$$
$$=\frac{41x}{41}$$
$$=x$$

Now, $g(f(x))=g\left(\frac{3x+4}{5x-7}\right)$

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$$= \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3}$$
$$= \frac{21x+28+20x-28}{15x+20-15x+21}$$
$$= \frac{41x}{41}$$
$$= x$$
$$\mathrm{f}(\mathrm{g}(x)) = \mathrm{g}(\mathrm{f}(x))$$

Question39

Let $\mathrm{f}(x) = \log(\sin x), 0 < x < \pi$ and $\mathrm{g}(x) = \sin^{-1}(\mathrm{e}^{-x}), x \geq 0$.If α is a positive real number such that $a = (f \circ g)'(\alpha)$ and $b = (f \circ g)(\alpha)$, then MHT CET 2023 (11 May Shift 1)

Options:

A. $a\alpha^2 - b\alpha - a = 0$

B. $a\alpha^2 - b\alpha - a = 1$

C. $\mathrm{a}\alpha^2 + \mathrm{b}\alpha - \mathrm{a} = -2\alpha^2$

D. $a\alpha^2 + b\alpha + a = 0$

Answer: B

Solution:

$$\mathrm{f}(x) = \log(\sin x), 0 < x < \pi \text{ and}$$
$$\mathrm{g}(x) = \sin^{-1}(\mathrm{e}^{-x}), x \geq 0$$
$$\therefore \quad (\text{fog})(x) = \log\left[\sin\left(\sin^{-1}\mathrm{e}^{-x}\right)\right] = \log(\mathrm{e}^{-x}) = -x$$
$$\therefore \quad (\text{ fog })'(x) = -1$$
$$\therefore \quad \mathrm{a} = (\text{ fog })'(\alpha) = -1 \text{ and } \mathrm{b} = (\text{fog})(\alpha) = -\alpha$$

These values satisfy only option (B).

$\therefore$ Option (B) is correct.

Question40

The domain of the function given by $2^x + 2^y = 2$ is MHT CET 2023 (11 May Shift 1)

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Options:

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Solution:

$$
\begin{aligned}
&2^x + 2^y = 2 \\
&\Rightarrow 2^y = 2 - 2^x \\
&\Rightarrow y = \log_2(2 - 2^x) \text{ is defined, if } 2 - 2^x > 0 \\
&\Rightarrow 2^x < 2 \\
&\Rightarrow 2^{x-1} < 1 \\
&\Rightarrow x - 1 < 0 \\
&\Rightarrow -\infty < x < 1
\end{aligned}
$$

Question41

Let $\mathrm{f}(x) = \mathrm{e}^x - x$ **and** $\mathrm{g}(x) = x^2 - x, \forall x \in \mathrm{R}$**, then the set of all** $x \in \mathrm{R}$**, where the function** $\mathrm{h}(x) = (\mathrm{fog})(x)$ **is increasing is MHT CET 2023 (10 May Shift 2)**

Options:

A. $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

B. $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

C. $[0, \infty)$

D. $\left[-\frac{1}{2}, 0\right] \cup [1, \infty)$

Answer: A

Solution:

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$$h(x) = (fog)(x)$$
$$\Rightarrow h(x) = f\left(x^2 - x\right)$$
$$\Rightarrow h(x) = e^{x^2-x} - x^2 + x$$
$$\therefore \quad h'(x) = e^{x^2-x}(2x-1) - 2x + 1$$
$$\Rightarrow h'(x) = \left(e^{x^2-x} - 1\right)(2x-1)$$

For function $h(x)$ to be increasing,

$$h'(x) \geq 0$$
$$\Rightarrow \left(e^{x^2-x} - 1\right)(2x-1) \geq 0$$
$$\Rightarrow x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$$

h' −	+	−	+
0	$\frac{1}{2}$	1	

Question42

Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$, for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to MHT CET 2023 (10 May Shift 2)

Options:

A. $4e^2$

B. $4e$

C. $2e$

D. $2e^2$

Answer: B

Solution:

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Given: $f'(x) = f(x)$ for all $x \in R'$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating on both sides, wwe get $\log|f(x)| = x + c$

$\Rightarrow f(x) = e^{x+c}$

$\Rightarrow f(x) = e^x \cdot e^c$

$\Rightarrow f(x) = e^x \cdot c_1 \ldots$ (i)$[e^c = c_1]$

As $f(1) = 2$

$$\therefore \quad c_1 \cdot e = 2$$
$$\Rightarrow c_1 = \frac{2}{e}$$

Equation (i) becomes

$f(x) = e^x \cdot \frac{2}{e}$

Now, $h(x) = f(f(x))$

$$\therefore \quad h'(x) = f'(f(x)) \times f'(x)$$
$$\therefore \quad h'(1) = f'(f(1)) \times f'(1)$$
$$\Rightarrow h'(1) = f'(2) \times f'(1)$$
$$\Rightarrow h'(1) = e^2 \times \frac{2}{e} \times 2$$
$$\Rightarrow h'(1) = 4e$$

Question43

The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to MHT CET 2023 (10 May Shift 2)

Options:

A. $\frac{\sqrt{17}}{2} + 1$

B. $\frac{\sqrt{17}-1}{2}$

C. $\frac{1+\sqrt{17}}{2}$

D. $\frac{\sqrt{17}}{2} - 1$

Answer: C

Solution:

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$f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$\mathrm{f}(x)$ is defined, if

$$
\begin{aligned}
& -1 \leq \frac{|x|+5}{x^2+1} \leq 1 \\
& \Rightarrow 0 \leq \frac{|x|+5}{x^2+1} \leq 1 \\
& \Rightarrow x^2-|x|-4 \geq 0 \\
& \Rightarrow\left(|x|-\frac{1-\sqrt{17}}{2}\right)\left(|x|-\frac{1+\sqrt{17}}{2}\right) \geq 0 \\
& \Rightarrow|x| \geq \frac{1+\sqrt{17}}{2} \\
& \text{or } |x| \leq \frac{1-\sqrt{17}}{2}, \text{ which is not possible} \\
& \Rightarrow x \in\left(-\infty,-\frac{1+\sqrt{17}}{2}\right] \cup\left[\frac{1+\sqrt{17}}{2}, \infty\right) \\
& \Rightarrow \mathrm{a}=\frac{1+\sqrt{17}}{2}
\end{aligned}
$$

Question44

If $\mathrm{f}(x)=\mathrm{e}^x,\ \mathrm{g}(x)=\sin^{-1} x$ and $\mathrm{h}(x)=\mathrm{f}(\mathrm{g}(x))$, then $\frac{\mathrm{h}'(x)}{\mathrm{h}(x)}$ is MHT CET 2023 (10 May Shift 1)

Options:

A. $\mathrm{e}^{\sin^{-1} x}$

B. $\frac{1}{\sqrt{1-x^2}}$

C. $\sin^{-1} x$

D. $\frac{\mathrm{e}^{\sin^{-1} x}}{\sqrt{1-x^2}}$

Answer: B

Solution:

$$
\begin{aligned}
\mathrm{h}(x) & =\mathrm{f}(\mathrm{g}(x)) \\
& =\mathrm{f}\left(\sin^{-1} x\right) \\
\therefore \quad \mathrm{h}(x) & =\mathrm{e}^{\sin^{-1} x}
\end{aligned}
$$

Differentiating w.r.t. x, we get

$$
\begin{aligned}
& \mathrm{h}'(x)=\mathrm{e}^{\sin^{-1} x} \cdot \frac{\mathrm{d}}{\mathrm{d} x}\left(\sin^{-1} x\right) \\
& \quad=\mathrm{e}^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} \\
& \text{Now, } \frac{\mathrm{h}'(x)}{\mathrm{h}(x)}=\frac{\mathrm{e}^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}}{\mathrm{e}^{\sin^{-1} x}}=\frac{1}{\sqrt{1-x^2}}
\end{aligned}
$$

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Question45

$f: R \rightarrow R ; g: R \rightarrow R$ **are two functions such that** $\mathrm{f}(x)=2 x-3, \ \mathrm{g}(x)=x^3+5$**, then** $(f \circ g)^{-1}(-9)$ **is MHT CET 2023 (10 May Shift 1)**

Options:

A. -2

B. 2

C. $-\sqrt{2}$

D. $\sqrt{2}$

Answer: A

Solution:

We have, $\mathrm{f}(x)=2 x-3, \ \mathrm{g}(x)=x^3+5$

$$\begin{aligned} \operatorname{fog}(x) & =\mathrm{f}(\mathrm{g}(x)) \\ & =2 \mathrm{~g}(x)-3 \\ & =2\left(x^3+5\right)-3=2 x^3+7 \end{aligned}$$

Let $(f \circ g)(x)=y=2 x^3+7$

$$\begin{aligned} & y=2 x^3+7 \\ & \Rightarrow y-7=2 x^3 \\ & \Rightarrow x^3=\frac{y-7}{2} \\ & \Rightarrow x=\left(\frac{y-7}{2}\right)^{\frac{1}{3}} \end{aligned}$$

$$\therefore \quad (\text{ fog })^{-1}(y)=\left(\frac{y-7}{2}\right)^{\frac{1}{3}}$$

$$\therefore \quad (\text{ fog })^{-1}(-9)=\left(\frac{-9-7}{2}\right)^{\frac{1}{3}}=(-8)^{\frac{1}{3}}=-2$$

Question46

$\mathrm{f}: \mathbb{R}-\left(-\frac{3}{5}\right) \rightarrow \mathbb{R}$ **is defined by** $f(x)=\frac{3 x-2}{5 x+3}$**, then** $f \circ f(1)$ **is MHT CET 2023 (09 May Shift 1)**

Options:

A. 1

B. $\frac{-13}{29}$

C. $\frac{13}{29}$

D. -1

Answer: B

Solution:

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$$f(x)=\frac{3x-2}{5x+3}$$

$$f(f(x))=\frac{3\left(\frac{3x-2}{5x+3}\right)-2}{5\left(\frac{3x-2}{5x+3}\right)+3}$$

$$=\frac{3(3x-2)-2(5x+3)}{5(3x-2)+3(5x+3)}$$

$$=\frac{9x-6-10x-6}{15x-10+15x+9}$$

$$=\frac{-x-12}{30x-1}$$

$$\text{fof }(1)=\frac{-1-12}{30-1}=\frac{-13}{29}$$

Question47

The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}\left(x^3-x\right)$ is MHT CET 2022 (11 Aug Shift 1)

Options:

A. $(-1,0)\cup(1,2)\cup(3,\infty)$

B. $(-1,0)\cup(1,2)\cup(2,\infty)$

C. $(-2,-1)\cup(-1,0)\cup(2,\infty)$

D. $(1,2)\cup(2,\infty)$

Answer: B

Solution:

$f(x)=\frac{1}{4-x^2}+\log_{10}\left(x^3-x\right)$

to define $f(x) 4-x^2\neq 0$ and $x^3-x>0$

$\Rightarrow x^2\neq 4$ and $x(x-1)(x+1)>0$

$\Rightarrow x\neq\pm 2$ and

$\Rightarrow x\in(-1,0)\cup(1,2)\cup(2,\infty)$

Question48

If the function $f:R-\{-1,1\}\rightarrow A$ defined by $f(x)=\frac{x^2}{1-x^2}$ is surjective, then A is equal to MHT CET 2022 (10 Aug Shift 2)

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Options:

A. $R-[-1,0)$

B. $R-\{-1\}$

C. $\{0,\infty)$

D. $R-(-1,0)$

Answer: A

Solution:

For $f(x)$ to be a surjective function $A=$ range of $f(x)$now, $y=\frac{x^2}{1-x^2}$

$$\begin{aligned}&\Rightarrow x^2=\frac{y}{1+y}\\&\Rightarrow x=\sqrt{\frac{y}{1+y}}\end{aligned}$$

for x to be real $\frac{y}{1+y}\geq 0$

$$\begin{aligned}&\Rightarrow y\in(-\infty,-1)\cup[0,\infty)\\&\Rightarrow \text{ Range of } f(x) \text{ is } R\sim[-1,0)\\&\Rightarrow A \text{ is } R\sim[-1,0)\end{aligned}$$

Question49

Let $f:R\rightarrow R$ be a differentiable function with $f(0)=1$ and satisfying the equation $f(x+y)=f(x)\cdot f'(y)+f'(x)\cdot f(y),\forall x,y\in R$, then the value of $\log(f(4))$ is MHT CET 2022 (10 Aug Shift 1)

Options:

A. 1

B. 4

C. 2

D. $\frac{1}{2}$

Answer: C

Solution:

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$f(x+y)=f(x)\cdot f'(y)+f'(x)\cdot f(y)\forall x,y\in R$

putting $x=y=0$ we get

$$f(0)=2f(0)\cdot f'(0)$$
$$\Rightarrow f'(0)=\frac{1}{2}[\because \text{ given } f(0)=1]$$

Now putting $x=x$ and $y=0$

$$f(x)=f(x)\cdot f'(0)+f'(x)\cdot f(0)$$
$$\Rightarrow f(x)=\frac{1}{2}f(x)+f'(x)\left[\because f(0)=1 \text{ and } f'(0)=\frac{1}{2}\right]$$
$$\Rightarrow \frac{1}{2}f(x)=f'(x)$$
$$\Rightarrow \int \frac{f'(x)}{f(x)}\mathrm{d}x=\int \frac{1}{2}\,\mathrm{d}x$$
$$\Rightarrow \log(f(x))=\frac{1}{2}x+c$$
$$\because f(0)=1$$
$$\Rightarrow c=0$$

i.e., $\log(f(x))=\frac{1}{2}x$ putting $x=4$

$\log(f(4))=\frac{1}{2}\times 4=2$

Question50

Let $A=\{x\in R/x$ is not a positive integer). Let a function f be defined as $f:A\rightarrow R$ so that $f(x)=\frac{2x}{x-1}$, then f is MHT CET 2022 (10 Aug Shift 1)

Options:

A. Not injective.

B. surjective but not injective.

C. neither injective nor surjective.

D. injective but not surjective.

Answer: D

Solution:

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$$f(x)=\frac{2x}{x-1}$$
$$\Rightarrow f'(x)=\frac{(x-1)2-2x(1-0)}{(x-1)^2}=\frac{-2}{(x-1)^2}<0$$

$f(x)$ is strictly decreasing so $f(x)$ is injective

But $f(x)=4$

$\Rightarrow \frac{2x}{x-1}=4$

$\Rightarrow x=2$ (which is a positive integer)

i.e., $4 \in R$ (co-domain of f) has no pre-image in A (domain of f)

So, ' f ' is not surjective

Question51

For a suitable chosen real constant a, let the function $f: R-\{-a\} \rightarrow R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further, suppose that for any real number $x \neq -a$ and $f(x) \neq -a, (fof)(x)=x$. Then, $f\left(-\frac{1}{2}\right)$ is equal to MHT CET 2022 (08 Aug Shift 2)

Options:

A. -3

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. 3

Answer: D

Solution:

$$\text{fof }(x)=f(f(x))=f\left(\frac{a-x}{a+x}\right)=x$$
$$\Rightarrow \frac{a-\frac{a-x}{a+x}}{a+\frac{a-x}{a+x}}=x$$
$$\Rightarrow \frac{a^2+ax-a+x}{a^2+ax+a-x}=x$$
$$\Rightarrow a^2+ax-a+x=a^2x+ax^2+ax-x^2$$

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$$\Rightarrow (a-1)x^2+\left(a^2-1\right)x-a(a-1)=0$$
$$\Rightarrow (a-1)(x+a)(x-1)=0$$
$$\Rightarrow a=1[\text{as } x\neq -a]$$
$$\Rightarrow f(x)=\frac{1-x}{1+x}$$
$$\Rightarrow f\left(-\frac{1}{2}\right)=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=3$$

Question52

The domain and range for the function $f(x)=e^{|x|\sin x}$ are domain $=$ IR MHT CET 2022 (08 Aug Shift 1)

Options:

A. range $=[0,\infty)$ domain $=$ IR

B. range $=[1,\infty)$ domain $=$ IR

C. range $=$ IR domain $=IR$

D. range $=(0,\infty)$

Answer: D

Solution:

$f(x)=e^{|x|\sin x}$ is defined every where

Hence domain of $f(x)$ is R

$$\therefore -\infty<|x|\sin x<\infty.$$
$$\Rightarrow 0<e^{|x|\sin x}<\infty$$
$$\Rightarrow \text{Range of } f(x) \text{ is } (0,\infty)$$

Question53

If $f(x)=e^{|x|}, g(x)=\log x$, then $(g \text{ of })(x)=$ MHT CET 2022 (07 Aug Shift 2)

Options:

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A. $|x|$

B. 1

C. $2x$

D. $-x^2$

Answer: A

Solution:

$$f(x)=e^{|x|}, g(x)=\log x$$
$$g \text{ of } (x)=g(f(x))=\log e^{|x|}=|x|$$

Question54

If $f(x)=\frac{a^x-a^{-x}}{a^x+a^{-x}}$, where a, x satisfy the necessary conditions, then $f^{-1}(x)=$ MHT CET 2022 (07 Aug Shift 1)

Options:

A. $\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$

B. $\frac{1}{2}\log_a\left(\frac{1+x}{x}\right)$

C. $\frac{1}{2}\log_a\left(\frac{2+x}{2-x}\right)$

D. $\frac{1}{2}\log_a\left(\frac{x}{1-x}\right)$

Answer: A

Solution:

$$y=\frac{a^x-a^{-x}}{a^x+a^{-x}}=\frac{a^{2x}-1}{a^{2x}+1}$$
$$\Rightarrow y\left(a^{2x}+1\right)=a^{2x}-1$$
$$\Rightarrow a^{2x}=\frac{1+y}{1-y}$$
$$\Rightarrow x=\frac{1}{2}\log_a\left(\frac{1+y}{1-y}\right)$$
$$\Rightarrow f^{-1}(x)=\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$$

Question55

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If $[x]$ is greatest integer function and $2[2x-5]-1=7$, then x lies in MHT CET 2022 (06 Aug Shift 2)

Options:

A. $\left[\frac{9}{2}, 5\right)$

B. $\left[\frac{9}{2}, 5\right]$

C. $\left(\frac{9}{2}, 5\right)$

D. $\left(\frac{9}{2}, 5\right]$

Answer: A

Solution:

$$\begin{aligned} & 2[2x-5]-1=7 \\ & \Rightarrow 2[2x]-10-1=7 \\ & \Rightarrow [2x]=9 \\ & \Rightarrow 9 \leq 2x<10 \\ & x \in\left[\frac{9}{2}, 5\right) \end{aligned}$$

Question56

If the function $f(x)=x^3-3(a-2)x^2+3ax+7$, for some $a \in R$ is increasing in $(0,1]$ and decreasing in $[0,5)$, then a root of the equation $\frac{f(x)-14}{(x-1)^2}=0(x \neq 1)$ is MHT CET 2022 (06 Aug Shift 2)

Options:

A. -7

B. -14

C. 7

D. 14

Answer: C

Solution:

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$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$
$f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(x)$ changes its behaving at $x = 1$

$f'(1) = 0$
$\Rightarrow 3 \times 1^2 - 6(a-2) \times 1 + 3a = 0$
$\Rightarrow a = b \Rightarrow f(x) = x^3 - 9x^2 + 15x + 7$
$\Rightarrow \frac{f(x)-14}{(x-1)^2} = 0 \Rightarrow \frac{(x^3-9x^2+15x+7)-14}{(x-1)^2} = 0$
$\Rightarrow x^3 - 9x^2 + 15x - 7 = 0$
$\Rightarrow (x-1)(x-1)(x-7)$
$\Rightarrow x = 1, 1, 7$

Question57

For a suitable chosen real constant a, let a function $f : R - [-a] \rightarrow R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further, suppose that for any real number $x \neq -a$ and $f(x) \neq -a, (fof)(x) = x$. Then $f\left(\frac{-1}{2}\right)$ is equal to MHT CET 2022 (06 Aug Shift 1)

Options:

A. $\frac{-1}{3}$

B. 3

C. $\frac{1}{3}$

D. -3

Answer: B

Solution:

$$(f \circ f)(x) = \frac{a - \frac{a-x}{a+x}}{a + \frac{a-x}{a+x}} = x$$
$$\Rightarrow \frac{a^2 + ax - a + x}{a^2 + ax + a - x} = x$$
$$\Rightarrow (a+1)x + \left(a^2 - a\right) = \left(a^2 + a\right)x + (a-1)x^2$$
$$\Rightarrow a + 1 = a^2 + a, a^2 - a = 0, a - 1 = 0$$
$$\Rightarrow a = 1$$
$$\Rightarrow f(x) = \frac{1-x}{1+x}$$
$$\Rightarrow f\left(-\frac{1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

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Question58

If R denotes the set of all real numbers then the function $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ defined by $f(x)=|x|$ is MHT CET 2022 (05 Aug Shift 2)

Options:

A. injective and surjective.

B. neither injective nor surjective.

C. injective.

D. surjective.

Answer: B

Solution:

$\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}, \mathrm{f}(\mathrm{x})=|\mathrm{x}|$
$\because \mathrm{f}(-1)=\mathrm{f}(1)=1$

i.e. not injection

and range of $\mathrm{f}(\mathrm{x})$ is $[0, \infty]$

Hence, not surjection

Question59

If $f(x)=\cos (\log x)$, then $f(x) \cdot f(y)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(x y)\right)$ has the value MHT CET 2022 (05 Aug Shift 1)

Options:

A. -2

B. -1

C. 0

D. $\frac{1}{2}$

Answer: C

Solution:

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$f(x)=\cos(\log x)$

Now, $f(x)\cdot f(y)-\frac{1}{2}\left(f\left(\frac{x}{y}\right)+f(xy)\right)$

$=\cos(\log x)\cdot\cos(\log y)-\frac{1}{2}\left(\cos\log\left(\frac{x}{y}\right)+\cos\log(xy)\right)$

$=\cos(\log x)\cdot\cos(\log y)-\frac{1}{2}\{\cos(\log x-\log y)+\cos(\log x+\log y)\}$

$=\cos(\log x)\cdot\cos(\log y)-\frac{1}{2}\times 2\cos(\log x)\cdot\cos(\log y)$

$=0$

Question60

If $\mathrm{f}(\mathrm{x})=2\{\mathrm{x}\}+5\mathrm{x}$, where $\{\mathrm{x}\}$ is fractional part function, then $f(-1.4)$ is MHT CET 2021 (24 Sep Shift 2)

Options:

A. 8.2

B. -8.2

C. -5.8

D. -5

Answer: C

Solution:

We know that $\mathrm{x}=[\mathrm{x}]+\{\mathrm{x}\}$

When $\mathrm{x}=-1.4$, we get $[\mathrm{x}]=-2$ and $\{\mathrm{x}\}=0.6$

We have $f(x)=2\{x\}+5x$

$\therefore \mathrm{f}(-1.4)=2(0.6)+5(-1.4)=-7+1.2=-5.8$

Question61

The domain of the function $\log_{10}\left(x^2-5x+6\right)$ is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $(-\infty,\infty)$

B. $(-\infty,2)\cup(3,\infty)$

C. $(2,3)$

D. None of these

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Answer: B

Solution:

$$f(x)=\log_{10}\left(x^2-5x+6\right)$$
$$\therefore x^2-5x+6>0 \Rightarrow (x-2)(x-3)>0$$
$$\Rightarrow x>3 \text{ or } x<2$$
$$\therefore x\in(-\infty,2)\cup(3,\infty)$$

Question62

Range of the function $f(x)=3+2^x+4^x$ is MHT CET 2021 (23 Sep Shift 2)

Options:

A. $(3,\infty)$

B. $(-\infty,\infty)$

C. $(3,\infty)$

D. $(-\infty,3)$

Answer: A

Solution:

$$f(x)=3+2^x+4^x=y$$

Let $2^x=a\Rightarrow 4^x=a^2$

$$\therefore a^2+a+(3-y)=0$$

As $a\in R$, we write

$$(1)^2-4(1)(3-y)\geq 0$$
$$\therefore 1-12+4y\geq 0 \quad \Rightarrow 4y\geq 11 \quad \Rightarrow y\geq\frac{11}{4}$$

Also $2^x+4^x>0\Rightarrow y\neq 3 \therefore$ Range from given option is $(3,\infty)$

Question63

If $f(x)=\frac{x}{2x+1}$ and $g(x)=\frac{x}{x+1}$, then $(f\circ g)(x)=$ MHT CET 2021 (23 Sep Shift 1)

Options:

A. $\frac{2x-1}{x+1}$

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B. $\frac{x}{3x+1}$

C. $\frac{x+1}{x+2}$

D. $\frac{x-1}{2x+1}$

Answer: B

Solution:

$$(f \circ g)(x) = f\left[\frac{x}{x+1}\right]$$
$$= \frac{\left(\frac{x}{x+1}\right)}{2\left(\frac{x}{x+1}\right)+1} = \frac{x}{x+1} \times \frac{x+1}{2x+x+1} = \frac{x}{3x+1}$$

$$(f \circ g)(x) = f\left[\frac{x}{x+1}\right]$$
$$= \frac{\left(\frac{x}{x+1}\right)}{2\left(\frac{x}{x+1}\right)+1} = \frac{x}{x+1} \times \frac{x+1}{2x+x+1} = \frac{x}{3x+1}$$

Question64

If $f(x) = [8x] - 3$, where $[x]$ is greatest integer function of x, then $\mathrm{f}(\pi) =$ MHT CET 2021 (22 Sep Shift 2)

Options:

A. 21

B. 25

C. 23

D. 22

Answer: D

Solution:

$f(x) = [8x] - 3$

$\therefore \mathrm{f}(\pi) = [8(3.14)] - 3 = [25.12] - 3 = 25 - 3 = 22$

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Question65

The domain of the function $f(x)=\sqrt{x-1}+\sqrt{6-x}$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $[1, \infty)$

B. $[1,6]$

C. $(-\infty, 6]$

D. $(-\infty, 6)$

Answer: B

Solution:

$$f(x)=\sqrt{x-1}+\sqrt{6-x}$$

Here $\mathrm{f}(\mathrm{x})$ is defined when

$x-1 \geq 0$ and $6-x \geq 0$ i.e.

$x \geq 1$ and $x-6 \leq 0$ i.e. $x \geq 1$ and $x \leq 6$

$\therefore$ Domain of $\mathrm{f}(\mathrm{x})$ is $[1,6]$

Question66

Let $\mathrm{A}=\{10,11,12,14,26\}$ and let $\mathrm{f}: \mathrm{A} \rightarrow \mathrm{N}$ be such that $\mathrm{f}(\mathrm{a})=$ highest prime factor of a, where $a \in A$, then range of $f=$ MHT CET 2021 (21 Sep Shift 2)

Options:

A. $\{5,7,13\}$

B. $\{5,7,11,13\}$

C. $\{3,5,7,11,13\}$

D. $\{3,7,11,13\}$

Answer: C

Solution:

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We have A = {10, 11, 12, 14, 26} and

f : A → n, where f(a) = Highest prime factor of a, when a ∈ A.

Now	$10 = 2 \times 5$	$\Rightarrow$	Highest prime factor is 5
	$11 = 1 \times 11$	$\Rightarrow$	Highest prime factor is 11
	$12 = 2 \times 2 \times 3$	$\Rightarrow$	Highest prime factor is 3
	$14 = 2 \times 7$	$\Rightarrow$	Highest prime factor is 7
	$26 = 2 \times 13$	$\Rightarrow$	Highest prime factor is 13

$\therefore$ f = {3, 5, 7, 11, 13}

Question67

If $f(x) = 3[x] + \{x+1\}$, where $[x]$ is greatest integer function of x and $\{x\}$ is fractional part function of x, then $f(-1.32) =$ MHT CET 2021 (21 Sep Shift 1)

Options:

A. -4.6

B. -2.6

C. -7.4

D. -3.4

Answer: B

Solution:

$$\begin{aligned}
& \mathrm{f}(\mathrm{x}) = 3[\mathrm{x}] + 5\{\mathrm{x}+1\} \\
& \mathrm{x} = -1.32 \Rightarrow [\mathrm{x}] = [-1.32] = -2 \\
& \text{ Also } \mathrm{x}+1 = -1.32+1 = -0.32 \\
& \therefore [\mathrm{x}+1] = [-0.32] = -1 \text{ and } \{\mathrm{x}+1\} = 0.68 \\
& \therefore \mathrm{f}(\mathrm{x}) = 3(-2) + 5(0.68) \\
& = -6 + 3.4 = -2.6
\end{aligned}$$

Question68

If $2f(x) - 3f\left(\frac{1}{x}\right) = x$, then $\int_1^e f(x)dx =$ MHT CET 2021 (20 Sep Shift 2)

Options:

A. $-\left(\frac{2+e^2}{5}\right)$

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B. $\frac{2+e}{5}$

C. $\frac{2+e^2}{5}$

D. $\frac{2-e^2}{5}$

Answer: A

Solution:

$2f(x) - 3f\left(\frac{1}{x}\right) = x$

...(1) and replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x}$$

[$2\times$ equation (1)]+[$3\times$ equation (2)] gives,

$$4f(x) - 9f(x) = 2x + \frac{3}{x} \quad \Rightarrow -5f(x) = 2x + \frac{3}{x}$$
$$\therefore f(x) = \frac{-2}{5}x - \frac{3}{5x}$$

$$\therefore \int_1^e f(x)dx = \int_1^e \left(\frac{-2}{5}x - \frac{3}{5x}\right)dx = \frac{-2}{5}\int_1^e xdx - \frac{3}{5}\int_1^e \frac{1}{x}dx$$
$$= \frac{-2}{5}\left[\frac{x^2}{2}\right]_1^e - \frac{3}{5}[\log x]_1^e = \frac{-1}{5}(e^2-1) - \frac{3}{5}(\log e - \log 1) = \frac{-1}{5}e^2 + \frac{1}{5} - \frac{3}{5}$$
$$= -\left(\frac{2+e^2}{5}\right)$$

Question69

The domain of the function $f(x) = \frac{1}{\sqrt{x+|x|}}$ is MHT CET 2021 (20 Sep Shift 1)

Options:

A. $(-\infty, 0)$

B. $(2, 5)$

C. $(0, \infty)$

D. $(-\infty, \infty)$

Answer: C

Solution:

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$$f(x)=\frac{1}{\sqrt{x+|x|}}$$

Here $x+|x|\geq 0$ and $\sqrt{x+|x|}\neq 0$

$$\therefore \mathrm{x}+|\mathrm{x}|>0$$

Now when $x>0, x+|x|=x+x\Rightarrow 2x>0$.

When $\mathrm{x}<0, \mathrm{x}+|\mathrm{x}|=\mathrm{x}-\mathrm{x}=0$

$\therefore \mathrm{x}>0$ is the required domain.

Question70

If $\mathrm{f}(\mathrm{x})=[\mathrm{x}]$, for $\mathrm{x}\in(-1,2)$, then f is discontinuous at (where $[\mathrm{x}]$ represents floor function) MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\mathrm{x}=-1,0,1,2$

B. $\mathrm{x}=-1,0,1$

C. $\mathrm{x}=0,1$

D. $\mathrm{x}=2$

Answer: C

Solution:

We have $\mathrm{f}(\mathrm{x})=[\mathrm{x}]$

Let $[\mathrm{x}]=\mathrm{K}$, an integer.

$$\therefore \lim_{x\to K^{+}} f(x)=K \text{ and } \lim_{x\to K^{-}} f(x)=K-1$$

Thus given function is not continuous at all integral values in its domain.

$\because \mathrm{f}$ is discontinuous at $\mathrm{x}=0,1$.

Question71

The domain and range of the relation R given by $\mathrm{R}=\left\{(x,y)/y=x+\frac{6}{x}, x,y\in\mathrm{N} \text{ and } x<6\right\}$ are MHT CET 2020 (20 Oct Shift 1)

Options:

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A. Domain $= \{2,3\}$, Range $= \{5\}$.

B. Domain $= \{1,2\}$, Range $= \{5,7\}$.

C. Domain $= \{1,2,3,4,5\}$, Range $= \left\{7,5,\frac{6}{4},\frac{6}{5}\right\}$.

D. Domain $= \{1,2,3\}$, Range $= \{5,7\}$.

Answer: D

Solution:

$y = x + \frac{6}{x}, x, y \in N$ and $x < 6$

When $x = 1, y = 1 + 6 = 7$

When $x = 2, y = 2 + 3 = 5$

When $x = 3, y = 3 + 2 = 5$

When $x = 4, y = 4 + 1.5 = 5.5$

When $x = 5, y = 5 + 1.2 = 6.2$

Since $x, y \in N$ and $x < 6$, we write Domain $= \{1,2,3,4,5\}$ and Range $= \{7,5\}$

Question72

$f(x) = \frac{3x+2}{5x-3}, x \in R - \left\{\frac{3}{5}\right\}$, **then MHT CET 2020 (20 Oct Shift 1)**

Options:

A. $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$

B. $\mathrm{f}^{-1}(x)$ does not exist.

C. $\mathrm{f}[\mathrm{f}(x)] = -x$

D. $\mathrm{f}^{-1}(x) = -\mathrm{f}(x)$

Answer: A

Solution:

Let $y = f(x) = \frac{3x+2}{5x-3}$

$\therefore y(5x-3) = 3x + 2 \Rightarrow 5xy - 3y = 3x + 2 \Rightarrow (5y-3)x = 3y + 2$

$x = \frac{3y+2}{5y-3}$

$\Rightarrow f^{-1}(y) = \frac{3y+2}{5y-3}$

$\Rightarrow f^{-1}(x) = \frac{3x+2}{5x-3}$

Thus $f^{-1}(x) = f(x)$

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Question73

If $\mathrm{R}=\{(a,\ \mathrm{b})/\mathrm{b}=a-1, a\in \mathrm{Z}, 5<a<9\}$, then the range of R is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $\{7,8,9\}$

B. $\{5,6,7\}$

C. $\{6,7,8\}$

D. $\{5,6,7,8,9\}$

Answer: B

Solution:

Given $\mathbf{R}=\{(\mathbf{a},\mathbf{b}),\mathbf{b}=\mathbf{a}-1,\mathbf{a}\in\mathbf{z},5<\mathbf{a}<9\}$

$\therefore \quad a=6,7,8$

$a=6, b=6-1=5$

$a=7, b=7-1=6$

$a=8, b=8-1=7$

$R=\{(6,5),(7,6),(8,7)\}$

Range of $R=\{5,6,7\}$

Question74

The range of the function $f(x)=\frac{x-3}{5-x}, x\neq 5$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $\mathrm{R}-\{1\}$

B. $\mathrm{R}-\{-5\}$

C. $\mathrm{R}-\{5\}$

D. $\mathrm{R}-\{-1\}$

Answer: D

Solution:

We have $y=\frac{x-3}{5-x} \therefore 5y-xy=x-3 \Rightarrow x+xy=5y+3 \therefore x=\frac{5y+3}{1+y}$ Hence Range $=R-\{-1\}$

Question75

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If a function $f: R \rightarrow R$ is defined by $f(x)=\frac{4x}{5}+3$, then $f^{-1}(x)=$ MHT CET 2020 (19 Oct Shift 1)

Options:

A. $\frac{5(x+3)}{4}$

B. $\frac{5(x-3)}{4}$

C. $\frac{4(x+3)}{5}$

D. $\frac{4(x-3)}{5}$

Answer: B

Solution:

Let $f(x)=\frac{4x}{5}+3=y$

$\therefore 4x=5y-15 \Rightarrow x=\frac{5y-15}{4}$

$\therefore f^{-1}(y)=\frac{5y-15}{4} \Rightarrow f^{-1}(x)=\frac{5x-15}{4}=\frac{5(x-3)}{4}$

Question76

If $f: R \rightarrow R$ is given by $f(x)=7x+8$ and $f^{-1}(12)=\frac{k}{7}$, then the value of k is MHT CET 2020 (19 Oct Shift 1)

Options:

A. 7

B. 1

C. 4

D. 8

Answer: C

Solution:

We have $f(x)=7x+8=y$...(let)

$\therefore \mathrm{x}=\frac{\mathrm{y}-8}{7} \Rightarrow \mathrm{f}^{-1}(\mathrm{y})=\frac{\mathrm{y}-8}{7} \therefore \mathrm{f}^{-1}(\mathrm{x})=\frac{\mathrm{x}-8}{7} \Rightarrow \mathrm{f}^{-1}(12)=\frac{12-8}{7}=\frac{4}{7} \Rightarrow \mathrm{k}=4$

Question77

The domain of the function $f(x)=\sqrt{x}$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. $R-\{0\}$

B. R^+

C. $\mathrm{R}^+ \cup \{0\}$

D. R

Answer: C

Solution:

For $\mathrm{f}(\mathrm{x})$ to be defined, the term under the square root should be greater than or equal to zero.

$x \geq 0$

So domain is $[0, \infty]$ i.e. $\mathbf{R}^+ \cup \{0\}$.

Question78

For $\mathrm{f}(x) = [x]$, whe inction. which of the following is tru very $x \in \mathbb{R}$ MHT CET 2020 (16 Oct Shift 2)

Options:

A. [x]+1=x

B. $[x] + 1 \leq x$

C. $[x] + 1 > x$

D. $[x] + 1 < x$

Answer: D

Solution:

[x] is the greatest integer function. This means, the greatest integer is less than or equal to x.

If x is an integer, then [x] = x.

If x is a non integer number, then $[x] < x$ and $0 < x - [x] < 1$

$\therefore x < [x] + 1$

Question79

If $f(x) = \frac{2x+3}{3x-2}, x \neq \frac{2}{3}$, then the function fof is MHT CET 2020 (16 Oct Shift 1)

Options:

A. an even function

B. an identity function

C. a constant function

D. an exponential function

Answer: B

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Solution:

$$f \circ f = f(f(x))$$

$$= \frac{2 \cdot \left(\frac{2x+3}{3x-2}\right) + 3}{3 \cdot \left(\frac{2x+3}{3x-2}\right) - 2} = \frac{\frac{4x+6}{3x-2} + 3}{\frac{6x+9}{3x-2} - 2}$$

$$= \frac{4x+6+9x-6}{6x+9-6x+4} = \frac{13x}{13} = x$$

Therefore, we can say that the composite function for for given function is an identity function.

Question80

If $f(x) = 2x^2 + bx + c$, $f(0) = 3$ and $f(2) = 1$, then $(fof)(1) =$ MHT CET 2020 (15 Oct Shift 2)

Options:

A. 0

B. 2

C. 1

D. 3

Answer: D

Solution:

$$\text{Here } f(x) = 2x^2 + bx + c$$
$$f(0) = 0 + 0 + c \quad \Rightarrow 3 = c$$
$$f(2) = 8 + 2b + 3$$

$$\therefore \quad f = 8 \quad +2b + 3 \Rightarrow 2b = -10 \Rightarrow b = -5$$
$$\therefore \quad f(x) = \quad 2x^2 - 5x + 3$$
$$f(1) = 2 - 5 + 3 = 0$$
$$(f0f)(x) = f[f(x)]$$
$$(f0f)(1) = f[f(1)]$$
$$= f(0) = 0 - 0 + 3 = 3$$

Question81

The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is MHT CET 2020 (15 Oct Shift 2)

Options:

A. 0

B. 2

C. 3

D. 1

Answer: B

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Solution:

Given

$\tan x+\sec x=2\cos x$

$\frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x \Rightarrow \sin x+1=2\cos^2 x$

$\sin +1=2\left(1-\sin^2 x\right) \Rightarrow 2\sin^2 x+\sin x-1=0$

$(2\sin x-1)(\sin x+1)=0 \Rightarrow \sin x=\frac{1}{2}, \sin x=-1$

If $\sin x=-1$, then $x=\frac{3\pi}{2}$ and $\cos\frac{3\pi}{2}=0$.

Hence given equation is not defined at $\sin x=-1$.

$\therefore \sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6}, \frac{5\pi}{6}$

Question82

If $f(x)=x^2-3x+4$ and $f(x)=f(2x+1)$, then $x=$ MHT CET 2020 (15 Oct Shift 1)

Options:

A. $-1, \frac{2}{3}$

B. $-1, \frac{3}{2}$

C. $1, \frac{3}{2}$

D. $1, \frac{2}{3}$

Answer: A

Solution:

$$\begin{aligned} \text{Here } f(2x+1) &= (2x+1)^2-3(2x+1)+4 \\ &= 4x^2+4x+1-6x-3+4 \\ \therefore \quad f(2x+1) &= 4x^2-2x+2 \end{aligned}$$

$$\text{Given } f(x)=f(2x+1)$$

$$\therefore x^2-3x+4=4x^2-2x+2$$

$$\therefore \quad 3x^2+x-2=0 \Rightarrow 3x^2+3x-2x-2=0$$

$$3x(x+1)-2(x+1)=0 \Rightarrow (x+1)(3x-2)=0$$

$$\therefore x=-1, \frac{2}{3}$$

Question83

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The domain of a function $f(y)=\frac{\cos^{-1}(y-5)}{\sqrt{25-y^2}}$ is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $(4,6]$

B. $(-5,5)$

C. $[4,5)$

D. $(4,5]$

Answer: C

Solution:

We have $f(y)=\frac{\cos^{-1}(y-5)}{\sqrt{25-y^2}}$ Here $-1\leq y-5\leq 1$ and $25-y^2>0 \therefore \quad 4\leq y\leq 6$ and $\quad -5<y<5$ Hence domain of $f(y)$ is $[4,5)$

Question84

If $f(x)=\frac{3x+4}{5x-7}, x\neq\frac{7}{5}$

$g(x)=\frac{7x+4}{5x-3}, x\neq\frac{3}{5}$ then

$(g\circ f)(3)=$

MHT CET 2020 (14 Oct Shift 2)

Options:

A. -3

B. $-\frac{1}{3}$

C. 3

D. $\frac{1}{3}$

Answer: C

Solution:

$$\begin{aligned}\therefore (g\circ f)(x) &= g[f(x)]\\ &= g\left[\frac{3x+4}{5x-7}\right]\\ &= \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3}=\frac{7(3x+4)+4(5x-7)}{5(3x+4)-3(5x-7)}=\frac{41x}{41}=x\end{aligned}$$

$$\therefore (g\circ f)(3)=3$$

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Question85

If $f(x)=\frac{2x+3}{3x-2}, x \neq \frac{2}{3}$ then f of is MHT CET 2020 (14 Oct Shift 1)

Options:

A. an even function

B. not defined for all $x \in R$

C. a constant function

D. an odd function

Answer: D

Solution:

We have, $\mathrm{f(x)} = \frac{2x+3}{3x-2}$

$\therefore (\mathrm{fof})(\mathrm{x}) = \mathrm{f}[\mathrm{f}(\mathrm{x})] = \mathrm{f}\left(\frac{2x+3}{3x-2}\right)$

$= \frac{2\left(\frac{2x+3}{3x-2}\right)+3}{3\left(\frac{2x+3}{3x-2}\right)-2} = \frac{2(2x+3)+3(3x-2)}{3(2x+3)-2(3x-2)}$

$= \frac{4x+6+9x-6}{6x+9-6x+4} = \frac{13x}{13} = x$ ... is an odd function

----

Question86

If $f(x)=[x]^2-5[x]+6=0$, where $[x]$ denotes greatest integer functionthen $x \in$ MHT CET 2020 (14 Oct Shift 1)

Options:

A. $(2,4]$

B. $[2,4]$

C. $[2,4)$

D. $(2,4)$

Answer: C

Solution:

We have, $[x]^2-5[x]+6=0$

$\therefore ([x]-3)([x]-2)=0 \Rightarrow [x]=2,3$

For $[x]=2, x \in [2,3)$ and for $[x]=3, x \in [3,4)$

$\therefore \quad x \in [2,4)$

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Question87

If f : $\mathrm{R} \rightarrow \mathrm{R}, g: \mathrm{R} \rightarrow \mathrm{R}$ defined by $f(x)=x^2-3x+4$ and $g(x)=2x+1$, then thevalue of x for which $f(x)=f \circ g)(x)$ is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $1, \frac{-2}{3}$

B. $-1, \frac{2}{3}$

C. $1, \frac{2}{3}$

D. $-1, \frac{-2}{3}$

Answer: B

Solution:

$(\text{ fog })(x)=f[g(x)]=f(2x+1)=(2x+1)^2-3(2x+1)+4$

$=4x^2+4x+1-6x-3+4=4x^2-2x+2$

Given $f(x)=(f \circ g)(x)$

$\therefore x^2-3x+4=4x^2-2x+2$

$3x^2+x-2=0 \Rightarrow 3x^2+3x-2x-2=0$

$3x(x+1)-2(x+1)=0 \Rightarrow (x+1)(3x-2)=0$

$\therefore x=-1, \frac{2}{3}$

Question88

If $f: \mathrm{R} \rightarrow \mathrm{R}, \mathrm{g}: \mathrm{R} \rightarrow \mathrm{R}$ are two functions defined by $f(x)=2x-3,\ \mathrm{g}(x)=x^3+5$then $(\mathrm{fog})^{-1}(x)=$ MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\left(\frac{2x+3}{2}\right)^{\frac{1}{2}}$

B. $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

C. $\left(\frac{x-7}{2}\right)^{\frac{1}{2}}$

D. $\left(\frac{x+7}{2}\right)^{\frac{1}{3}}$

Answer: B

Solution:

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Given $f(x) = 2x - 3, \quad g(x) = x^3 + 5$

$$\therefore \quad (fog)(x) = f[g(x)] = f(x^3 + 5)$$
$$= 2(x^3 + 5) - 3 = 2x^3 + 7$$

$$\text{Let } y = 2x^3 + 7 \Rightarrow \frac{y-7}{2} = x^3$$
$$\therefore \left(\frac{y-7}{2}\right)^{\frac{1}{3}} = x \Rightarrow f^{-1}(y) = \left(\frac{y-7}{2}\right)^{\frac{1}{3}}$$
$$(f \circ g)^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$$

Question89

If f : R → R, such that $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, then f is MHT CET 2020 (13 Oct Shift 1)

Options:

A. a periodic function

B. an even function

C. an odd function

D. a neither even nor odd function

Answer: C

Solution:

$$f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \frac{e^x + \frac{1}{e^x}}{e^x - \frac{1}{e^x}} = \frac{e^{2x} + 1}{e^{2x} - 1}$$

$$f(-x) = \frac{e^{-x} + e^x}{e^{-x} - e^x}$$

$$= \frac{\frac{1}{e^x} + e^x}{\frac{1}{e^x} - e^x} = \frac{1 + e^{2x}}{1 - e^{2x}} = \frac{1 + e^{2x}}{-(e^{2x} - 1)}$$

$$\therefore f(-x) = -f(x)$$

$\therefore f(x)$ is an odd function.

Question90

Domain of the real valued function $f(x) = \frac{x+2}{9-x^2}$ is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $-3 \leq x \leq 3$

B. $R - \{-3, 3\}$

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C. R

D. $R-\{3\}$

Answer: B

Solution:

$f(x)=\frac{x+2}{9-x^2}$ is not defined if $9-x^2=0 \Rightarrow x=\pm 3$

$\therefore$ Required domain $=\mathrm{R}-\{\pm 3\}$

Question91

If $f(x)=\frac{4x+7}{7x-4}$, then the value of $f\{f[f(2)]\}=$ MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{35}{39}$

D. $\frac{39}{35}$

Answer: A

Solution:

$f(x)=\frac{4x+7}{7x-4}$

$f(2)=\frac{8+7}{14-4}=\frac{15}{10}=\frac{3}{2}$

$f[f(2)]=f\left(\frac{3}{2}\right)=\frac{\left(4\times\frac{3}{2}\right)+7}{\left(7\times\frac{3}{2}\right)-4}=\frac{6+7}{\left(\frac{21-8}{2}\right)}=\frac{13\times 2}{13}=2$

$f\{f[f(2)]\}=f(2)=\frac{3}{2}$

This problem can also be solved as follows :

$$\mathrm{f(x)} = \frac{4x+7}{7x-4} \Rightarrow \mathrm{f[f(x)]} = \mathrm{f}\left[\frac{4x+7}{7x-4}\right]$$

$$\therefore \mathrm{f}\left[\frac{4x+7}{7x-4}\right] = \frac{4\left(\frac{4x+7}{7x-4}\right)+7}{7\left(\frac{4x+7}{7x-4}\right)-4}$$

$$= \frac{16x+28+49x-28}{28x+49-28x+16} = \frac{65x}{65} = x$$

$$\therefore \quad \mathrm{f\{f[f(x)]\}} = \mathrm{f\{x\}} = \frac{4x+7}{7x-4}$$

$$\therefore \quad \mathrm{f\{f[f(2)]\}} = \frac{4(2)+7}{7(2)-4} = \frac{15}{10} = \frac{3}{2}$$

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Question92

If $f(x) = 3x - 2$ and $g(x) = x^2$, then $fog(x) =$ ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $3x^2 - 2$

B. $3x^2 + 2$

C. $3x - 2$

D. $2 - 3x^2$

Answer: A

Solution:

Given: $f(x) = 3x - 2 \quad g(x) = x^2$
Now, $fog(x) = f(g(x)) = f(x^2)$
$= 3x^2 - 2$

Question93

If $f(x) = [x]$, where $[x]$ is the greatest integer not greater than x, then $f'(1^+)$ = …. MHT CET 2019 (Shift 2)

Options:

A. 1

B. 2

C. 0

D. -1

Answer: C

Solution:

We have, $f(x) = [x]$
$\therefore f'(1^+) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$
$\lim_{h \to 0} \frac{[1+h] - [1]}{h} \Rightarrow \lim_{h \to 0} \frac{1-1}{h} = 0$

Question94

If $f(x) = 3x + 6, g(x) = 4x + k$ and $fog(x) = gof(x)$ then k = MHT CET 2019 (Shift 2)

Options:

A. -9

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B. 18

C. $\frac{1}{9}$

D. 9

Answer: D

Solution:

We have,
$f(x) = 3x + 6, g(x) = 4x + k$
Since, $fog(x) = gof(x)$
$\Rightarrow f(g)x = g(f(x))$
$\Rightarrow f(4x + k) = g(3x + 6)$
$\Rightarrow 3(4x + k) + 6 = 4(3x + 6) + K$
$\Rightarrow 3k - k = 24 - 6$
$\Rightarrow 2k = 18$
$\Rightarrow k = 9$

Question95

The domain of the real valued function $f(x) = \sqrt{\frac{x-2}{3-x}}$ is… MHT CET 2019 (Shift 1)

Options:

A. (2,3]

B. [2,3)

C. (2,3)

D. [2, 3]

Answer: B

Solution:

We have,
$f(x) = \sqrt{\frac{x-2}{3-x}}$
Clearly, $f(x)$ will be defined when
$3 - x > 0 \Rightarrow x < 3$
And $x - 2 \geq 0 \Rightarrow x \geq 2$
$\therefore$ Domain of $f(x)$ is $[2,3)$.

Question96

If $f : R - \{2\} \rightarrow R$ is a function defined by $f(x) = \frac{x^2-4}{x-2}$, then range is MHT CET 2018

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Options:

A. R

B. $R-\{2\}$

C. $R-\{4\}$

D. $R-\{-2,\ 2\}$

Answer: C

Solution:

Given $f(x)=\frac{(x-2)(x+2)}{(x-2)}$; $\Rightarrow D_f : R-\{2\}$
$\Rightarrow R_f : R-\{4\}$

Question97

$f: R \rightarrow R$, then $f(x)=x|x|$ will be MHT CET 2012

Options:

A. many-one-onto

B. one-one-onto

C. many-one-into

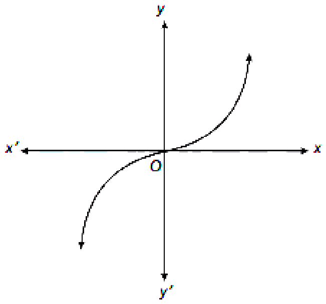
D. one-one-into

Answer: B

Solution:

Given, $f: R \rightarrow R, \quad f(x)=x|x|$Redefined the function,

$$f(x)=\begin{cases}-x^2, x<0 \\ 0, x=0 \\ x^2, x>0\end{cases}$$



The graph of $f(x)$ shows that it is a bijective (one-one-onto) function.

Question98

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If $f(x)=\frac{2^x+2^{-x}}{2}$, then $f(x+y)\cdot f(x-y)$ is MHT CET 2012

Options:

A. $\frac{1}{4}[f(2x)-f(2y)]$

B. $\frac{1}{2}[f(2x)-f(2y)]$

C. $\frac{1}{4}[f(2x)+f(2y)]$

D. $\frac{1}{2}[f(2x)+f(2y)]$

Answer: D

Solution:

Given $f(x)=\frac{2^x+2^{-x}}{2}$

Now, $f(x+y)=\frac{2^{x+y}+2^{-x-y}}{2}$

and $f(x-y)=\frac{2^{x-y}+2^{-x+y}}{2}$

$\therefore f(x+y)\cdot f(x-y)$

$=\frac{(2^{x+y}+2^{-x-y})}{2}\cdot\frac{(2^{x-y}+2^{-x+y})}{2}$

$=\frac{2^{2x}+2^{-2y}+2^{2y}+2^{-2x}}{4}$

$=\frac{1}{2}\left\{\left(\frac{2^{2x}+2^{-2x}}{2}\right)+\left(\frac{2^{2y}+2^{-2y}}{2}\right)\right\}$

$=\frac{1}{2}\{f(2x)+f(2y)\}$

Question99

The period of $|\cos x|$ will be MHT CET 2012

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

Answer: C

Solution:

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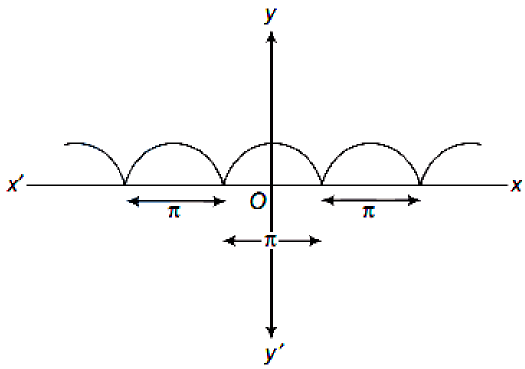

Let $f(x)=|\cos x|$

From figure, it is clear that the period of $|\cos x|$ is π.

Question100

If $f(x)=e^x g(x), g(0)=2, g'(0)=1$, then $f'(0)$is MHT CET 2011

Options:

A. 1

B. 3

C. 2

D. 0

Answer: B

Solution:

$f'(x)=e^x g'(x)+e^x g(x)$

$$\begin{aligned} \Rightarrow \quad f'(0) &= e^0 \cdot g'(0)+e^0 g(0) \\ &= 1 \cdot 1+1 \cdot 2 \\ [\because \{g'(0)=1 \text{ and } g(0)&=2\}] \\ &= 1+2=3 \end{aligned}$$

Question101

Let $f(x+y)=f(x) \cdot f(y), \forall x, y \in R$, suppose that $f(3)=3$ and $f'(0)=11$, then $f'(3)$ is given by MHT CET 2011

Options:

A. 22

B. 44

C. 28

D. 33

Answer: D

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Solution:

$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$$=\lim_{h\to 0}\frac{f(3)\cdot f(h)-f(3)}{h}$$
$$\Rightarrow\quad f'(3)=3\lim_{h\to 0}\frac{f(h)-1}{h}$$

$[\because f(3)=3]$Use L'Hospital rule,

$$\Rightarrow\quad f'(3)=3\lim_{h\to 0}\frac{f'(h)}{1}$$
$$=3f'(0)$$

$[\because f'(0)=11]$

$$=3\times 11=33$$

Question102

Missing term in the following table is $\begin{array}{lllllll} x & : & 0 & 1 & 2 & 3 & 4 \\ y=f(x) & : & 1 & 3 & 9 & ? & 81\end{array}$ **MHT CET 2010**

Options:

A. 27

B. 30

C. 31

D. 34

Answer: C

Solution:

Here, 4 values are given, therefore

$$\Delta^4 f(x)=0\forall x$$
$$\Rightarrow(E-1)^4 f(x)=0$$
$$\Rightarrow\left(E^4-4E^3+6E^2-4E+1\right)f(x)=0$$
$$\Rightarrow E^4 f(x)-4E^3 f(x)+6E^2 f(x)$$
$$-4Ef(x)+f(x)=0$$
$$\Rightarrow f(x+4)-4f(x+3)+6f(x+2)$$
$$-4f(x+1)+f(x)=0$$

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On putting $x=0$, we get

$$f(4)-4f(3)+6f(2)-4f(1)+f(0)=0 \quad \ldots \text{(i)}$$

On substituting the values of $f(0), f(1), f(2), f(4)$ in Eq. (i), we get

$$81-4f(3)+6\times 9-4\times 3+1=0$$
$$\Rightarrow \quad 4f(3)=124$$

$$\Rightarrow \quad f(3)=31$$

Question103

If g is the inverse of f and $f'(x)=\frac{1}{1+x^2}$, then $g'(x)$ is equal to MHT CET 2010

Options:

A. $1+[g(x)]^2$

B. $\frac{-1}{1+[g(x)]^2}$

C. $\frac{1}{2(1+x^2)}$

D. None of these

Answer: A

Solution:

Given, $g=$ inverse of $f=f^{-1}$

$$\Rightarrow \quad g(x)=f^{-1}(x)$$
$$\Rightarrow \quad f[g(x)]=x$$

On differentiating w.r.t. x, we get

$$f'[g(x)]\cdot g'(x)=1$$
$$\Rightarrow \quad g'(x)=\frac{1}{f'[g(x)]}$$
$$=\frac{1}{\frac{1}{1+[g(x)]^2}}$$
$$\Rightarrow g'(x)=1+[g(x)]^2$$

Question104

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If D_{30} is the set of all divisors of $30, x, y \in D_{30}$, we define $x+y=\mathrm{LCM}(x, y), x \cdot y=\mathrm{GCD}(x, y)$,$x'=\frac{30}{x}$ and $f(x, y, z)=(x+y) \cdot(y'+z)$, then$f(2,5,15)$ is equal to MHT CET 2009

Options:

A. 2

B. 5

C. 10

D. 15

Answer: C

Solution:

$D_{30}=\{1,2,3,5,6,10,15,30\}$

$$\begin{aligned} f(2,5,15) &= (2+5) \cdot (5'+15) \\ &= 10 \cdot \left(\frac{30}{5}+15\right) \\ &= 10(6+15) \\ &= 10 \cdot 30 \\ &= 10 \end{aligned}$$

Question105

Find the function $f(x_1, x_2, x_3)$ satisfying $f(x_1, x_2, x_3)=1$ at $x_1=1, x_2=x_3=0$ MHT CET 2009

Options:

A. $x_1' \cdot x_2$

B. $x_1 \cdot {x_2}'$

C. $(x_1+x_2+x_3)' \cdot x_2$

D. $(x_1'+x_3) \cdot x_3$

Answer: B

Solution:

Given, $x_1=1, x_2=x_3=0$

$x_1 \cdot x_2'=1(0)'=1$

Question106

For a certain function u_x, given that $u_0=3u_1=12, u_2=81, \quad u_3=200, u_4=100$

$u_5=8$, then $\Delta^5 u_x$ is equal to

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Options:

A. 750

B. 778

C. 765

D. 755

Answer: D

Solution:

Given, $u_0 = 3, u_1 = 12, u_2 = 81, u_3 = 200,$

$u_4 = 100, u_5 = 8$

Then,

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$	$\Delta^4 u_x$	$\Delta^5 u_x$
0	3					
		9				
1	12		60			
		69		−10		
2	81		50		−259	
		119		−269		755
3	200		−219		496	
		−100		227		
4	100		8			
		−92				
5	8					

Hence, $\Delta^5 u_x = 755$

Question107

Find a polynomial $f(x)$ of degree 2 where $f(0) = 8, f(1) = 12, f(2) = 18$ MHT CET 2009

Options:

A. $x^2 + 3x - 8$

B. $x^2 - 3x + 8$

C. $2x^2 - x + 3$

D. $x^2 + 3x + 8$

Answer: D

Solution:

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Let the polynomial is $ax^2 + bx + c$.

Now, $f(0) = 8$

$\Rightarrow$

$c = 8$

$\therefore$ Equation is $ax^2 + bx + 8$.

Again, $f(1) = 12 \Rightarrow a + b + 8 = 12$

$\Rightarrow \quad a + b = 4 \quad \ldots$ (i)

and $f(2) = 18$

$\Rightarrow \quad 4a + 2b + 8 = 18$

$\Rightarrow \quad 2a + b = 5 \quad \ldots$ (ii)

On solving Eqs. (i) and (ii), we get

$a = 1, b = 3$

$\therefore$ Required equation is $x^2 + 3x + 8$.

Question108

The value of $\Delta \log f(x) + \Delta^2 (3^x)$ is MHT CET 2008

Options:

A. $\log\left[1 + \frac{\Delta f(x)}{f(x)}\right] + 4 \cdot 3^x$

B. $\log\left[1 + \frac{\Delta f(x)}{f(x)}\right] + 3^x$

C. $\log\left[\frac{\Delta f(x)}{1+f(x)}\right] + 4 \cdot 3^x$

D. $\log\left[\frac{\Delta f(x)}{1+f(x)}\right] + 3^x$

Answer: A

Solution:

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$$
\begin{aligned}
&\Delta \log f(x)+\Delta^2\left(3^x\right) \\
&\quad=\log f(x+h)-\log f(x)+(E-1)^2 3^x \\
&\quad=\log \left[\frac{f(x+h)}{f(x)}\right]+\left(E^2-2 E+1\right) 3^x \\
&\quad=\log \left[\frac{E f(x)]}{f(x)}\right]+E^2\left(3^x\right)-2 E\left(3^x\right)+3^x \\
&\quad=\log \left[\frac{(1+\Delta) f(x)}{f(x)}\right]+3^{x+2}-2 \cdot 3^{x+1}+3^x \\
&\quad=\log \left[1+\frac{\Delta f(x)}{f(x)}\right]+3^x(9-6+1) \\
&\quad=\log \left[1+\frac{\Delta f(x)}{f(x)}\right]+4 \cdot 3^x
\end{aligned}
$$

Question109

If $f: R \rightarrow R$ be mapping defined by $f(x)=x^3+5$, then $f^{-1}(x)$ is equal to MHT CET 2007

Options:

A. $(x+5)^{1/3}$

B. $(x-5)^{1/3}$

C. $(5-x)^{1/3}$

D. $5-x$

Answer: B

Solution:

Let

$$y=f(x)=x^3+5$$

$$
\begin{aligned}
&\Rightarrow \quad x=(y-5)^{1/3} \\
&\therefore \quad f^{-1}(x)=(x-5)^{1/3} \\
&\text{Given,} \qquad\qquad\qquad f(x)=\tfrac{ax+b}{cx+d}
\end{aligned}
$$

Question110

Let $f(x)=\frac{ax+b}{cx+d}$. Then, $fof(x)=x$ providedthat MHT CET 2007

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Options:

A. $d=-a$

B. $d=a$

C. $a=b=c=d=1$

D. $a=b=1$

Answer: A

Solution:

Given,and $\quad fof(x)=x$

$$\Rightarrow \quad f\left(\frac{ax+b}{cx+d}\right)=x$$

$$\Rightarrow \quad \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}=x$$

$$\Rightarrow \quad \frac{x\left(a^2+bc\right)+ab+bd}{x(ac+cd)+bc+d^2}=x$$

$$\Rightarrow \quad d=-a$$

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