Determinants

Question1

The lines $x+2ay+a=0, x+3by+b=0, x+4cy+c=0$ are concurrent then $a,$ b, c are in MHT CET 2025 (22 Apr Shift 2)

Options:

A. Harmonic progression

B. Geometric progression

C. Arithmetic progression

D. Arithmetico geometric progression

Answer: A

Solution:

Harmonic progression.

For concurrency of three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$, $a_3x+b_3y+c_3=0$, the determinant must vanish:

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Row-reducing (subtract row 1 from rows 2 and 3) gives

$$(3b-2a)(c-a)-(b-a)(4c-2a)=0$$

$$\Rightarrow \ -bc+2ac-ab=0 \ \Rightarrow \ 2ac=ab+bc.$$

Divide by $abc \neq 0$:

$$\frac{2}{b}=\frac{1}{a}+\frac{1}{c} \quad \Rightarrow \quad \frac{1}{b}=\frac{\frac{1}{a}+\frac{1}{c}}{2}.$$

Thus $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in arithmetic progression, so a, b, c are in **harmonic progression.**

Question2

If $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$, then the value of $x^2+y^2+z^2=$ MHT CET 2025 (20 Apr Shift 2)

Options:

A. 6

B. 12

C. 3

D. 14

Answer: D

Solution:

$$x + 3y + 3z = 12$$
$$x + 4y + 4z = 15$$
$$x + 3y + 4z = 13$$

Subtract first from second: $y + z = 3$.
Subtract first from third: $z = 1 \Rightarrow y = 2$.
Then from the first: $x = 12 - 3(2) - 3(1) = 3$.

So $x^2 + y^2 + z^2 = 3^2 + 2^2 + 1^2 = 9 + 4 + 1 = 14$.

Question3

If $[2\bar{p} - 3\bar{r}\ \bar{q}\ \bar{s}] + [3\bar{p} + 2\bar{q}\ \bar{r}\ \bar{s}] = m[\bar{p}\ \bar{r}\ \bar{s}] + n[\bar{q}\ \bar{r}\ \bar{s}] + t[\bar{p}\ \bar{q}\ \bar{s}]$,, then the values of m, n, t respectively are ....

MHT CET 2025 (19 Apr Shift 1)

Options:

A. $2, 3, 3$

B. $3, 4, 5$

C. $1, 2, 3$

D. $3, 5, 2$

Answer: D

Solution:

We're given:

$$[\mathbf{2p} - \mathbf{3r}\ \mathbf{q}\ \mathbf{s}] + [\mathbf{3p} + \mathbf{2q}\ \mathbf{r}\ \mathbf{s}] = m[\mathbf{p}\ \mathbf{r}\ \mathbf{s}] + n[\mathbf{q}\ \mathbf{r}\ \mathbf{s}] + t[\mathbf{p}\ \mathbf{q}\ \mathbf{s}]$$

Expanding determinants linearly in their columns, we can group coefficients of each vector combination.

Step 1: Expand the first term

$$[\mathbf{2p} - \mathbf{3r}, \mathbf{q}, \mathbf{s}] = 2[\mathbf{p}, \mathbf{q}, \mathbf{s}] - 3[\mathbf{r}, \mathbf{q}, \mathbf{s}]$$

But $[\mathbf{r}, \mathbf{q}, \mathbf{s}] = -[\mathbf{q}, \mathbf{r}, \mathbf{s}]$, so

$$= 2[\mathbf{p}, \mathbf{q}, \mathbf{s}] + 3[\mathbf{q}, \mathbf{r}, \mathbf{s}]$$

Step 2: Expand the second term

$$[\mathbf{3p} + \mathbf{2q}, \mathbf{r}, \mathbf{s}] = 3[\mathbf{p}, \mathbf{r}, \mathbf{s}] + 2[\mathbf{q}, \mathbf{r}, \mathbf{s}]$$

Step 3: Add both results

$$(2[\mathbf{p}, \mathbf{q}, \mathbf{s}] + 3[\mathbf{q}, \mathbf{r}, \mathbf{s}]) + (3[\mathbf{p}, \mathbf{r}, \mathbf{s}] + 2[\mathbf{q}, \mathbf{r}, \mathbf{s}])$$

$$= 3[\mathbf{p}, \mathbf{r}, \mathbf{s}] + 5[\mathbf{q}, \mathbf{r}, \mathbf{s}] + 2[\mathbf{p}, \mathbf{q}, \mathbf{s}]$$

✅ Therefore,

$$m = 3,\ n = 5,\ t = 2$$

Answer: $m, n, t = 3, 5, 2$ ✅

Question4

If $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$,where A_{21}, A_{22}, A_{23} are cofactors of a_{21}, a_{22}, a_{23} respectively, then the value of $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} =$ MHT CET 2025 (19 Apr Shift 1)

Options:

A. 1

B. -1

C. 0

D. 2

Answer: A

Solution:

$$a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

is the cofactor expansion of $\det A$ along the 2nd row.

For

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

the top-left 2×2 block has determinant $\cos^2\theta + \sin^2\theta = 1$, and with the bottom-right 1, we get $\det A = 1$.

Hence $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 1$.

Question5

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ such that $\mathrm{AX} = \mathrm{B}$, then $\mathrm{X} =$ MHT CET 2024 (11 May Shift 1)

Options:

A. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$

Answer: B

Solution:

$\mathrm{AX} = \mathrm{B}$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

Applying $\mathrm{R}_2 \to \mathrm{R}_2 - 2\mathrm{R}_1$ and $\mathrm{R}_3 \to \mathrm{R}_3 - \mathrm{R}_1$,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$

$$\therefore \quad x_1 - x_2 + x_3 = 4 \ldots (i)$$
$$3x_2 - 5x_3 = -8 \ldots (ii)$$
$$\therefore \quad 2x_2 = -2 \Rightarrow x_2 = -1$$

From (ii),

$$3(-1) - 5x_3 = -8 \Rightarrow x_3 = 1$$

From (i),

$$x_1 + 1 + 1 = 4 \Rightarrow x_1 = 2$$
$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Question6

If $w = \frac{-1-i\sqrt{3}}{2}$ where $i = \sqrt{-1}$, then the value of $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$ is MHT CET 2024 (10 May Shift 2)

Options:

A. -1

B. 0

C. 1

D. 3

Answer: B

Solution:

$$\omega = \frac{-1 - i\sqrt{3}}{2}, \omega^2 = \frac{1 - 3 + 2i\sqrt{3}}{4} = \frac{-1 + i\sqrt{3}}{2},$$
$$\omega^3 = 1, \omega^6 = 1$$
$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$
$$= 1\left(\omega^3 - 1\right) - \omega\left(\omega^2 - \omega^2\right) + \omega^2\left(\omega - \omega^4\right)$$
$$= 1\left(\omega^3 - 1\right) + 0 + \omega^2\left(\omega - \omega^4\right).$$

$$= \omega^3 - 1 + \omega^3 - \omega^6$$
$$= 2\omega^3 - \omega^6 - 1$$
$$= 2(1) - 1 - 1$$
$$= 0$$

Question7

For the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$**, the matrix of cofactors is MHT CET 2024 (09 May Shift 2)**

Options:

A. $\begin{bmatrix} 0 & 8 & -4 \\ -1 & 3 & 2 \\ 1 & -7 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -8 & 4 \\ -1 & 3 & -2 \\ 1 & -7 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 8 & -4 \\ 1 & -3 & 2 \\ -1 & 7 & -2 \end{bmatrix}$

D. $\begin{bmatrix} 0 & -8 & 4 \\ -1 & 3 & 2 \\ -1 & -7 & 2 \end{bmatrix}$

Answer: B

Solution:

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 1(0) = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ -1 & 2 \end{vmatrix} = (-1)(8) = -8$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = (1)(4) = 4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (-1)(1) = -1$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = (1)(3) = 3$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = (-1)(2) = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (1)(1) = 1$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = (-1)(7) = -7$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} = (1)(2) = 2$$

$\therefore$ The matrix of the cofactors is $\begin{bmatrix} 0 & -8 & 4 \\ -1 & 3 & -2 \\ 1 & -7 & 2 \end{bmatrix}$

Question8

Let $X = \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix}$, $\mathrm{A} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$.If $AX = B$, then the value of $2a - 3b + 4c$ will be MHT CET 2024 (02 May Shift 2)

Options:

A. 0

B. -4

C. 6

D. 4

Answer: D

Solution:

$$\mathrm{AX} = \mathrm{B}$$

$$\therefore \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{a} \\ \mathrm{b} \\ \mathrm{c} \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

$$\therefore \quad \begin{aligned} & a - b + 2c = 3 \ldots (i) \\ & 2a + c = 1 \ldots (ii) \\ & 3a + 2b + c = 4 \ldots (iii) \end{aligned}$$

Solving (i), (ii) and (iii), we get

$$\begin{aligned} & a = -1, b = 2, c = 3 \\ \therefore \quad 2a - 3b + 4c & = 2(-1) - 3(2) + 4(3) \\ & = 4 \end{aligned}$$

Question9

If $\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos(2B) \\ \sin A & \cos A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix} = 0$, then the value of B is MHT CET 2023 (09 May Shift 2)

Options:

A. $n\pi, n \in \mathbb{Z}$

B. $(2n+1)\frac{\pi}{2}, n \in Z$

C. $(2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$

D. $2n\frac{\pi}{3}, n \in \mathbb{Z}$

Answer: B

Solution:

$$\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos(2B) \\ \sin A & \cos A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix} = 0$$

$$\therefore \quad \cos(A+B)[(\cos A \cos B - \sin A \sin B)]$$
$$+\sin(A+B)[\sin A \cos B + \sin B \cos A]$$
$$+\cos 2B\left[\sin^2 A + \cos^2 A\right] = 0$$
$$\therefore \quad \cos(A+B) \cdot \cos(A+B)$$
$$+\sin(A+B) \cdot \sin(A+B) + \cos 2B = 0$$
$$\cos^2(A+B) + \sin^2(A+B) + \cos 2B = 0$$
$$1 + \cos 2B = 0$$
$$2\cos^2 B = 0$$
$$\therefore \quad \cos B = 0$$
$$\therefore \quad B = (2n+1)\frac{\pi}{2} \text{ for } (n \in Z)$$

Question10

If $A = [a_{ij}]_{3\times 3} = \begin{bmatrix} 1 & 3 & 3 \\ -1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix}$ **and** A_{ij} **is a cofactor of** a_{ij} **then the value of** $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ **is equal to MHT CET 2022 (10 Aug Shift 2)**

Options:

A. 5

B. 15

C. 20

D. 0

Answer: B

Solution:

$$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A|$$
$$= 1 \times (8-2) - 3(-4-2) + 3(-1-2) = 6 + 18 - 9 = 15$$

Question11

If $A = [a_{ij}]_{3\times3} = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$ and A_{ij} is a cofactor of a_{ij}, then the value of $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$ is equal to MHT CET 2022 (08 Aug Shift 2)

Options:

A. 18

B. 8

C. -8

D. 0

Answer: B

Solution:

$$a_{21}A_{21}, a_{22}A_{22} + a_{23}A_{23} = |A| = 3 \times (12-6) - 2(3-2) + 4(6-8) = 8$$

Question12

If $\mathrm{A} = [\mathrm{a_{ij}}]_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ and Aij is a cofactor of $\mathrm{a_{ij}}$, then $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is equal to MHT CET 2022 (05 Aug Shift 1)

Options:

A. -1

B. 2

C. 0

D. 1

Answer: C

Solution:

If we multiply elements of one row to the corresponding cofactors of another row and add them then we get zero hence, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$

Question13

If $A = \begin{bmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then cofactors of all elements of second row are respectively. MHT CET 2021 (24 Sep Shift 1)

Options:

A. $-39, 3, 11$

B. $-39, 27, 11$

C. $39, -3 - 11$

D. $-39, -27, 11$

Answer: B

Solution:

$$A = \begin{bmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$$

Cofactors of elements in second row are

Cofactor of -4

Cofactor of 3

$$= (-1)^{2+1} \begin{vmatrix} 6 & 3 \\ -7 & 3 \end{vmatrix} = -(18 + 21) = -39$$
$$= (-1)^{2+2} \begin{vmatrix} 5 & 3 \\ -4 & 3 \end{vmatrix} = 15 + 12 = 27$$
$$= (-1)^{2+3} \begin{vmatrix} 5 & 6 \\ -4 & -7 \end{vmatrix} = -(-35 + 24) = 11$$

Question14

The sum of three numbers is 6 . Thrice the third number when added to the first number given 7 . On adding three time first number to the sum of second and third number we get 12 . The product of these numbers is MHT CET 2021 (22 Sep Shift 1)

Options:

A. 20

B. 3

C. $\frac{20}{3}$

D. $\frac{5}{3}$

Answer: C

Solution:

Let the numbers be x, y, z

$$\begin{aligned}
& x+y+z=6 . .(1) \\
& x+3 z==7 . .(2) \\
& 3 x+y+z=12 . .(3) \\
& \text { Eq. (3)-(1), gives } \\
& 2 x=6 \Rightarrow x=3
\end{aligned}$$

$$2 \mathrm{x}=6 \Rightarrow \mathrm{x}=3$$

From eq. (2), we get

$$3+3 z+7 \Rightarrow z=\frac{4}{3}$$

From eq. (1), we get

$$\begin{gathered}
3+y+\frac{4}{3}=6 \quad \Rightarrow \quad y=\frac{5}{3} \\
\therefore x y z=(3)\left(\frac{5}{3}\right)\left(\frac{4}{3}\right)=\frac{20}{3}
\end{gathered}$$

Question15

The co-factors of the elements of second column of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ are MHT CET 2021 (20 Sep Shift 2)

Options:

A. $-13, 6, 5$

B. $13, 5, 6$

C. $13, -6, -5$

D. $-13, -6, 5$

Answer: A

Solution:

Required cofactors are as follows:

$$\mathrm{A}_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} = -(12+1) = -13$$
$$\mathrm{A}_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} = (4+2) = 6$$
$$\mathrm{A}_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -(-1-6) = 5$$

Question16

If $A\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 1 \\ 3 & 2 & 6 \end{bmatrix}$ and A_{ij} are cofactors of the elements a_{ij} of A, then $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ is equal to MHT CET 2021 (20 Sep Shift 1)

Options:

A. 8

B. 6

C. 4

D. 0

Answer: A

Solution:

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$$

$$\therefore |A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & 1 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= 3(12-2) - 2(6-3) + 4(2-6) = 30 - 6 - 16 = 8$$

Question17

If $AX = B$, where $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$, then $x^2 + y^2 + z^2 =$ MHT CET 2020 (19 Oct Shift 1)

Options:

A. 14

B. 19

C. 21

D. 6

Answer: A

Solution:

Given $\mathrm{AX} = \mathrm{B}$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$$

$\mathrm{R_2 \rightarrow R_2 - R_1}$ and $\mathrm{R_3 \rightarrow R_3 - R_1}$

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x + 3y + 3z \\ y + z \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore x + 3y + 3z = 12$$
$$y + z = 3$$
$$z = 1$$

Thus $z=1, y=2, x=3$

$\therefore x^2+y^2+z^2=9+4+1=14$

Question18

If $\mathrm{AX}=\mathrm{B}$, where $\mathrm{A}=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 3 & 3 & -4\end{bmatrix}$, $\mathrm{B}=\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$ and $\mathrm{X}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, then $x+y+z=$ MHT CET 2020 (15 Oct Shift 1)

Options:

A. 2

B. 3

C. 6

D. 1

Answer: B

Solution:

Given $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 3 & 3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $AX=B$

$\therefore \mathrm{AA}^{-1}\mathrm{X}=\mathrm{A}^{-1}\mathrm{B} \Rightarrow \mathrm{IX}=\mathrm{A}^{-1}\mathrm{B} \Rightarrow \mathrm{X}=\mathrm{A}^{-1}\mathrm{B}$Now $|\mathrm{A}|=4+(-8)+9=5$

$$\therefore A^{-1}=\begin{bmatrix}4 & -1 & 1\\ 8 & -7 & 2\\ 9 & -6 & 1\end{bmatrix}\times\frac{1}{5}$$

$$\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{5}\begin{bmatrix}4 & -1 & 1\\ 8 & -7 & 2\\ 9 & -6 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\\ 2\end{bmatrix}$$

$$=\frac{1}{5}\begin{bmatrix}8-1+2\\ 8-7+4\\ 9-6+2\end{bmatrix}=\frac{1}{5}\begin{bmatrix}5\\ 5\\ 5\end{bmatrix}$$

$$y=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

On comparing both side, we get

$$x=y=z=1 \Rightarrow x+y+z=1+1+1=3$$

Question19

If $A=\begin{bmatrix}1 & 0 & 2\\2 & 1 & 3\\0 & 3 & -5\end{bmatrix}$, where A_{ij} is the cofactor of the element a_{ij} of matrix A, then $a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}=$ MHT CET 2020 (14 Oct Shift 1)

Options:

A. -26

B. 0

C. -2

D. 26

Answer: C

Solution:

$$\begin{aligned}&a_{21}A_{21}+a_{22}A_{22}+a_{23}A_{23}\\&\quad=-2(0-6)+1(-5-0)-3(3-0)\\&\quad=12-5-9=-2\end{aligned}$$

Question20

If A and B are square matrices of order 3 such that $|A|=2,|B|=4$, then $|A(adj\,B)|=$ ... MHT CET 2019 (Shift 1)

Options:

A. 16

B. 8

C. 64

D. 32

Answer: D

Solution:

We have, A and B are square matrices of order 3 such that
$|A| = 2, |B| = 4$
Now, $|A(adj)B| = |A||adj\, B| \qquad (\because |AB| = |A||B|)$
$|A||B|^{3-1}$
$= |A||B|^2 = (2)(4)^2 = 32$

Question21

The value of $'a$ for which the system of equations

$$\begin{aligned} a^3x + (a+1)^3y + (a+2)^3z = 0 \\ ax + (a+1)y + (a+2)z = 0 \\ x + y + z = 0 \end{aligned}$$

has a non-zero solution is MHT CET 2011

Options:

A. 1

B. 0

C. -1

D. None of these

Answer: C

Solution:

For non-zero solution,

$$\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & (a+1) & (a+2) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad -\begin{vmatrix} 1 & 1 & 1 \\ a & (a+1) & (a+2) \\ a^3 & (a+1)^3 & (a+2)^3 \end{vmatrix} = 0$$

$$\Rightarrow \quad -(a-a-1)(a+1-a-2)(a+2-a) \times (a+a+1+a+2) = 0$$

$$\Rightarrow \quad -2(3a+3) = 0$$

$$\Rightarrow \quad a = -1$$

$$\left[\begin{matrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{matrix}\right| = (x-y)(y-z)(z-x)(x+y+z)$$

Question22

For the equations

$$x + 2y + 3z = 1, 2x + y + 3z = 2$$

and $5x + 5y + 9z = 4$ **MHT CET 2007**

Options:

A. there is only one solution

B. there exists infinitely many solution

C. there is no solution

D. None of the above

Answer: A

Solution:

Step 1: Eliminate one variable

Subtract the first equation from the second:

$$(2x + y + 3z) - (x + 2y + 3z) = 2 - 1 \Rightarrow x - y = 1 \Rightarrow x = y + 1$$

Step 2: Substitute $x = y + 1$ into the first equation

$$(y+1) + 2y + 3z = 1 \Rightarrow 3y + 3z = 0 \Rightarrow y + z = 0 \Rightarrow y = -z$$

Then $x = y + 1 = -z + 1 = 1 - z$.

Step 3: Substitute into the third equation

$$5(1-z)+5(-z)+9z=4$$

Simplify:

$$5-5z-5z+9z=4 \Rightarrow 5-z=4 \Rightarrow z=1$$

Then $y=-1, x=0$.

✅ **Solution:** $x=0, y=-1, z=1$

Since all variables have a unique value, **there is only one solution.**

Question23

If the vectors $\vec{\mathbf{a}}=\hat{\mathbf{i}}+a\hat{\mathbf{j}}+a^2\hat{\mathbf{k}}$,

$\vec{\mathbf{b}}=\hat{\mathbf{i}}+b\hat{\mathbf{j}}+b^2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}}=\hat{\mathbf{i}}+c\hat{\mathbf{j}}+c^2\hat{\mathbf{k}}$ are three

non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$,

then the value of abc is

MHT CET 2007

Options:

A. 0

B. 1

C. 2

D. -1

Answer: D

Solution:

Since, $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are non-coplanar vectors,

therefore $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]\neq 0$

$$\Rightarrow \Delta=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}\neq 0$$

$\Rightarrow$

$\Delta \neq 0$

Now, $\begin{pmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{pmatrix}| = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$\Rightarrow$ $\Delta(1 + abc) = 0$

$\Rightarrow$

$$abc = -1$$
