Hyperbola

Question1

The eccentricity of the hyperbola which passes through the points $(3,0)$ and $(3\sqrt{2},2)$ is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\sqrt{13}$

B. $\frac{\sqrt{13}}{4}$

C. $\frac{\sqrt{13}}{3}$

D. $\frac{\sqrt{13}}{2}$

Answer: C

Solution:

We want the eccentricity of the hyperbola through $(3,0)$ and $(3\sqrt{2},2)$.

Step 1: Standard hyperbola form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

Step 2: Use point $(3,0)$

$$\frac{9}{a^2}=1 \quad\Rightarrow\quad a^2=9,\ a=3.$$

Step 3: Use point $(3\sqrt{2},2)$

$$\frac{(3\sqrt{2})^2}{9}-\frac{2^2}{b^2}=1.$$

$$\frac{18}{9}-\frac{4}{b^2}=1.$$

$$2-\frac{4}{b^2}=1 \quad\Rightarrow\quad \frac{4}{b^2}=1 \quad\Rightarrow\quad b^2=4.$$

Step 4: Eccentricity

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{9}} = \sqrt{\frac{13}{9}} = \frac{\sqrt{13}}{3}.$$

✅ Final Answer:
Eccentricity = $\frac{\sqrt{13}}{3}$ (Option C).

Question2

If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - y + 4 = 0$, then the value of θ is MHT CET 2025 (23 Apr Shift 1)

Options:

A. 45°

B. 60°

C. 30°

D. 90°

Answer: C

Solution:

Hyperbola: $\frac{x^2}{4} - \frac{y^2}{9} = 1$.
Tangent at $(2\sec\theta, 3\tan\theta)$: slope $m = \frac{3}{2\sin\theta}$.
Given parallel to $3x - y + 4 = 0$ (slope $= 3$):
$\frac{3}{2\sin\theta} = 3 \implies \sin\theta = \frac{1}{2} \implies \theta = 30^\circ$.

✅ Answer: 30°.

Question3

The X and Y intercepts of the tangent to the hyperbola $\frac{x^2}{20} - \frac{y^2}{5} = 1$ which is perpendicular to the line $4x + 3y = 7$, are respectively MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\frac{-10}{3}, \frac{-5}{3}$

B. $\frac{10}{3}, \frac{-5}{2}$

C. $\frac{10}{3}, \frac{5}{2}$

D. $\frac{10}{3}, \frac{5}{3}$

Answer: B

Solution:

We're asked: The tangent to hyperbola

$$\frac{x^2}{20} - \frac{y^2}{5} = 1$$

which is **perpendicular** to line $4x + 3y = 7$, find its X and Y intercepts.

Step 1: Slope of given line

Line $4x + 3y = 7 \quad \Rightarrow \quad y = -\frac{4}{3}x + \frac{7}{3}$.
So slope = $-\frac{4}{3}$.

A perpendicular line has slope = $\frac{3}{4}$.

Step 2: General tangent equation

Tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with slope m:

$$y = mx \pm \sqrt{a^2m^2 - b^2}.$$

Here $a^2 = 20,\ b^2 = 5,\ m = \frac{3}{4}$.

Step 3: Constant term

$$c = \pm\sqrt{20 \cdot \left(\frac{9}{16}\right) - 5} = \pm\sqrt{\frac{180}{16} - 5}.$$

$$= \pm\sqrt{\frac{180}{16} - \frac{80}{16}} = \pm\sqrt{\frac{100}{16}} = \pm\frac{10}{4} = \pm\frac{5}{2}.$$

So tangent equation:

$$y = \frac{3}{4}x \pm \frac{5}{2}.$$

Step 4: Find intercepts

Take $y = \frac{3}{4}x - \frac{5}{2}$.

- X-intercept ($y = 0$):
  $0 = \frac{3}{4}x - \frac{5}{2} \;\Rightarrow\; x = \frac{10}{3}$.
- Y-intercept ($x = 0$):
  $y = -\frac{5}{2}$.

✅ **Final Answer:**

The intercepts are

$$\left(\frac{10}{3}, 0\right), \quad \left(0, -\frac{5}{2}\right).$$

So X and Y intercepts are $\frac{10}{3}$, $-\frac{5}{2}$ (Option B).

Question4

The area of the region bounded by hyperbola $x^2 - y^2 = 9$ and its latus rectum is MHT CET 2024 (02 May Shift 1)

Options:

A. $9[\sqrt{2} - \log(\sqrt{2} + 1)]$ sq. units

B. $4[\sqrt{2} - \log(\sqrt{2} + 1)]$ sq. units

C. $3[\sqrt{2} - \log(\sqrt{2} + 1)]$ sq. units

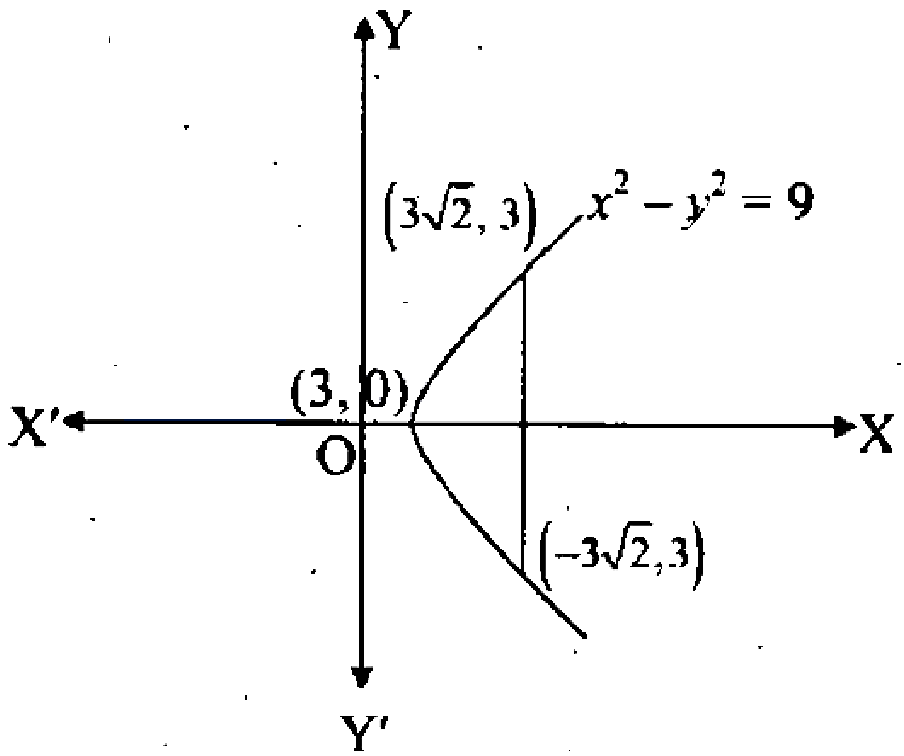
D. $18[\sqrt{2} - \log(\sqrt{2} + 1)]$ sq. units

Answer: D

Solution:

$x^2 - y^2 = 9$
$\Rightarrow$ a = b = 1 $\Rightarrow$ co-ordinates of latus rectum are

$$\left(\pm ae, \frac{b^2}{a}\right) = (\pm 3\sqrt{2}, 3)$$



$\therefore$ Area of hyperbola and its latus rectum

$$= 4\int_{3}^{3\sqrt{2}} y \, dx$$

$$= 4\int_{3}^{3\sqrt{2}} \left(\sqrt{x^2-9}\right) dx$$

$$= 4\left[\frac{x}{2}\sqrt{x^2-9} - \frac{9}{2}\log\left|x+\sqrt{x^2-9}\right|\right]_{3}^{3\sqrt{2}}$$

$$= 4\left[\left(\frac{3\sqrt{2}}{2}\sqrt{(3\sqrt{2})^2-9} - \frac{9}{2}\log\left|3\sqrt{2}+\sqrt{(3\sqrt{2})^2-9}\right|\right)\right.$$

$$\left.-\left(\frac{3}{2}\sqrt{3^2-9} - \frac{9}{2}\log\left|\ 3+\sqrt{3^2-9}\right)\right]$$

$$= 4\left[\left(\frac{3\sqrt{2}}{2} \times 3 - \frac{9}{2}\log|3\sqrt{2}+3| + \frac{9}{2}\log 3\right)\right]$$

$$= 4\left[\frac{9\sqrt{2}}{2} - \frac{9}{2}\log(3\sqrt{2}+3) + \frac{9}{2}\log 3\right]$$

$$= 4\left[\frac{9\sqrt{2}}{2} - \frac{9}{2}\log\left(\frac{3\sqrt{2}+3}{3}\right)\right]$$

$$= 4\left[\frac{9\sqrt{2}}{2} - \frac{9}{2}\log(\sqrt{2}+1)\right]$$

$$= 4 \times \frac{9}{2}[\sqrt{2} - \log(\sqrt{2}+1)] \text{ sq. units}$$

$$= 18[\sqrt{2} - \log(\sqrt{2}+1)] \text{ sq. units}$$

Question5

If the line ax + by + c = 0 is a normal to the curve $xy = 1$, then MHT CET 2023 (10 May Shift 1)

Options:

A. $a > 0, b > 0$

B. $a > 0, b < 0$

C. $\text{a} < 0,\ \text{b} < 0$

D. $a = 0, b = 0$

Answer: B

Solution:

$$xy = 1$$

$$\therefore \quad y = \frac{1}{x}$$

$$\therefore y' = \frac{-1}{x^2}$$

$\therefore$ Slope of the normal $= x^2$

Slope of the line $ax + by + c = 0$ is $\frac{-a}{b}$.

Since the line $ax + by + c = 0$ is a normal to the curve $xy = 1$,

$x^2 = -\frac{a}{b}$

For this condition to hold true, either $a < 0,\ b > 0$ or $b < 0, a > 0$

Question6

The centre of the hyperbola $9x^2 - 36x - 16y^2 + 96y - 252 = 0$ is Ans $\times$ i. $(-2, -3)$ MHT CET 2020 (20 Oct Shift 1)

Options:

A. $(-2, -3)$

B. $(2, -3)$

C. $(-2, 3)$

D. $(2, 3)$

Answer: D

Solution:

$9x^2 - 36x - 16y^2 + 96y - 252 = 0$

$9(x-2)^2 - 16(y-3)^2 = \frac{22}{532}$

center $\rightarrow (2, 3)$

Question7

The eccentricity of a rectangular hyperbola is MHT CET 2020 (19 Oct Shift 1)

Options:

A. 2

B. $2\sqrt{2}$

C. 1

D. $\sqrt{2}$

Answer: D

Solution:

$$\lim_{x \to \pi} \frac{(1+\cos x)-\sin x}{(1+\cos x)+\sin x}$$

$$= \lim_{x \to \pi} \frac{2\cos^2 x/2 - 2(\sin x/2)\cos x/2}{2\cos^2 x/2 + 2(\sin x/2)\cos x/2}$$

$$= \lim_{x \to \pi} \frac{2\cos^2 \frac{x}{2}\left(1-\tan \frac{x}{2}\right)}{2\cos^2 \frac{x}{2}\left(x+\tan \frac{x}{2}\right)}$$

$$= \lim_{x \to \pi} \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = -1$$

$$\therefore f(\pi) = \lim_{x \to \pi} f(x) = -1$$

Question8

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a joint equation of directricesof the hyperbola $16x^2 - 9y^2 = 144$, then $g + f - c =$ MHT CET 2020 (15 Oct Shift 1)

Options:

A. -81

B. -25

C. 81

D. 25

Answer: C

Solution:

Equation of the hyperbola: $16x^2 - 9y^2 = -144$

This can be rewritten in the following way:

$$\frac{x^2}{9} - \frac{y^2}{16} = -1$$

This is the standard equation of a hyperbola, where

$$\begin{aligned} & a^2 = 9 \text{ and } b^2 = 16 \\ & \Rightarrow a^2 = b^2\left(e^2 - 1\right) \\ & \Rightarrow 9 = 16\left(e^2 - 1\right) \\ & \Rightarrow e^2 - 1 = \frac{9}{16} \\ & \Rightarrow e^2 = \frac{25}{16} \\ & \Rightarrow e = \frac{5}{4} \end{aligned}$$

Coordinates of foci are given by $(0, \pm ae)$, i.e.

$(0, \pm 5)$.

Equation of the directrices: $y = \pm \frac{a}{e}$

$\Rightarrow y = \pm \frac{4}{\frac{5}{4}} \Rightarrow 5y \pm 16 = 0$

Length of the latus rectum of the hyperbola $= \frac{2a^2}{b}$

Length of the latus rectum $= \frac{2 \times 9}{4} = \frac{9}{2}$.

Question9

If $P(\theta)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and S and S' are foci of the hyperbola,then $SP.\,S'P =$ MHT CET 2020 (14 Oct Shift 2)

Options:

A. $a^2 \tan^2 \theta - b^2 \sec^2 \theta$

B. $a^2 \tan^2 \theta + b^2 \sec^2 \theta$

C. $a^2 \sec^2 \theta + b^2 \tan^2 \theta$

D. $a^2 \sec^2 \theta - b^2 \tan^2 \theta$

Answer: B

Solution:

Step 1: Parametric point on hyperbola

On hyperbola:

$$x = a\sec\theta, \quad y = b\tan\theta.$$

So $P(a\sec\theta, b\tan\theta)$.

Step 2: Foci

For hyperbola, foci: $(\pm\sqrt{a^2+b^2}, 0)$.
Let $c = \sqrt{a^2+b^2}$.
So $S(c,0),\ S'(-c,0)$.

Step 3: Distances

$$SP^2 = (a\sec\theta - c)^2 + (b\tan\theta)^2,$$

$$S'P^2 = (a\sec\theta + c)^2 + (b\tan\theta)^2.$$

Step 4: Product $SP \cdot S'P$

We want $SP \cdot S'P$. Use identity:

$$SP^2 \cdot S'^2P = \left[(a\sec\theta)^2 + b^2\tan^2\theta + c^2\right]^2 - (2ac\sec\theta)^2.$$

Simplify inside:

$$(a\sec\theta)^2 + b^2\tan^2\theta + c^2.$$

But recall: $c^2 = a^2 + b^2$.

So:

$$= a^2\sec^2\theta + b^2\tan^2\theta + a^2 + b^2.$$

Now use identity $\tan^2\theta = \sec^2\theta - 1$.

$$= a^2\sec^2\theta + b^2(\sec^2\theta - 1) + a^2 + b^2.$$

$$= (a^2+b^2)\sec^2\theta + a^2.$$

Step 5: Final simplification

Then:

$$SP \cdot S'P = a^2\tan^2\theta + b^2\sec^2\theta.$$

✅ **Final Answer:**

$$SP \cdot S'P = a^2\tan^2\theta + b^2\sec^2\theta$$

(Option **B**).

Question10

The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ is MHT CET 2020 (14 Oct Shift 1)

Options:

A. $\sqrt{\frac{19}{3}}$

B. $\sqrt{\frac{13}{19}}$

C. $\frac{\sqrt{19}}{3}$

D. $\frac{13}{\sqrt{19}}$

Answer: A

Solution:

Equation: $16x^2 - 3y^2 - 32x - 12y - 444 = 0$

$$16\left(x^2 - 2x\right) - 3\left(y^2 - 4y\right) = 44$$

$$16(x+1)^2 - 3(y-2)^2 = 48$$

$$\frac{(x+1)^2}{3} - \frac{(y-2)^2}{16} = 1$$

Centre $(1, -2)$ bength of ransuene axis $= 2a$

Length y con jugateaxis $= 2b = 2\sqrt{3}$

$$e = \sqrt{\frac{1+b^2}{a^2}} = \sqrt{1 + \left(\frac{4}{\sqrt{3}}\right)^2} = \sqrt{\frac{1+16}{3}}$$

$$e = \sqrt{\frac{19}{3}}$$

Question11

If $P(x_1, y_1)$ is a point on the hyperbola $x^2 - y^2 = a^2$, then $SP.S'P =$ ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $\frac{x_1^2-y_1^2}{a^2}$

B. $\frac{x_1^2+y_1^2}{a^2}$

C. $x_1^2-y_1^2$

D. $x_1^2+y_1^2$

Answer: D

Solution:

Given Hyperbola $x^2-y^2=a^2$ is
Rectangular hyperbola, its $e=\sqrt{2}$
Then $S\left(\sqrt{2}a,0\right)$ and S'$\left(-\sqrt{2}a,0\right)$
Also S'P $=\left(\sqrt{2}x_1+a\right)$ $and\ SP=\sqrt{2}x_1-a$
Then $(SP)\left(S'P\right)=\left(2x_1^2-a^2\right)\because x_1^2-y_1^2=a^2$
Hence, $(SP)\left(S'P\right)=x_1^2+y_1^2$

Question12

If the lengths of the transverse axis and the latus rectum of a hyperbola are 6 and $\frac{8}{3}$ respectively, then, the equation of the hyperbola is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $4x^2-9y^2=72$

B. $4x^2-9y^2=36$

C. $9x^2-4y^2=72$

D. $9x^2-4y^2=36$

Answer: B

Solution:

Given: $2a = 6$ and $\frac{2b^2}{a} = \frac{8}{3}$
Hence, $a = 3, b = 2$
Then, equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Question13

The eccentricity of hyperbola $25x^2 - 9y^2 = 225$ is... MHT CET 2019 (Shift 1)

Options:

A. $\frac{\sqrt{34}}{3}$

B. 4

C. $\sqrt{34}$

D. $\frac{\sqrt{34}}{5}$

Answer: A

Solution:

We have equation of hyperbola
$25x^2 - 9y^2 = 225$
$\Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$
On comparing with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get
$a^2 = 9, b^2 = 25$
$\therefore$ Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}}$
$= \sqrt{1 + \frac{25}{9}}$
$= \frac{\sqrt{34}}{3}$

Question14

The normal to the rectangular hyperbola $xy = c^2$ at the point ' ' ' meets the curve again at a point $'t''$, such that MHT CET 2011

Options:

A. $t^2t' = -1$

B. $t^3t' = -1$

C. $tt' = -1$

D. None of these

Answer: B

Solution:

Equation of normal at $(ct, c/ct)$ is $yt - xt^3 - c + ct^4 = 0$

If it passes through any point say Q having its coordinates as ' t ' i.e., $(\mathrm{ct}'\mathrm{c}/\mathrm{t}')$

The coordinates must satisfy the equation. Hence, we get $\frac{c}{t'} \cdot t - ct't^3 - c + ct^4 = 0$

$$\Rightarrow \quad t - t'^2t^3 - t' + t't^4 = 0$$

Factorizing, we get, $(t't^3 + 1)(t - t') = 0$

$$\text{Hence} t'{t^3} + 1 = 0 \text{ or } t't^3 = -1$$

as $t \neq t'$

Question15

If e_1 and e_2 represent the eccentricity of the curves $16x^2 - 9y^2 = 144$ and $9x^2 - 16y^2 = 144$respectively. Then, $\frac{1}{e_1^2} + \frac{1}{e_2^2}$ is equal to MHT CET 2010

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Given curves are $\frac{x^2}{9} - \frac{y^2}{16} = 1$ and $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\therefore$

$e_1 = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

and $e_2 = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Now, $\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{9}{25} + \frac{16}{25} = 1$

Question16

Focus of hyperbola is $(\pm 3, 0)$ and equation of tangent is $2x + y - 4 = 0$, find the equation of hyperbola. MHT CET 2009

Options:

A. $4x^2 - 5y^2 = 20$

B. $5x^2 - 4y^2 = 20$

C. $4x^2 - 5y^2 = 1$

D. $5x^2 - 4y^2 = 1$

Answer: A

Solution:

Given,
$$\begin{aligned} &(\pm ae, 0) = (\pm 3, 0) \\ &\Rightarrow \quad ae = 3 \\ &\quad \Rightarrow a^2e^2 \quad = 9 \\ &\quad \Rightarrow b^2 + a^2 \quad = 9 \\ &\quad \because 2x + y - 4 \quad = 0 \\ &\quad \Rightarrow y = \quad 2x + 4 \end{aligned}$$

is the tangent to the hyperbola. $\therefore$

$(4)^2 = a^2(-2)^2 - b^2$

$\Rightarrow \quad 4a^2 - b^2 = 16$

On solving Eqs. (i) and (ii), we get $a^2 = 5, b^2 = 4$

$\therefore$ Equation of hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1$

$\Rightarrow$

$4x^2 - 5y^2 = 20$
