# Ellipse

## Question1

**The eccentric angle of the point $\mathrm{P}(-6, 2)$ of the ellipse $\frac{x^2}{48} + \frac{y^2}{16} = 1$ is MHT CET 2025 (5 May Shift 2)**

**Options:**

A. $30^{\circ}$

B. $135^{\circ}$

C. $150^{\circ}$

D. $120^{\circ}$

**Answer: C**

## Solution:

### Step 1: Recall the definition

For an ellipse in standard form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

any point on it can be written parametrically as:

$$x = a\cos\theta, \quad y = b\sin\theta,$$

where $\theta$ is called the **eccentric angle.**

Here, $a^2 = 48 \implies a = \sqrt{48} = 4\sqrt{3}, \quad b^2 = 16 \implies b = 4.$

---

**Step 2: Plug in the coordinates**

Point is $P(-6, 2)$.
So,

$$-6 = a\cos\theta = 4\sqrt{3}\cos\theta,$$

$$2 = b\sin\theta = 4\sin\theta.$$

---

**Step 3: Solve for $\cos\theta$ and $\sin\theta$**

$$\cos\theta = \frac{-6}{4\sqrt{3}} = \frac{-6}{4\cdot 1.732...} = \frac{-6}{6.928...} \approx -0.866 \approx -\frac{\sqrt{3}}{2},$$

$$\sin\theta = \frac{2}{4} = 0.5.$$

So:

$$\cos\theta = -\frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2}.$$

---

**Step 4: Identify the quadrant**

- $\cos\theta < 0, \sin\theta > 0$ → **Quadrant II.**
- The reference angle for $\sin = 1/2$ and $\cos = \sqrt{3}/2$ is $30^\circ$.
- In Quadrant II, angle is:

$$\theta = 180^\circ - 30^\circ = 150^\circ.$$

✅ **Final Answer:**

$$\theta = 150^\circ$$

The correct eccentric angle is **150°** (Option C), not 30°.

---------------------------------------------------------------------------------------------------

# Question2

**The tangent to the ellipse $9x^2 + 16y^2 = 288$ making equal intercepts on the co-ordinate axes intersects the X -axis and the Y -axis in the**

**points $A$ and $B$ respectively. Then $A(\triangle OAB) =$ (where O is origin) MHT CET 2025 (26 Apr Shift 2)**

**Options:**

A. $\frac{25}{2}$ sq. units

B. 25 sq. units

C. $\frac{25\sqrt{5}}{2}$ sq. units

D. $25\sqrt{5}$ sq. units

**Answer: B**

## Solution:

Ellipse:

$$9x^2 + 16y^2 = 288 \quad \Rightarrow \quad \frac{x^2}{32} + \frac{y^2}{18} = 1$$

Line with equal intercepts: $x + y = k$.

Substitute $y = k - x$ → quadratic in $x$.
For tangency, discriminant = 0 → gives $k^2 = 50 \implies k = 5\sqrt{2}$.

Intercepts: $(k, 0)$, $(0, k)$.
Area of $\triangle OAB = \frac{1}{2}k^2 = \frac{1}{2} \cdot 50 = 25$.

✅ Final Answer: **25 sq. units.**

--------------------------------------------------------------------------------------------------

# Question3

**The eccentricity of the curve represented by $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ is MHT CET 2025 (25 Apr Shift 1)**

**Options:**

A. $\frac{\sqrt{7}}{4}$

B. $\frac{7}{16}$

C. $\frac{\sqrt{7}}{3}$

D. $\frac{\sqrt{8}}{4}$

**Answer: A**

## Solution:

Given:

$$x = 3(\cos t + \sin t),\ y = 4(\cos t - \sin t)$$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 2$
$\Rightarrow \frac{x^2}{18} + \frac{y^2}{32} = 1.$

So ellipse: $a^2 = 32,\ b^2 = 18.$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{18}{32}} = \frac{\sqrt{7}}{4}.$$

✅ Answer: $\frac{\sqrt{7}}{4}$.

----------------------------------------------------------------------------------------------------

# Question4

**The foci of the conic $25x^2 + 16y^2 - 150x = 175$ are MHT CET 2025 (23 Apr Shift 2)**

**Options:**

A. $(0, \pm 3)$

B. $(3, \pm 3)$

C. $(0, \pm 5)$

D. $(5, \pm 5)$

**Answer: B**

## Solution:

**Step 1: Arrange in ellipse form**

$$25x^2 - 150x + 16y^2 = 175.$$

Factor $25$:

$$25(x^2 - 6x) + 16y^2 = 175.$$

Complete square:

$$x^2 - 6x = (x-3)^2 - 9.$$

So:

$$25\left((x-3)^2 - 9\right) + 16y^2 = 175.$$

$$25(x-3)^2 - 225 + 16y^2 = 175.$$

$$25(x-3)^2 + 16y^2 = 400.$$

---

**Step 2: Put in standard ellipse form**

$$\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1.$$

This is ellipse centered at $(3, 0)$, with **major axis along y-axis** since $25 > 16$.

So:

$$a^2 = 25,\ b^2 = 16,\ a = 5,\ b = 4.$$

---

**Step 3: Find eccentricity and foci**

$$c^2 = a^2 - b^2 = 25 - 16 = 9 \quad \Rightarrow \quad c = 3.$$

Foci (along y-axis):

$$(3, \pm c) = (3, \pm 3).$$

---

✅ **Final Answer:**

The foci are $(3, \pm 3)$ (Option B).

---------------------------------------------------------------------------------------------------

# Question5

**The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. The equation of the hyperbola with eccentricity 2 is MHT CET 2025 (22 Apr Shift 2)**

**Options:**

A. $\frac{x^2}{12} - \frac{y^2}{4} = 1$

B. $\frac{x^2}{4} - \frac{y^2}{12} = 1$

C. $\frac{x^2}{12} - \frac{y^2}{16} = 1$

D. $\frac{x^2}{16} - \frac{y^2}{12} = 1$

**Answer: B**

## Solution:

### Step 1: Foci of the ellipse

For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

- $a^2 = 25 \implies a = 5$,
- $b^2 = 9 \implies b = 3$.

Foci: $c^2 = a^2 - b^2 = 25 - 9 = 16 \implies c = 4$.

So ellipse foci: $(\pm 4, 0)$.

---

**Step 2: Hyperbola with same foci**

Equation of hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
Here focus distance: $c = ae$.

Given eccentricity $e = 2$, $c = 4$:

$$a = \frac{c}{e} = \frac{4}{2} = 2.$$

So $a^2 = 4$.

---

**Step 3: Find $b^2$**

$$c^2 = a^2 + b^2 \quad \Rightarrow \quad 16 = 4 + b^2 \quad \Rightarrow \quad b^2 = 12.$$

---

**Step 4: Final equation**

$$\frac{x^2}{4} - \frac{y^2}{12} = 1.$$

---

✅ Final Answer:

The hyperbola's equation is

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

(Option **B**).

------------------------------------------------------------------------------------------------

# Question6

**The equations of two ellipses are be $\frac{x^2}{4} + \frac{y^2}{2} = 1$ and $\frac{x^2}{36} + \frac{y^2}{b^2} = 1$. If the product of their eccentricities is $\frac{\sqrt{2}}{3}$, then the product of the length of the major axis and minor axis of the second ellipse is**
**MHT CET 2025 (20 Apr Shift 2)**

**Options:**

A. $12\sqrt{5}$

B. $720$

C. $6\sqrt{20}$

D. $48\sqrt{5}$

**Answer: D**

## Solution:

Ellipse 1: $\frac{x^2}{4} + \frac{y^2}{2} = 1 \rightarrow a = 2,\ b = \sqrt{2},\ e_1 = \frac{1}{\sqrt{2}}.$

Ellipse 2: $\frac{x^2}{36} + \frac{y^2}{b^2} = 1 \rightarrow a = 6,\ e_2 = \sqrt{1 - \frac{b^2}{36}}.$

Given $e_1 e_2 = \frac{\sqrt{2}}{3}$:
$\frac{1}{\sqrt{2}} \cdot e_2 = \frac{\sqrt{2}}{3} \Rightarrow e_2 = \frac{2}{3}.$

So $1 - \frac{b^2}{36} = \frac{4}{9} \Rightarrow b^2 = 20,\ b = 2\sqrt{5}.$

Major axis $= 12$, minor axis $= 4\sqrt{5}.$
Product $= 48\sqrt{5}.$

✅ Answer: $48\sqrt{5}.$

---------------------------------------------------------------------------------------------------------

# Question7

## The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is MHT CET 2025 (19 Apr Shift 2)

**Options:**

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{7}$

D. $\frac{4}{9}$

**Answer: B**

## Solution:

**Step 1: Rearrange**

$$9x^2 + 5(y^2 - 6y) = 0.$$

Complete the square:

$$y^2 - 6y = (y - 3)^2 - 9.$$

So:

$$9x^2 + 5\big((y - 3)^2 - 9\big) = 0$$

$$9x^2 + 5(y - 3)^2 - 45 = 0$$

$$9x^2 + 5(y - 3)^2 = 45.$$

---

**Step 2: Standard ellipse form**

$$\frac{x^2}{5} + \frac{(y - 3)^2}{9} = 1.$$

So $a^2 = 9,\ b^2 = 5.$

Major axis along $y$-axis.

---

**Step 3: Eccentricity**

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{5}{9}} = \sqrt{\tfrac{4}{9}} = \tfrac{2}{3}.$$

---

✅ **Final Answer:**

Eccentricity = $\frac{2}{3}$ (Option B).

-------------------------------------------------------------------------------------------------------

# Question8

## An ellipse has $OB$ as semi-minor axis, $S$ and $S'$ are foci and angle $SBS'$ is a right angle. Then the eccentricity of the ellipse is MHT CET 2025 (19 Apr Shift 1)

**Options:**

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $\frac{1}{3}$

**Answer: B**

## Solution:

### Step 1: Recall ellipse properties

Ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b.$$

Foci: $(\pm c, 0)$, with $c^2 = a^2 - b^2$.
Eccentricity: $e = \frac{c}{a}$.

Here, $OB = b$ (semi-minor axis).

---

### Step 2: Condition from geometry

At $B(0, b)$, angle $\angle SBS' = 90^\circ$.
That means triangle $SBS'$ is right-angled at $B$.

By Pythagoras:

$$SB^2 + S'B^2 = SS'^2.$$

But ellipse symmetry → $SB = S'B$. So:

$$2SB^2 = (2c)^2 \quad \Rightarrow \quad SB^2 = 2c^2.$$

---

**Step 3: Express $SB$**

Coordinates: $S(c, 0)$, $B(0, b)$.

$$SB^2 = c^2 + b^2.$$

So:

$$c^2 + b^2 = 2c^2 \quad \Rightarrow \quad b^2 = c^2.$$

---

**Step 4: Relation**

From ellipse: $c^2 = a^2 - b^2$.
But we also found $c^2 = b^2$.

So:

$$a^2 = b^2 + c^2 = 2b^2.$$

**Step 5: Eccentricity**

$$e = \frac{c}{a} = \frac{b}{\sqrt{2}b} = \frac{1}{\sqrt{2}}.$$

---

✅ **Final Answer:**

Eccentricity = $\frac{1}{\sqrt{2}}$ (Option B).

---

# Question9

**If $e_1$ is the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $e_2$ is the**

**eccentricityof the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $e_1^2 + e_2^2 =$ MHT CET 2020 (20 Oct Shift 2)**

**Options:**

A. 2

B. 4

C. 1

D. 3

**Answer: A**

## Solution:

$e_1 = \sqrt{1 - \frac{b^2}{a^2}} \quad e_2 = \sqrt{1 + \frac{b^2}{a^2}}$

$e_1^2 + e_2^2 = 2$

----------------------------------------------------------------------------------------------------

# Question10

**If $\mathrm{B}$ is end point of minor axis of the ellipse $b^2x^2 + a^2y^2 = a^2b^2 (a > b)$ and $\mathrm{S}$ andS' are foci of ellipse such that $\Delta \mathrm{SBS}'$ is an equilateral triangle, then eccentricity $\mathrm{e} =$ MHT CET 2020 (19 Oct Shift 1)**

**Options:**

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

**Answer: A**

## Solution:

Correct option is A

Given $\mathrm{S}(-\mathrm{ae}, 0), \mathrm{T}(\mathrm{ae}, 0), \mathrm{B}(0,\ \mathrm{b})$

As STB is an equilateral triangle

In $\triangle \mathrm{TOB}$

$\tan 60^{\circ} = \frac{\mathrm{OB}}{\mathrm{OS}}$

$\sqrt{3} = \frac{\mathrm{OB}}{\mathrm{OS}} = \frac{\mathrm{b}}{\mathrm{ae}}$

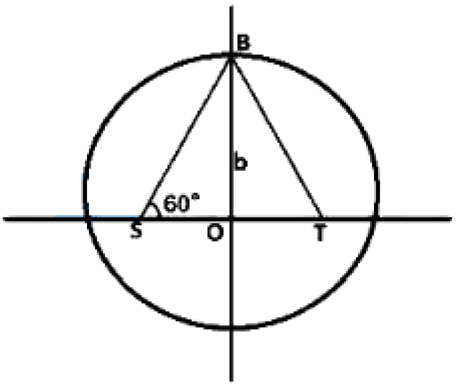
$(\sqrt{3}ae)^2 = b^2$

$3a^2\left(1 - \frac{b^2}{a^2}\right) = b^2$

$3\left(1 - \frac{b^2}{a^2}\right) = \frac{b^2}{a^2}$

$\frac{3}{4} = \frac{b^2}{a^2}$

$e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$



---

# Question11

## The eccentricity of the ellipse $y^2 + 4x^2 - 12x + 6y + 14 = 0$ is MHT CET 2020 (16 Oct Shift 2)

**Options:**

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

**Answer: A**

## Solution:

The eccentricity of the ellipse $y^2 + 4x^2 - 12x + 6y + 14 = 0$ is $\frac{\sqrt{3}}{2}$.

Given ellipse,

$$y^2 + 4x^2 - 12x + 6y + 14 = 0$$
$$4x^2 - 12x + y^2 + 6y + 14 = 0$$
$$4\left(x^2 - 3x + \frac{9}{4}\right) + y^2 + 6y + 9 = -14 + 9 + 9$$
$$4\left(x - \frac{3}{2}\right)^2 + (y+3)^2 = 4$$
$$\frac{\left(x - \frac{3}{2}\right)^2}{1} + \frac{(y+3)^2}{4} = 1$$

Here, $a^2 = 1, b^2 = 4$

$$\therefore e = \sqrt{1 - \frac{a^2}{b^2}}$$
$$e = \sqrt{1 - \frac{1}{4}}$$

---

# Question12

**If CP and CD is a pair of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $CP^2 + CD^2 =$ MHT CET 2020 (15 Oct Shift 2)**

**Options:**

A. $\frac{a^2+b^2}{2}$

B. $a^2 + b^2$

C. $a^2 - b^2$

D. $\frac{a^2-b^2}{2}$

**Answer: B**

## Solution:

The coordinates of $P$ and $D$ are $(a\cos\theta, b\sin\theta)$ and $\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$ respectively.

$\therefore CP^2 + CD^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$\Rightarrow \quad CP^2 + CD^2 = a^2 + b^2$

---

# Question13

**If foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1\ (b^2 < 16)$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide,then the value of $b^2$ is MHT CET 2020 (15 Oct Shift 1)**

**Options:**

A. 4

B. 9

C. 14

D. 7

**Answer: D**

## Solution:

Given hyperbola is, $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$$\Rightarrow \frac{x^2}{144/25} - \frac{y^2}{81/25} = 1$$

$$\Rightarrow a^2 = \frac{144}{25},\ b^2 = \frac{81}{25}, e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$$

$\therefore$ foci of hyperbola are $(\pm ae, 0)$ i.e $(\pm 3, 0)$Now given ellipse is $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$$\Rightarrow a^2 = 16$$

Assume eccentricity of this ellipse is e then its foci are $(\pm ae', 0)$ i. e $(\pm 4e', 0)$ Given foci of given hyperbola and ellipse coincide

$$\Rightarrow 4e' = 3 \Rightarrow e' = \frac{3}{4}$$

For ellipse, using eccentricity relationship, $e'^2 = 1 - \frac{b^2}{a^2}$

$$\Rightarrow \frac{9}{16} = 1 - \frac{b^2}{16}$$

$$\therefore b^2 = 7$$

---

# Question14

**The co-ordinates of foci of the ellipse $16x^2 + 9y^2 = 144$ are MHT CET 2020 (13 Oct Shift 1)**

**Options:**

A. $(\pm 7, 0)$

B. $(0, \pm\sqrt{7})$

C. $(\pm\sqrt{7}, 0)$

D. $(0, \pm 7)$

**Answer: B**

## Solution:

Center is $(0,0)$, focii are $(0, \sqrt{7})$ and $(0, -\sqrt{7})$, major axis is along $y$-axis and is 8 , and minor axis is along $x$-axis and is 6 .

Explanation:

Note that equation $16x^2 + 9y^2 = 144$ by dividing each term by 144 can be written as $\frac{x^2}{9} + \frac{y^2}{16} = 1$

As the standard equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, center is $(0,0)$.

In this equation major axis is along $y$-axis and is $2b = 8$, and minor axis is along $x$ axis and is $2a = 6$.

As $a = 3$ and $b = 4$, focii are given by $\pm\sqrt{4^2 - 3^2} = \pm\sqrt{7}$ i.e. $(0, \sqrt{7})$ and $(0, -\sqrt{7})$

-----------------------------------------------------------------------------------------------

# Question15

**The eccentricity of the ellipse given by the equation $9x^2+16y^2=144$ is MHT CET 2020 (12 Oct Shift 2)**

**Options:**

A. $\frac{\sqrt{7}}{4}$

B. $\frac{1}{4}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{5}}{4}$

**Answer: A**

## Solution:

$$9x^2+16y^2=144$$

Given equation of an ellipse is $\Rightarrow \frac{9x^2}{144}+\frac{16y^2}{144}=\frac{144}{144}$

$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

Here, $a^2=16, b^2=9 \quad [\because a^2>b^2]$

$\therefore$ Eccentricity $(e)=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\sqrt{\frac{7}{16}}=\frac{\sqrt{7}}{4}$.

-----------------------------------------------------------------------------------------------

# Question16

**The length of the latus rectum of an ellipse is $\frac{18}{5}$ and eccentricity is $\frac{4}{5}$, then equation of the ellipse is... MHT CET 2019 (Shift 1)**

**Options:**

A. $\frac{x^2}{25}+\frac{y^2}{8}=1$

B. $\frac{x^2}{25}+\frac{y^2}{9}=1$

C. $\frac{x^2}{25}+\frac{y^2}{16}=1$

D. $\frac{x^2}{16}+\frac{y^2}{9}=1$

**Answer: B**

## Solution:

Given, $\frac{2b^2}{a}=\frac{18}{5} \Rightarrow \frac{b^2}{a}=\frac{9}{5}$

$\Rightarrow b^2=\frac{9}{5}a \quad \ldots\text{(i)}$

and, $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{4}{5} \Rightarrow 1-\frac{b^2}{a^2}=\frac{16}{25}$

$\Rightarrow \frac{b^2}{a^2}=1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow \frac{\left(\frac{9}{5}\right)a}{a^2}=\frac{9}{25} \Rightarrow \frac{9}{5a}=\frac{9}{25}$

$\Rightarrow a=5$

$\therefore b^2=\frac{9}{5}(5)=9$ [using Eq. (i)]

$\therefore$ Required equation of ellipse,

$\frac{x^2}{25}+\frac{y^2}{9}=1$

---

# Question17

**From point $P(8,27)$ tangents $PQ$ and $PR$ are drawn to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$. Then, angle subtended by $QR$ at origin is MHT CET 2011**

**Options:**

A. $\tan^{-1} \frac{2\sqrt{6}}{65}$

B. $\tan^{-1} \frac{4\sqrt{6}}{65}$

C. $\tan^{-1} \frac{8\sqrt{2}}{65}$

D. None of these

**Answer: D**

## Solution:

Equation of chord of contact $QR$ is

$$\begin{aligned} & 8 \cdot \frac{x}{4}+27 \cdot \frac{y}{9}=1 \\ \Rightarrow \quad & 2x+3y=1 \end{aligned}$$

Now, equation of the pair of lines passing through origin and points $Q, R$ given by

$$\begin{aligned} & \left(\frac{x^2}{4}+\frac{y^2}{9}\right)=(2x+3y)^2 \\ \Rightarrow \quad & 9x^2+4y^2=36\left(4x^2+12xy+9y^2\right) \\ \Rightarrow \quad & 135x^2+432xy+320y^2=0 \end{aligned}$$

∴ Required angle is

$$= \tan^{-1} \frac{2\sqrt{(216)^2 - 135 \cdot 320}}{455}$$

$$= \tan^{-1} \frac{8\sqrt{2916 - 2700}}{455}$$

$$= \tan^{-1} \frac{8\sqrt{216}}{455}$$

$$= \tan^{-1} \frac{48\sqrt{6}}{455}$$

---

# Question18

**The sum of focal radii of the curve $90x^2 + 25y^2 = 225$ is MHT CET 2010**

**Options:**

A. 5

B. 10

C. 6

D. 3

**Answer: B**

## Solution:

Given curve is $9x^2 + 25y^2 = 225 \Rightarrow$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Here, $a=5, b=3 \therefore \quad$ Sum of focal radii $=2a=10$

---

# Question19

**The equation of tangent to the curve $9x^2+16y^2=144$ which makes equal intercepts with coordinate axes, is MHT CET 2010**

**Options:**

A. $x+y=5$

B. $x+y=16$

C. $x+y=15$

D. None of these

**Answer: A**

**Solution:**

Given curve is $9x^2+16y^2=144$

$$\Rightarrow \quad \frac{x^2}{16}+\frac{y^2}{9}=1$$

Let the equation of tangent is $x+y=k \Rightarrow \quad y=-x+k$It is a tangent to the given curve if

$$c^2 = a^2m^2 + b^2$$
$$\Rightarrow \quad k^2 = 16(-1)^2 + 9$$

$$\Rightarrow \quad k^2 = 25$$

$$\Rightarrow \quad k = \pm 5$$

$\therefore$ Required equations of tangent are

$$x + y = \pm 5$$

---

# Question20

**If $4x - 3y + k = 0$ touches the ellipse $5x^2 + 9y^2 = 45$, then $k$ is equal to MHT CET 2009**

**Options:**

A. $\pm 3\sqrt{21}$

B. $3\sqrt{21}$

C. $-3\sqrt{21}$

D. $2\sqrt{21}$

**Answer: A**

## Solution:

If line $4x - 3y + k = 0$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, then

$\frac{k}{3} = \sqrt{9 \times \left(\frac{4}{3}\right)^2 + 5} = \pm\sqrt{21}$

$\Rightarrow$

$k = \pm 3\sqrt{21}$

---

# Question21

**Tangent to the ellipse $\frac{x^2}{32}+\frac{y^2}{18}=1$ having slope $-\frac{3}{4}$ meet the coordinate axes in $A$ and $B$. Find the area of the $\Delta AOB$, where $O$ is the origin. MHT CET 2009**

**Options:**

A. 12 sq unit

B. 8sq unit

C. 24 sq unit

D. 32 sq unit

**Answer: C**

**Solution:**

Equation of tangent with slope $-\frac{3}{4}$ is

$$y=-\frac{3}{4}x+c$$

According to condition of tengency.∴

$$\begin{aligned} c &= \sqrt{32\times\left(\frac{-3}{4}\right)^2+18} \\ &= \sqrt{18+18} \\ &= 6 \end{aligned}$$

$$y=-\frac{3}{4}x+6$$

$$\Rightarrow 4y+3x=24$$

It meets the coordinate axes in $A$ and $B$. $\therefore A\equiv(8,0)$ and $B\equiv(0,6)$Required area $=\frac{1}{2}\times8\times6=24$ sq unit

---------------------------------------------------------------------------------------------------

## Question22

**If the line $x\cos\alpha+y\sin\alpha=p$ be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then MHT CET 2008**

**Options:**

A. $p^2\left(a^2\cos^2\alpha+b^2\sin^2\alpha\right)=a^2-b^2$

B. $p^2\left(a^2\cos^2\alpha+b^2\sin^2\alpha\right)=\left(a^2-b^2\right)^2$

C. $p^2\left(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha\right)=a^2-b^2$

D. $p^2\left(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha\right)=\left(a^2-b^2\right)^2$

**Answer: D**

### Solution:

The equation of any normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $ax\sec\phi-by\operatorname{cosec}\phi=a^2-b^2 \quad \ldots$ (i)

The straight line $x\cos\alpha+y\sin\alpha=p$ will be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if Eq. (i) and $x\cos\alpha+y\sin\alpha=p$ represent the same line.

$$\therefore \quad \frac{a\sec\phi}{\cos\alpha}=\frac{-b\operatorname{cosec}\phi}{\sin\alpha}=\frac{a^2-b^2}{p}$$

$$\Rightarrow \quad \cos\phi=\frac{ap}{(a^2-b^2)\cos\alpha},$$

$$\sin\phi=\frac{-bp}{(a^2-b^2)\sin\alpha}$$

$$\because \quad \sin^2\phi+\cos^2\phi=1$$

$$\Rightarrow \frac{b^2p^2}{(a^2-b^2)^2\sin^2\alpha}+\frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha}=1$$

$$\Rightarrow p^2\left(b^2\operatorname{cosec}^2\alpha+a^2\sec^2\alpha\right)=\left(a^2-b^2\right)^2$$

---

# Question23

**The parametric representation of a point on the ellipse whose foci are $(-1,0)$ and $(7,0)$ and eccentricity $1/2$, is MHT CET 2007**

**Options:**

A. $(3+8\cos\theta, 4\sqrt{3}\sin\theta)$

B. $(8\cos\theta, 4\sqrt{3}\sin\theta)$

C. $(3+4\sqrt{3}\cos\theta, 8\sin\theta)$

D. None of the above

**Answer: A**

**Solution:**

Distance between two foci, $2ae=7+1=8 \therefore$

$$ae=4$$

$$\Rightarrow$$

$a=8$

$$\left(\because e=\frac{1}{2}\text{ given}\right)$$

Now,

$$b^2=a^2\left(1-e^2\right)=64\left(1-\tfrac{1}{4}\right)$$
$$b^2=48\Rightarrow b=4\sqrt{3}$$

$\therefore$Since, the centre of the ellipse is the mid point of the line joining two foci, therefore the coordinates of the centre are $(3,0)$. $\therefore$ Its equation is

$$\frac{(x-3)^2}{8^2}+\frac{(y-0)^2}{(4\sqrt{3})^2}=1$$

Hence, the parametric coordinates of a point on Eq. (i) are $(3+8\cos\theta, 4\sqrt{3}\sin\theta)$.

---

# Question24

**A common tangent to $9x^2-16y^2=144$ and $x^2+y^2=9$, is MHT CET 2007**

**Options:**

A. $y=\frac{3}{\sqrt{7}}x+\frac{15}{\sqrt{7}}$

B. $y=3\sqrt{\frac{2}{7}}x+\frac{15}{\sqrt{7}}$

C. $y=2\sqrt{\frac{3}{7}}x+15\sqrt{7}$

D. None of the above

**Answer: B**

## Solution:

Let $y=mx+c$ be a common tangent to $9x^2-16y^2=144$ and $x^2+y^2=9$.

Since, $y=mx+c$ is a tangent to $9x^2-16y^2=144$

$\therefore c^2=a^2m^2-b^2 \Rightarrow c^2=16m^2-9 \quad \ldots$ (i)

Now, $y=mx+c$ is a tangent to $x^2+y^2=9 \therefore \frac{c}{\sqrt{m^2+1}}=3 \Rightarrow c^2=9\left(1+m^2\right)$

From Eqs. (i) and (ii), we get $16m^2-9=9+9m^2$

$\Rightarrow \quad m=\pm 3\sqrt{\frac{2}{7}}$

On putting the value of $m$ in $\mathrm{Eq}$ (ii), we get $c=\pm\frac{15}{\sqrt{7}}$

Hence, $y=3\sqrt{\frac{2}{7}}x+\frac{15}{\sqrt{7}}$ is a common tangent.

---

# Question25

**The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of $b^2$ is MHT CET 2007**

**Options:**

A. 1

B. 5

C. 7

D. 9

**Answer: C**

**Solution:**

Given equation of ellipse is

$$\frac{x^2}{16}+\frac{y^2}{b^2}=1$$

Here, $a^2=16 \Rightarrow a=4$

$$e=\sqrt{1-\frac{b^2}{16}}=\frac{\sqrt{16-b^2}}{4}$$

$\therefore$ Foci of ellipse are $(\pm ae, 0) ie,\left(\pm\sqrt{16-b^2}, 0\right)$. Also, given equation of hyperbola is $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ Here,
$a^2=\left(\frac{12}{5}\right)^2, b^2=\left(\frac{9}{5}\right)^2 \therefore$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{81}{144}}=\frac{5}{4}$$

$\therefore$ Foci of the hyperbola are $(\pm ae, 0) ie,(\pm 3,0)$. According to the given condition, foci of ellipse $=$ foci of hyperbola
$\therefore \sqrt{16-b^2}=3 \Rightarrow \quad b^2=7$

-----------------------------------------------------------------------------------------------