Parabola

Question1

The length of latus rectum of the parabola whose focus is $(3, 3)$ and directrix is $3x - 4y - 2 = 0$ is ______ units. MHT CET 2025 (27 Apr Shift 2)

Options:

A. 8

B. 6

C. $\frac{1}{2}$

D. 2

Answer: D

Solution:

Step 1: Distance from focus to directrix

Formula for distance from point (x_1, y_1) to line $ax + by + c = 0$:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Here:

- Focus: $(3, 3)$
- Line: $3x - 4y - 2 = 0 \implies a = 3, b = -4, c = -2$

$$d = \frac{|3(3) - 4(3) - 2|}{\sqrt{3^2 + (-4)^2}} = \frac{|9 - 12 - 2|}{\sqrt{9 + 16}} = \frac{|-5|}{\sqrt{25}} = \frac{5}{5} = 1$$

So, the **distance between focus and directrix = 1**.

Step 2: Relation to focal length a

For a parabola, the **vertex** lies midway between the focus and the directrix.

Thus, the **focal length** a = half of this distance:

$$a = \frac{1}{2}$$

Step 3: Length of latus rectum

By standard parabola property:

$$\text{Length of latus rectum} = 4a$$

$$= 4 \times \frac{1}{2} = 2$$

✅ Correct Answer = 2 (Option D)

Question2

The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $x = -1$

B. $x = 1$

C. $x = \frac{-3}{2}$

D. $x = \frac{3}{2}$

Answer: D

Solution:

Step 1: Complete the square in y

$$y^2 + 4y = (y+2)^2 - 4$$

So the equation becomes:

$$(y+2)^2 - 4 + 4x + 2 = 0$$

$$(y+2)^2 + 4x - 2 = 0$$

$$(y+2)^2 = -4(x - \tfrac{1}{2})$$

Step 2: Identify parabola form

This matches:

$$(y-k)^2 = 4a(x-h)$$

Here:

- Vertex $(h, k) = \left(\frac{1}{2}, -2\right)$
- $4a = -4 \implies a = -1$

So it's a **left-opening parabola**.

Step 3: Directrix

For parabola $(y-k)^2 = 4a(x-h)$:

- Focus: $(h+a, k)$
- Directrix: $x = h - a$

Here:

$$x = \tfrac{1}{2} - (-1) = \tfrac{1}{2} + 1 = \tfrac{3}{2}$$

✅ Correct Answer: $x = \frac{3}{2}$ (Option D)

Question3

The angle, between the tangents drawn from the point $(1, 4)$ to the parabola $y^2 = 4x$, is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Step 1: Equation of tangents from an external point

General tangent to parabola $y^2 = 4ax$ is

$$y = mx + \frac{a}{m}, \quad (m \neq 0)$$

Here, $a = 1$. So:

$$y = mx + \frac{1}{m}$$

Step 2: Condition for tangency from external point (x_1, y_1)

Substitute $(1, 4)$:

$$4 = m(1) + \frac{1}{m}$$

$$4m = m^2 + 1$$

$$m^2 - 4m + 1 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

So the slopes are:

$$m_1 = 2 + \sqrt{3}, \quad m_2 = 2 - \sqrt{3}$$

Step 3: Angle between tangents

Formula:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$m_1 - m_2 = (2 + \sqrt{3}) - (2 - \sqrt{3}) = 2\sqrt{3}$$

$$m_1 m_2 = (2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$$

So:

$$\tan\theta = \frac{2\sqrt{3}}{1+1} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Correct Answer: π/3 (Option C)

Question4

The line y $= mx + 3$ is tangent to the parabola y$^2 = 4x$, if the value of m is MHT CET 2025 (22 Apr Shift 1)

Options:

A. 3

B. $\frac{1}{3}$

C. 4

D. $\frac{1}{4}$

Answer: B

Solution:

Step 1: Substitute line into parabola

$$(mx+3)^2 = 4x$$

$$m^2x^2 + 6mx + 9 = 4x$$

$$m^2x^2 + (6m-4)x + 9 = 0$$

Step 2: Condition for tangency

For tangency, the quadratic in x has **discriminant = 0.**

$$\Delta = (6m-4)^2 - 4(m^2)(9) = 0$$

Step 3: Simplify discriminant

$$(6m-4)^2 - 36m^2 = 0$$

$$36m^2 - 48m + 16 - 36m^2 = 0$$

$$-48m + 16 = 0$$

$$48m = 16 \quad \Rightarrow \quad m = \frac{1}{3}$$

✅ **Final Answer:**

$$m = \frac{1}{3}$$

Correct option: **B** ✔

Question5

Angle between the parabola $y^2 = 4(x-1)$ and $x^2 + 4(y-3) = 0$ at the common end of their latus rectum is MHT CET 2025 (21 Apr Shift 2)

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Answer: $\boxed{\frac{\pi}{2}}$

Quick proof:

Parabola 1: $y^2 = 4(x-1) \Rightarrow$ vertex $(1,0)$, $a = 1$.
Differentiate: $2y\,y' = 4 \Rightarrow y' = \dfrac{2}{y}$.
At $(2,2)$: $m_1 = 1$.

Parabola 2: $x^2 + 4(y-3) = 0 \iff x^2 = -4(y-3) \Rightarrow$ vertex $(0,3)$, $a = -1$.
Differentiate: $2x + 4y' = 0 \Rightarrow y' = -\dfrac{x}{2}$.
At $(2,2)$: $m_2 = -1$.

Their latus recta share the endpoint $(2,2)$ (ends are $(2,\pm 2)$ for the first, $(\pm 2, 2)$ for the second).

Angle between tangents:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1-(-1)}{1+(-1)}\right| = \infty \Rightarrow \theta = \frac{\pi}{2}.$$

Question6

The Y-intercept of the common tangent to the parabola $y^2 = 32x$ and $x^2 = 108y$ is MHT CET 2025 (21 Apr Shift 1)

Options:

A. 3

B. -12

C. -3

D. 2

Answer: B

Solution:

For $y^2 = 32x \Rightarrow a = 8$: tangent $y = mx + \frac{8}{m}$.
For $x^2 = 108y \Rightarrow a = 27$: tangent $y = mx - 27m^2$.

Common tangent ⇒ $\frac{8}{m} = -27m^2 \Rightarrow m^3 = -\frac{8}{27} \Rightarrow m = -\frac{2}{3}$.

Intercept $c = \frac{8}{m} = -12$.

Answer: -12.

Question7

The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 20y$ to the end of its Latus rectum is MHT CET 2025 (20 Apr Shift 1)

Options:

A. 100 sq.units

B. 20 sq.units

C. 40 sq.units

D. 50 sq.units

Answer: D

Solution:

- The vertex of the parabola $x^2 = 20y$ is at $(0, 0)$.
- Standard form: $x^2 = 4ay$, so $4a = 20 \implies a = 5$.
- The ends of the latus rectum are at $(5, 5)$ and $(-5, 5)$.
- The base of the triangle is the distance between the ends: $|5 - (-5)| = 10$ units.
- The height (from vertex to latus rectum): $y = 5$.
- Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 10 \times 5 = 25$.
- But for the parabola $x^2 = 20y$, the correct coordinates for the ends are $(10, 5)$ and $(-10, 5)$: base 20, height 5, so
- Area = $\frac{1}{2} \times 20 \times 5 = 50$ sq. units.

Thus, the correct answer is **D: 50 sq. units.**

Question8

The equation of the tangent to the parabola $y^2 = 8x$, which is parallel to the line $4x - y + 3 = 0$ is MHT CET 2024 (15 May Shift 1)

Options:

A. $2x - 8y + 1 = 0$

B. $8x - 2y + 1 = 0$

C. $8x + 2y + 1 = 0$

D. $2x - 8y - 1 = 0$

Answer: B

Solution:

$y^2 = 8x$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 8$$
$$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$$

Slope of the line $4x - y + 3 = 0$ is 4 .

Since the tangent is parallel to $4x - y + 3 = 0$, their slopes are equal

$$\therefore \quad \frac{4}{y} = 4$$
$$\Rightarrow y = 1$$

When $y = 1, x = \frac{1}{8}$

$\therefore$ Equation of the tangent at $\left(\frac{1}{8}, 1\right)$ is

$$y - 1 = 4\left(x - \frac{1}{8}\right)$$
$$\Rightarrow 2y - 2 = 8x - 1$$
$$\Rightarrow 8x - 2y + 1 = 0$$

Question9

The equation of the directrix of the parabola $3x^2 = 16y$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $3y + 4 = 0$

B. $3x + 4 = 0$

C. $3y - 4 = 0$

D. $3x - 4 = 0$

Answer: A

Solution:

$3x^2 = 16y$

$x^2 = \frac{16}{3} \quad ; a = \frac{4}{3}$

Digectin $\rightarrow$

$y + \frac{4}{3} = 0$

$3y + 4 = 0$

Question10

The length of latus -rectum of the parabola $x^2 + 2y = 8x - 7$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. 8

B. 2

C. 6

D. 4

Answer: B

Solution:

We have,

$$x^2 + 2y = 8x - 7$$
$$x^2 - 8x = -2y - 7$$
$$x^2 - 8x + 16 = -2y - 7 + 16$$
$$(x-4)^2 = -2y + 9$$
$$(x-4)^2 = -2(y - 9/2)$$

$\therefore$ Length of latus rectum $= 2$

Question11

The cartesian co-ordinates of the point on the parabola $y^2 = x$ whose parameter is $\frac{-4}{3}$ are MHT CET 2020 (16 Oct Shift 1)

Options:

A. $\left(\frac{4}{9}, \frac{4}{3}\right)$

B. $\left(\frac{4}{3}, \frac{-4}{3}\right)$

C. $\left(\frac{4}{3}, \frac{4}{9}\right)$

D. $\left(\frac{4}{9}, \frac{-2}{3}\right)$

Answer: D

Solution:

$y^2 = x \quad t = -4/3$

$a = \frac{1}{4}$

$(at^2, 2at)$

$= \left(\frac{1}{4} \cdot \frac{16}{9} 4, \frac{-2}{3}\right) = \left(\frac{4}{9}, \frac{-2}{3}\right)$

Question12

The focal distance of the point $(4, 4)$ on the parabola with vertex at $((0, 0)$ andsymmetric about y-axis is MHT CET 2020 (13 Oct Shift 2)

Options:

A. 4

B. 5

C. $5\sqrt{2}$

D. $4\sqrt{2}$

Answer: B

Solution:

The equation of the parabola is $x^2 = 4ay$, with vertex at (0,0) and symmetric about the y-axis. For point (4,4) on this parabola, the focus is at (0, a). Here, $4^2 = 4a \times 4 \implies a = 1$, so the focus is at (0,1). The focal distance is the distance from (4,4) to (0,1):

$$\sqrt{(4-0)^2 + (4-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

So, the answer is 5 (option B).

Question13

The length of latus rectum of the parabola whose focus is at $(1, -2)$ and directrix isthe line $x + y + 3 = 0$ is MHT CET 2020 (13 Oct

Shift 1)

Options:

A. $8\sqrt{2}$ units

B. $2\sqrt{2}$ units

C. $\sqrt{2}$ units

D. $4\sqrt{2}$ units

Answer: B

Solution:

$\hat{f}^{(1,-2)}$

$x+y+3=0$

Distance of focus from

Dizectrix $=22$

1^2 distance

$=\frac{|1-2+3|}{\sqrt{2}}$

$=\sqrt{2}.$

Length of L.R

$=2\sqrt{2}$

Question14

The cartesian co-ordinates of the point on the parabola $y^2=-16x$, whose parameter is $\frac{1}{2}$ are MHT CET 2018

Options:

A. $(-2,4)$

B. $(4,-1)$

C. $(-1,-4)$

D. $(-1,4)$

Answer: D

Solution:

Given $t=\frac{1}{2}; y^2=-16x$
Comparing with standard parabola,
$y^2=-4ax$
We get $a=4$
$P(t)=\left(-at^2,2at\right)=\left(-4\times\frac{1}{4},2\left(\frac{1}{2}4\right)\right)$
$P(t)=(-1,4)$

Question15

The pole of the line $Lx+my+n=0$ w.r.t. the parabola $y^2=4ax$ will be MHT CET 2012

Options:

A. $\left(\frac{-n}{l},\frac{-2am}{l}\right)$

B. $\left(\frac{-n}{l},\frac{2am}{l}\right)$

C. $\left(\frac{n}{l},\frac{-2am}{l}\right)$

D. $\left(\frac{n}{l},\frac{2am}{l}\right)$

Answer: C

Solution:

Tangent of the parabola $y^2=4ax$ at point $P(x_1,y_1)$ is

$$yy_1 = 2a(x + x_1)$$

$$\Rightarrow \quad 2ax_1 - yy_1 + 2\alpha x = 0 \ldots \text{(i)}$$

which is also equation of the polar of the parabola $y^2 = 4ax$.
Same as the line

$$bx + my + n = 0...(ii)$$

On comparing both lines, we get

$$\frac{2a}{l} = \frac{-y}{m} = \frac{2ax}{n}$$

Taking first two parts

$$\left(y = -\frac{2am}{l}\right)$$

Taking first and last parts,

$$\left(x = \frac{n}{l}\right)$$

$\therefore$ Required pole of the line is $\left\{\frac{n}{l}, \frac{-2am}{l}\right\}$.

Question16

If $2x + y + \lambda = 0$ is normal to the parabola $y^2 = 8x$, then λ is MHT CET 2012

Options:

A. -24

B. 8

C. -16

D. 24

Answer: A

Solution:

Given, parabola, $y^2 = 8x$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 8$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{y}$$

Slope of normal of parabola $= -\frac{y}{4} \ldots (\mathrm{i})$

Given, $2x + y + \lambda = 0$ is a equation of normal of the parabola $y^2 = 8x$.

$\therefore$ Slope of normal $= -2 \ldots$ (ii)

From Eqs. (i) and (ii), we get

$\therefore$

$-\frac{y}{4} = -2 \Rightarrow y = 8$

$(8)^2 = 8x \Rightarrow x = 8$

Now, putting the values of x and y in the equation of normal

$$2(8) + 8 + \lambda = 0$$
$$\Rightarrow \lambda = -24$$

Question17

If the line $bx + my + n = 0$ is tangent to the parabola $y^2 = 4ax$, then MHT CET 2012

Options:

A. $mn = al^2$

B. $\mathrm{lm} = \mathrm{an}^2$

C. $\ln = am^2$

D. None of the above

Answer: C

Solution:

Given, parabola, $y^2 = 4ax$

$\Rightarrow \quad 2\text{y} \frac{dy}{dx} = 4a$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$

which is the slope of tangent.

Given, $lx + my + n = 0$

is an equation of tangent of the parabola $y^2 = 4ax$

$\therefore$ Slope of tangent $= -\frac{l}{m}$

From Eqs. (i) and (ii)

$$\frac{2a}{y} = -\frac{l}{m} \Rightarrow y = \frac{-2am}{l}$$

$$\Rightarrow \quad y^2 = 4ax$$

$$\Rightarrow \quad \frac{4a^2m^2}{l^2} = 4ax$$

$$\Rightarrow x = \frac{am^2}{l^2}$$

On putting the values of x and y in the following equation

$$bx + my + n = 0$$
$$l\left(\frac{am^2}{l^2}\right) + m\left(\frac{-2am}{l}\right) + n = 0$$
$$\frac{am^2}{l} - \frac{2am^2}{l} + n = 0$$
$$\Rightarrow \quad \frac{am^2}{l} = n \Rightarrow am^2 = nl$$

which is the required relation.

Question18

The intercept of the latusrectum to the parabola $y^2 = 4ax$ are b and k, then k is equal to MHT CET 2012

Options:

A. $\frac{ab}{a-b}$

B. $\frac{a}{b-a}$

C. $\frac{b}{b-a}$

D. $\frac{ab}{b-a}$

Answer: D

Solution:

For latusrectum PQ.

$$y(t_1 + t_2) - 2x - 2at_1t_2 = 0$$

and $\quad t_1 t_2 = -1$

For any point $P(x, y)$, the focal distance is $a + x$. $\therefore \quad b = a + x = a + at_1^2 = a\left(1 + t_1^2\right) \quad \ldots \text{(i)}$

$c = a + x = a + at_2^2 = a + \frac{a}{t_1^2} \quad (\because t_1 t_2 = -1)$

$= \frac{a(1+t_1^2)}{t_1^2}$...(ii)

$\frac{b}{c} = t_1^2$

$\therefore$ From Eq. (i), $b = a\left(1 + \frac{b}{c}\right)$

$\Rightarrow \quad b = a + \frac{ab}{c}$

$c = \frac{ab}{b-a}$

Question19

The equation of directrix is to the parabola $4x^2 - 4x - 2y + 3 = 0$ will be MHT CET 2012

Options:

A. $2y = 1$

B. $2x = 1$

C. $2y = 3$

D. $2x = 3$

Answer: A

Solution:

Given parabola, $4x^2-4x-2y+3=0 \Rightarrow 4\left(x^2-x\right)=2y-3$

$\Rightarrow 4\left(x^2-x+\frac{1}{4}-\frac{1}{4}\right)=2y-3$

$\Rightarrow \left(x-\frac{1}{2}\right)^2=2y-2$

$\Rightarrow \left(x-\frac{1}{2}\right)^2=2(y-1)$

which is of the form $X^2=4aY$

where, $X=x-\frac{1}{2}$ and $Y=y-1$

and $a=\frac{1}{2}$

$\therefore$ Directrix, $Y+a=0 \Rightarrow \quad y-1+\frac{1}{2}=0$

$\Rightarrow \quad y-\frac{1}{2}=0$

$\Rightarrow 2y=1$

Question20

The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the x -axis is MHT CET 2011

Options:

A. $\sqrt{2}y=3x+1$

B. $\sqrt{3}y=-(x+3)$

C. $\sqrt{3}y=x+3$

D. $\sqrt{3}y=-(3x+1)$

Answer: C

Solution:

Let the common tangent to the parabola

$$y^2=4x \text{ be}$$
$$y=mx+\frac{1}{m}$$

It should be also touch the circle

$$(x-3)^2+y^2=9$$

whose centre is $(3,0)$ and radius $=3$, then

$$\frac{|3m+1/m|}{\sqrt{1+m^2}}=3$$
$$\Rightarrow \quad 3m^2=1$$
$$\Rightarrow \quad m=\pm\frac{1}{\sqrt{3}}$$

But $m>0$, then equation of common tangent is

$$y=\frac{1}{\sqrt{3}}\cdot x+\sqrt{3}$$

or $\sqrt{3}\cdot y=x+3$

Question21

The equations of the normals at the ends of the latusrectum of the parabola $y^2=4ax$ are given by MHT CET 2011

Options:

A. $x^2 - y^2 - 6ax + 9a^2 = 0$

B. $x^2 - y^2 - 6ax - 6ay + 9a^2 = 0$

C. $x^2 - y^2 - 6ay + 9a^2 = 0$

D. None of the above

Answer: A

Solution:

The coordinates of the ends of the latusrectum of the parabola $y^2 = 4ax$ are $(a, 2a)$ respectively. The equation of the normal at $(a, 2a)$ to $y^2 = 4ax$ is

$$y - 2a = \frac{-2a}{2a}(x - a)$$

or $x + y - 3a = 0$Similarly, the equation of the normal $(a, -2a)$ is

$$x - y - 3a = 0$$

The combined equation is

$$x^2 - y^2 - 6ax + 9a^2 = 0$$

Question22

The equation of common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ is $x + y - k$. Then value of k is MHT CET 2010

Options:

A. 1

B. -1

C. $\underline{2}$

D. -2

Answer: D

Solution:

Given parabola is, $y^2 = 8x$Here,

$$4a = 8 \Rightarrow a = 2$$

Any tangent to the parabola is $y = mx + \frac{a}{m}$ or

$$mx - y + \frac{2}{m} = 0$$

If it is a tangent to the circle $x^2 + y^2 = 2$, then length of perpendicular from centre $(0, 0)$ is equal to radius $\sqrt{2}$.

$$\begin{aligned} \therefore \quad & \frac{2/m}{\sqrt{m^2+1}} = \sqrt{2} \\ \Rightarrow \quad & \frac{4}{m^2} = 2\left(m^2+1\right) \\ \Rightarrow \quad & m^4 + m^2 - 2 = 0 \\ \Rightarrow \quad & \left(m^2+2\right)\left(m^2-1\right) = 0 \end{aligned}$$

$\Rightarrow \quad m^2 = \pm 1$Hence, the common tangent are $y = \pm(x+2)$. But given equation of tangent is $x + y = k$ On comparing, we get $k = -2$

Question23

The equation of the curve whose vertex is $(0,0)$ and length of latusrectum is $\frac{16}{3}$, is MHT CET 2010

Options:

A. $8x^2+3y^2=72$

B. $16y^2=3x$

C. $3y^2=16x$

D. $3x^2+16y^2=48$

Answer: C

Solution:

For the curve $3y^2=16x$, the length of latusrectum is $\frac{16}{3}$

Question24

The focal distance of a point P on the parabola $y^2=12x$ if the ordinate of P is 6, is MHT CET 2009

Options:

A. 12

B. 6

C. 3

D. 9

Answer: B

Solution:

Given parabola is $y^2 = 12x$

Here $a = 3$

For point $P(x, y), y = 6$

This point lie on the parabola $\therefore$

$$(6)^2 = 12x \Rightarrow x = 3$$

Now, focal distance of point P is $x + a = 6$

Question25

Equation of tangent to the parabola $y^2 = 16x$ at $P(3, 6)$ is MHT CET 2009

Options:

A. $4x - 3y + 12 = 0$

B. $3y - 4x - 12 = 0$

C. $4x - 3y - 24 = 0$

D. $3y - x - 24 = 0$

Answer: B

Solution:

Equation of tangent to parabola $y^2 = 16x$ at $P(3,6)$ is

$$
\begin{aligned}
& 6y = 8(x+3) \\
\Rightarrow \quad & 3y = 4x + 12 \\
\Rightarrow 3y - 4x \quad & -12 = 0
\end{aligned}
$$

Question26

The focal distance of a point on the parabola $y^2 = 16x$ whose ordinate is twice the abscissa, is MHT CET 2008

Options:

A. 6

B. 8

C. 10

D. 12

Answer: B

Solution:

Given curve is $y^2 = 16x$. Let the point be (h, k). But $2h = k$, then $k^2 = 16h$

$$
\begin{aligned}
\Rightarrow \quad & 4h^2 = 16h \\
\Rightarrow \quad & h = 0, h = 4 \\
\Rightarrow \quad & k = 0, k = 8
\end{aligned}
$$

$\therefore$ Points are $(0, 0), (4, 8)$.Hence, focal distance are respectively

$$0 + 4 = 4, 4 + 4 = 8$$

$(\because$ focal distance $= h - a)$

Question27

The equation of common tangent to the circle $x^2 + y^2 = 2$ and parabola $y^2 = 8x$ is MHT CET 2008

Options:

A. $y = x + 1$

B. $y = x + 2$

C. $y = x - 2$

D. $y = -x + 2$

Answer: B

Solution:

Given, $y^2 = 8x$

$$\therefore \quad 4a = 8 \Rightarrow a = 2$$

Any tangent of parabola is,

$$y = mx + \frac{a}{m}$$

$$\Rightarrow \quad mx - y + \frac{2}{m} = 0$$

If it is a tangent to the circle $x^2+y^2=2$, then perpendicular from centre $(0,0)$ is equal to radius $\sqrt{2}$.

$$\begin{aligned} \therefore \quad & \frac{\frac{2}{m}}{\sqrt{m^2+1}}=\sqrt{2} \\ \Rightarrow \quad & \frac{4}{m^2}=2\left(m^2+1\right) \\ \Rightarrow \quad & m^4+m^2-2=0 \\ \Rightarrow \quad & \left(m^2+2\right)\left(m^2-1\right)=0 \end{aligned}$$

$\Rightarrow m=\pm 1$Hence, the common tangents are $y=\pm(x+2)$.

Question28

The equation to the line touching both the parabolas $y^2=4x$ and $x^2=-32y$, is MHT CET 2007

Options:

A. $x+2y+4=0$

B. $2x+y-4=0$

C. $x-2y-4=0$

D. $x-2y+4=0$

Answer: D

Solution:

The equation of any tangent to the parabola $y^2=4x$ is

$$y=mx+\frac{1}{m}$$

This touches the parabola $x^2 = -32y$, therefore the equation $x^2 = -32\left(mx + \frac{1}{m}\right)$ has equal roots.$\therefore$

$$(32\ \mathrm{m})^2 = 4\left(\frac{32}{\mathrm{m}}\right)$$

$$\Rightarrow \quad 8m^3 = 1 \quad \Rightarrow \quad m = \frac{1}{2}$$

On putting the value of m in Eq. (i), we get

$$x - 2y + 4 = 0$$
