Gravitation

Question1

The acceleration due to gravity at a height of $(\sqrt{2}-1)\mathrm{R}$ from the surface of the Earth is

(Acceleration due to gravity on the surface of the Earth $= 10\ \mathrm{ms}^{-2}$ and R is radius of the Earth)

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Options:

A.

$2.5\ \mathrm{ms}^{-2}$

B.

$7.5\ \mathrm{ms}^{-2}$

C.

$5\ \mathrm{ms}^{-2}$

D.

$10\ \mathrm{ms}^{-2}$

Answer: C

Solution:

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left[1+\frac{(\sqrt{2}-1)R}{R}\right]^2}$$
$$= \frac{g}{(\sqrt{2})^2} = \frac{g}{2} = \frac{10}{2} = 5\ \mathrm{m/s^2}$$

Question2

The potential energy of a satellite of mass ' m ' revolving around the Earth at a height of R_e from the surface of the Earth is

($R_e =$ Radius of Earth, g $=$ acceleration due to gravity)

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Options:

A.

$-0.5 mgR_e$

B.

$-mgR_e$

C.

$-2 mgR_e$

D.

$-4 mgR_e$

Answer: A

Solution:

Step 1: Formula for Potential Energy

The potential energy (U) of a satellite of mass m at a distance r from the center of Earth is given by: $U = -\frac{GM_e m}{r}$ where G is the gravitational constant, and M_e is the mass of the Earth.

Step 2: Substitute the Satellite's Height

The satellite is at a height $h = R_e$ above the surface of the Earth. The total distance from the Earth's center is $r = R_e + h = R_e + R_e = 2R_e$.

Step 3: Put this Distance Into the Formula

So, the potential energy is:$U = -\frac{GM_e m}{2R_e}$

Step 4: Use the Relationship $GM_e = gR_e^2$

We know that $GM_e = gR_e^2$, where g is the acceleration due to gravity. Substituting, we get:$U = -\frac{gR_e^2 m}{2R_e}$

Step 5: Simplify

Divide R_e^2 by R_e:$U = -\frac{gR_e m}{2}$

Final Answer

The potential energy of the satellite is:$U = -0.5mgR_e$

Question3

The time period of a simple pendulum on the surface of the Earth is T. If the pendulum is taken to a height equal to half of the radius of the Earth, then its time period is

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Options:

A.

$\frac{T}{2}$

B.

$\frac{3T}{2}$

C.

$2T$

D.

$3T$

Answer: B

Solution:

Time period of pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

At a height $h = \frac{R}{2}$ above Earth's surface,

$$g_h = g \cdot \frac{R^2}{(R+h)^2} = \frac{g \cdot R^2}{\frac{9}{4}R^2} = \frac{4}{9}g$$

Time period at this height is,

$$T' = 2\pi\sqrt{\frac{l}{gh}} = 2\pi\sqrt{\frac{l}{\frac{4}{9}g}}$$

$$= \frac{3}{2}\left(2\pi\sqrt{\frac{l}{g}}\right) = \frac{3}{2}T$$

Question4

If the escape velocity of a body from the surface of the Earth is 11.2 km s^{-1}, then the orbital velocity of a satellite in an orbit which is at a height equal to the radius of the Earth is

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Options:

A.

11.2 km s^{-1}

B.

2.8 km s^{-1}

C.

22.4 km s^{-1}

D.

5.6 km s^{-1}

Answer: D

Solution:

Step 1: Escape velocity from Earth

The escape velocity from the Earth's surface is given by:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Here, G is the gravitational constant, M is Earth's mass, and R is Earth's radius.

Step 2: Orbital velocity at height $h = R$

The orbital velocity for a satellite at a distance r from the center of the Earth is:

$$v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$$

If the satellite is at a height equal to the Earth's radius ($h = R$), the total distance from Earth's center is $r = R + R = 2R$.

So, the orbital velocity becomes:

$$v_{\text{orbit}} = \sqrt{\frac{GM}{2R}}$$

Step 3: Find the relationship between escape velocity and orbital velocity

Let's compare the orbital and escape velocities:

$$\frac{v_{\text{orbit}}}{v_{\text{escape}}} = \frac{\sqrt{\frac{GM}{2R}}}{\sqrt{\frac{2GM}{R}}}$$

Simplifying this gives:

$$\frac{v_{\text{orbit}}}{v_{\text{escape}}} = \frac{1}{2}$$

Step 4: Calculate the final answer

This means the orbital velocity is half the escape velocity. Given that the escape velocity is 11.2 km s^{-1}:

$$v_{\text{orbit}} = \frac{11.2}{2} = 5.6 \text{ km s}^{-1}$$

Question5

An artificial satellite is revolving around a planet of radius R in a circular orbit of radius ' a '. If the time period of revolution of the satellite. $T \propto a^{3/2} g^{x} R^{y}$, then the values of x and y are respectively [$g =$ acceleration due to gravity]

Options:

A.

$1, \frac{1}{2}$

B.

$\frac{1}{2}, 1$

C.

$-\frac{1}{2}, \frac{1}{2}$

D.

$\frac{-1}{2}, -1$

Answer: D

Solution:

$$
\begin{aligned}
& T \propto a^{\frac{3}{2}} g^x R^y \Rightarrow T = k a^{\frac{3}{2}} g^x R^y \\
& \left[\mathrm{M}^0 \mathrm{~L}^0 \mathrm{~T}^1\right] = \left[\mathrm{L}^{3/2}\right]\left[\mathrm{LT}^2\right]^x\left[\mathrm{~L}^y\right] \\
& \left[\mathrm{M}^0 \mathrm{~L}^0 \mathrm{~T}^1\right] = \left[\mathrm{L}^{\frac{3}{2}+x+y} \mathrm{~T}^{-2x}\right] \\
& \therefore -2x = 1 \\
& \Rightarrow \quad x = \frac{-1}{2} \\
& \frac{3}{2} + x + y = 0 \\
& \frac{3}{2} - \frac{1}{2} + y = 0 \\
& 1 + y = 0 \Rightarrow y = -1
\end{aligned}
$$

Question6

A mass of 6×10^{24} kg is to be compressed in the form of a solid sphere such that the escape velocity from its surface is $3 \times 10^4 \mathrm{~ms}^{-1}$. The radius of the sphere is

(Universal gravitational constant $= 6.66 \times 10^{-11}$ N m^2 kg^{-2})

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Options:

A.

483 km

B.

575 km

C.

789 km

D.

888 km

Answer: D

Solution:

Escape velocity is given as

$v=\sqrt{2 g R}=\sqrt{\frac{2 G M}{R}}$

$M \rightarrow$ Mass of sphere, $R \rightarrow$ Radius of sphere

$$3 \times 10^4=\sqrt{\frac{2 \times 6.66 \times 10^{-11} \times 6 \times 10^{24}}{R}}$$

$$R=\frac{2 \times 6.66 \times 10^{-11} \times 6 \times 10^{24}}{9 \times 10^8}$$

$R=8.88 \times 10^5$ metre

$R=888 \times 10^3$ metre $=888$ km

Question7

Two satellites A and B are revolving around the Earth in orbits of heights $1.25 R_E$ and $19.25 R_E$ from the surface of Earth respectively,

where R_E is the radius of the Earth. The ratio of the orbital speeds of the satellites A and B is

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Options:

A.

5 : 1

B.

4 : 1

C.

9 : 1

D.

3 : 1

Answer: D

Solution:

Given data

- Height of satellite A above Earth's surface: $h_A = 1.25R_E$
- Height of satellite B above Earth's surface: $h_B = 19.25R_E$

Hence,

the orbital radii (distance from Earth's center):

$r_A = R_E + h_A = R_E + 1.25R_E = 2.25R_E$

$r_B = R_E + h_B = R_E + 19.25R_E = 20.25R_E$

Formula for orbital speed

For a satellite in a circular orbit around Earth:

$v = \sqrt{\frac{GM_E}{r}}$

where G is the gravitational constant and M_E is Earth's mass.

Thus, for two satellites A and B:

$\frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}}$

Substitute r_A and r_B

$\frac{v_A}{v_B} = \sqrt{\frac{20.25R_E}{2.25R_E}} = \sqrt{\frac{20.25}{2.25}} = \sqrt{9} = 3$

✅ **Therefore,**

$\boxed{v_A : v_B = 3 : 1}$

Correct Option: D) 3 : 1

Question8

Two solid spheres each of radius ' R ' made of same material are placed in contact with each other. If the gravitational force acting between them is F, then

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Options:

A.

$F \alpha R^4$

B.

$F \alpha R^3$

C.

$F \alpha R^2$

D.

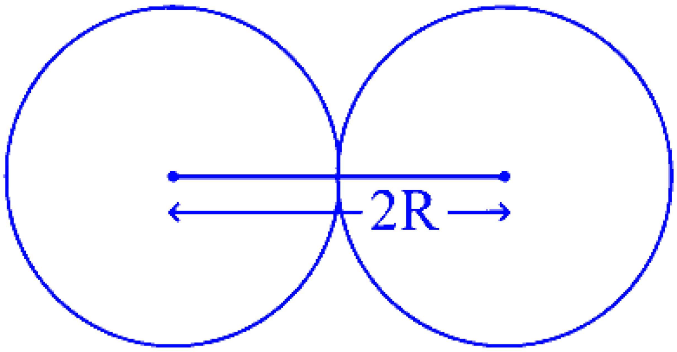
$F \alpha R$

Answer: A

Solution:

Mass of each, sphere

$M = \frac{4}{2}\pi R^3 \times \rho$



$\Rightarrow \quad M \propto R^3$

$\therefore \quad F \propto \frac{M_1 M_2}{(2R)^2}$

$\quad \propto \frac{R^3 \times R^3}{R^2}$

$\Rightarrow \quad \propto R^4$

Question9

If the angular velocity of a planet about its axis is halved, the distance of the stationary satellite of this planet from the centre of the planet becomes 2^n times the initial distance. Then, the value of ' n ' is

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Options:

A.

$\frac{2}{3}$

B.

$\frac{3}{2}$

C.

$\frac{1}{3}$

D.

$\frac{4}{3}$

Answer: A

Solution:

$T^2 \propto R^3$

$\Rightarrow \quad \left(\frac{2\pi}{\omega}\right)^2 \propto R^3$

$\Rightarrow \quad \frac{\omega^2}{4\pi^2} \propto \frac{1}{R^3}$

$\Rightarrow \quad \omega^2 \propto \frac{1}{R^3}$

$\Rightarrow \quad \left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \quad \left(\frac{\omega_{1/2}}{\omega_1}\right)^2 = \left(\frac{R_1}{2^n R_1}\right)^3$

$\Rightarrow \quad \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2^n}\right)^3$

$\Rightarrow \quad \frac{1}{2^2} = \frac{1}{2^{3n}}$

$\Rightarrow \quad 2^2 = 2^{3n} \Rightarrow 2 = 3n$

$n = \frac{2}{3}$

Question10

An infinite number of objects each 1 kg mass are placed on the X-axis on both sides of $x = 0$ at ± 1 m, ± 2 m, ± 4 m, ± 8 m . . . . . . and so on. The magnitude of the resultant gravitational potential (in SI units) at $x = 0$ is

($G =$ Universal gravitational constant)

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Options:

A.

$-G$

B.

$-2G$

C.

$-3G$

D.

$-4G$

Answer: D

Solution:

Gravitational potential

$V=-\frac{Gm}{r}$

Since, the mass of each object is 1 kg and the distance are $1, 2, 4, 8, \ldots$ on both sides of $x=0$. Thus, the total potential at $x=0$

$$\begin{aligned} V_{\text{total}} & =-G\left(\frac{m_1}{r_1}+\frac{m_2}{r_2}+\frac{m_3}{r_3}+\ldots\right) \\ & \quad -G\left(\frac{m_1}{r_1}+\frac{m_2}{r_2}+\frac{m_3}{r_3}+\ldots\right) \\ & =-2G\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right) \\ & =-2G\left(\frac{1}{1-\frac{1}{2}}\right)=-4G \end{aligned}$$

Question11

The time period of revolution of a satellite close to planet's surfaces is 80 min . The time period of another satellite, which is at a height of 3 times the radius of the planet from surface is

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Options:

A. 64 min

B. 640 min

C. 320 min

D. 240 min

Answer: B

Solution:

According to Kepler's law, $T^2 \propto r^3$, where T is the orbital period and r is the orbital radius.

For satellites orbiting close to the surface of a planet, we use:

T_1: the orbital time period of the first satellite.

R: the radius of the planet.

Since the first satellite is close to the planet's surface, its orbital radius is approximately equal to the planet's radius, R.

For the second satellite, which orbits at a height of 3 times the planet's radius, its orbital radius becomes $R + 3R = 4R$.

Let T_2 denote the orbital period of this second satellite. From Kepler's law, we have:

$\frac{T_2^2}{T_1^2} = \frac{(4R)^3}{R^3} \quad \Rightarrow \quad \frac{T_2^2}{T_1^2} = 4^3 = 64$

Thus,

$\frac{T_2}{T_1} = 8$

Given T_1 is 80 minutes, we can calculate T_2:

$T_2 = 8 \times T_1 = 8 \times 80 \text{ minutes} = 640 \text{ minutes}$

Therefore, the orbital period of the second satellite, which is at a height of 3 times the planet's radius from its surface, is 640 minutes.

Question12

The gravitational potential energy of a body on the surface of the earth is E. If the body is taken from the surface of the earth to a height equal to 150% of the radius of the earth. Its gravitational potential energy is

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Options:

A. $0.4E$

B. $0.2E$

C. $0.6E$

D. $0.3E$

Answer: A

Solution:

The gravitational potential energy U at a distance r from the center of the Earth is given by the formula:

$U=-\frac{GMm}{r}$

where:

M = mass of the Earth

m = mass of the body

r = distance from the center of the Earth

On the surface of the Earth ($r=R$), the gravitational potential energy is expressed as:

$U=E=-\frac{GMm}{R}$ ...(i)

When the body is taken to a height equal to 150% of the Earth's radius, the new distance from the center of the Earth, r', becomes:

$r'=R+\frac{150}{100}\times R=R+1.5R=2.5R$

The new gravitational potential energy U' at this height is calculated as:

$U'=-\frac{GMm}{r'}=-\frac{GMm}{2.5R}$

Rewriting this in terms of E, we have:

$U'=\frac{1}{2.5}\left(-\frac{GMm}{R}\right)$

Substituting from Eq. (i):

$U'=\frac{E}{2.5}$

Simplifying this gives:

$U'=0.4E$

Question13

A satellite moving round the earth in a circular orbit has kinetic energy E. Then, the minimum amount of energy to be added so that it escapes from the earth.

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Options:

A. $\frac{E}{4}$

B. E

C. $\frac{E}{2}$

D. $2F$

Answer: B

Solution:

To understand the energy required for a satellite to escape Earth's gravitational pull, let's break down the process:

Orbital Velocity of the Satellite:

The orbital velocity (v_0) of a satellite moving in a circular orbit around Earth is given by:

$v_0 = \sqrt{\frac{GM}{r}}$

where G is the gravitational constant, M is the mass of the Earth, and r is the radius of the orbit.

Kinetic Energy of the Satellite:

The kinetic energy (E) of the satellite in orbit is:

$E = \frac{1}{2}mv_0^2 = \frac{1}{2} \cdot \frac{GMm}{r}$

where m is the mass of the satellite.

Kinetic Energy for Escape Velocity:

To escape Earth's gravitational field, the satellite needs to achieve escape velocity (v_e), which is calculated as:

$v_e = \sqrt{\frac{2GM}{r}}$

Therefore, the kinetic energy required for escape (E') is:

$E' = \frac{1}{2}mv_e^2 = \frac{1}{2}m \cdot \frac{2GM}{r} = \frac{GMm}{r}$

Simplifying, we find:

$E' = 2E$

Additional Energy Required:

The additional kinetic energy needed to achieve escape velocity is:

$E' - E = 2E - E$

Therefore, the minimum additional energy required is:

E

Question14

A particle is projected from the surface of the earth with a velocity equal to twice the escape velocity. When particle is very far from the earth. Its speed would be

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Options:

A. v_e

B. $2v_e$

C. $\sqrt{3}v_e$

D. $\sqrt{2}v_e$

Answer: C

Solution:

Given:

Particle velocity, $v_p = 2v_{\text{escape}}$

The total energy of the projectile on Earth is expressed as:

$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{\text{escape}}^2$

When the particle is very far from the Earth, its gravitational potential energy becomes zero. Therefore, the total energy of the particle far from Earth is:

$\frac{1}{2}mv_f^2$

where v_f is the velocity of the particle when it is very far away.

By the conservation of energy, we have:

$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{\text{escape}}^2 = \frac{1}{2}mv_f^2$

Solving for v_f:

$v_f = \sqrt{v_p^2 - v_{\text{escape}}^2}$

Substituting $v_p = 2v_{\text{escape}}$:

$v_f = \sqrt{(2v_{\text{escape}})^2 - v_{\text{escape}}^2}$

$v_f = \sqrt{4v_{\text{escape}}^2 - v_{\text{escape}}^2}$

$v_f = \sqrt{3v_{\text{escape}}^2}$

$v_f = \sqrt{3}v_{\text{escape}}$

Question15

The time period of revolution of a satellite T around the carth depends on the radius of the circular orbit R. mass of the earth M and universal gravitational constant G. The expression for T, using dimensional analysis is (K is constant of proportionality)

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Options:

A. $K\sqrt{\frac{R^2}{GM}}$

B. $K\sqrt{\frac{R}{GM}}$

C. $K\sqrt{\frac{R^3}{GM}}$

D. $K\sqrt{\frac{R^3}{GM^2}}$

Answer: C

Solution:

The time period of a satellite's revolution, denoted as T, depends on the radius of the circular orbit R, the mass of the Earth M, and the universal gravitational constant G. Using dimensional analysis, we derive the expression for T as follows:

Given:

Time period of revolution is T.

Radius of circular orbit is R.

We use the relation:

$T \propto G^a M^b R^c$

This can be expressed as:

$T = KG^a M^b R^c$

where K is a constant of proportionality.

The dimensional expressions for the relevant quantities are:

$[T] = [T]$

$[G] = [M^{-1} L^3 T^{-2}]$

$[M] = [M]$

$[R] = [L]$

Thus, the dimensional equation becomes:

$[T] = [M^{-a} L^{3a+c} T^{2a}]$

Comparing dimensions on both sides, we obtain the following equations:

$-a + b = 0$

$3a + c = 0$

$2a = -1$

Solving these equations:

From $2a = -1$, we find $a = -\frac{1}{2}$.

Using $-a + b = 0$, we substitute for a to get $b = -\frac{1}{2}$.

From $3a + c = 0$, substituting for a gives $c = \frac{3}{2}$.

Substituting these values back into the relation for T:

$T = KG^{-\frac{1}{2}} M^{-\frac{1}{2}} R^{\frac{3}{2}}$

This simplifies to:

$T = K \frac{R^{\frac{3}{2}}}{G^{\frac{1}{2}} M^{\frac{1}{2}}}$

Therefore:

$T = K \sqrt{\frac{R^3}{GM}}$

Question16

If the time period of revolution of a satellite is T, the its kinetic energy is proportional to

Options:

A. T^{-1}

B. T^{-2}

C. T^{-3}

D. $T^{-2/3}$

Answer: D

Solution:

To solve this, let's start with the time period of the satellite's revolution, denoted as T. According to Kepler's third law, the relationship between the time period T and the radius r of the satellite's orbit can be expressed as:

$T^2 \propto r^3$

This implies:

$T \propto r^{3/2}$

Thus, rearranging gives:

$r \propto T^{2/3}$

Next, let's consider the kinetic energy (KE) of the satellite. It's known that the kinetic energy is inversely proportional to the radius r:

$\mathrm{KE} \propto \frac{1}{r}$

Substituting the expression for r from above, we find:

$\mathrm{KE} \propto \frac{1}{T^{2/3}}$

Therefore, the kinetic energy is proportional to:

$\mathrm{KE} \propto T^{-2/3}$

Question17

What is the height from the surface of earth, where acceleration due to gravity will be $1/4$ of that of the earth? $(R_E = 6400 \text{ km})$

Options:

A. 6400 km

B. 3200 km

C. 1600 km

D. 640 km

Answer: A

Solution:

To find the height above the Earth's surface where the acceleration due to gravity is one-fourth of its value at the surface, we can follow these steps:

Given Condition:

$g_h = \frac{g_0}{4} \quad \text{(equation 1)}$

Formula for Gravity at Height h:

$g_h = g\left(\frac{R_E}{R_E+h}\right)^2$

Here, $R_E = 6400\,\text{km}$.

Set the Equation:

$\frac{g}{4} = g\left(\frac{R_E}{R_E+h}\right)^2$

Simplify the Equation:

$\frac{1}{2} = \frac{R_E}{R_E+h}$

Solve for h:

$R_E + h = 2R_E$

$h = R_E$

Therefore, $h = 6400\,\text{km}$.

Thus, the height where the acceleration due to gravity is one-fourth of its surface value is 6400 km.

Question18

The acceleration due to gravity at a height of 6400 km from the surface of the earth is 2.5 ms^{-2}. The acceleration due to gravity at a height of 12800 km from the surface of the earth is (Radius of the earth= 6400 km)

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Options:

A. 1.11 ms^{-2}

B. 1.5 ms^{-2}

C. 2.22 ms^{-2}

D. 1.25 ms^{-2}

Answer: A

Solution:

We know the gravitational acceleration follows an inverse-square law, meaning :

$g \propto \frac{1}{r^2}$

So for two positions with distances r_1 and r_2 from the Earth's center, we have :

$\frac{g_2}{g_1} = \left(\frac{r_1}{r_2}\right)^2$

The Earth's radius is given as 6400 km.

At a height of 6400 km above the Earth's surface, the distance from the center is :

$r_1 = 6400 \text{ km} + 6400 \text{ km} = 12800 \text{ km}$

At a height of 12800 km above the Earth's surface, the distance from the center is :

$r_2 = 6400 \text{ km} + 12800 \text{ km} = 19200 \text{ km}$

Given that the acceleration at r_1 is $g_1 = 2.5 \text{ m/s}^2$, we can find g_2 by :

$g_2 = g_1 \times \left(\frac{r_1}{r_2}\right)^2$

Substitute the values :

$g_2 = 2.5 \times \left(\frac{12800}{19200}\right)^2$

Notice that :

$\frac{12800}{19200} = \frac{2}{3}$

So,

$g_2 = 2.5 \times \left(\frac{2}{3}\right)^2 = 2.5 \times \frac{4}{9} = \frac{10}{9} \approx 1.11 \mathrm{~m/s}^2$.

Thus, the acceleration due to gravity at a height of 12800 km from the Earth's surface is approximately $1.11 \mathrm{~m/s}^2$.

Answer: Option A.

Question19

Maximum height reached by a rocket fired with a speed equal to 50% of the escape speed from the surface of the earth is ($R =$ Radius of the earth)

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Options:

A. $\frac{R}{2}$

B. $\frac{16R}{9}$

C. $\frac{R}{3}$

D. $\frac{R}{8}$

Answer: C

Solution:

The maximum height reached by the rocket occurs when its kinetic energy is completely converted into gravitational potential energy, meaning the rocket momentarily stops.

Using the conservation of energy principle:

$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{R+H}$

Here, v is the velocity of the rocket when launched, given as $\frac{v_e}{2}$, where v_e is the escape velocity. The escape velocity is given by:

$v_e = \sqrt{\frac{2GM}{R}}$

Thus, the initial velocity of the rocket is:

$v = \frac{v_e}{2} = \frac{1}{2}\sqrt{\frac{2GM}{R}} = \sqrt{\frac{GM}{2R}}$

Substituting into the energy equation:

$$\frac{-GMm}{R} + \frac{1}{2}m\left(\sqrt{\frac{GM}{2R}}\right)^2 = \frac{-GMm}{R+H}$$

Solving this equation:

$$\frac{-GMm}{R} + \frac{1}{2}m \cdot \frac{GM}{2R} = \frac{-GMm}{R+H}$$

This simplifies to:

$$\frac{-GMm}{R} + \frac{GMm}{4R} = \frac{-GMm}{R+H}$$

$$\frac{-3GMm}{4R} = \frac{-GMm}{R+H}$$

Setting the fractions equal:

$$\frac{3}{4} \cdot \frac{1}{R} = \frac{1}{R+H}$$

Cross-multiplying gives:

$$3(R+H) = 4R$$

Solving for H:

$$3R + 3H = 4R$$

$$3H = R$$

Thus:

$$H = \frac{R}{3}$$

Therefore, the maximum height H attained by the rocket is $\frac{R}{3}$.

Question20

Two satellites of masses m and 1.5 m are revolving around the earth with different speeds in two circular orbits of heights R_E and $2R_E$ respectively, where R_F is the radius of the earth. The ratio of the minimum and maximum gravitational forces on the earth due to the two satellites is

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Options:

A. 2 : 5

B. 2 : 3

C. 1 : 2

D. 1 : 5

Answer: B

Solution:

Given:

Mass of first satellite, $m_1 = m$

Mass of second satellite, $m_2 = 1.5m$

Radius of Earth, R_E

Calculating Distances:

For the first satellite:

Distance from Earth's center, $r_1 = R_E + R_E = 2R_E$

For the second satellite:

Distance from Earth's center, $r_2 = R_E + 2R_E = 3R_E$

Gravitational Forces:

Gravitational force by the first satellite:

$F_1 = \frac{GM \cdot m}{(2R_E)^2} = \frac{GMm}{4R_E^2}$

Gravitational force by the second satellite:

$F_2 = \frac{1.5 \times GMm}{(3R_E)^2} = \frac{1.5GMm}{9R_E^2}$

Since $r_2 > r_1$, it follows that $F_1 > F_2$.

Ratio of Forces:

$\frac{F_2}{F_1} = \frac{\frac{1.5 \cdot GMm}{9R_E^2}}{\frac{GMm}{4R_E^2}} = \frac{1.5}{9} \times \frac{4}{1} = \frac{6}{9} = \frac{2}{3}$

Thus, the ratio $F_2 : F_1 = 2 : 3$.

Question21

Statement (A) Two artificial satellites revolving in the same circular orbit have same period of revolution.

Statement (B) The orbital velocity is inversely proportional to the square root of radius of the orbit.

Statement (C) The escape velocity of the body is independent of the altitude of the point of projection.

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Options:

A. A, B, C are true

B. A, B, C are false

C. A, C true B false

D. B, C true A false

Answer: B

Solution:

Time period of satellite moving around a planet with angular velocity ω is given as,

$T = \frac{2\pi}{\omega}$

Since, the two artificial satellites revolving in the same circular orbit, then their angular velocities will be same.

Hence, both satellites revolve with same period of revolution.

Orbital velocity of revolving satellite, $v = \sqrt{\frac{Gm}{r}}$ i.e. $v \propto \frac{1}{\sqrt{r}}$

Where, $m =$ mass of the planet

$G =$ Universal gravitational constant

$r =$ radius of circular orbit.

Escape velocity depends on the gravitational potential at the point from where the body is launched. Since, this potential depends on the height of the point of projection, hence escape velocity also depends on the height (altitude) of the point of projection.

Question22

A uniform solid sphere of radius R produces a gravitational acceleration of a_0 on its surface. The distance of the point from the centre of the sphere where the gravitational acceleration becomes $\frac{a_0}{4}$ is

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Options:

A. 4R

B. $\frac{3}{2}$R

C. 2R

D. 3R

Answer: C

Solution:

Since, uniform solid sphere of radius R produces a gravitational acceleration a_0 on its surface, hence $a_0 = \frac{Gm}{R^2}$

where, m is the mass of solid sphere.

Let x be the distance of the point from the surface of sphere where the gravitational acceleration becomes $\frac{a_0}{4}$.

i. e. $\frac{a_0}{4} = \frac{a_0}{\left(1+\frac{x}{R}\right)^2} \Rightarrow \left(\frac{1}{2}\right)^2 = \frac{1}{\left(1+\frac{x}{R}\right)^2}$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{1+\frac{x}{R}} \Rightarrow 1 + \frac{x}{R} = 2$$
$$\Rightarrow \quad x = R$$

Hence, distance of the point from the centre of sphere $= x + R = R + R = 2R$

Question23

A projectile is thrown straight upward from the earth's surface with an initial speed $v = \alpha v_e$ where α is a constant and v_e is the escape speed. The projectile

travels upto a height 800 km from earth's surface, before it comes to rest. The value of the constant α is (radius of the earth $= 6400$ km)

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Options:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: A

Solution:

Initial speed of projectile from the surface of earth, $v = \alpha v_e$

Where, α is constant and v_e is escape velocity of projectile.

Radius of earth, $R_E = 6400$ km

$= 6.4 \times 10^6$ m

Maximum height attained by the projectile,

$h = 800 \text{ km} = 8 \times 10^5 \text{ m}$

We know that, escape velocity on the surface of earth, $v_e = 11.2 \text{ km/s}$

$= 11.2 \times 10^3 \text{ m/s}$
$v = \alpha v_e$
$v = \alpha \times 11.2 \times 10^3$ ... (i)

According to given situation, applying the law of conservation of energy

$$\frac{1}{2} m v^2 = mgh$$
$$\Rightarrow v^2 = 2gh \Rightarrow (\alpha v_e)^2 = 2gh$$
$$\Rightarrow \quad \alpha = \frac{\sqrt{2gh}}{v_e} = \frac{\sqrt{2 \times 10 \times 8 \times 10^5}}{11.2 \times 10^3} = \frac{4}{11.2} \approx \frac{1}{3}$$

Question24

If the Earth stops rotating in its orbit aboutthe sun, there will be variation in the weightof our bodies at

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Options:

A. equator

B. 60° latitude

C. poles

D. No where

Answer: D

Solution:

As we know that,

$g' = g - \omega^2 r \cos \theta$

where, g' is effective acceleration, g is acceleration due to gravity, ω is angular frequency, r is radius of earth and θ is angle between equator and body.

If $\omega = 0 \text{ rads}^{-1}$ (i.e. earth stops rotation)

$g' = g$

i.e., weight remains same everywhere.

Question25

At what depth below surface of the Earth, the acceleration due to gravity will be half of its value that at 1600 km above the surface of the Earth?

Options:

A. 4.8×10^6 m

B. 3.19×10^6 m

C. 1.59×10^6 m

D. 5.5×10^6 m

Answer: A

Solution:

Given, height, H = 1600 km

Let depth be d.

Acceleration due to gravity at height, depth and surface be g_h, g_d and g.

Also, $g_d = \frac{g_h}{2}$ ...... (i)

As we know that,

$$\begin{aligned} g_h &= g\left(1 - \frac{2h}{R}\right) \\ \Rightarrow \quad g_h &= g\left(1 - \frac{2 \times 1600}{6400}\right) = \frac{g}{2} \end{aligned}$$

and $g_d = g\left(1 - \frac{d}{R}\right)$

From Eq. (i),

$$\begin{aligned} \frac{1}{2} \times \frac{g}{2} &= g\left(1 - \frac{d}{R}\right) \\ \Rightarrow \quad \frac{d}{R} &= \frac{3}{4} \\ \Rightarrow \quad d &= \frac{3}{4}R = \frac{3}{4} \times 6400 = 4.8 \times 10^6 \text{ m} \end{aligned}$$

Question26

The gravitational potential energy is maximum at

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Options:

A. infinity

B. the earth's surface

C. the centre of the earth

D. twice the radius of the earth

Answer: A

Solution:

As we know that,

Gravitational potential energy, $(U)=-\frac{GM_e m}{r}$

If $\gamma=\infty, U=0$ which is maximum.

Question27

A geostationary satellite is taken to a neworbit, such that its distance from centre ofthe earth is doubled. Then, find the timeperiod of this satellite in the new orbit.

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Options:

A. 24 h

B. 4.8 h

C. $48\sqrt{2}$ h

D. $24\sqrt{2}$ h

Answer: C

Solution:

Let, Initial radius $= R$

Final radius $= 2R$

Initial time period, $T = 24$ h

Final time period, $T' = ?$

By using third Kepler's law of planetary motion,

$T^2 \propto R^3$

$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{24}{T'}\right)^2 = \left(\frac{R}{2R}\right)^3$

$\Rightarrow \quad \frac{24\times24}{T'^2} = \frac{1}{8} \Rightarrow T'^2 = 24 \times 24 \times 8$

$\Rightarrow \quad T' = 24 \times 2\sqrt{2} = 48\sqrt{2}$ h

Question28

The distance through which one has to digthe Earth from its surface, so as to reach thepoint where the acceleration due to gravity isreduced by 40% of that at the surface of theEarth, is (radius of Earth is 6400 km)

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Options:

A. 2560 km

B. 3000 km

C. 3260 km

D. 1560 km

Answer: A

Solution:

Given, radius of earth, $R = 6400$ km

Let gravity on surface of earth be g.

$\therefore$ Gravity at depth d be $g_d = g - 40\%$ of $g = \frac{60}{100} g$

As we know that, $g_d = g\left(1 - \frac{d}{R}\right)$

$$\therefore \quad \frac{60}{100} g = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow \quad 0.6 = 1 - \frac{d}{R} \Rightarrow \frac{d}{R} = 0.4$$

$$\Rightarrow \quad d = 0.4 \times R \Rightarrow d = \frac{4}{10} \times 6400$$

$$= 2560 \text{ km}$$

Question29

Infinite number of masses each of 3kg areplaced along a straight line at the distancesof 1 m, 2m, 4m, 8m, ... from a point O onthe same line. If G is the universal gravitational constant, then the magnitude ofgravitational field intensity at O is

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Options:

A. 1.0 G

B. 2.0 G

C. 3.0 G

D. 4.0 G

Answer: D

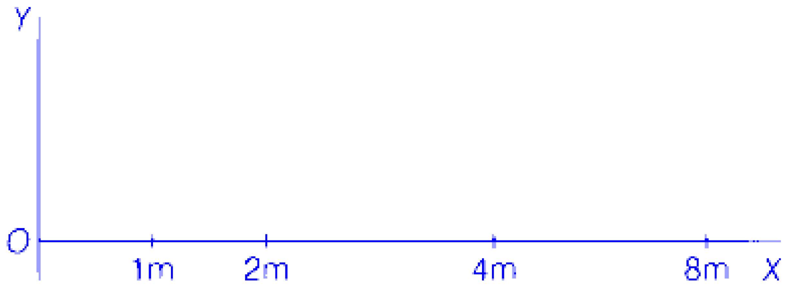
Solution:

Given, mass of body, m = 3 kg

Position of masses are shown below.

Let, gravitational field intensity be E.

As we know that, $E = \frac{Gm}{R^2}$



$$E = \frac{3G}{1^2} + \frac{3G}{2^2} + \frac{3G}{4^2} + \frac{3G}{8^2} + \ldots\ldots$$

$$= 3G\left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\right) \quad \ldots.(\mathrm{i})$$

$$= 3G\left[\frac{1}{1-\frac{1}{4}}\right]\left[\because S = \frac{a}{1-r}\right]$$

$$E = 3G\frac{4}{3} = 4G$$

Question30

A particle is kept on the surface of a uniformsphere of mass 1000 kg and radius 1 m. Thework done per unit mass against thegravitational force between them is

$[\mathrm{G} = 6.67 \times 10^{-11}\ \mathrm{Nm}^2\ \mathrm{kg}^{-2}]$

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Options:

A. $3.35 \times 10^{-10}\ \mathrm{Jkg}^{-1}$

B. $-3.35 \times 10^{-10}\ \mathrm{Jkg}^{-1}$

C. $6.67 \times 10^{-8}\ \mathrm{Jkg}^{-1}$

D. $-3.35 \times 10^{-8}\ \mathrm{Jkg}^{-1}$

Answer: C

Solution:

Given, mass of sphere = 1000 kg

Radius of sphere = 3m

Since, work (W) = $\frac{GM_1M_2}{r}$

where, G is gravitational constant i.e., $6.67 \times 10^{-11}\ \mathrm{N}-\mathrm{m}^2\mathrm{kg}^{-1}$

M_1, M_2 are mass of particle and sphere respectively.

$\therefore \frac{W}{m_1} = \frac{GM_2}{r}$

$= \frac{6.67 \times 10^{-11} \times 1000}{1}$

$= 6.67 \times 10^{-8}$ J/kg

Question31

The acceleration due to gravity at a height (1/20)th of the radius of Earth above the Earth's surface is 9 ms^{-2}. Its value at an equal depth below the surface of earth is

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Options:

A. 9 ms^{-2}

B. 9.25 ms^{-2}

C. 9.5 ms^{-2}

D. 9.8 ms^{-2}

Answer: C

Solution:

Given, let the radius of earth = R

Height above earth, H = R/20

Acceleration due to gravity at H, $g_H = 9$ ms^{-2}

Acceleration due to gravity on the surface of earth, $g = 10\ ms^{-2}$

Acceleration due to gravity at depth d from the surface of earth,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

$$\Rightarrow \quad g_d = 10\left(1 - \frac{R}{20 \times R}\right) = 10\left(\frac{19}{20}\right) = 9.5\ ms^{-2}$$
