Differentiation

Question1

If $\sin x\sqrt{\cos y}-\cos y\sqrt{\sin x}=0$**, then** $\frac{dy}{dx}=$

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Options:

A.

$\tan x$

B.

1

C.

-1

D.

$-\cot x$

Answer: C

Solution:

$$\because \sin x\sqrt{\cos y}-\cos y\sqrt{\sin x}=0 \quad \ldots \text{(i)}$$
$$\Rightarrow \frac{d}{dx}[\sin x\sqrt{\cos y}-\cos y\cdot\sqrt{\sin x}]=0$$
$$\Rightarrow \cos x\sqrt{\cos y}-\frac{\sin x\sin y}{2\sqrt{\cos y}}\cdot\frac{dy}{dx}+\sin y\cdot\sqrt{\sin x}\cdot\frac{dy}{dx}-\cos y\cdot\frac{\cos x}{2\sqrt{\sin x}}=0$$
$$\Rightarrow \frac{dy}{dx}\left(\sin y\cdot\sqrt{\sin x}-\frac{\sin x\cdot\sin y}{2\sqrt{\cos y}}\right)$$
$$=\frac{\cos x\cdot\cos y}{2\sqrt{\sin x}}-\cos x\cdot\sqrt{\cos y}$$

$(\cos x \cdot \cos y - 2\cos x\sqrt{\sin x \cos y})$

$\Rightarrow \frac{dy}{dx} = \frac{(2\sqrt{\cos y})}{(2\sqrt{\sin x})(2\sin y\sqrt{\cos y \cdot \sin x}}$

$- \sin x \cdot \sin y)$

By Eq. (i), we get

$\sin x\sqrt{\cos y} = \cos y\sqrt{\sin x}$

$\Rightarrow \cos y = \sin x$

$$\therefore \frac{dy}{dx} = \frac{\sin x \cos x - 2\cos x\sqrt{\sin^2 x}}{2\sin y\sqrt{\sin^2 x} - \sin x \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2\cos x \sin x - \sin x \cos x)}{(2\sin x \cos x - \sin x \cos x)} = -1$$

Question2

If $y = (\log_x \sin x)^x$, then $\frac{dy}{dx} =$

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Options:

A.

$y\left[\frac{x \sin x}{\log \cos x} + \log(\log \sin x) + \frac{1}{\log x} - \log(\log x)\right]$

B.

$y\left[\frac{x \cos x}{\log \sin x} - \log(\log \sin x) + \frac{1}{\log x} + \log(\log x)\right]$

C.

$y\left[\frac{x \cot x}{\log \sin x} + \log(\log \sin x) - \frac{1}{\log x} - \log(\log x)\right]$

D.

$y\left[\frac{x \cot x}{\log \sin x} - \log(\log \sin x) + \frac{1}{\log x} - (\log x)\right]$

Answer: C

Solution:

$$y=(\log_x \sin x)^x=\left(\frac{\log(\sin x)}{\log x}\right)^x$$
$$\Rightarrow \quad \ln y=x\cdot\ln\left(\frac{\log(\sin x)}{\log x}\right)$$
$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\ln\left(\frac{\log(\sin x)}{\log x}\right)+x\cdot\frac{d}{dx}\left[\ln\left(\frac{\log(\sin x)}{\log x}\right)\right]$$
$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\ln\left(\frac{\ln(\sin x)}{\ln x}\right)+x\frac{d}{dx}\left[\ln\left(\frac{\ln(\sin x)}{\ln x}\right)\right]$$
$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\ln\left(\frac{\ln(\sin x)}{\ln x}\right)+x\left[\frac{\ln x}{\ln(\sin x)}\left\{\frac{\ln x\cdot\cot x-\frac{\ln(\sin x)}{x}}{(\ln x)^2}\right\}\right]$$
$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\ln\left(\frac{\ln(\sin x)}{\ln x}\right)+\frac{x}{\ln(\sin x)}\left\{\frac{\ln x\cdot\cot x-\frac{\ln(\sin x)}{x}}{(\ln x)}\right\}$$
$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\ln\left(\frac{\ln(\sin x)}{\ln x}\right)+\frac{x\cdot\cot x}{\ln(\sin x)}-\frac{1}{\ln x}$$
$$\Rightarrow \frac{dy}{dx}=y\left[\begin{array}{l}\frac{x\cdot\cot x}{\ln(\sin x)}+\ln(\ln(\sin x))\\ -\ln(\ln x)-\frac{1}{\ln x}\end{array}\right]$$
$$\Rightarrow \frac{dy}{dx}=y\left[\begin{array}{l}\frac{x\cdot\cot x}{\log(\sin x)}+\log(\log(\sin x))\\ -\frac{1}{\log x}-\log(\log x)\end{array}\right]$$

Question3

If $f(x)=x^{\sec^{-1}x}$, then $f'(2)=$

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Options:

A.

$\frac{2^{\pi/3}}{6}(\pi-\sqrt{3}\log 2)$

B.

$\frac{2^{\pi/6}}{6}(\pi+\sqrt{3}\log 2)$

C.

$\frac{2^{\pi/3}}{6}(\pi+\sqrt{3}\log 2)$

D.

$\frac{2^{\pi/6}}{6}(\pi-\sqrt{3}\log 2)$

Answer: B

Solution:

We have,

$f(x)=x^{\sec^{-1}x}\Rightarrow \ln f=\sec^{-1}x\cdot\ln x$

Differentiate w.r.t. x, we get

$$\begin{aligned}
&\frac{1}{f}\cdot f'(x)=\frac{\sec^{-1}x}{x}+\ln x\left(\frac{1}{x\sqrt{x^2-1}}\right)\\
&\Rightarrow\frac{f'(2)}{f(2)}=\frac{\sec^{-1}2}{2}+\ln 2\left(\frac{1}{2\sqrt{4-1}}\right)\\
&\Rightarrow f'(2)=f(2)\left(\frac{\pi}{6}+\frac{\ln 2}{2\sqrt{3}}\right)\\
&\qquad=2^{\pi/6}\left(\frac{\pi}{6}+\frac{\ln 2}{2\sqrt{3}}\right)\\
&\qquad=\frac{2^{\pi/6}}{6}(\pi+\sqrt{3}\log 2
\end{aligned}$$

Question4

If $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)+\tan^{-1}\left(\frac{7x}{1-12x^2}\right)$, then at $x=0$, $\frac{dy}{dx}=$

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Options:

A.

6

B.

7

C.

9

D.

10

Answer: D

Solution:

Given, $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)+\tan^{-1}\left(\frac{7x}{1-12x^2}\right)$

Let $x=\tan\theta$

$$So, =\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)+\tan^{-1}\left(\frac{7\tan\theta}{1-12\tan^2\theta}\right)$$
$$=3\tan^{-1}x+\tan^{-1}\left(\frac{7x}{1-12x^2}\right)$$

Now, differentiate y w.r.t x, we get

$$\frac{dy}{dx}=3\cdot\frac{1}{1+x^2}+\frac{1}{1+\left(\frac{7x}{1-12x^2}\right)^2}\cdot\frac{d}{dx}\left(\frac{7x}{1-12x^2}\right)$$
$$=\frac{3}{1+x^2}+\frac{1}{1+\frac{49x^2}{(1-12x^2)^2}}\cdot\frac{(1-12x^2)\cdot 7-7x(-24x)}{(1-12x^2)^2}$$
$$=\frac{3}{1+x^2}+\frac{(1-12x^2)^2}{(1-12x^2)^2+49x^2}\cdot\frac{7-84x^2+168x^2}{(1-12x^2)^2}$$
$$=\frac{3}{1+x^2}+\frac{7+84x^2}{(1-12x^2)^2+49x^2}$$

Now, At $x=0$

$$\left.\frac{dy}{dx}\right|_{x=0}=\frac{3}{1+0}+\frac{7+84(0)}{(1-12(0)^2)^2+49.0^2}=3+\frac{7}{1+0}=3+7=10$$

Question5

If $y=\sqrt{\frac{x^4\sqrt{3x-5}}{(x^2-3)(2x-3)}}$, then $\left(\frac{dy}{dx}\right)_{x=2}=$

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Options:

A.

5

B.

0

C.

1

D.

-5

Answer: D

Solution:

$y=\sqrt{\frac{x^4\sqrt{3x-5}}{(x^2-3)(2x-3)}}$

Take the natural log to both sides, we get

$$\ln(y)=\frac{1}{2}\ln\left(\frac{x^4\sqrt{3x-5}}{(x^2-3)(2x-3)}\right)$$
$$\ln(y)=\frac{1}{2}\left[\ln\left(x^4\right)+\ln(\sqrt{3x-5})-\ln\left(x^2-3\right)-\ln(2x-3)\right]$$

$=\frac{1}{2}\left[4\ln(x)+\frac{1}{2}\ln(3x-5)-\ln\left(x^2-3\right)-\ln(2x-3)\right]$

Now, differentiating w.r.t x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{2}\left[\frac{4}{x}+\frac{1}{2}\cdot\frac{3}{3x-5}-\frac{2x}{x^2-3}-\frac{2}{2x-3}\right]$$
$$\frac{dy}{dx}=\frac{y}{2}\left[\frac{4}{x}+\frac{3}{2(3x-5)}-\frac{2x}{x^2-3}-\frac{2}{2x-3}\right]$$

Now, at $x=2$

$y=\sqrt{\frac{2^4\sqrt{3\cdot2-5}}{(2^2-3)(4-3)}}=\sqrt{\frac{16\cdot1}{1\cdot1}}=4$

So, $\left.\frac{dy}{dx}\right|_{x=2}=\frac{4}{2}\left[\frac{4}{2}+\frac{3}{2(3\cdot2-5)}-\frac{2.2}{2^2-3}-\frac{2}{2\cdot2-3}\right]$

$=2\left[2+\frac{3}{2}-\frac{4}{1}-2\right]=2\left[\frac{3-8}{2}\right]=-5$

$\therefore\quad\left.\frac{dy}{dx}\right|_{x=2}=-5$

Question6

If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at $x = -2$ is

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Options:

A.

-30

B.

-34

C.

-32

D.

-18

Answer: B

Solution:

Given, $x^2 + y^2 + \sin y = 4$

Differentiate both sides w.r.t x, we get

$$2x + 2y \cdot \frac{dy}{dx} + \cos y \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}(2y + \cos y) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y + \cos y}$$

Again, differentiate both sides w.r.t x, we get

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx}\left(\frac{-2x}{2y+\cos y}\right)\\ &= \frac{(2y+\cos y)\cdot(-2)-(-2x)\cdot(2-\sin y)\cdot\frac{dy}{dx}}{(2y+\cos y)^2}\\ &= \frac{-4y-2\cos y+2x\cdot(2-\sin y)\cdot\left(\frac{-2x}{2y+\cos y}\right)}{(2y+\cos y)^2}\\ &= \frac{-4y-2\cos y-\frac{4x^2(2-\sin y)}{2y+\cos y}}{(2y+\cos y)^2} \qquad \ldots(i)\end{aligned}$$

Put $x=-2$ in original equation, we get

$$(-2)^2+y^2+\sin y=4$$

$$\Rightarrow \quad y^2+\sin y=0 \Rightarrow y=0$$

Now, from Eq. (i), we get

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{-4(0)-2\cos(0)-\frac{4(-2)^2(2-\sin(0))}{2\cdot(0)+\cos(0)}}{(2(0)+\cos(0))^2}\\ &= \frac{-2-\frac{16(2-0)}{1}}{1^2}=\frac{-2-32}{1}=-34\end{aligned}$$

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right|_{x=-2}=-34$$

Question7

If $y=\sqrt{\cosh x+\sqrt{\cosh x}}$, then $\frac{dy}{dx}=$

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Options:

A.

$\frac{\sinh x\left(2y^2+2\cosh x+1\right)}{4y(y^2+\cosh x)}$

B.

$\frac{\sinh x\left(2y^2-2\cosh x-1\right)}{4y(y^2-\cosh x)}$

C.

$\frac{\sinh x(1-2\sqrt{\cosh x})}{4y\sqrt{\cosh x}}$

D.

$\frac{\sinh x(1+2\sqrt{\cosh x})}{4y\sqrt{\cosh x)}}$

Answer: D

Solution:

$y=\sqrt{\cos hx+\sqrt{\cos hx}}$

Let $\sqrt{\cos hx}=u \Rightarrow y=\sqrt{u^2+u}$

Now, $\frac{dy}{dx}=\frac{1}{2\sqrt{u^2+u}}\cdot(2u+1)=\frac{2u+1}{2\sqrt{u^2+u}}$

$\frac{dy}{du}=\frac{2u+1}{2y}$

Now, $\frac{du}{dx}=\frac{d}{dx}(\sqrt{\cos hx})=\frac{1}{2\sqrt{\cosh x}}(\sin hx)$

$=\frac{\sin hx}{2u}$

Thus, $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{2u+1}{2y}\cdot\frac{\sin hx}{2u}$

$=\frac{(2u+1)\sin hx}{4yu}$

put $u=\sqrt{\cos hx}$

Therefore, $\frac{dy}{dx}=\frac{\sin hx(1+2\sqrt{\cos hx})}{4y\sqrt{\cos hx}}$

Question8

If $y=\tan^{-1}\sqrt{x^2-1}+\sinh^{-1}\sqrt{x^2-1}, x>1$, then $\frac{dy}{dx}=$

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Options:

A.

$\frac{1}{x\sqrt{x^2-1}}$

B.

$\frac{x+1}{x\sqrt{x^2-1}}$

C.

$\frac{x+1}{x^2\sqrt{x^2-1}}$

D.

$\frac{x}{\sqrt{x^2-1}}$

Answer: B

Solution:

$$y=\tan^{-1}\sqrt{x^2-1}+\sin h^{-1}\sqrt{x^2-1}$$
$$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(\tan^{-1}\sqrt{x^2-1}\right)+\frac{d}{dx}\left(\sin h^{-1}\sqrt{x^2-1}\right)$$
$$=\frac{1}{1+\left(\sqrt{x^2-1}\right)^2}\left(\frac{(2x)}{2\sqrt{x^2-1}}\right)+\frac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}}\left(\frac{2x}{2\sqrt{x^2-1}}\right)$$
$$=\frac{2x}{x^2\left(2\sqrt{x^2-1}\right)}+\frac{1}{x}\left(\frac{2x}{2\sqrt{x^2-1}}\right)$$
$$=\frac{1+x}{x\sqrt{x^2-1}}$$

Question9

If $y=(\log x)^{1/x}+x^{\log x}$, at $x=e$, $\frac{dy}{dx}=$

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Options:

A.

$2+\frac{1}{e}$

B.

$e^2+\frac{1}{2}$

C.

$\frac{1}{e^2}+2$

D.

$e+\frac{1}{e}$

Answer: C

Solution:

$$y=\left(\log_{\downarrow} x\right)^{\frac{1}{x}}+\underset{v\,v}{x^{\log x}}$$

$$\text{So, } \ln(u)=\frac{1}{x}\log(\log x)$$

$$\frac{1}{u}\cdot\frac{du}{dx}=\frac{1}{x}\cdot\frac{1}{x\log x}+\log(\log x)\cdot\left(-\frac{1}{x^2}\right)$$

$$=\frac{1}{x^2\log x}-\frac{\log(\log x)}{x^2}$$

$$\Rightarrow \left.\frac{du}{dx}\right|_{x=e}=(\log e)^{\frac{1}{e}}\left[\frac{1}{e^2\log e}-\frac{\log(\log e)}{e^2}\right]$$

$$=1\left[\frac{1}{e^2}-0\right]=\frac{1}{e^2}$$

and $\ln(y)=\log x\cdot\log(x)=(\log x)^2$

$$\Rightarrow \frac{1}{v}\left(\frac{dv}{dx}\right)=2\log(x)\cdot\frac{1}{x}$$

$$\Rightarrow \quad \frac{dv}{dx}=\eta\left[\frac{2\log x}{x}\right]$$

$at\ x=e$

$$=e^{\log x}\left[\frac{2\log x}{e}\right]$$

$$=e\cdot\frac{2}{e}=2$$

Hence, $\left.\frac{dy}{dx}\right|_{x=e}=\frac{1}{e^2}+2$

Question10

If $x=\sqrt{2}e^t(\sin t-\cos t)$ and $y=\sqrt{2}e^t(\sin t+\cos t)$, then $\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}}=$

Options:

A.

$-e^{\frac{-\pi}{4}}$

B.

$\sqrt{2}e^{\frac{\pi}{4}}$

C.

$\sqrt{2}e^{\frac{-\pi}{4}}$

D.

$e^{\frac{-\pi}{4}}$

Answer: A

Solution:

Given, $x=\sqrt{2}e^t(\sin t-\cos t)$

and $y=\sqrt{2}e^t(\sin t+\cos t)$

On differentiating both side, we get

$\therefore \quad \frac{dx}{dt}=\sqrt{2}e^t[2\sin t]$ and $\frac{dy}{dt}=\sqrt{2}e^t(2\cos t)$

$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\cot t$

Again, differentiating both side,

$$\begin{aligned}\frac{d^2y}{dx^2}&=\frac{d\cot t}{dx}=\frac{d\cot t}{dt}\times\frac{dt}{dx}\\&=-\operatorname{cosec}^2 t\times\frac{1}{\sqrt{2}e^t(2\sin t)}=\frac{-1}{2\sqrt{2}e^t\cdot\sin^3 t}\end{aligned}$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}}=\frac{-1}{2\sqrt{2}\times e^{\frac{\pi}{4}}\cdot\left(\frac{1}{\sqrt{2}}\right)^3}=-e^{\frac{-\pi}{4}}$$

Question11

If g is the inverse of the function $f(x)$ and $g(x) = x + \tan x$, then $f'(x) =$

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Options:

A.

$1 + \sec^2 x$

B.

$\frac{1}{1+\sec^2 f(x)}$

C.

$\frac{1}{1+\sec^2 g(x)}$

D.

$1 + \sec^2 f(x)$

Answer: B

Solution:

$g(x) = x + \tan x$
$g'(x) = 1 + \sec^2 x$

and g is the inverse of $f(x)$, then

$$\Rightarrow f'(x) = \frac{1}{g'(f(x))}$$
$$g(x) = x + \tan x, g'(x) = 1 + \sec^2 x$$
$$f'(x) = \frac{1}{1 + \sec^2(f(x))}$$

Question12

If $\sqrt{x - xy} + \sqrt{y - xy} = 1$, then $\frac{dy}{dx} =$

Options:

A.

$-\sqrt{\frac{y-y^2}{x-x^2}}$

B.

$-\sqrt{\frac{1-y^2}{1-x^2}}$

C.

$-\sqrt{\frac{1-y}{1-x}}$

D.

$-\sqrt{\frac{x-y}{x+y}}$

Answer: A

Solution:

Given, $\sqrt{x(1-y)}+\sqrt{y(1-x)}=1$

$$\begin{aligned}
&\text{Let } x=\sin^2 A \text{ and } y=\sin^2 B \\
&\Rightarrow \sqrt{\sin^2 A\left(1-\sin^2 B\right)}+\sqrt{\sin^2 B\left(1-\sin^2 A\right)} \\
&\Rightarrow \quad \sin A \cos B+\cos A \sin B=1 \\
&\Rightarrow \quad \sin(A+B)=1=\sin\frac{\pi}{2} \\
&\Rightarrow \quad A+B=\frac{\pi}{2} \qquad\qquad \ldots(i)
\end{aligned}$$

Now, $A=\sin^{-1}\sqrt{x}, B=\sin^{-1}\sqrt{y}$

From Eqs. (i), we get

$\sin^{-1}\sqrt{x}+\sin^{-1}\sqrt{y}=\frac{\pi}{2}$

On differentiating,

$$\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{1-y}} \cdot \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{2\sqrt{y}\sqrt{1-y}} \frac{dy}{dx} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}\sqrt{1-y}}{\sqrt{x}} \sqrt{1-x} = -\sqrt{\frac{y-y^2}{x-x^2}}$$

Question13

If $x = 2\cos^3\theta$ and $y = 3\sin^2\theta$, then $\frac{dy}{dx} =$

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Options:

A.

$-\sec\theta$

B.

$\cos\theta$

C.

$-\operatorname{cosec}\theta$

D.

$\sin\theta$

Answer: A

Solution:

$$x = 2\cos^3\theta, \dot{y} = 3\sin^2\theta$$

$$\Rightarrow \frac{dx}{dy} = \frac{\frac{dx}{d\theta}}{\frac{dy}{d\theta}} \Rightarrow \frac{dx}{dy} = \frac{6\cos^2\theta(-\sin\theta)}{6\sin\theta(\cos\theta)}$$

$$\Rightarrow \frac{dy}{dx} = -\sec\theta$$

Question14

Assertion (A) If $y = f(x) = (|x| - |x - 1|)^2$, then $\left(\frac{dy}{dx}\right)_{x=1} = 1$

Reason (R) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exist, then it is called derivative of $f(x)$ at $x = a$.

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Options:

A.

(A) is true, (R) is true, (R) is correct explanation to (A)

B.

(A) is true, (R) is true, (R) is not the correct explanation to (A)

C.

(A) is true, (R) is false

D.

(A) if false, (R) is true

Answer: D

Solution:

$y = f(x) = \left(|x| - |x - 1|^2\right)$

Here, $f(x)$ is not differentiable at $x = 0$,

$\therefore$ A is false.

R is definition of derivative $f(x)$ at $x = a$

$\therefore$ R is true.

Question15

If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then $\frac{dy}{dx}=$

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Options:

A.

$\frac{y}{x}$

B.

$\frac{y^2}{x^2}$

C.

$\sqrt{\frac{y}{x}}$

D.

$-\frac{y}{x}$

Answer: D

Solution:

Given $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$

$$\begin{aligned} \because \quad & \left(x^2+y^2\right)^2=\left(t-\frac{1}{t}\right)^2 \\ \Rightarrow \quad & x^4+y^4+2x^2y^2=t^2+\frac{1}{t^2}-2 \\ \Rightarrow \quad & x^4+y^4+2x^2y^2=x^4+y^4-2 \\ \Rightarrow \quad & x^2y^2=-1 \\ \Rightarrow \quad & x^2 2yy'+y^2\cdot 2x=0 \\ \Rightarrow \quad & xy'+y=0 \\ \Rightarrow \quad & y'=-\frac{y}{x} \\ \therefore \quad & \frac{dy}{dx}=-\frac{y}{x} \end{aligned}$$

Question16

If $y=(ax+b)\cos x$, then

$$y_2+y_1\sin 2x+y\left(1+\sin^2 x\right)=$$

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Options:

A.

$y_2\cos^2 x$

B.

$y_2\sin^2 x$

C.

$y_1\sin^2 x$

D.

$y\sin^2 x$

Answer: B

Solution:

We have, $y=(ax+b)\cos x$

$y\sec x=ax+b$

$y\sec x\tan x+\sec x y_1=a$

Again, differentiating w.r.t. x, we get
$\Rightarrow y\sec x\sec^2 x+\sec x\tan x\cdot y_1+y\tan x\sec x\tan x+\sec x y_2+y_1\sec x\tan x=0$

$\Rightarrow y\sec^3 x+y_1\sec x\tan x+y\sec x\tan^2 x+y_1\sec x\tan x+\sec x\cdot y_2=0$

$\Rightarrow y_2\sec x+2y_1\sec x\tan x+y\left(\sec^2 x+\tan^2 x\right)\sec x=0$

On dividing both sides by $\sec^3 x$, we get $y_2\cos^2 x+2y_1\sin x\cdot\cos x+y\left(1+\sin^2 x\right)=0$

$y_2-y_1\sin^2 x+y_1\sin 2x+y\left(1+\sin^2 x\right)=0$

$\therefore y_2+y_1\sin 2x+y\left(1+\sin^2 x\right)=y_2\sin^2 x$

Question17

If $5f(x)+3f\left(\frac{1}{x}\right)=x+2$ and $y=xf(x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to

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Options:

A.

14

B.

$\frac{7}{8}$

C.

1

D.

7

Answer: B

Solution:

Given, $5f(x)+3f\left(\frac{1}{x}\right)=x+2 \qquad \ldots(i)$

replacing x by $\frac{1}{x}$

$5f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x}+2 \qquad \ldots(ii)$

Multiplying Eq. (i) by 5 and Eq. (ii) by 3, we get

$$25f(x)+15f\left(\frac{1}{x}\right)=5x+10$$

$$15f\left(\frac{1}{x}\right)+9f(x)=\frac{3}{x}+6$$

$$-\qquad +\qquad -$$

$$16f(x)=5x-\frac{3}{x}+4$$

$$\because \quad f(x)=\frac{1}{16}\left(5x-\frac{3}{x}+4\right)$$

$$y=xf(x)=\frac{x}{16}\left(5x-\frac{3}{x}+4\right)$$

$$y=\frac{1}{16}\left(5x^2-3+4x\right)$$

$$\frac{dy}{dx}=\frac{1}{16}(10x+4)$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1}=\frac{10+4}{16}$$

$$=\frac{14}{16}=\frac{7}{8}$$

Question18

If $y=\sinh^{-1}\left(\frac{1-x}{1+x}\right)$, then $\frac{dy}{dx}$ is equal to

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Options:

A. $\frac{-\sqrt{2}}{|1+x|\sqrt{1+x^2}}$

B. $\frac{-1}{(1+x)\sqrt{x}}$

C. $\frac{1}{(1+x^2)\sqrt{1+x}}$

D. $\frac{-\sqrt{2}}{(1+x)\sqrt{1-x}}$

Answer: A

Solution:

To find the derivative $\frac{dy}{dx}$ for $y=\sinh^{-1}\left(\frac{1-x}{1+x}\right)$, we use the formula for the derivative of the inverse hyperbolic sine function:

$\frac{d}{dx}\left(\sinh^{-1}x\right)=\frac{1}{\sqrt{1+x^2}}$

Applying this to our function:

Identify $u=\frac{1-x}{1+x}$.

Differentiate u with respect to x:

$\frac{d}{dx}\left(\frac{1-x}{1+x}\right) = \frac{(1+x)(-1)-(1-x)}{(1+x)^2} = \frac{-1-x-1+x}{(1+x)^2} = \frac{-2}{(1+x)^2}$

Now apply the chain rule:

$\frac{dy}{dx} = \frac{1}{\sqrt{1+\left(\frac{1-x}{1+x}\right)^2}} \times \frac{du}{dx}$

Simplify the expression:

Compute $1+\left(\frac{1-x}{1+x}\right)^2$:

$1+\left(\frac{1-x}{1+x}\right)^2 = \frac{(1+x)^2+(1-x)^2}{(1+x)^2}$

Note that $(1+x)^2+(1-x)^2 = 2(1+x^2)$

Plug this into the derivative formula:

$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{2(1+x^2)}{(1+x)^2}}} \times \frac{-2}{(1+x)^2}$

Simplify further to get the final derivative:

$= \frac{-2}{\sqrt{2(1+x^2)}\cdot(1+x)} = \frac{-\sqrt{2}}{|1+x|\sqrt{1+x^2}}$

Thus, the derivative $\frac{dy}{dx}$ is:

$\frac{-\sqrt{2}}{|1+x|\sqrt{1+x^2}}$

Question19

If $y = (x-1)(x+2)\left(x^2+5\right)\left(x^4+8\right)$, then $\lim_{x\to-1}\left(\frac{dy}{dx}\right)$ is equal to

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Options:

A. -30

B. 30

C. 52

D. -52

Answer: B

Solution:

Given,

$$y=(x-1)(x+2)\left(x^2+5\right)\left(x^4+8\right)$$
$$\Rightarrow y=\left(x^4+x^3+3x^2+5x-10\right)\left(x^4+8\right)$$
$$\Rightarrow \frac{dy}{dx}=\left(x^4+x^3+3x^2+5x-10\right)\left(4x^3\right)$$
$$+\left(4x^3+3x^2+6x+5\right)\left(x^4+8\right)$$
$$\therefore \lim_{x\rightarrow-1}\left(\frac{dy}{dx}\right)=(1-1+3-5-10)(-4)$$
$$+(-4+3-6+5)(1+8)$$
$$=(-12)(-4)+(-2)(9)$$
$$=48-18=30$$

Question20

If $y=\left(\tan^{-1}2x\right)^2+\left(\cot^{-1}2x\right)^2$, then $\left(1+4x^2\right)^2y''-16$ is equal to

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Options:

A. $8xy'$

B. $-8x\left(1+4x^2\right)y'$

C. $8x\left(1+4x^2\right)y'$

D. $-8xy'$

Answer: B

Solution:

To find the expression for $(1+4x^2)^2y''-16$, we start with

$y=\left(\tan^{-1}2x\right)^2+\left(\cot^{-1}2x\right)^2.$

Using the identity $\cot^{-1}u=\frac{\pi}{2}-\tan^{-1}u$, we can rewrite the equation as

$y=\left(\tan^{-1}2x\right)^2+\left(\frac{\pi}{2}-\tan^{-1}2x\right)^2.$

Simplifying,

$y=2\left(\tan^{-1}2x\right)^2+\frac{\pi^2}{4}-\pi\tan^{-1}2x.$

Next, we differentiate y with respect to x:

$y' = \frac{4\tan^{-1}2x}{1+4x^2} \cdot 2 - \frac{\pi}{1+4x^2} \cdot 2.$

This simplifies to

$y' = \frac{8\tan^{-1}2x - 2\pi}{1+4x^2}.$

Multiplying both sides by $(1+4x^2)$, we get

$(1+4x^2)y' = 8\tan^{-1}2x - 2\pi.$

Differentiating this equation with respect to x gives:

$(1+4x^2)y'' + y'(8x) = \frac{16}{1+4x^2},$

Rearranging this result, we find:

$(1+4x^2)^2y'' + 8xy'(1+4x^2) = 16.$

Thus,

$(1+4x^2)^2y'' - 16 = -8xy'(1+4x^2).$

Question21

If $y = \tan^{-1}\frac{x}{1+2x^2} + \tan^{-1}\frac{x}{1+6x^2} + \tan^{-1}\frac{x}{1+12x^2}$, then $\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}}$ is equal to

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Options:

A. 1

B. -1

C. 0

D. 1/2

Answer: C

Solution:

Given,
$y=\tan^{-1}\frac{x}{1+2x^2}+\tan^{-1}\frac{x}{1+6x^2}$

$+\tan^{-1}\frac{x}{1+12x^2}$
$\Rightarrow \frac{dy}{dx}=\frac{1}{1+\left(\frac{x}{1+2x^2}\right)^2}\cdot\frac{(1+2x^2)\cdot 1-x\cdot 4x}{(1+2x^2)^2}$

$$+\frac{1}{1+\left(\frac{x}{1+6x^2}\right)^2}\times\frac{(1+6x^2)\cdot 1-\times(12,x)}{(1+6x^2)^2}$$
$$+\frac{1}{1+\left(\frac{x}{1+12x^2}\right)^2}\cdot\frac{(1+12x^2)\cdot 1-x\cdot 24x}{(1+12x^2)^2}$$
put, $x=\frac{1}{2}$ and simplify
$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}}=0$$

Question22

If $f(x)=5\cos^3 x-3\sin^2 x$ and $g(x)=4\sin^3 x+\cos^2 x$, then the derivative of $f(x)$ with respect to $g(x)$ is

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Options:

A. $\frac{5\cos+2}{6\cos x-1}$

B. $-\left(\frac{5\cos x+2}{6\cos x-1}\right)$

C. $\frac{15\cos x-6}{12\sin x+2}$

D. $-\left(\frac{15\cos x+6}{12\sin x-2}\right)$

Answer: D

Solution:

Given that

$f(x)=5\cos^3 x-3\sin^2 x$

and $g(x) = 4\sin^3 x + \cos^2 x$

Now, $\frac{df(x)}{dx} = -15\cos^2 x \cdot \sin x - 6\sin x\cos x$

and $\frac{dg(x)}{dx} = 12\sin^2 x \cdot \cos x - 2\cos x\sin x$
Now, $\frac{df(x)}{dg(x)} = \frac{df(x)}{dx} \times \frac{dx}{dg(x)}$

$$= -\frac{15\cos^2 x\sin x - 6\sin x\cos x}{12\sin^2 x\cos x - 2\cos x\sin x}$$
$$= \frac{-\cos x\sin x(15\cos x + 6)}{-\cos x\sin x(-12\sin x + 2)}$$
$$= -\left(\frac{15\cos x + 6}{12\sin x - 2}\right)$$

Question23

If $y = 1 + x + x^2 + x^3 + \ldots\ldots\infty$ and $|x| < 1$, then y'' is equal to

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Options:

A. $2y'$

B. $\frac{2y}{y'}$

C. $\frac{y'}{2y}$

D. $2y^2y'$

Answer: A

Solution:

Given the series:

$y = 1 + x + x^2 + x^3 + \ldots$

and noting that $|x| < 1$, we recognize this as a geometric series. The sum of this series can be expressed in closed form as:

$y = \frac{1}{1-x}$

Now, let's find the first derivative of y with respect to x:

$y' = \left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}$

Next, differentiate y' to find the second derivative y'':

$$y'' = \left(\frac{1}{(1-x)^2}\right)' = \frac{2}{(1-x)^3}$$

Since we have $y = \frac{1}{1-x}$, it follows that:

$$y'' = 2 \cdot \frac{1}{1-x} \cdot \frac{1}{(1-x)^2}$$

This can be rewritten using the previously calculated derivatives:

$$y'' = 2y \cdot y'$$

Thus, the second derivative of y, y'', is equal to:

$$y'' = 2y \cdot y'$$

Question24

If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, then the value of $\frac{d^2y}{dx^2}$ at the point $(\pi, 1)$ is

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Options:

A. 2

B. -2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer: B

Solution:

Given the expression:

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}$$

We will evaluate $\frac{d^2y}{dx^2}$ at the point $(\pi, 1)$.

Step 1: Determine y

Squaring both sides of the equation:

$$y^2 = \sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}$$

Therefore, from the self-similarity of the expression, we have:

$$y^2 - \sin x = y \quad \Rightarrow \quad y^2 - y = \sin x$$

Step 2: First Derivative $\frac{dy}{dx}$

Differentiating both sides with respect to x:

$$2y\frac{dy}{dx} - \cos x = \frac{dy}{dx}$$

Rearranging, we get:

$$\frac{dy}{dx}(2y-1) = \cos x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Step 3: Second Derivative $\frac{d^2y}{dx^2}$

Differentiating the equation for $\frac{dy}{dx}$ again:

$$2\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) + \sin x = \frac{d^2y}{dx^2}$$

Rearranging:

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{\cos x}{2y-1}\right)^2 + \sin x = \frac{d^2y}{dx^2}$$

Simplifying:

$$(2y-1)\frac{d^2y}{dx^2} = -\sin x - \frac{2\cos^2 x}{(2y-1)^2}$$

Thus:

$$\frac{d^2y}{dx^2} = \frac{-\sin x - \frac{2\cos^2 x}{(2y-1)^2}}{2y-1}$$

Final Calculation at $(\pi, 1)$

At the point $(\pi, 1)$:

$$\sin \pi = 0$$

$$\cos \pi = -1$$

$$y = 1$$

Substituting these values into the formula for the second derivative:

$$\frac{d^2y}{dx^2} = \frac{0 - \frac{2\times(-1)^2}{(2\times1-1)^2}}{1} = \frac{-2}{1} = -2$$

Therefore, $\frac{d^2y}{dx^2}$ at $(\pi, 1)$ is -2.

Question25

If $f(0)=0, f'(0)=3$, then the derivative of $y=f(f(f(f(f(x)))))$ at $x=0$ is

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Options:

A.

16

B.

32

C.

81

D.

243

Answer: D

Solution:

We are given the function $y=f(f(f(f(f(x)))))$ and need to find its derivative at $x=0$, where we know $f(0)=0$ and $f'(0)=3$.

To find $y'(0)$, apply the chain rule:

$y'=f'(f(f(f(f(x))))) \cdot f'(f(f(f(x)))) \cdot f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x)$

Substituting $x=0$, we find:

$y'(0)=f'(f(f(f(f(0))))) \cdot f'(f(f(f(0)))) \cdot f'(f(f(0))) \cdot f'(f(0)) \cdot f'(0)$

Since $f(0)=0$, this simplifies to:

$y'(0)=f'(0) \cdot f'(0) \cdot f'(0) \cdot f'(0) \cdot f'(0)$

Given $f'(0)=3$, this becomes:

$y'(0) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5 = 243$

Thus, the derivative of y at $x = 0$ is 243.

Question26

If $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = ax + b$, then $(a, b) =$

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Options:

A. $(-1, 2)$

B. $(-2, 1)$

C. $(2, -1)$

D. $(1, 2)$

Answer: C

Solution:

To find (a, b) such that

$\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = ax + b$

we start by simplifying the expression in the numerator.

Simplification

The numerator $1 + x^2 + x^4$ can be factored:

$$\begin{aligned} 1 + x^2 + x^4 &= x^4 + 2x^2 + 1 - x^2 \\ &= (x^2 + 1)^2 - x^2 \\ &= (x^2 + 1 - x)(x^2 + 1 + x). \end{aligned}$$

Substituting Back

Using the factorized form of the numerator, rewrite the original expression:

$\frac{d}{dx}\left(\frac{(x^2-x+1)(x^2+x+1)}{1+x+x^2}\right) = ax + b.$

We notice that $(x^2 + x + 1)$ cancels out with the denominator:

$\frac{d}{dx}(x^2 - x + 1) = ax + b.$

Differentiation

Calculating the derivative of the simplified expression $x^2 - x + 1$:

$\frac{d}{dx}(x^2 - x + 1) = 2x - 1.$

Identifying Constants

Set the expression for derivative equal to $ax + b$:

$2x - 1 = ax + b.$

By comparing coefficients, we get:

$a = 2, \quad b = -1.$

Thus, the values are:

$(a, b) = (2, -1).$

Question27

The rate of change of $x^{\sin x}$ with respect to $(\sin x)^x$ is

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Options:

A. $\frac{x^{\sin x}\left(\frac{\sin x}{x}+\cos x \cdot \log x\right)}{(\sin x)^x(x \cdot \cot x+\log \sin x)}$

B. $\frac{(\sin x)^x(x \cot x+\log \sin x)}{x^{\sin x}\left(\frac{\sin x}{x}+\cos x \cdot \log x\right)}$

C. $y\left(\frac{\sin x}{x}+\cos x \log x\right)$

D. $(\sin x)^x(x \cot x+\log \sin x)$

Answer: A

Solution:

To find the rate of change of $x^{\sin x}$ with respect to $(\sin x)^x$, we proceed as follows:

Let $y = x^{\sin x}$.

Taking the natural logarithm on both sides, we have:

$\log y = \sin x \log x$

Differentiating with respect to x, we get:

$\frac{1}{y} \frac{dy}{dx} = \cos x \log x + \frac{\sin x}{x}$

Let $z = (\sin x)^x$.

Taking the logarithm similarly, we have:

$\log z = x \log \sin x$

Differentiating with respect to x, we have:

$\frac{1}{z} \frac{dz}{dx} = \log \sin x + x \frac{\cos x}{\sin x}$

Simplifying this gives:

$\frac{1}{z} \frac{dz}{dx} = \log \sin x + x \cot x$

Next, we find the derivative of y with respect to z by dividing the derivatives:

$\frac{dz}{dy} = \frac{yddx}{zddx} = \frac{\cos x \log x + \frac{\sin x}{x}}{\log \sin x + x \cot x}$

Thus, the rate of change is:

$\frac{dy}{dz} = \frac{x^{\sin x}}{(\sin x)^x} \left(\frac{\log x \cos x + \frac{1}{x} \sin x}{\log \sin x + x \cot x} \right)$

Question28

If $y = \frac{\alpha x + \beta}{\gamma \alpha + \delta}$, then $2 y_1 y_3 =$

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Options:

A. $2{y_2}^3$

B. $3{y_2}^2$

C. ${y_2}^2$

D. $3{y_3}^2$

Answer: B

Solution:

$y=\frac{\alpha x+\beta}{\gamma x+\delta}$ differentiation w.r.t. x, we get

$y_1=\frac{\alpha\delta-\beta\gamma}{(\gamma x+\delta)^2}$

Again, differentiation with respect to x, we get

$y_2=(\alpha\delta-\beta\gamma)\left(\frac{-2\gamma}{(\gamma x+\delta)^3}\right)$

Again, differentiation with respect to x, we get

$$y_3=(\alpha\delta-\beta\gamma)\left(\frac{6\gamma^2}{(\gamma x+\delta)^4}\right)$$
$$y_3=\frac{(\alpha\delta-\beta\gamma)}{(\gamma x+\delta)^2}\cdot\frac{6\gamma}{(\gamma x+\delta)^2}$$

On squaring both sides of Eq. (ii), we get

$$y_2^2=(\alpha\delta-\beta\gamma)^2\cdot\frac{4\gamma^2}{(\gamma x+\delta)^6}$$
$$y_2^2=\frac{(\alpha\delta-\beta\gamma)}{(\gamma x+\delta)^2}\cdot\frac{4\gamma^2}{(\gamma x+\delta)^4}$$
$$y_2^2=\frac{(\alpha\delta-\beta\gamma)}{(\gamma\alpha+\delta)^2}\cdot\frac{(\alpha\delta-\beta\gamma)\cdot 6\gamma^2}{(\gamma x+\delta)^4}\cdot\frac{4}{6}$$

Here, from Eqs. (i) and (iii)

$y_2^2=y_1y_3\cdot\left(\frac{2}{3}\right)$

$$y_2^2=\frac{2}{3}y_1y_3$$
$$y_2^2=\frac{2}{3}y_1y_3$$

Now, $2y_1y_3=3y_2^2$

Question29

Which one of the following is false ?

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Options:

A. $\frac{d}{dx}\left[\sec^{-1}(\cosh x)\right]=\operatorname{sech} x$

B. $\frac{d}{dx}\left[\cos^{-1}(\operatorname{sech} x)\right]=\operatorname{sech} x$

C. $\frac{d}{dx}\left[\tan^{-1}(\sinh x)\right]=\operatorname{sech} x$

D. $\frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{x}{2}\right)\right]=\sec x$

Answer: D

Solution:

The following explanations clarify the derivatives being calculated:

(A) $\frac{d}{dx}\left[\sec^{-1}(\cosh x)\right]=\operatorname{sech}(x)$

For the inverse secant function, the derivative is given by:

$$\frac{d}{dx}\left[\sec^{-1}(\theta)\right]=\frac{1}{|\theta|\sqrt{\theta^2-1}}$$

Substituting $\theta=\cosh x$:

$$\frac{d}{dx}\left[\sec^{-1}(\cosh x)\right]=\frac{1}{|\cosh x|\sqrt{(\cosh x)^2-1}}\cdot\sinh x$$

Since $|\cosh x|=\cosh x$ for all $x\in\mathbb{R}$ and $(\cosh x)^2-1=\sinh^2 x$, we find:

$$\frac{d}{dx}\left[\sec^{-1}(\cosh x)\right]=\frac{\sinh x}{\cosh x\sinh x}=\frac{1}{\cosh x}=\operatorname{sech}(x)$$

This statement is true.

(B) $\frac{d}{dx}\left[\cos^{-1}(\operatorname{sech} x)\right]=\operatorname{sech} x$

The derivative of the inverse cosine function is:

$$\frac{d}{dx}\left[\cos^{-1}(\theta)\right]=-\frac{1}{\sqrt{1-\theta^2}}$$

Substituting $\theta=\operatorname{sech} x$:

$$\frac{d}{dx}\left[\cos^{-1}(\operatorname{sech} x)\right]=-\frac{\operatorname{sech} x\tanh x}{\sqrt{1-(\operatorname{sech} x)^2}}$$

Since $1-\operatorname{sech}^2 x=\tanh^2 x$, we have:

$$\frac{d}{dx}\left[\cos^{-1}(\operatorname{sech} x)\right]=\operatorname{sech} x$$

This statement is true.

(C) $\frac{d}{dx}\left[\tan^{-1}(\sinh x)\right]=\operatorname{sech} x$

The derivative of the inverse tangent function is:

$$\frac{d}{dx}\tan^{-1}(\theta)=\frac{1}{1+\theta^2}$$

Substituting $\theta=\sinh x$:

$$\frac{d}{dx}\left[\tan^{-1}(\sinh x)\right]=\frac{1}{1+(\sinh x)^2}$$

Knowing that $\cosh^2 x-\sinh^2 x=1$, it simplifies to:

$$\frac{\cosh x}{1+\sinh^2 x}=\operatorname{sech} x$$

This statement is true.

(D) $\frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{x}{2}\right)\right] = \sec x$

The expression simplifies as:

$\tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2} \quad \text{for } x \in (-\pi, \pi)$

Thus, the derivative is:

$\frac{d}{dx}\left[\tan^{-1}\left(\tan\frac{x}{2}\right)\right] = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$

And $\frac{1}{2} \neq \sec x$, rendering this statement false.

Question30

If $y = t^2 + t^3$ and $x = t - t^4$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is

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Options:

A. $\frac{-2}{3}$

B. $\frac{-4}{3}$

C. $\frac{8}{3}$

D. 4

Answer: B

Solution:

Given the functions $y = t^2 + t^3$ and $x = t - t^4$, we need to find $\frac{d^2y}{dx^2}$ at $t = 1$.

First, find the derivatives with respect to t:

$\frac{dy}{dt} = 2t + 3t^2$

$\frac{dx}{dt} = 1 - 4t^3$

Next, use these derivatives to find $\frac{dy}{dx}$:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t+3t^2}{1-4t^3}$

To find $\frac{d^2y}{dx^2}$, differentiate $\frac{dy}{dx}$ with respect to t and multiply by $\frac{dt}{dx}$:

$$\frac{d^2y}{dx^2} = \frac{(1-4t^3)(2+6t)-(2t+3t^2)(-12t^2)}{(1-4t^3)^2} \cdot \frac{dt}{dx}$$

Substitute $\frac{dt}{dx} = \frac{1}{\frac{dx}{dt}}$:

$$= \frac{(2+6t)(1-4t^3)+12t^2(2t+3t^2)}{(1-4t^3)^2} \cdot \frac{1}{1-4t^3}$$

Evaluate at $t = 1$:

$$\frac{d^2y}{dx^2} = \frac{(2+6)(1-4)+12(2+3)}{(1-4)^2} \cdot \frac{1}{1-4}$$

Simplify the expression:

$$= \frac{(-24+60)}{-27} = \frac{36}{-27} = -\frac{4}{3}$$

Therefore, $\frac{d^2y}{dx^2}$ at $t = 1$ is $-\frac{4}{3}$.

Question31

If $y = \tan(\log x)$, then $\frac{d^2y}{dx^2} =$

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Options:

A. $\frac{-\sec^2(\log x)[1+2\tan x]}{x^2}$

B. $\frac{\sec^2(\log x)[1+\tan(\log x)]}{x^2}$

C. $\frac{\sec(\log x)[2\tan(\log x)-1]}{x^2}$

D. $\frac{\sec^2(\log x)[2\tan(\log x)-1]}{x^2}$

Answer: D

Solution:

To find the second derivative of $y = \tan(\log x)$, follow these steps:

First Derivative:

Start by differentiating $y = \tan(\log x)$ with respect to x. Apply the chain rule:

$\frac{dy}{dx} = \frac{d}{dx}[\tan(\log x)] = \sec^2(\log x) \cdot \frac{d}{dx}[\log x] = \frac{\sec^2(\log x)}{x}$

Second Derivative:

To find the second derivative, apply the quotient rule to $\frac{dy}{dx} = \frac{\sec^2(\log x)}{x}$. The quotient rule is given by:

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

Here, let $u = \sec^2(\log x)$ and $v = x$.

Then,

$\frac{du}{dx} = 2\sec^2(\log x)\tan(\log x) \cdot \frac{1}{x} = \frac{2\sec^2(\log x)\tan(\log x)}{x}$

$\frac{dv}{dx} = 1$

Substitute into the quotient rule:

$\frac{d^2y}{dx^2} = \frac{x \cdot \left(\frac{2\sec^2(\log x)\tan(\log x)}{x}\right) - \sec^2(\log x) \cdot 1}{x^2}$

Simplify the expression:

$\frac{d^2y}{dx^2} = \frac{2\sec^2(\log x)\tan(\log x) - \sec^2(\log x)}{x^2}$

$= \frac{\sec^2(\log x)[2\tan(\log x) - 1]}{x^2}$

Thus, the second derivative is:

$\frac{d^2y}{dx^2} = \frac{\sec^2(\log x)[2\tan(\log x) - 1]}{x^2}$

Question32

For $x < 0$, $\frac{d}{dx}[|x|^x] =$

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Options:

A. $(-x)^x[-1 + \log(-x)]$

B. $(-x)^x[1 + \log(-x)]$

C. $(-x)^x[1 - \log(-x)]$

D. $(-x)^x[-1 - \log(-x)]$

Answer: B

Solution:

For $x < 0$, we need to find the derivative $\frac{d}{dx}[|x|^x]$.

Start by letting $y = (-x)^x$. Since $x < 0$, we have $|x| = -x$.

We proceed by taking the natural logarithm of both sides:

$\log y = x \log(-x)$

Now, differentiate both sides with respect to x:

$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(x \log(-x))$

Use the product rule to differentiate the right side:

$\frac{d}{dx}(x \log(-x)) = \log(-x) + x \cdot \frac{d}{dx}(\log(-x))$

Since $\frac{d}{dx}(\log(-x)) = \frac{-1}{-x} = \frac{1}{x}$, we have:

$\frac{d}{dx}(x \log(-x)) = \log(-x) - 1$

Therefore:

$\frac{1}{y} \frac{dy}{dx} = \log(-x) - 1$

Multiplying both sides by $y = (-x)^x$:

$\frac{dy}{dx} = (-x)^x(\log(-x) - 1)$

Simplify to:

$\frac{dy}{dx} = (-x)^x[1 + \log(-x)]$

Question33

If $y = x - x^2$, then the rate of change of y^2 with respect to x^2 at $x = 2$ is

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Options:

A. 0

B. -1

C. 3

D. 9

Answer: C

Solution:

To find the rate of change of y^2 with respect to x^2, we begin with the given function:

$y = x - x^2$

We introduce new variables $u = y^2$ and $v = x^2$. Thus:

$u = (x - x^2)^2$

First, we find the derivatives $\frac{du}{dx}$ and $\frac{dv}{dx}$:

$$\frac{du}{dx} = 2(x - x^2)(1 - 2x)$$
$$\frac{dv}{dx} = 2x$$

Now, the rate of change of y^2 with respect to x^2 is:

$\frac{dy^2}{dx^2} = \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2(x-x^2)(1-2x)}{2x} = (1 - x)(1 - 2x)$

We evaluate this expression at $x = 2$:

$(1 - 2)(1 - 4) = (-1)(-3) = 3$

Thus, the derivative of y^2 with respect to x^2 at $x = 2$ is 3.

Question34

If $y = f(x)$ is a thrice differentiable function and a bijection, then $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} =$

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Options:

A. y

B. $-y$

C. x

D. 0

Answer: D

Solution:

If $y = f(x)$ is a thrice differentiable function and a bijection, we want to evaluate the expression $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2}$.

First, we know:

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

To find the second derivative of x with respect to y, we differentiate:

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{dy}{dx}\right)^{-1}$$

Applying the chain rule:

$$= -\left(\frac{dy}{dx}\right)^{-2} \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) \cdot \frac{1}{\frac{dy}{dx}}$$

Rearranging terms, we have:

$$= -\left(\frac{dx}{dy}\right)^2 \cdot \frac{d^2y}{dx^2} \cdot \left(\frac{dx}{dy}\right)$$

Thus, simplifying further:

$$= -\left(\frac{dx}{dy}\right)^3 \cdot \frac{d^2y}{dx^2}$$

This simplifies our original expression:

$$\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 = -\frac{d^2y}{dx^2}$$

Therefore:

$$\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = 0$$

Question35

If $y = \tan^{-1}\left(\frac{2-3\sin x}{3-2\sin x}\right)$, then $\frac{dy}{dx} =$

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Options:

A. $\frac{(3-2\sin x)^2}{13\sin^2 x-24\sin x+13}$

B. $\frac{-5 \cos x}{13 \sin^2 x-24 \sin x+19}$

C. $\frac{5 \sin x}{13 \sin^2 x-24 \sin x+13}$

D. $\frac{-5 \sin x}{13 \sin^2 x-24 \sin x+13}$

Answer: B

Solution:

$y=\tan^{-1}\left(\frac{2-3 \sin x}{3-2 \sin x}\right)$

$\frac{dy}{dx}=\frac{1}{1+\left(\frac{2-3 \sin x}{3-2 \sin x}\right)^2}$

$\left\{\frac{(3-2 \sin x)(-3 \cos x)-(2-3 \sin x)(-2 \cos x)}{(3-2 \sin x)^2}\right\}$

$=\frac{-9 \cos x+6 \sin x \cos x+4 \cos x-6 \sin x \cos x}{(3-2 \sin x)^2+(2-3 \sin x)^2}$

$=\frac{-5 \cos x}{9+4 \sin^2 x-12 \sin x+4+9 \sin^2 x-12 \sin x}$

$=\frac{-5 \cos x}{13+13 \sin^2 x-24 \sin x}$

Question36

If $x=3\left[\sin t-\log \left(\cot \frac{t}{2}\right)\right]$ **and** $y=6\left[\cos t+\log \left(\tin \frac{t}{2}\right)\right]$ **then** $\frac{dy}{dx}=$

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Options:

A. $\frac{2 \sin^2 t}{1+\sin t \cos t}$

B. $\frac{2 \cos^2 t}{1+\sin 2t}$

C. $\frac{2 \cos^2 t}{1+\sin t \cos t}$

D. $\frac{1+\cos g}{1+\sin a}$

Answer: C

Solution:

To determine $\frac{dy}{dx}$, we begin with the expressions given for x and y.

$x = 3\left[\sin t - \log\left(\cot \frac{t}{2}\right)\right]$

Differentiating x with respect to t:

$$\begin{aligned}\frac{dx}{dt} &= 3\left[\cos t - \frac{1}{\cot \frac{t}{2}} \cdot \left(-\csc^2 \frac{t}{2}\right) \cdot \frac{1}{2}\right] \\ &= 3\left[\cos t + \frac{1}{\sin t}\right] \\ &= 3\left[\frac{\cos t \sin t + 1}{\sin t}\right]\end{aligned}$$

Next, differentiating y with respect to t:

$y = 6\left[\cos t + \log\left(\tan \frac{t}{2}\right)\right]$

$$\begin{aligned}\frac{dy}{dt} &= 6\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \cdot \frac{1}{2}\right] \\ &= 6\left[-\sin t + \frac{1}{\sin t}\right] \\ &= 6\left[\frac{-\sin^2 t + 1}{\sin t}\right]\end{aligned}$$

Now, to find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6\left[\frac{-\sin^2 t+1}{\sin t}\right]}{3\left[\frac{\cos t \sin t+1}{\sin t}\right]}$$

The $\sin t$ terms cancel out:

$$= \frac{6(-\sin^2 t+1)}{3(\cos t \sin t+1)}$$

Simplifying further:

$$= \frac{2(-\sin^2 t+1)}{\cos t \sin t+1}$$

Rewriting the numerator, $1 - \sin^2 t = \cos^2 t$:

$$= \frac{2\cos^2 t}{\cos t \sin t+1}$$

Therefore, the expression for $\frac{dy}{dx}$ is:

$$\frac{dy}{dx} = \frac{2\cos^2 t}{1+\sin t \cos t}$$

Question37

The length of the tangent drawn at the point $P\left(\frac{\pi}{4}\right)$ on the curve $x^{2/3}+y^{2/3}=2^{2/3}$ is

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Options:

A. $\frac{2}{3}$

B. 1

C. $\frac{4}{3}$

D. 2

Answer: B

Solution:

To find the length of the tangent at the point $P\left(\frac{\pi}{4}\right)$ on the curve $x^{2/3}+y^{2/3}=2^{2/3}$, we can proceed as follows:

The given equation of the curve can be parameterized as:

$x=2\cos^3\theta, \quad y=2\sin^3\theta$

For the point P, we substitute $\theta=\frac{\pi}{4}$:

$x=2\cos^3\frac{\pi}{4}, \quad y=2\sin^3\frac{\pi}{4}$

Calculating the coordinates:

$x=2\left(\frac{1}{\sqrt{2}}\right)^3=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}, \quad y=2\left(\frac{1}{\sqrt{2}}\right)^3=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$

So the point is $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$.

Next, differentiate the curve equation $x^{2/3}+y^{2/3}=2^{2/3}$ with respect to x:

$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$

Solving for $\frac{dy}{dx}$:

$\frac{dy}{dx}=-\left(\frac{y}{x}\right)^{1/3}$

At the point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$,

$\frac{dy}{dx}=-1$

The formula to find the length of the tangent at a point is:

$L = y\sqrt{1+\left(\frac{dx}{dy}\right)^2}$

Given $\frac{dx}{dy} = -1$, the length of the tangent at the point is calculated as:

$L = \frac{1}{\sqrt{2}}\sqrt{1+(-1)^2} = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$

Thus, the length of the tangent is 1.

Question38

Assertion (A) $\frac{d}{dx}\left(\frac{x^2 \sin x}{\log x}\right) = \frac{x^2 \sin x}{\log x}\left(\cot x + \frac{2}{x} - \frac{1}{x \log x}\right)$

Reason (R) $\frac{d}{dx}\left(\frac{uv}{w}\right) = \frac{uv}{w}\left[\frac{u'}{u} + \frac{v'}{v} + \frac{w'}{w}\right]$

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Options:

A. A is true, R is true and R is correct explanation of A

B. A is true, R is true and R is not correct explanation of A

C. A is true, R is not correct

D. A is not correct, R is correct

Answer: A

Solution:

$$\text{Assertion}: \frac{d}{dx}\left(\frac{x^2 \sin x}{\log x}\right)$$
$$= \frac{(\log x)\left[x^2 \cos x + 2x \sin x\right] - x \sin x}{(\log x)^2}$$
$$= \frac{x^2 \cos x \log x + 2x \sin x \log x - x \sin x}{(\log x)^2}$$
$$= \frac{x^2 \sin x}{\log x}\left[\cot x + \frac{2}{x} - \frac{1}{x \log x}\right]$$

Reason : $\frac{d}{dx}\left(\frac{uv}{w}\right)=\frac{uw}{w}\left[\frac{u'}{u}+\frac{v'}{v}+\frac{w'}{w}\right]$

Question39

If $x=f(\theta)$ and $y=g(\theta)$, then $\frac{d^2y}{dx^2}=$

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Options:

A. $\frac{g''(\theta)}{f'(\theta)}$

B. $\frac{f''(\theta)}{x(\theta)}$

C. $\frac{f'(\theta)g''(\theta)-g'(\theta)f''(\theta)}{(f'(\theta))^3}$

D. $\frac{g'(\theta)f''(\theta)-g''(\theta)f''(\theta)}{(g''(\theta))^3}$

Answer: C

Solution:

Given,

$$x=f(\theta)$$
$$\text{and } y=g(\theta)$$
$$\Rightarrow \frac{dx}{d\theta}=f'(\theta), \frac{dy}{d\theta}=g'(\theta)$$
$$\Rightarrow \frac{dy}{dx}=\frac{g'(\theta)}{f'(\theta)}$$
$$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{g''(\theta)f'(\theta)-f''(\theta)g'(\theta)}{(f'(\theta)^2)}\right)\frac{d\theta}{dx}$$
$$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{g''(\theta)f'(\theta)-f''(\theta)g'(\theta)}{(f'(\theta))^3}\right)$$

Question40

$y=x^3-ax^2+48x+7$ is an increasing function for all real values of x, then a lies in the interval

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Options:

A. $(-14,14)$

B. $(-12,12)$

C. $(-16,16)$

D. $(-21,-21)$

Answer: B

Solution:

Given, equation $y=x^3-ax^2+48x+7$

Differentiating the above equation with respect to the variable x, we have

$\frac{dy}{dx}=3x^2-2ax+48$

Thus, above function is a quadratic function.

So, $D<0$.

In $y'=3x^2-2ax+48, A=3, B=-2a$ and $c=48$

$\therefore \quad D<0$

$\Rightarrow(-2a)^2-4\cdot3\cdot48<0\Rightarrow 4a^2-4\cdot3\cdot48<0$

$\Rightarrow 4\left(a^2-144\right)<0\Rightarrow a^2-(12)^2<0$

$\Rightarrow(a-12)(a+12)<0$

$\Rightarrow a\in(-12,12)$

Question41

If $x\neq0$ and $f(x)$ satisfies $8f(x)+6f(1/x)=x+5$, then $\frac{d}{dx}\left(x^2f(x)\right)$ at $x=1$ is

Options:

A. $-1/14$

B. $25/14$

C. $9/14$

D. $19/14$

Answer: D

Solution:

Given the equation:

$8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$

First, let's determine the value of $f(1)$ by setting $x = 1$:

$8f(1) + 6f(1) = 1 + 5$

$14f(1) = 6$

$f(1) = \frac{6}{14} = \frac{3}{7}$

Next, we'll differentiate the given equation with respect to x:

$8f'(x) + 6\left(-\frac{1}{x^2}\right)f'\left(\frac{1}{x}\right) = 1$

$8f'(x) - \frac{6}{x^2}f'\left(\frac{1}{x}\right) = 1$

By setting $x = 1$:

$8f'(1) - 6f'(1) = 1$

$2f'(1) = 1$

$f'(1) = \frac{1}{2}$

Now, let us find the derivative of $x^2f(x)$ with respect to x:

$y = x^2f(x)$

$\frac{dy}{dx} = 2xf(x) + x^2f'(x)$

Evaluating at $x = 1$:

$\left.\frac{dy}{dx}\right|_{x=1} = 2(1)f(1) + (1)^2 f'(1)$

$= 2\left(\frac{3}{7}\right) + \frac{1}{2}$

$= \frac{6}{7} + \frac{1}{2} = \frac{12}{14} + \frac{7}{14}$

$= \frac{19}{14}$

Therefore, the value of $\frac{d}{dx}(x^2 f(x))$ at $x = 1$ is $\frac{19}{14}$.

Question42

If $f(x) = \cot^{-1}\left(\frac{x^x + x^{-x}}{2}\right)$, then $f'(1) =$

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Options:

A. 1

B. -1

C. 2

D. -2

Answer: B

Solution:

$f(x) = \cot^{-1}\left(\frac{x^x + x^{-x}}{2}\right)$

Let $u = x^x$

$\Rightarrow \quad \log u = x \log x$

On differentiating w.r.t. x, we get

$$\frac{1}{u}\frac{du}{dx} = x \cdot \frac{1}{x} + \log x$$
$$\Rightarrow \quad \frac{du}{dx} = x^x(1 + \log x)$$

Similarly, $v = x^{-x}$ gives

$$\frac{dv}{dx} = -x^{-x}(1+\log x)$$

$$\therefore \quad f'(x) = -\frac{1}{1+\left(\frac{x^x-x^{-x}}{2}\right)^2} \cdot \frac{d}{dx}\left(\frac{x^x-x^{-x}}{2}\right)$$

$$= -\frac{4}{(x^x)^2+(x^{-x})^2+2}\left[\frac{1}{2}(1+\log x)\left(x^x-(-x^{-x})\right]\right.$$

$$= -\frac{2}{x^{2x}+x^{-2x}+2}\left[(1+\log x)\left(x^x+x^{-x}\right)\right]$$

$$\therefore f'(1) = -\frac{2}{1+1+2}[(1+0)(1+1)]$$

$$= \left(-\frac{2}{4}\right)(2) = -1$$

Question43

If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then $\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2$ is equal to

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Options:

A. $n(y+4)$

B. $n^2\left(y^2+4\right)$

C. $n(y+2)$

D. $n^2\left(y^2+2\right)$

Answer: B

Solution:

$x=\sec\theta-\cos\theta$, then

$\frac{dx}{d\theta}=\sec\theta\tan\theta+\sin\theta=\tan\theta(\sec\theta+\cos\theta)$

$y=\sec^n\theta-\cos^n\theta$, then

$\frac{dy}{d\theta}=n\sec^{n-1}\theta\sec\theta\tan\theta+n\cos^{n-1}\theta\sin\theta$

$=n\tan\theta\left(\sec^n\theta+\cos^n\theta\right)$

$\therefore\ \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{n\tan\theta\left(\sec^n\theta+\cos^n\theta\right)}{\tan\theta(\sec\theta+\cos\theta)}$

$=\frac{n\left(\sec^n\theta+\cos\theta\right)}{\sec\theta+\cos\theta}$

$\left(\frac{dy}{dx}\right)^2=\frac{n^2(\sec\theta+\cos\theta)^2}{(\sec\theta+\cos\theta)^2}$

$=\frac{n^2\left\{(\sec\theta-\cos\theta)^2+4\right\}}{(\sec\theta-\cos\theta)^2+4}=\frac{n^2\left(y^2+4\right)}{x^2+4}$

$\Rightarrow\quad\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2=n^2\left(y^2+4\right)$

Question44

If $y=\log_{\cot x}\tan x-\log_{\tan x}\cot x+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$, then $\frac{dy}{dx}$ is equal to

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Options:

A. $\frac{1}{4+x^2}$

B. $\frac{4}{4+x^2}$

C. $\frac{1}{4-x^2}$

D. $\frac{4}{4-x^2}$

Answer: B

Solution:

$$y=\log_{\cot x}\tan x-\log_{\tan x}\cot x+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$
$$=\frac{\log_e\tan x}{\log_e\cot x}-\frac{\log_e\cot x}{\log_e\tan x}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$
$$=\frac{\log_e\tan x}{\log_e\left(\frac{1}{\tan x}\right)}-\frac{\log_e\left(\frac{1}{\tan x}\right)}{\log_e(\tan x)}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$
$$=\frac{\log_e\tan x}{-\log_e\tan x}+\frac{\log_e\tan x}{\log_e\tan x}+\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$
$$=-1+1+\tan^{-1}\left(\frac{4x}{4-x^2}\right)=\tan^{-1}\left(\frac{4x}{4-x^2}\right)$$

Now, $\frac{dy}{dx}=\frac{1}{1+\left(\frac{4x}{4-x^2}\right)^2}\frac{d}{dx}\left(\frac{4x}{4-x^2}\right)$

$$\frac{dy}{dx}=\frac{\left(4-x^2\right)^2}{(4-x^2)^2+16x^2}\left[\frac{(4-x^2)4+(4x)(2x)}{(4-x^2)^2}\right]$$
$$=\frac{\left(4-x^2\right)^2}{16+x^4+8x^2}\left[\frac{16-4x^2+8x^2}{(4-x^2)^2}\right]$$
$$=\frac{16+4x^2}{(x^2+4)^2}=\frac{4\left(x^2+4\right)}{(x^2+4)^2}=\frac{4}{x^2+4}$$
$$\frac{dy}{dx}=\frac{4}{4+x^2}$$

Question45

If $f(x)=2x^2+3x-5$, then the value of $f'(0)+3f'(-1)$ is equal to

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Options:

A. 1

B. 0

C. 3

D. 2

Answer: B

Solution:

$f'(x) = 4x + 3$

$f'(0) = 3, f'(-1) = -1$

$\Rightarrow f'(0) + 3f'(1) = 3 - 3 = 0$

Question46

If $y = \left(1 + \frac{1}{x}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{3}{x}\right)\ldots\left(1 + \frac{n}{x}\right)$ and $x \neq 0$. When $x = -1$, $\frac{dy}{dx}$ is equal to

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Options:

A. $n!$

B. $(n-1)!$

C. $(-1)^n(n-1)!$

D. $(-1)^n n!$

Answer: C

Solution:

$$y = \left(1 + \frac{1}{x}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{3}{x}\right) + \ldots + \left(1 + \frac{n}{x}\right)$$
$$\frac{dy}{dx} = \left(-\frac{1}{x^2}\right)\left(1 + \frac{2}{x}\right)\ldots\left(1 + \frac{n}{x}\right)$$
$$+\left(1 + \frac{1}{x}\right)\left(-\frac{2}{x^2}\right)\ldots\left(1 + \frac{n}{x}\right) + \ldots$$

At $x = -1$

$dy/dx = -1(-1)(-2)(-n+1) + 0 = (-1)^n(n-1)!$

Question47

If $\log\left(\sqrt{1+x^2}-x\right)=y\left(\sqrt{1+x^2}\right)$, then $\left(1+x^2\right)\frac{dy}{dx}+xy$ is equal to

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Options:

A. 0

B. 1

C. 2

D. -1

Answer: D

Solution:

Given, $y\sqrt{1+x^2}=\log\left[\sqrt{1+x^2}-x\right]$

On differentiating both sides with respect to x, we get,

$$y\cdot\frac{1}{2\sqrt{1+x^2}}\cdot 2x+\sqrt{1+x^2}\frac{dy}{dx}$$
$$=\frac{1}{\sqrt{1+x^2}-x}\left\{\frac{1}{2\sqrt{1+x^2}}\cdot 2x-1\right\}$$
$$\Rightarrow\frac{xy}{\sqrt{1+x^2}}+\left(\sqrt{1+x^2}\right)\frac{dy}{dx}$$
$$=\frac{1}{\sqrt{1+x^2}-x}\cdot\left\{\frac{x}{\sqrt{1+x^2}}-1\right\}$$
$$\Rightarrow\frac{xy}{\sqrt{1+x^2}}+\sqrt{1+x^2}\frac{dy}{dx}$$
$$=\frac{-\left(\sqrt{1+x^2}-x\right)}{\sqrt{1+x^2}-x}\cdot\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow xy+\left(1+x^2\right)\frac{dy}{dx}=-1\text{ or }\left(1+x^2\right)\frac{dy}{dx}+xy=-1$$

Question48

If $y = e^{x^2 + e^{x^2 + e^{x^2 + \cdots \infty}}}$, then $\frac{dy}{dx}$ is equal to

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Options:

A. $\frac{2x}{1-y}$

B. $\frac{2xy}{y-1}$

C. $\frac{2xy}{1-y}$

D. $\frac{2y}{y-1}$

Answer: C

Solution:

$$y = e^{x^2 + e^{x^2 + e^{x^2} + \ldots \infty}}$$
$$y = e^{x^2 + y}$$

Taking log on both sides

$\log y = x^2 + y$

Differentiating w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$$
$$\Rightarrow \quad \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{2xy}{1-y}$$

Question49

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$ **is equal to**

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Options:

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. -1

Answer: B

Solution:

$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right)$

$=\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\pi}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)}\right]$

$=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\right]$

$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{\pi}{2}$

$\therefore \frac{d}{dx}\left(\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right)=\frac{d}{dx}\left(\frac{\pi}{4}-\frac{\pi}{2}\right)=\frac{-1}{2}$

Question50

If $x^2+y^2=1$, then

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Options:

A. $y\left(y''\right)-4(y')^2+1=0$

B. $y\left(y''\right)+(y')^2+1=0$

C. $y\left(y''\right)-(y')^2-1=0$

D. $y\left(y''\right)+2(y')^2+1=0$

Answer: B

Solution:

$x^2 + y^2 = 1$

Again differentiating w.r.t. x

$2x + 2yy' = 0$

or $x + yy' = 0$

Again differentiating w.r.t. x

$1 + y\left(y''\right) + \left(y'\right)^2 = 0$
$y\left(y''\right) + \left(y'\right)^2 + 1 = 0$

Question51

If $y = x + \frac{1}{x}$, then which among the following holds?

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Options:

A. $x^2y' + xy = 0$

B. $x^2y' + xy + 2 = 0$

C. $x^2y' - xy + 2 = 0$

D. $x^2y' + xy - 2 = 0$

Answer: C

Solution:

$y = x + \frac{1}{x}$ ..... (i)

Differentiating with respect to x, we get

$y' = 1 - \frac{1}{x^2}$

or $x^2y' = x^2 - 1$ .... (ii)

From Eq. (i), we get

$x^2 = xy - 1$

$\therefore$ From Eq. (ii), we have

$x^2 y' = xy - 1 - 1$ or $x^2 y' - xy + 2 = 0$

Question52

If $3 \sin xy + 4 \cos xy = 5$, then $\frac{dy}{dx}$ is equal to

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Options:

A. $\frac{3 \sin xy + 4 \cos xy}{3 \cos xy - 4 \sin xy}$

B. $\frac{3 \cos xy + 4 \sin xy}{4 \cos xy - 3 \sin xy}$

C. $\frac{-y}{x}$

D. $\frac{x}{y}$

Answer: C

Solution:

$3 \sin xy + 4 \cos xy = 5$

Differentiating with respect to x, we get

$$3 \cos xy \cdot \left(x \frac{dy}{dx} + y\right) - 4 \sin xy \left(x \frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow x \frac{dy}{dx}(3 \cos xy - 4 \sin xy)$$
$$+ y(3 \cos xy - 4 \sin xy) = 0$$
$$\Rightarrow (3 \cos xy - 4 \sin xy)\left(x \frac{dy}{dx} + y\right) = 0$$
$$\Rightarrow \quad x \frac{dy}{dx} + y = 0$$
$$\text{or} \quad \frac{dy}{dx} = \frac{-y}{x}$$

Question53

$f(x)=\sqrt{x^2+1}: g(x)=\frac{x+1}{x^2+1}: h(x)=2x-3$, **then the value of** $f'\left[h'\left(g'(x)\right)\right]$ **is equal to**

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Options:

A. $\sqrt{5}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{1}{\sqrt{5}}$

Answer: B

Solution:

$$\begin{aligned} & f(x)=\sqrt{x^2+1} \\ \Rightarrow & f'(x)=\frac{2x}{2\sqrt{x^2+1}}=\frac{x}{\sqrt{x^2+1}} \\ & h(x)=2x-3 \Rightarrow h'(x)=2 \end{aligned}$$

So, $\quad h'\left(g'(x)\right)=2$

$\therefore f'\left(h'\left(g'(x)\right)\right)=f'(2)=\frac{2}{\sqrt{2^2+1}}=\frac{2}{\sqrt{5}}$

Question54

For which value(s) of a $f(x)=-x^3+4ax^2+2x-5$ **is decreasing for every** x **?**

Options:

A. (1, 2)

B. (3, 4)

C. R

D. no value of a

Answer: D

Solution:

$f(x) = -x^3 + 4ax^2 + 2x - 5$

$f'(x) = -3x^2 + 8ax + 2$

Discriminant of $f'(x) = (8a)^2 - 4(-3)(2)$

$= 64a^2 + 24 > 0, \forall a \in R$

Hence, $f(x)$ is increasing for every real value of a.
