Circle

Question1

$A(4,3), B(2,5)$ are two points. If P is a variable point on the same side as that of the origin with respect to the line AB and is at most at a distance of 5 units from the mid-point of AB, then the locus of P is

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Options:

A.

$x^2+y^2-6x-8y=0$

B.

$x^2+y^2-6x-8y \leq 0, x+y-7<0$

C.

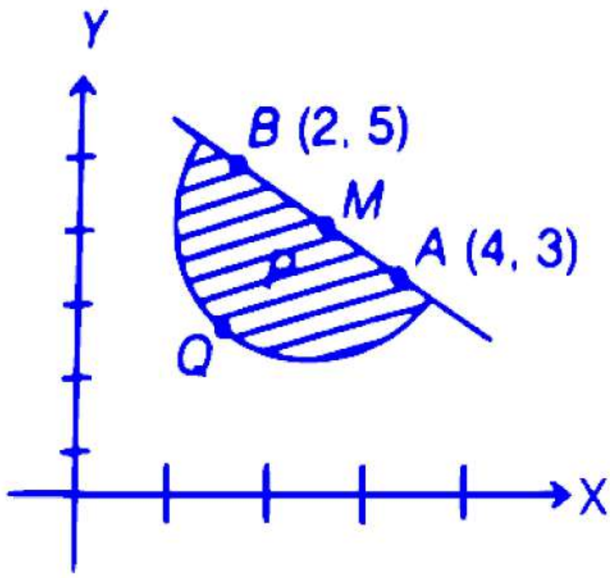
$x^2+y^2+6x+8y-25=0, x+y-7 \leq 0$

D.

$x^2+y^2-6x+8y \geq 0, x+y-7<0$

Answer: B

Solution:



$m = \left(\frac{2+4}{2}, \frac{5+3}{2}\right) = (3, 4)$

and $mQ \leq 5$

So, P should be in the shaded region thus, $AB : (y-3) = \frac{3-5}{4-2}(x-4)$

$\Rightarrow \quad y = -x + 4 + 3$
$\Rightarrow \quad x + y - 7 = 0$

and equation of the semi-circle

$(x-3)^2 + (y-4)^2 = (5)^2$
$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 = 25$
$\Rightarrow \quad x^2 + y^2 - 6x - 8y = 0$

Hence, shaded region is

$x^2 + y^2 - 6x - 8y \leq 0, x + y - 7 < 0$

Question2

The circles $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 + 2x + 4y - 11 = 0$

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Options:

A.

cut each other orthogonally

B.

do not meet

C.

intersect at the points lying on the line $4x + 8y - 7 = 0$

D.

touch each other at the point lying on the line $4x + 8y - 7 = 0$

Answer: C

Solution:

$x^2 + y^2 - 2x - 4y - 4 = 0 \quad \ldots(i)$

$c_1 = (1, 2)$

and $r_1 = \sqrt{(1)^2 + (2)^2 - (-4)} = 3$

$x^2 + y^2 + 2x + 4y - 11 = 0 \quad \ldots(ii)$

$c_2 = (-1, -2)$ and

$r_2 = \sqrt{(-1)^2 + (-2)^2 - (-11)} = 4$

$c_1c_2 = \sqrt{(-1-1)^2 + (-2-2)^2}$

$= \sqrt{18} < r_1 + r_2 = 7$

So, both circles intersect, each others.

Subracting Eq. (i) from Eq. (ii), we get $4x + 8x - 7 = 0$

Hence, intersects at the points lying on the line $4x + 8y - 7 = 0$

Question3

If the line $4x - 3y + 7 = 0$ touches the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ at (α, β), then $\alpha + 2\beta =$

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Options:

A.

3

B.

-1

C.

1

D.

-3

Answer: C

Solution:

$$x^2 + y^2 - 6x + 4y - 12 = 0$$
$$\Rightarrow (x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$
$$\Rightarrow (x-3)^2 + (y+2)^2 = (5)^2$$

Centre $= (3, -2)$ and radius $= 5$

The line $4x - 3y + 7 = 0$ touches the circle at the point (α, β)

Foot of perpendicular from $(3, -2)$ to the line

$4x - 3y + 7 = 0$ is

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = -\frac{ax_0 + by_0 + c}{a^2 + b^2}$$
$$\Rightarrow \frac{x-3}{4} = \frac{y+2}{-3} = -\frac{4(3) - 3(-2) + 7}{3^2 + 4^2} = -1$$

Thus, $\frac{x-3}{4} = -1 \Rightarrow x = -1$

and $\frac{y+2}{-3} = -1 \Rightarrow y = +1$

so, $(\alpha, \beta) = (-1, 1)$

Hence, $\alpha + 2\beta = -1 + 2 = 1$

Question4

The slope of the common tangent drawn to the circles $x^2+y^2-4x+12y-216=0$ and $x^2+y^2+6x-12y+36=0$ is

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Options:

A.

1

B.

-1

C.

$5/12$

D.

$12/7$

Answer: C

Solution:

$$x^2-4x+y^2+12y=216$$
$$\Rightarrow (x-2)^2+(y+6)^2=256$$
$$c_1=(2,-6), r_1=16$$
$$\text{and } x^2+6x+y^2-12y=-36$$
$$\Rightarrow (x+3)^2+(y-6)^2=9$$
$$\therefore c_2=(-3,6), r_2=3$$
$$c_1c_2=\sqrt{(2+3)^2+(-6-6)^2}$$
$$=\sqrt{169}=13$$

and $\quad r_1-r_2=16-3=13$

$\because c_1c_2=|r_1-r_2|$ the circle touches internally.

then, the slope of line joining centers

$=\frac{6-(-6)}{3-2}=-12/5$

$\therefore$ the slope of tangent

$=\frac{-1}{(-12/5)}=5/12$

Question5

If r_1 and r_2 are radii of two circles touching all the four circles $(x \pm r)^2 + (y \pm r)^2 = r^2$, then $\frac{r_1+r_2}{r} =$

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Options:

A.

$\frac{\sqrt{2}+1}{2}$

B.

$3\sqrt{2}$

C.

$2\sqrt{2}$

D.

$\frac{3+\sqrt{2}}{4}$

Answer: C

Solution:

The four given circles are

$(x \pm r)^2 + (y \pm r)^2 = r^2$

centers are $(r, r), (r, -r), (-r, -r), (-r, r)$ the circle touches all four given centered at the origin $(0, 0)$, due to the symmetry.

One circle is internally tangent to the region by the four circles, with radius $(r_1) = r(\sqrt{2} - 1)$

Other circles is externally tangent to the region, with radius $(r_2) = r(\sqrt{2} + 1)$

$\therefore r_1 + r_2 = r(\sqrt{2} - 1 + \sqrt{2} + 1) = 2\sqrt{2}r$

Hence, $\frac{r_1+r_2}{r} = \frac{2\sqrt{2}r}{r} = 2\sqrt{2}$

Question6

If the equation of the circle having the common chord to the circles $x^2+y^2+x-3y-10=0$ and $x^2+y^2+2x-y-20=0$ as its diameter is $x^2+y^2+\alpha x+\beta y+\gamma=0$, then $\alpha+2\beta+\gamma=$

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Options:

A.

0

B.

1

C.

-1

D.

2

Answer: A

Solution:

$\because x^2+y^2+x-3y-10=0$ and $x^2+y^2+2x-y-20=0$

So, common chord will be

$$\left(x^2+y^2+x-3y-10\right)-\left(x^2+y^2+2x-y-20\right)=0$$
$$\Rightarrow x+2y-10=0 \Rightarrow x=10-2y$$

put the ' x ' in first circle

$$(10-2y)^2+y^2+(10-2y)-3y-10=0$$
$$\Rightarrow 100+4y^2-40y+y^2-2y+10-3y-10=0$$

$$\Rightarrow y^2-9y+20=0$$
$$\Rightarrow (y-5)(y-4)=0$$
when $y=4 \Rightarrow x=2$
when $y=5 \Rightarrow x=0$

so, intersection points are $(2,4)$ and $(0,3)$ which are end-points of diameters.

So, equation of the circle

$$\begin{aligned} & (x-0)(x-2)+(y-4)(y-5)=0 \\ \Rightarrow \quad & x(x-2)+(y-4)(y-5)=0 \\ \Rightarrow \quad & x^2+y^2-2x-9y+20=0 \end{aligned}$$

Comparing with

$x^2+y^2+\alpha x+\beta y+\gamma=0$
$\alpha=-2, \beta=-9$ and $\gamma=20$

Therefore, $\alpha+2\beta+\gamma$

$=-2-18+20=0$

Question7

If $A(\cos\alpha, \sin\alpha), B(\sin\alpha, -\cos\alpha), C(1,2)$ are the vertices of a $\triangle ABC$, then the locus of its centroid is

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Options:

A.

$3\left(x^2+y^2\right)-2x-4y+1=0$

B.

$x^2+y^2-2x-4y+1=0$

C.

$x^2+y^2-2x-4y+3=0$

D.

$2\left(x^2+y^2\right)-2x-4y+5=0$

Answer: A

Solution:

Let the centroid of the triangle be at (h, k).

Step 1: Finding the Centroid Coordinates

The centroid's x-coordinate (h) is the average of all three x-coordinates:$h = \frac{\cos\alpha+\sin\alpha+1}{3}$ So,
$3h - 1 = \cos\alpha + \sin\alpha$

The centroid's y-coordinate (k) is the average of all three y-coordinates:$k = \frac{\sin\alpha-\cos\alpha+2}{3}$ So,
$3k - 2 = \sin\alpha - \cos\alpha$

Step 2: Squaring Both Sides

Square both sides of each equation to remove the trigonometric functions:

$(3h-1)^2 = (\cos\alpha + \sin\alpha)^2 \quad \cdots (i)$

$(3k-2)^2 = (\sin\alpha - \cos\alpha)^2 \quad \cdots (ii)$

Step 3: Add the Two Equations

Add equations (i) and (ii):$(3h-1)^2 + (3k-2)^2 = (\cos\alpha + \sin\alpha)^2 + (\sin\alpha - \cos\alpha)^2$

Notice that$(\cos\alpha + \sin\alpha)^2 + (\sin\alpha - \cos\alpha)^2 = 2\sin^2\alpha + 2\cos^2\alpha = 2$

So,$(3h-1)^2 + (3k-2)^2 = 2$

Step 4: Locus Equation

Replace h by x and k by y because the locus is about every possible point (x, y):
$(3x-1)^2 + (3y-2)^2 - 2 = 0$

Step 5: Expand to Standard Form

Expanding and simplifying:

$$(3x-1)^2 + (3y-2)^2 - 2 = 0$$
$$9x^2 - 6x + 1 + 9y^2 - 12y + 4 - 2 = 0$$
$$9x^2 + 9y^2 - 6x - 12y + 3 = 0$$
$$3(x^2+y^2) - 2x - 4y + 1 = 0$$

Question8

A circle passing through origin cuts the coordinate axes is A and B. If the straight line AB passes through a fixed point (x_1, y_1), then the locus of the centre of the circle is

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Options:

A.

$\frac{x_1}{x}+\frac{y_1}{y}=1$

B.

$x_1 y=x y_1$

C.

$x y_1+y x_1=2$

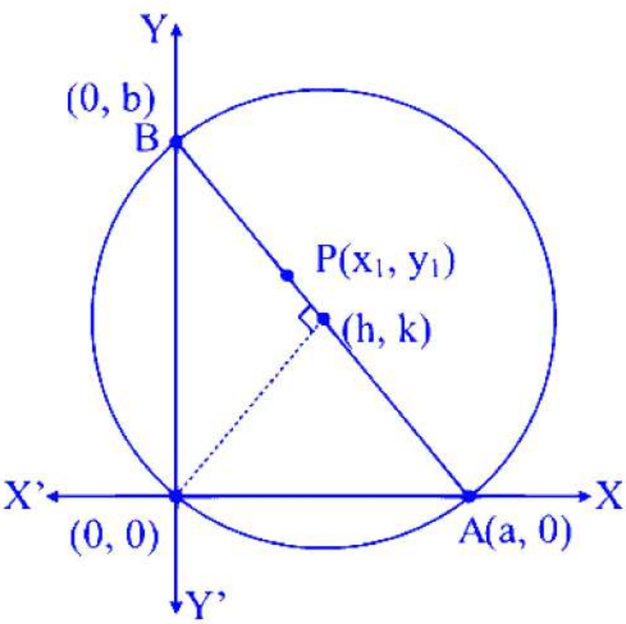
D.

$\frac{x_1}{x}+\frac{y_1}{y}=2$

Answer: D

Solution:

$\because AB$ will be the diameter



$\therefore$ Let equation of circle
$x^2+y^2+2gx+2fy=0$

$(a, 0)$ lies on the circle

$\therefore \quad a^2+2ga=0 \Rightarrow a=-2g$

$(0, b)$ lies on the circle

$\therefore \quad b^2+2fb \Rightarrow b=-2f$

∴ Equation of line AB

$$\frac{x}{-2g}+\frac{y}{-2f}=1\ [\because (x_1, y_1) \text{ lies on } L]$$
$$\Rightarrow \quad \frac{x}{-g}+\frac{y}{-f}=2 \Rightarrow \frac{x_1}{-g}+\frac{y_1}{-f}=2$$

∴ Locus is centre $(-g,-f)$ is $\frac{x_1}{x}+\frac{y_1}{y}=2$

Question9

If (α, β) is the external centre of similitude of the circles $x^2+y^2=3$ and $x^2+y^2-2x+4y+4=0$, then $\frac{\beta}{\alpha}=$

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Options:

A.

-3

B.

-2

C.

2

D.

3

Answer: B

Solution:

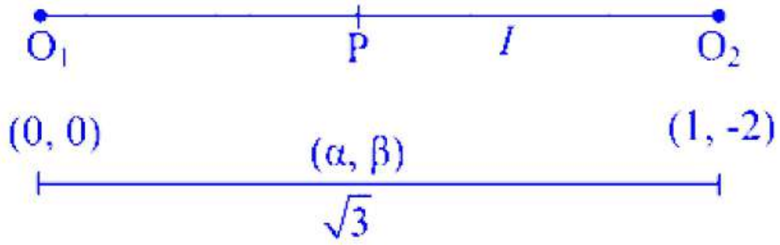
Given, $C_1: x^2+y^2=3$,

$O_1(0,0), r_1=\sqrt{3}$

and $C_2: x^2+y^2-2x+4y+4=0$

$O_2(1,-2), r_2=1$

Point P divides O_1O_2 externally in $r_1 : r_2$



$$\therefore \alpha = \frac{0 \times \sqrt{3} - 1(1)}{\sqrt{3}+1} = \frac{-1}{\sqrt{3}+1} \text{ and}$$

$$\beta = \frac{0 \times \sqrt{3} - (-2)}{\sqrt{3}+1} = \frac{2}{\sqrt{3}+1}$$

$$\Rightarrow \quad \frac{\beta}{\alpha} = -2$$

Question10

The equation of the circle touching the lines $|x-2|+|y-3|=4$ is

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Options:

A.

$x^2+y^2-6x-4y+5=0$

B.

$x^2+y^2-4x-6y+5=0$

C.

$x^2+y^2-x-2y-5=0$

D.

$x^2+y^2-2x-y-5=0$

Answer: B

Solution:

Given, $|x-2|+|y-3|=4$

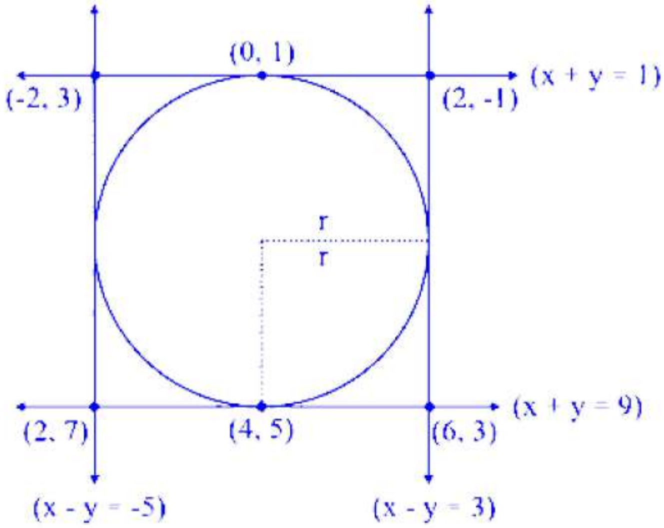
$\therefore \quad x-2+y-3=4$

$\Rightarrow \quad x+y=9$ and $x-2-y+3=4$

$\Rightarrow \quad x-y=3$ and $-x+2+y-3=4$

$\Rightarrow \quad x-y=-5$ and $-x+2-y+3=4$

$\Rightarrow \quad x+y=1$



$\therefore$ Centre $\left(\frac{4+0}{2}, \frac{5+1}{2}\right)=(2,3)$

and $r=\frac{\sqrt{4^2+4^2}}{2}=2\sqrt{2}$

$\therefore$ Equation of circle :

$(x-2)^2+(y-3)^2=(2\sqrt{2})^2$

$\Rightarrow \quad x^2+y^2-4x-6y+5=0$

Question11

If the chord joining the points $(1,2)$ and $(2,-1)$ on a circle subtends an angle of $\frac{\pi}{4}$ at any point on its circumference, then the equation of such a circle is

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Options:

A.

$x^2+y^2+6x-2y+5=0$

B.

$x^2+y^2-6x-2y+5=0$

C.

$x^2+y^2-6x+2y+5=0$

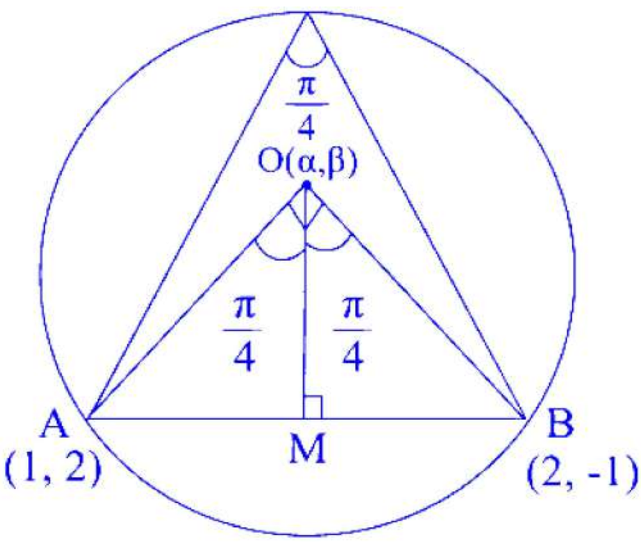
D.

$x^2+y^2+6x+2y+5=0$

Answer: B

Solution:

$$AB=\sqrt{1^2+9}=\sqrt{10}$$
$$AM=\frac{\sqrt{10}}{2}$$



$$\therefore OA=r=\sqrt{\frac{10}{4}+\frac{10}{4}}=\sqrt{\frac{20}{4}}=\sqrt{5}$$

Let $O(\alpha,\beta)$

$$OA = OB$$
$$(\alpha-1)^2+(\beta-2)^2=(\alpha-2)^2+(\beta+1)^2$$
$$\Rightarrow -2\alpha+1-4\beta+4=-4\alpha+4+2\beta-1$$
$$\Rightarrow 2\alpha-6\beta=0$$
$$\Rightarrow \alpha=3\beta \quad \ldots(i)$$
$$\because m_{OA}\cdot m_{OB}=-1$$
$$\Rightarrow \frac{\beta-2}{\alpha-1}\cdot\frac{\beta+1}{\alpha-2}=-1$$
$$\Rightarrow (\beta-2)(\beta+1)+(\alpha-1)(\alpha-2)=0$$
$$\Rightarrow \beta^2-\beta-2+\alpha^2-3\alpha+2=0$$
$$\Rightarrow \beta^2-\beta+9\beta^2-9\beta=0$$
$$\Rightarrow 10\beta^2=10\beta$$
$$\Rightarrow \beta=0,1$$
$$\alpha=0,3$$

$\therefore$ Centre $(0,0)$ or $(3,1)$

$\therefore$ Equation of circle

$x^2+y^2=5$ or $(x-3)^2+(y-1)^2=5$
$\Rightarrow x^2+y^2-6x-2y+5=0$

Question12

The equation of the circle which cuts all the three circles $4(x-1)^2+4(y-1)^2=1, 4(x+1)^2+4(y-1)^2$ and $4(x+1)^2+4(y+1)^2=1$ orthogonally is

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Options:

A.

$4x^2+4y^2=49$

B.

$4(x-1)^2+4(y+1)^2=1$

C.

$(x-1)^2+(y+1)^2=4$

D.

$4x^2 + 4y^2 = 7$

Answer: D

Solution:

Given,

$$S_1 : 4(x-1)^2 + 4(y-1)^2 = 1$$
$$\Rightarrow (x-1)^2 + (y-1)^2 = \frac{1}{4}$$
$$O_1(1,1), r_1 = \frac{1}{2}$$
$$S_2 : 4(x+1)^2 + 4(y-1)^2 = 1$$
$$\Rightarrow (x+1)^2 + (y-1)^2 = \frac{1}{4}$$
$$O_2(-1,1), r_2 = 1/2$$
$$\text{and } S_3 : 4(x+1)^2 + 4(y+1)^2 = 1$$
$$\Rightarrow (x+1)^2 + (y+1)^2 = \frac{1}{4}$$
$$O_3(-1,-1), r_3 = \frac{1}{2}$$
$$\because s_1 - s_2 = 0$$
$$\Rightarrow -2x - 2y + 2x - 2y = 0$$
$$x = 0$$
$$S_2 - s_3 = 0$$
$$\Rightarrow \quad 2y = 0$$
$$\Rightarrow \quad y = 0$$

$\therefore$ Centre of required circle $= (0,0)$

radius $= L = \sqrt{s_1}$

$$= \sqrt{1 + 1 - \frac{1}{4}} = \sqrt{\frac{7}{4}}$$

$\therefore$ Equation of circle

$$x^2 + y^2 = \frac{7}{4}$$
$$\Rightarrow 4x^2 + 4y^2 = 7$$

Question13

$A(a,0)$ is a fixed point and θ is a parameter such that $0 < \theta < 2\pi$. If $P(a\cos\theta, a\sin\theta)$ is a point on the circle $x^2 + y^2 = a^2$ and $Q(b\sin\theta, -b\cos\theta)$ is a point on the circle $x^2 + y^2 = b^2$, then the locus of the centroid of the $\triangle APQ$ is

Options:

A.

a circle with centre at $\left(\frac{a}{3}, 0\right)$ and radius $\left(\frac{\sqrt{a^2+b^2}}{3}\right)$

B.

a circle with centre at $(a, 0)$ and radius $\left(\frac{\sqrt{a^2+b^2}}{3}\right)$

C.

a parabola with focus at $\left(\frac{a}{3}, 0\right)$

D.

a parabola with focus at $(a, 0)$

Answer: A

Solution:

Let the coordinates of the centroid be $G(x, y)$ of the $\triangle APQ$ is

$x = \frac{x_A+x_P+x_Q}{3}, y = \frac{y_A+y_P+y_Q}{3}$

Given the points $A(a, 0), P(a\cos\theta, a\sin\theta)$ and $Q(b\sin\theta, -b\cos\theta)$

The coordinates of the centroid are

$$x = \frac{a + a\cos\theta + b\sin\theta}{3}$$
$$y = \frac{0 + a\sin\theta - b\cos\theta}{3}$$

Let $x = h$ and $y = k$, then,

$$h = \frac{a + a\cos\theta + b\sin\theta}{3}, k = \frac{a\sin\theta - b\cos\theta}{3}$$
$$\Rightarrow 3h - a = a\cos\theta + b\sin\theta$$
$$3k = a\sin\theta - b\cos\theta$$

Squaring and adding these equations

$(3h - a)^2 + (3k)^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$=a^2\cos^2\theta+b^2\sin^2\theta+2ab\cos\theta\sin\theta+a^2\sin^2\theta+b^2\cos^2\theta-2ab\cos\theta\sin\theta$
$=a^2+b^2$

Replacing h with x and k with y, we get the locus of the centroid as

$$
\begin{aligned}
&(3x-a)^2+(3y)^2=a^2+b^2\\
\Rightarrow\quad &\left(x-\frac{a}{3}\right)^2+y^2=\frac{a^2+b^2}{9}\\
\Rightarrow\quad &\left(x-\frac{a}{3}\right)^2+y^2=\left(\frac{\sqrt{a^2+b^2}}{3}\right)^2
\end{aligned}
$$

So, the locus of the centroid is

$\left(x-\frac{a}{3}\right)^2+y^2=\left(\frac{\sqrt{a^2+b^2}}{3}\right)^2$

It is a circle with centre at $\left(\frac{a}{3},0\right)$ and radius $\frac{\sqrt{a^2+b^2}}{3}$

Question14

If the equation of the circle passing through the point $(8,8)$ and having the lines $x+2y-2=0$ and $2x+3y-1=0$ as its diameters is $x^2+y^2+px+qy+r=0$, then $p^2+q^2+r=$

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Options:

A.

244

B.

100

C.

-44

D.

44

Answer: C

Solution:

Given, lines are $x+2y-2=0$ and

$2x+3y-1=0$

Solving these two lines, we get

$x=-4$ and $y=3$

Thus, the center of circle is $(-4,3)$.

Since, the general eq. of circle is $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius.

$$\begin{aligned} &\text{So, } (x-(-4))^2+(y-3)^2=r^2 \\ \Rightarrow \quad &(x+4)^2+(y-3)^2=r^2 \qquad \ldots(i) \end{aligned}$$

But, the circle passes through the point $(8,8)$,

$$\begin{aligned} &\text{So, } (8+4)^2+(8-3)^2=r^2 \\ &12^2+5^2=r^2 \\ &169=r^2 \\ \Rightarrow \quad &r=13 \end{aligned}$$

Now, from Eq. (i), we get

$$\begin{aligned} &(x+4)^2+(y-3)^2=r^2=169 \\ \Rightarrow \quad &x^2+8x+16+y^2-6y+9=169 \\ \Rightarrow \quad &x^2+y^2+8x-6y-144=0 \end{aligned}$$

Comparing with the given equation

$x^2+y^2+px+qx+r=0$

We get $p=8, q=-6, r=-144$

$$\begin{aligned} \therefore \quad p^2+q^2+r &= 8^2+(-6)^2+(-144) \\ &= 64+36-144 \\ &= -44 \end{aligned}$$

Question15

If $2x-3y+1=0$ is the equation of the polar of a point $P(x_1,y_1)$ with respect to the circle $x^2+y^2-2x+4y+3=0$, then $3x_1-y_1=$

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Options:

A.

$\frac{1}{3}$

B.

-3

C.

3

D.

$-\frac{1}{3}$

Answer: C

Solution:

Given circle is

$x^2 + y^2 - 2x + 4y + 3 = 0$

So, center $= (-g, -f) = \left(\frac{-2}{2}, \frac{4}{2}\right)$

$= (-1, 2)$ and $c = 3$

Also, the equation of the polar of a point (x_1, y_1) w.r.t the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$$\Rightarrow xx_1 + yy_1 - (x + x_1) + 2(y + y_1) + 3 = 0$$
$$\Rightarrow xx_1 + yy_1 - x - x_1 + 2y + 2y_1 + 3 = 0$$
$$\Rightarrow (x_1 - 1)x + (y_1 + 2)y - x_1 + 2y_1 + 3 = 0 \quad \ldots(i)$$

Given, polar equation is

$$2x - 3y + 1 = 0 \quad \ldots(ii)$$

By comparing the coefficients of two equations, we get

$$\frac{x_1 - 1}{2} = \frac{y_1 + 2}{-3} = \frac{-x_1 + 2y_1 + 3}{1}$$
$$\Rightarrow \frac{x_1 - 1}{2} = \frac{y_1 + 2}{-3} \text{ and } \frac{x_1 - 1}{2} = \frac{-x_1 + 2y_1 + 3}{1}$$
$$\Rightarrow -3x_1 + 3 = 2y_1 + 4 \text{ and } x_1 - 1 = -2x_1 + 4y_1 + 6$$
$$\Rightarrow 3x_1 + 2y_1 = -1 \text{ and } 3x_1 - 4y_1 - 7 = 0$$

By solving these two equations, we get $x_1 = \frac{5}{9}$ and $y_1 = \frac{-4}{3}$

$$
\begin{aligned}
Now,\ 3x_1 - y_1 &= 3\left(\frac{5}{9}\right) - \left(\frac{-4}{3}\right) \\
&= \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3 \\
\therefore \quad 3x_1 - y_1 &= 3
\end{aligned}
$$

Question16

If a unit circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ touches the circle $S' \equiv x^2 + y^2 - 6x + 6y + 2 = 0$ externally at the point $(-1, -3)$, then $g + f + c =$

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Options:

A.

0

B.

1

C.

15

D.

17

Answer: D

Solution:

Given,

$$
\begin{aligned}
& s \equiv x^2 + y^2 + 2gx + 2fy + c = 0 \\
& \text{And } s' \equiv x^2 + y^2 - 6x + 6y + 2 = 0 \\
\Rightarrow \quad & \left(x^2 - 6x\right) + \left(y^2 + 6y\right) = -2 \\
\Rightarrow \quad & \left(x^2 - 6x + 9\right) + \left(y^2 + 6y + 9\right) \\
& = -2 + 9 + 9 \\
\Rightarrow \quad & (x-3)^2 + (y+3)^2 = 16
\end{aligned}
$$

Thus, the center of circle S' is $(3,-3)$ and radius, $r'=4$

$$\begin{aligned}&\text{Now, } S\equiv x^2+y^2+2gx+2fy+c=0\\ \Rightarrow\quad &(x+g)^2+(y+f)^2=g^2+f^2-c\end{aligned}$$

So, the center of circle S is $(-g,-f)$ and radius,

$$r=\sqrt{g^2+f^2-c}$$

Since, radius of circle S is 1 .

$$\begin{aligned}\therefore\quad &\sqrt{g^2+f^2-c}=1\\ \Rightarrow\quad &g^2+f^2-c=1\end{aligned}$$

Also, two circle touch externally,

So, distance between $(-g,-f)$ and

$$\begin{aligned}&(3,-3)=r+r'\\ \Rightarrow\quad &\sqrt{(-g-3)^2+(-f+3)^2}=4+1\\ \Rightarrow\quad &(g+3)^2+(f-3)^2=25\\ \Rightarrow\quad &g^2+6g+9+f^2-6f+9=25\\ \Rightarrow\quad &g^2+f^2+6g-6f+18-25=0\\ \Rightarrow\quad &g^2+f^2+6g-6f=7 \qquad \ldots(i)\end{aligned}$$

Since, point $(-1,-3)$ lies on both circles, it satisfies both circle.

$\therefore$ For circles

$$\begin{aligned}&(-1)^2+(-3)^2+2g(-1)+2f(-3)+c=0\\ \Rightarrow\quad &1+9-2g-6f+c=0\\ \Rightarrow\quad &-2g-6f+c=-10 \qquad \ldots(ii)\end{aligned}$$

For circle S',

$$\begin{aligned}&(-1)^2+(-3)^2-6(-1)+6(-3)+2=0\\ \Rightarrow\quad &1+9+6-18+2=0\\ \Rightarrow\quad &0=0\end{aligned}$$

Now, the slope of line joining $(-g,-f)$ and $(-1,-3)$ is equal to slope of line joining $(-1,-3)$ and $(3,-3)$

$$\begin{aligned}&(-f+3)=\left(\frac{-3+3}{3+1}\right)(-g+1)\\ \Rightarrow\quad &(-f+3)=0\\ \Rightarrow\quad &f=3\end{aligned}$$

Put $f=3$ into Eq. (i), we get

$$\begin{aligned}&g^2+9+6g-6(3)\\ \Rightarrow\quad &g^2+9+6g-18=7\\ \Rightarrow\quad &g^2+6g-16=0\\ \Rightarrow\quad &(g+8)(g-2)=0\\ \Rightarrow\quad &g=-8 \text{ or } g=2(\textit{not possible})\end{aligned}$$

Now, put $g=2, f=3$ into Eq. (ii), we get

$$-2(2)-6(3)+c=-10$$
$$\Rightarrow \quad c=-10+4+18=12$$

Now, $g+f+c=2+3+12=17$

Question17

$3x+4y-43=0$ is a tangent to the circle $S \equiv x^2+y^2-6x+8y+k=0$ at a point P. If C is the centre of the circle and Q is a point which divides CP in the ratio $-1:2$, then the power of the point Q with respect to the circle $S=0$ is

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Options:

A.

50

B.

21

C.

0

D.

5

Answer: C

Solution:

$S \equiv x^2+y^2-6x+8y+k=0$
$\Rightarrow (x-3)^2+(y+4)^2=25-k$

Thus, the center of the circle C is at $(3,-4)=(x_1, y_1)$.

Let point P as (x_2, y_2) and point Q is the point that divides the line segment CP in the ratio $-1:2$

Also, $3x + 4y - 43 = 0$

$$\Rightarrow \quad y = \frac{-3}{4}x + \frac{43}{3}$$

So, slope of tangent line $= \frac{-3}{4}$

And slope of line $CP = -\left(\frac{-4}{3}\right) = \frac{4}{3}$

Thus, equation of line CP is

$$\begin{aligned} \Rightarrow \quad y - (-4) &= \tfrac{4}{3}(x - 3) \\ \Rightarrow \quad y + 4 &= \tfrac{4}{3}(x - 3) \\ \Rightarrow \quad y &= \tfrac{4}{3}x - 8 \end{aligned}$$

Since, P lies on both the lines $3x + 4y - 43 = 0$ and the line CP,

$$\therefore \quad 3x + 4\left(\tfrac{4}{3}x - 8\right) - 43 = 0$$
$$\Rightarrow \quad 3x + \tfrac{16}{3}x - 32 - 43 = 0$$

$$\Rightarrow \quad \frac{25}{3}x = 75$$
$$\Rightarrow \quad x = 9$$

Now, $y = \dfrac{4}{3}x - 8 = \dfrac{4}{3}(9) - 8$
$= 12 - 8 = 4$

So, $\quad P = (9, 4)$

Using section formula, we find the coordinates of $Q(x, y)$.

$$x = \frac{2x_1 - x_2}{2 - 1} = \frac{2(3) - 9}{1} = -3$$
$$y = \frac{2y_1 - y_2}{2 - 1} = \frac{2(-4) - 4}{1} = -12$$

So, $Q = (x, y) = (-3, -12)$

The power of point Q w.r.t to circle, $x^2 + y^2 - 6x + 8y + k = 0$ is

$$\begin{aligned} &(-3)^2 + (-12)^2 - 6(-3) + 8(-12) + k \\ &\quad = 9 + 144 + 18 - 96 + k \\ &\quad = 75 + k \qquad \ldots (i) \end{aligned}$$

Now, the line $3x + 4y - 43 = 0$ is tangent to the circle, the radius of the circle is the distance from the center $C(3, -4)$ to the line $3x + 4y - 43 = 0$

$$\begin{aligned} So,\ r &= \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|3(3) + 4(-4) - 43|}{\sqrt{3^2 + 4^2}} \\ &= \frac{|9 - 16 - 43|}{5} = \frac{50}{5} = 10 \end{aligned}$$

Now, $r = \sqrt{25 - k}$

$$\Rightarrow \quad 10=\sqrt{25-k} \Rightarrow 25-k=100$$
$$\Rightarrow \quad k=-75$$

Put $k=-75$ in Eq. (i), we get The power of Q w.r.t the circle

$=75+k=75-75=0$

Question18

If the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$ touches the circle $x^2+y^2+2x+2y+1=0$, then

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Options:

A.

either $g=\frac{3}{2}$ or $f \neq 2$

B.

either $g \neq \frac{3}{4}$ or $f=\frac{1}{2}$

C.

either $g=\frac{3}{4}$ or $f=2$

D.

either $g=\frac{1}{2}$ or $f=\frac{3}{4}$

Answer: C

Solution:

$$s_1: x^2+y^2+2gx+2fy+c=0$$
$$s_2: 2x^2+2y^2+3x+8y+2c=0$$
$$\Rightarrow \quad x^2+y^2+\frac{3}{2}x+4y+c=0$$
$$s_3: x^2+y^2+2x+2y+1=0$$

Radical axis $s_1-s_2=0$

$\Rightarrow \quad (x^2+y^2+2gx+2fy+c)$
$\quad -(x^2+y^2+\frac{3}{2}x+4y+c)+0$
$\Rightarrow \quad 2gx+2fy-\frac{3x}{2}-4y=0$
$\Rightarrow \quad (2g-\frac{3}{2})x+(2f-4)y=0$
$\Rightarrow \quad (4g-3)x+(4f-8)y=0$

The radical axis touches the circle s_3, where radius of s_3.

$=\sqrt{(-1)^2+(-1)^2-1}=1$ and center is $(-1,-1)$

The perpendicular distance from the center of s_3 to the radical axis $=$ radius of s_3

$$\frac{|(4g-3)\cdot(-1)+(4f-8)(-1)|}{\sqrt{(4g-3)^2+(4f-8)^2}}=1$$

$$\Rightarrow \quad \frac{|-4g+3-4f+8|}{\sqrt{(4g-3)^2+(4f-8)^2}}=1$$

$\Rightarrow \quad |-4g-4f+11|$
$=\sqrt{(4g-3)^2+(4f-8)^2}$
$\Rightarrow \quad (-4g-4f+11)^2$
$=(4g-3)^2+(4f-8)^2$
$\Rightarrow \quad 16g^2+16f^2+121+32gf-88g-88f$
$=16g^2-24g+9+16f^2-64f+64$
$\Rightarrow \quad 32gf-64g-24f+48=0$
$\Rightarrow \quad 8gf-16g-6f+12=0$
$\Rightarrow \quad 8g(f-2)-6(f-2)=0$
$\Rightarrow \quad (8g-6)(f-2)=0$
$\Rightarrow \quad (4g-3)(f-2)=0$

$\Rightarrow \quad 4g-3=0$ or $f-2=0$
$\Rightarrow \quad g=\frac{3}{4}$ or $f=2$

Either $g=\frac{3}{4}$ or $f=2$ is the required condition.

Question19

After the coordinate axes are rotated through an angle $\frac{\pi}{4}$ in the anti-clockwise direction without shifting the origin, if the equation $x^2+y^2-2x-4y-20=0$ transforms to $ax^2+2hxy+by^2+2gx+2fy+c=0$ in the new coordinate system, then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} =$$

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Options:

A.

-20

B.

-25

C.

-30

D.

-35

Answer: B

Solution:

$\because x^2 + y^2 - 2x - 4y - 20 = 0$

After rotation of axes through $\theta = \pi/4$, it transforms to $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ in the given equation

$a = 1, h = 0, b = 1, g = -1, f = -2, c = -20$

After rotation of $\theta = \pi/4$

$$\begin{aligned} a' &= a\cos^2\theta + b\sin^2\theta + 2h\cos\theta\sin\theta \\ &= 1\left(\frac{1}{\sqrt{2}}\right)^2 + 1\left(\frac{1}{\sqrt{2}}\right)^2 + 0 = 1 \end{aligned}$$

$$\begin{aligned} b' &= a\sin^2\theta + b\cos^2\theta - 2h\sin\theta\cos\theta \\ &= 1\left(\frac{1}{\sqrt{2}}\right)^2 + 1\left(\frac{1}{\sqrt{2}}\right)^2 - 0 = 1 \end{aligned}$$

also,

$h' = (b-a)\sin\theta\cos\theta + h\left(\cos^2\theta - \sin^2\theta\right)$
$= 0 + 0 = 0$

Similarly, $x' = x\cos\theta - y\sin\theta = \frac{1}{\sqrt{2}}(x-y)$

and $y' = x\sin\theta + y\sin\theta = \frac{1}{\sqrt{2}}(x+y)$
then $-2x' - 4y' = \frac{-2x+2y-4x-4y}{\sqrt{2}}$

$$= \frac{-6x-2y}{\sqrt{2}}$$
$$= -3\sqrt{2}x - \sqrt{2}y = 2gx + 2fy$$

then $g = -3/\sqrt{2}, f = -1/\sqrt{2}, c = -20$

So,

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3/\sqrt{2} \\ 0 & 1 & -1/\sqrt{2} \\ -3/\sqrt{2} & -1/\sqrt{2} & -20 \end{vmatrix}$$
$$= 1\left(-20 - \frac{1}{2}\right) - \frac{3}{\sqrt{2}}\left(+\frac{3}{\sqrt{2}}\right)$$
$$= -41/2 - 9/2 = -25$$

Question20

If the circles $x^2 + y^2 + 5kx + 2y + k = 0$ and $2x^2 + 2y^2 + 2kx + 3y - 1 = 0, k \in R$ intersect at points P and Q then the line $4x + 5y - k = 0$ passes through P and Q for

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Options:

A.

exactly one value of k

B.

exactly two values of k

C.

no value of k

D.

infinitely many value of k

Answer: C

Solution:

Given equation of circle

$x^2 + y^2 + 5kx + 2y + k = 0$

and $2x^2 + 2y^2 + 2kx + 3y - 1 = 0$

Equation of common chord

$8kx + y + 2k + 1 = 0$

Since, this chord is coincide with

$$4x + 5y - k = 0$$
$$\frac{8k}{4} = \frac{1}{5} = \frac{2k+1}{-k}$$

Solving $\frac{8k}{4} = \frac{1}{5}$

we get $k = \frac{1}{10}$

Solving $8\frac{k}{4} = \frac{2k+1}{-k} \Rightarrow 2k^2 + 2k + 1 = 0$

No solution

$\therefore$ No values of k possible

Question21

The slope of one of the direct common tangents drawn to the circles $x^2 + y^2 - 2x + 4y + 1 = 0$ and $x^2 + y^2 - 4x - 2y + 4 = 0$ is

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Options:

A.

0

B.

$\frac{4}{3}$

C.

$\frac{3}{4}$

D.

1

Answer: B

Solution:

$$\begin{aligned}
& c_1: x^2+y^2-2x+4y+1=0 \\
& \Rightarrow (x-1)^2+(y+2)^2=4 \\
& \text{Centre } (c_1)=(1,-2), r_1=2 \\
& \text{and } c_2: x^2+y^2-4x-2y+4=0 \\
& \Rightarrow (x-2)^2+(y-1)^2=1 \\
& \text{Centre } (c_2)=(2,1), r_2=1
\end{aligned}$$

$$c_1c_2=d=\sqrt{(2-1)^2+(1+2^2}=\sqrt{10}$$

Let the slope of direct common tangent be m the general line is

$y=mx+c$

this line has perpendicular distance r_1 from c_1 and r_2 from c_2

$$\begin{aligned}
& \therefore \left|\frac{m(1)-(-2)+c}{\sqrt{1+m^2}}\right|=2 \\
& \Rightarrow |m+2+c|=2\sqrt{1+m^2} \qquad \ldots(i) \\
& \text{and } \left|\frac{m(2)-1+c}{\sqrt{1+m^2}}\right|=1 \\
& \Rightarrow |2m+c-1|=\sqrt{1+m^2} \qquad \ldots(ii)
\end{aligned}$$

Put the value of ' c ' from Eq. (i) to Eq. (ii)

$$2m-1+\left(-m-2+2\sqrt{m^2+1}\right)=\sqrt{m^2+1}$$

$$\begin{aligned}
& \Rightarrow m=3-\sqrt{m^2+1} \\
& \text{Squaring both sides, we get} \\
& \quad m^2= \quad m^2+10 \quad -6\sqrt{m^2+1} \\
\Rightarrow & \quad \sqrt{m^2+1} \quad =5/3 \\
\Rightarrow & \quad m^2+1 \quad =\tfrac{25}{9} \\
\Rightarrow & \quad m^2=16/9 \\
\Rightarrow & \quad m=4/3
\end{aligned}$$

Question22

If $(1, a), (b, 2)$ are conjugate points with respect to the circle $x^2 + y^2 = 25$, then $4a + 2b =$

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Options:

A.

25

B.

50

C.

100

D.

150

Answer: B

Solution:

We are told that $(1, a)$ and $(b, 2)$ are *conjugate points* with respect to the circle

$x^2 + y^2 = 25.$

Step 1. Condition for conjugate points

If (x_1, y_1) and (x_2, y_2) are conjugate points with respect to the circle $x^2 + y^2 = r^2$,

then the polar of one point passes through the other.

Equation of the *polar* of (x_1, y_1) w.r.t. $x^2 + y^2 = r^2$ is:

$x_1 x + y_1 y = r^2$

Step 2. The point $(1, a)$

For the point $(1, a)$, the polar is:

$1 \cdot x + a \cdot y = 25$

That is:

$x + ay = 25.$

Step 3. The point $(b, 2)$ lies on this polar

Since the points $(1, a)$ and $(b, 2)$ are conjugate, the point $(b, 2)$ **lies on the polar** of $(1, a)$.

So:

$b + a(2) = 25$

$b + 2a = 25 \quad (1)$

Step 4. Symmetry of conjugate points

Similarly, the polar of $(b, 2)$ is:

$bx + 2y = 25.$

And $(1, a)$ must lie on this polar:

$b(1) + 2(a) = 25$

Wait—that would give same, yes so far matches (1). Let's check the other way:

Substitute $(x, y) = (1, a)$:

$b \cdot 1 + 2a = 25$

This is the *same* as (1). So it's consistent.

Step 5. We need $4a + 2b = ?$

From (1): $b + 2a = 25.$

Multiply both sides by 2:

$2b + 4a = 50$

So,

$\boxed{4a + 2b = 50.}$

✅ **Final Answer: 50**

Option B

Question23

If the pole of the line $x + 2by - 5 = 0$ with respect to the circle $S \equiv x^2 + y^2 - 4x - 6y + 4 = 0$ lies on the line $x + by + 1 = 0$, then the polar of the point $(b, -b)$ with respect to the circle $S = 0$ is

Options:

A.

$5y-6=0$

B.

$y-6=0$

C.

$x+5y-6=0$

D.

$5x+y-6=0$

Answer: A

Solution:

Let (h, k) be the pole of the line

$x+2by-5=0$

with respect to the circle

$$s \equiv x^2+y^2-4x-6y+4=0$$
$$\therefore \quad hx+ky-2(x+h)-3(y+k)+4=0$$
$$(h-2)x+(k-3)y-2h-3k+4=0$$

is coincide with $x+2by-5=0$

$$\frac{h-2}{1}=\frac{k-3}{2b}=\frac{2h+3k-4}{5}$$
$$\Rightarrow \frac{h-2}{1}=\frac{k-3}{2b}$$

$2bh-k=4b-3 \qquad \ldots(i)$

and $\frac{h-2}{1}=\frac{2h+3k-4}{5}$

$h-k=2 \qquad \ldots(ii)$

Solving Eqs. (i) and (ii), we get

$h=\frac{4b-5}{2b-1}, k=\frac{-3}{2b-1}$

(h, k) lie on line $x+by+1=0$

$$\frac{4b-5}{2b-1}-\frac{3b}{2b-1}+1=0$$
$$4b-5-3b+2b-1=0$$
$$=2$$

$\therefore$ Pole of the line from pole $(2,-2)$ is

$(2-2)x+(-2-3)y-2(2)-3(-2)+4=0$

$\Rightarrow \quad -5y+6=0$

$\Rightarrow \quad 5y-6=0$

Question24

If $P(\alpha, \beta)$ is the radical centre of the circles $S \equiv x^2+y^2+4x+7=0, S'=2x^2+2y^2+3x+5y+9=0$ and $S'' \equiv x^2+y^2+y=0$, then the length of the tangent drawn from P to $S'=0$ is

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Options:

A.

5

B.

8

C.

4

D.

2

Answer: D

Solution:

The given equations are

$s : x^2 + y^2 + 4x + 7 = 0$
$s' : 2x^2 + 2y^2 + 3x + 5y + 9 = 0$
$\Rightarrow x^2 + y^2 + \frac{3}{2}x + \frac{5}{2}y + \frac{9}{2} = 0$
$s'' : x^2 + y^2 + y = 0$

Radical axis of s and $s' : s - s' = 0$

$\Rightarrow \quad 4x - \frac{3}{2}x - \frac{5}{2}y + 7 - \frac{9}{2} = 0$

$\Rightarrow \quad \frac{5}{2}x - \frac{5}{2}y + \frac{5}{2} = 0$

$\Rightarrow \quad x - y + 1 = 0 \quad \quad \ldots (i)$

Radical axis of s and $s'' : s - s'' = 0$

$\Rightarrow \quad 4x - y + 7 = 0 \quad \quad \ldots (ii)$

From Eqs. (i) and (ii), we have

$x = -2, y = -1$

$\therefore$ Radical centre is $P(-2, -1)$

Length of tangent from P to s'

$= \sqrt{(-2)^2 + (-1)^2 + \frac{3}{2}(-2) + \frac{5}{2}(-1) + \frac{9}{2}}$
$= \sqrt{4 + 1 - 3 + 2} = \sqrt{4} = 2$

Question25

When the axes are rotated through an angle θ about origin in anti-clockwise direction and then translated to the new origin $(2, -2)$, if the transformed equation the equation of $x^2 + y^2 = 4$ is $X^2 + Y^2 + aX + bY + c = 0$ then $a + b + c =$

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Options:

A.

4

B.

8

C.

0

D.

12

Answer: A

Solution:

Substitute $x = X + 2$ and $y = Y - 2$ into the original equation, $x^2 + y^2 = 4$ we get;

$$\begin{aligned} & (X+2)^2 + (Y-2)^2 = 4 \\ \Rightarrow & X^2 + 4 + 4X + Y^2 + 4 - 4Y = 4 \\ \Rightarrow & X^2 + Y^2 + 4X - 4Y + 4 = 0 \\ & \therefore a + b + c = 4 - 4 + 4 = 4 \end{aligned}$$

Question26

From a point $P(-4, 0)$, two tangents are drawn to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ touching the circle at A and B. If the equation of the circle passing through P, A and B is $x^2 + y^2 + 2gx + 2fy + c = 0$, then $(g, f) =$

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Options:

A.

$\left(-1, \frac{3}{2}\right)$

B.

$\left(\frac{3}{2}, -1\right)$

C.

$\left(\frac{1}{2}, \frac{-3}{2}\right)$

D.

$\left(1, \frac{-3}{2}\right)$

Answer: D

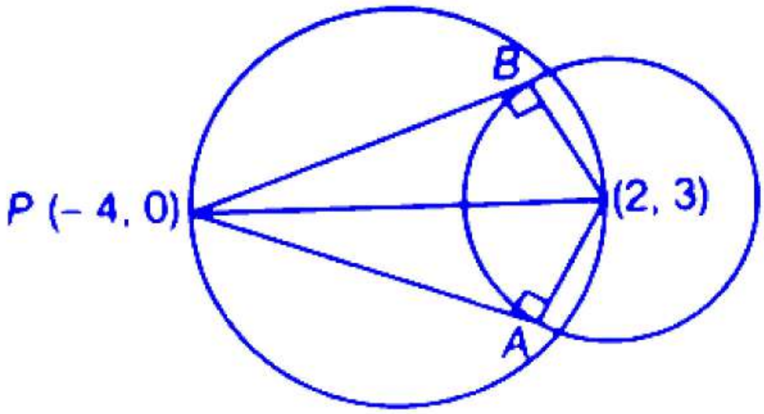
Solution:

Given, point $P(-4, 0)$

Equation of circle

$x^2 + y^2 - 4x - 6y - 12 = 0$

Centre $(2, 3)$



Centre of circle passing through P, A, B is mid point of OP

$\therefore \left(\frac{-4+2}{2}, \frac{0+3}{2}\right) = \left(-1, \frac{3}{2}\right)$
$\therefore (g, f) = \left(1, \frac{-3}{2}\right)$

Question27

If the equation of the polar of the point $(\alpha, -1)$ with respect to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ is $y = \beta$, then $4(\alpha + \beta) =$

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Options:

A.

-5

B.

7

C.

-6

D.

0

Answer: A

Solution:

Equation of polar is $T=0$

i.e., $\alpha x+y(-1)-4\cdot\frac{1}{2}(x+\alpha)$

$-6\cdot\frac{1}{2}(y+(-1))-12=0$

$\Rightarrow \quad \alpha x-y-2x-2\alpha-3y+3-12=0$

$\Rightarrow \quad (\alpha-2x-4y-2\alpha-9=0 \qquad \ldots(i)$

But equation of polar is $y=\beta$ given

So, coefficient of x must be zero.

i.e. $\alpha-2=0\Rightarrow\alpha=2$

Thus, $0-4y-2\times(2)-9=0 \quad (fromEq.(i))$

$\Rightarrow \quad -4y=13$

$\Rightarrow \quad y=\frac{-13}{4}=\beta \ (given)$

$\therefore 4(\alpha+\beta)=4\left(2-\frac{13}{4}\right)=-5$

Question28

If θ is the angle between the tangents drawn from the point $(-1,-1)$ to the circle $x^2+y^2-4x-6y+c=0$ and $\cos\theta=-\frac{7}{25}$, then the radius of the circle is

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Options:

A.

4

B.

1

C.

2

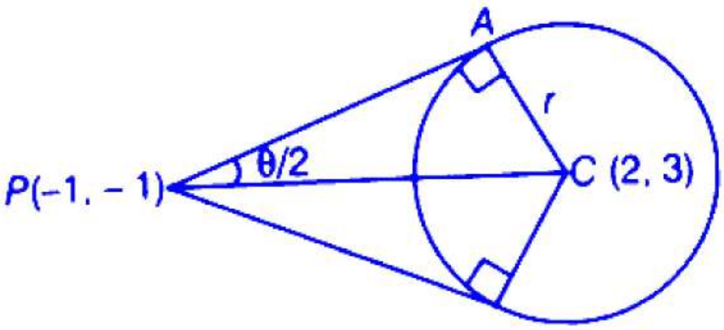
D.

3

Answer: A

Solution:

Here centre of circle $= C = (2, 3)$ let $P \equiv (-1, -1)$ and $CP = 5$



Let radius $= r$

$$\therefore \sin\frac{\theta}{2} = \frac{r}{CP}$$

$$\Rightarrow \quad r^2 = (5)^2 \sin^2\frac{\theta}{2}$$

$$\because \cos\theta = \frac{-7}{25} \quad (given)$$

$$1 - 2\sin^2\frac{\theta}{2} = \frac{-7}{25} \Rightarrow 2\sin^2\frac{\theta}{2} = 1 + \frac{7}{25}$$

$$\Rightarrow \quad \sin^2\frac{\theta}{2} = \frac{16}{25} = 25 \cdot \frac{16}{25} = 16$$

$$\therefore \quad r = 4 \quad (\because r > 0)$$

Question29

If the power of the point $(1, 6)$ with respect to the circle $x^2 + y^2 + 4x - 6y - a = 0$ is -16 , then $a =$

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Options:

A.

7

B.

11

C.

13

D.

21

Answer: D

Solution:

As we know that, power of a point

W.r.t a circle $= \left(\sqrt{S_1}\right)^2 = S_1$

$\therefore 1 + 36 + 4 - 36 - a = -16$

$\Rightarrow a = 5 + 16 = 21$

Question30

The radius of the circle passing through the points of intersection of the circles $x^2 + y^2 + 2x + 4y + 1 = 0$, $x^2 + y^2 - 2x - 4y - 4 = 0$ and intersecting the circle $x^2 + y^2 = 6$ orthogonally is

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Options:

A.

$\sqrt{19}$

B.

5

C.

$\sqrt{39}$

D.

4

Answer: C

Solution:

The equation of a circle passing through the intersection of two circles

$x^2+y^2+2x+4y+1=0$ and
$x^2+y^2-2x-4y-4=0$ is
$(x^2+y^2+2x+4y+1)$
$+\lambda(x^2+y^2-2x-4y-4)=0,$ where λ

is a parameter.
$\Rightarrow (1+\lambda)x^2+(1+\lambda)y^2+(2-2\lambda)+(4-4\lambda)y+1-4\lambda=0$

$$\Rightarrow x^2+y^2+\frac{2(1-\lambda)}{(1+\lambda)}x+\frac{4(1-\lambda)}{(1+\lambda)}y+\frac{1-4\lambda}{1+\lambda}=0\ldots\text{ (i)}$$

Applying the orthogonal condition,

$$2\left(\frac{1-\lambda}{1+\lambda}\right)\times 0+2\times\frac{2(1-\lambda)}{(1+\lambda)}\times 0 = \frac{1-4\lambda}{1+\lambda}-6$$

$$\Rightarrow \frac{1-4\lambda}{1+\lambda}=6$$

$$\Rightarrow 6+6\lambda=1-4\lambda\Rightarrow\lambda=\frac{-1}{2}$$

Substituting $\lambda=\frac{-1}{2}$ in Eq. (i) and simplifying we get

$x^2+y^2+6x+12y+6=0$

$\therefore$ Radius of this circle

$=\sqrt{3^2+6^2-6}=\sqrt{39}$

Question31

A circle passing through the point $(1, 0)$ makes an intercept of length 4 units on X-axis and an intercept of length $2\sqrt{11}$ units on Y-axis. If the centre of the circle lies in the fourth quadrant, then the radius of the circle is

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Options:

A.

$4\sqrt{5}$

B.

3

C.

$2\sqrt{5}$

D.

5

Answer: C

Solution:

Let the x-intercepts be $(a, 0)$ and

$(a+4, 0)$

Let the y-intercepts be $(0, b)$ and

$(0, b+2\sqrt{11})$

Center of circle $= (a+2, b+\sqrt{11})$

Since, the center is in the fourth quadrant $h > 0$ and $k < 0$

$\Rightarrow a+2 > 0$ and $b+\sqrt{11} < 0$

Equation of circle $(x-h)^2+(y-k)^2=r^2$

$$\Rightarrow (1-(a+2))^2+(0-(b+\sqrt{11}))^2=r^2$$
$$\Rightarrow (-a-1)^2+(-b-\sqrt{11})^2=r^2$$

Substituting $(a, 0)$ gives

$$(a-(a+2))^2+(0-(b+\sqrt{11}))^2=r^2$$
$$\Rightarrow (-2)^2+(-b-\sqrt{11})^2=r^2$$

substituting ($a+4, 0$) gives

$$\Rightarrow (a+4-(a+2))^2+(0-(b+\sqrt{11}))^2=r^2$$
$$\Rightarrow 2^2+(-b-\sqrt{11})^2=r^2$$

Similarly, $(0, b)$ gives

$$(0-(a+2))^2+(b-(b+\sqrt{11}))^2=r^2$$

substituting $(0, b+2\sqrt{11})$

$$(0-(a+2))^2+(b+2\sqrt{11}-(b+\sqrt{11}))^2=r^2$$
$$(-a-2)^2+(\sqrt{11})^2=r^2$$

Solving above equation, we get

$r^2=20$

$r=2\sqrt{5}$

Question32

If $\left(\frac{1}{10}, \frac{-1}{5}\right)$ is the inverse point of a point $(-1, 2)$ with respect to the circle $x^2+y^2-2x+4y+c=0$ then $c=$

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Options:

A.

4

B.

-4

C.

2

D.

-2

Answer: B

Solution:

The inverse point is $\left(\frac{1}{10}, \frac{-1}{5}\right)$

The original point is $(-1, 2)$

The circle equation is

$x^2 + y^2 - 2x + 4y + c = 0$

Centre of the circle is $(1, -2)$

The original point is $(-1, 2)$

$$\begin{aligned} \text{The distance } &= \sqrt{(-1-1)^2 + (2-(-2))^2} \\ &= \sqrt{4+16} = \sqrt{20} \end{aligned}$$

The centre of the circle is $(1, -2)$

The inverse point is $\left(\frac{1}{10}, \frac{-1}{5}\right)$

$$\begin{aligned} \text{The distance } &= \sqrt{\left(\frac{1}{10} - 1\right)^2 + \left(\frac{-1}{5} + 2\right)^2} \\ &= \frac{9\sqrt{5}}{10} \end{aligned}$$

$$\begin{aligned} &\Rightarrow \quad r^2 = d_1 d_2 = \sqrt{20} \times \tfrac{9\sqrt{5}}{10} = 9 \\ &\Rightarrow \quad 5 - c = 9 \Rightarrow c = -4 \end{aligned}$$

Question33

If the equation of the circle lying in the first quadrant, touching both the coordinate axes and the line $\frac{x}{3}+\frac{y}{4}=1$ is $(x-c)^2+(y-c)^2=c^2$, then $c=$

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Options:

A.

1 or 4

B.

2 or 3

C.

1 or 6

D.

2 or 5

Answer: C

Solution:

$$\begin{aligned} & \frac{x}{3}+\frac{y}{4}=1 \\ \Rightarrow \quad & 4x+3y=12 \\ \Rightarrow \quad & 4x+3y-12=0 \end{aligned}$$

distance from the centre (c, c) to the line

$$4x+3y-12=0$$

$$d=\frac{|4c+3c-12|}{\sqrt{25}}$$

$$=\frac{|4c+3c-12|}{5}$$

$$\Rightarrow \frac{|7c-12|}{5}=c$$

$$\Rightarrow |7c-12|=5c$$

$$\Rightarrow 7c-12=5c \text{ and } 7c-12=-5c$$

$c=6, c=1$

Question 34

If the point of contact of the circles $x^2 + y^2 - 6x - 4y + 9 = 0$ and $x^2 + y^2 + 2x + 2y - 7 = 0$ is (α, β), then $7\beta =$

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Options:

A.

5α

B.

2α

C.

3α

D.

4α

Answer: D

Solution:

Circle I : $x^2 + y^2 - 6x - 4y + 9 = 0$

Circle II : $x^2 + y^2 + 2x + 2y - 7 = 0$

Point of contact : (α, β)

$\Rightarrow (x-3)^2 + (y-2)^2 = 4$

Center $= (3, 2)$

$(x+1)^2 + (y+1)^2 = 9$

centre $(-1, -1)$

Slope of line $(m) = \frac{3}{4}$

Equation of line $y - 2 = \frac{3}{4}(x - 3)$

$\Rightarrow 3x - 4y - 1 = 0$

The point of contact (α, β) lies on the line $3x - 4y - 1 = 0$

$\Rightarrow$ Radii are $r_1 = 2$ and $r_2 = 3$

Ratio $= 2 : 3$

Using section formula

$$\alpha = \frac{2(-1)+3(3)}{2+3} = \frac{7}{5}, \beta = \frac{2(-1)+3(2)}{2+3}$$
$$\Rightarrow \beta = \frac{4}{5} \Rightarrow 7\beta = 7 \times \frac{4}{5} = 4 \cdot \frac{7}{5}$$
$$\therefore 7\beta = 4\alpha$$

Question35

If the circles $x^2 + y^2 - 2\lambda x - 2y - 7 = 0$ and $3\left(x^2 + y^2\right) - 8x + 29y = 0$ are orthogonal, then $\lambda =$

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Options:

A.

4

B.

3

C.

2

D.

1

Answer: D

Solution:

$x^2 + y^2 - \frac{8}{3}x + \frac{29}{3}y = 0$

For the first circle, $g_1 = -\lambda, f_1 = -1, c_1 = -7$ for the second circle $g_2 = \frac{-4}{3}$,

$f_2 = \frac{29}{6}, c_2 = 0$

Orthogonality condition.

$$\begin{aligned} & 2 g_1 g_2+2 f_1 f_2=c_1+c_2 \\ \Rightarrow \quad & 2(-\lambda)\left(\frac{-4}{3}\right)+2(-1)\left(\frac{29}{6}\right)=-7+0 \\ \Rightarrow \quad & \frac{8}{3} \lambda-\frac{29}{3}=-7 \\ \Rightarrow \quad & \frac{8}{3} \lambda=\frac{29}{3}-7 \\ \Rightarrow \quad & \frac{8}{3} \lambda=\frac{8}{3} \Rightarrow \lambda=1 \end{aligned}$$

Question36

If Q is the inverse point of $P(-1,1)$ with respect to the circle $x^2+y^2-2x+2y=0$, then the line containing Q is

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Options:

A.

$x-3y-2=0$

B.

$x-y+1=0$

C.

$x+y-2=0$

D.

$2x-3y+5=0$

Answer: A

Solution:

Here Q is the inverse point of $P(-1,1)$ with respect to the circle

$$\begin{aligned} & x^2+y^2-2x+2y=0 \\ \Rightarrow \quad & (x-1)^2+(y+1)^2=2 \end{aligned}$$

Centre of circle $O \equiv (1, -1)$

$$\therefore \quad OP \cdot OQ = r^2$$
$$\Rightarrow OQ = \frac{r^2}{OP} = \frac{2}{\sqrt{(1+1)^2 + (-1-1)^2}}$$
$$= \frac{2}{\sqrt{8}} = \frac{1}{\sqrt{2}}$$

OP, OQ are collinear

$\therefore$ Slope of $OP = -1$

Q point lie on OPQ

$$\therefore \quad \frac{x-1}{\cos\theta} = \frac{y+1}{\sin\theta} = OQ$$
$$\Rightarrow \quad \frac{x-1}{\frac{-1}{\sqrt{2}}} = \frac{y+1}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$
$$x - 1 = \frac{-1}{2}, \quad y + 1 = \frac{1}{2}$$
$$x = \frac{1}{2}, y = \frac{-1}{2}$$
$$\therefore \quad Q \equiv \left(\frac{1}{2}, \frac{-1}{2}\right) \text{ lie on line}$$
$$x - 3y - 2 = 0$$

Question37

If the circle passing through $(3, 5), (5, 5)$ and $(3, -3)$ cuts the circle $x^2 + y^2 + 2x + 2fy = 0$ orthogonally, then $f =$

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Options:

A.

-12

B.

-3

C.

-15

D.

-4

Answer: D

Solution:

Equation of circle passes through $(3,5)$, $(5,5)$ and $(3,-3)$ is

Let equation be

$x^2+y^2+2gx+2fy+c=0 \qquad \ldots(i)$

This is passes through $(3,5)$, $(5,5)$ and $(3,-3)$

$$\begin{aligned} 6g+10f+c&=-34 && \ldots(ii)\\ 10g+10f+c&=50 && \ldots(iii)\\ 6g-6f+c&=-18 && \ldots(iv)\end{aligned}$$

From Eqs. (i), (ii) and (iii) $g=-4$,

$f=-1$ and $c=0$

Required equation of circle

$x^2+y^2-8x-2y=0 \qquad \ldots(v)$

and given circle is

$x^2+y^2+2x+2fy=0$

are orthogonal

$$\begin{aligned} 2(-4)(1)+2(-1)f&=0+0\\ f&=-4\end{aligned}$$

Question38

Length of the common chord of two circles of same radius is $2\sqrt{17}$. If one of the two circles is $x^2+y^2+6x+4y-12=0$, then acute angle between the two circles is

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Options:

A.

$\frac{\pi}{2}$

B.

$\sin^{-1}\left(\frac{3}{5}\right)$

C.

$\cos^{-1}\left(\frac{9}{25}\right)$

D.

$\tan^{-1}\left(\frac{9}{17}\right)$

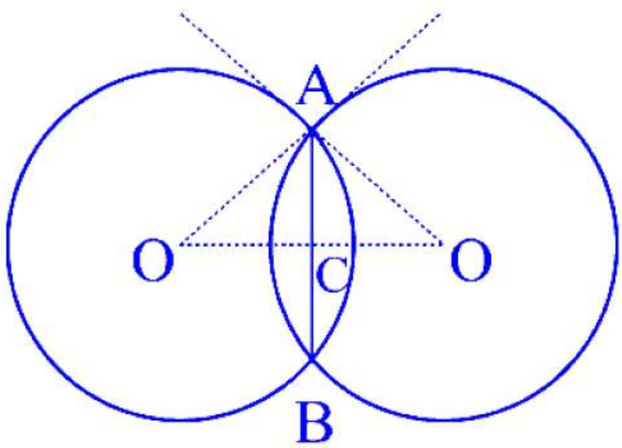
Answer: C

Solution:

Given that length of common chord of two circles $= 2\sqrt{17}$, with same radii one circle is

$$\Rightarrow \quad x^2 + y^2 + 6x + 4y - 12 = 0$$
$$r = \sqrt{3^2 + 2^2 + 12} = 5$$

both circles have same radii $= 5$



$AB = 2\sqrt{17}$
$AC = \sqrt{17}$
In $\triangle AO'C$

$$O'C = O'A^2 - AC^2$$
$$= 25 - 17 \Rightarrow O'C = 2\sqrt{2}$$
$$OO' = 4\sqrt{2}$$
$$\Rightarrow \quad \cos\theta = \frac{O'A^2 + OA^2 - O'O^2}{2\,(OA')(OA)}$$
$$\Rightarrow \quad \cos\theta = \frac{25 + 25 - 32}{50} \Rightarrow \cos\theta = \frac{9}{25}$$
$$\theta = \cos^{-1}\left(\frac{9}{25}\right)$$

Question39

A circle $S \equiv x^2 + y^2 - 16 = 0$ intersects another circle $S' = 0$ of radius 5 units such that their common chord is of maximum length. If the slope of that chord is $\frac{3}{4}$, then the centre of such a circle $S' = 0$ is

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Options:

A.

$\left(\frac{9}{5}, \frac{12}{5}\right)$

B.

$\left(\frac{5}{9}, \frac{-12}{5}\right)$

C.

$\left(\frac{-9}{5}, \frac{12}{5}\right)$

D.

$\left(\frac{3}{5}, \frac{4}{5}\right)$

Answer: C

Solution:

Given that

$S \equiv x^2 + y^2 - 16 = 0$

Let equation of

$S' \equiv (x-h)^2 + (y-k)^2 = 5^2$

Common chord $S_2 - S_1 = 0$

$2xh + 2ky + 9 = k^2 + h^2$

Given slope $= \frac{3}{4}$

$\frac{-2h}{2k} = \frac{3}{4} \Rightarrow h = \frac{-3}{4}k$

The chord of maximum length is diameter

$\therefore (0,0)$ lies on common chord

$h^2 + k^2 = 9$
$\frac{9}{16}k^2 + k^2 = 9 \Rightarrow k = \pm\frac{12}{5}$
$h = \mp\frac{9}{5}$
re $\equiv \left(\frac{-9}{5}, \frac{12}{5}\right), \left(\frac{9}{5}, \frac{-12}{5}\right)$

Question40

Let θ be the angle between the circles $S \equiv x^2 + y^2 + 2x - 2y + c = 0$ and $S' \equiv x^2 + y^2 - 6x - 8y + 9 = 0$. If c is an integer and $\cos\theta = \frac{5}{16}$, then the radius of the circle $S = 0$ is

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Options:

A.

2

B.

4

C.

3

D.

1

Answer: A

Solution:

Given equation of circles,

$$S \equiv x^2 + y^2 + 2x - 2y + c = 0$$
$$S' = x^2 + y^2 - 6x - 8y + 9 = 0$$
$$(g_1, f_1) \equiv -1, 1, c_1 = c$$
$$(g_2, f_2) \equiv (3, 4), c_2 = 9$$

Angle between circle S and S' be θ

$$\cos\theta = \frac{2\left(g_1 g_2 + f_1 f_2\right) - \left(c_1 + c_2\right)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}$$
$$\Rightarrow \quad \cos\theta = \frac{2(-3+4) - c - 9}{2\sqrt{2-c}\sqrt{25-9}}$$
$$\Rightarrow \quad \frac{5}{16} = \frac{-c-7}{2\sqrt{2-c}(4)}$$
$$\Rightarrow \quad 5 \times \sqrt{2-c} = -2(c+7)$$
$$\Rightarrow \quad 25(2-c) = 4\left(c^2 + 14c + 49\right)$$
$$\Rightarrow \quad 4c^2 + 81c + 146 = 0$$
$$\Rightarrow \quad c = -2 \frac{-146}{8}$$
$$\therefore \quad c = -2$$
$$\text{Radius } = \sqrt{1+1+2} = 2$$

Question41

If a circle S passes through the origin and makes an intercept of length 4 units on the line $x = 2$, then the equation of the curve on which the centre of S lies is

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Options:

A.

$y^2 - 4x = 8$

B.

$y^2 + 4x = 8$

C.

$x^2 + 4y = 8$

D.

$x^2 - 4y = 8$

Answer: B

Solution:

Let centre (h, k) and radius be r of circle S

$\because S \equiv (x-h)^2 + (y-k)^2 = r^2$

Circle passes through origin

$\because \quad r^2 = h^2 + k^2$

Given, the line $x = 2$ intersects the circle at two points let these points be ($2, y_1$) and ($2, y_2$) mid-points of these points is ($2, k$)

Now, the distance of $(2, k)$ to $(2, y_1)$ is 2 $y_1 = k + 2$ and $y_2 = k - 2$

($2, k+2$) and ($2, k-2$) are intersection point radius of the circle.

$$h^2 + k^2 = (h-2)^2 + (k-(k+2))^2$$
$$h^2 + k^2 = h^2 + 4 - 4h + 4$$
$$k^2 = -4h + 8 \Rightarrow k^2 + 4h = 8$$

Taking locus, we get

$y^2 + 4x = 8$

Question42

A circle touches the line $2x + y - 10 = 0$ at $(3, 4)$ and passes through the point $(1, -2)$. Then, a point that lies on the circle is

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Options:

A.

$(5, 4)$

B.

$(4, 5)$

C.

$(-5, 4)$

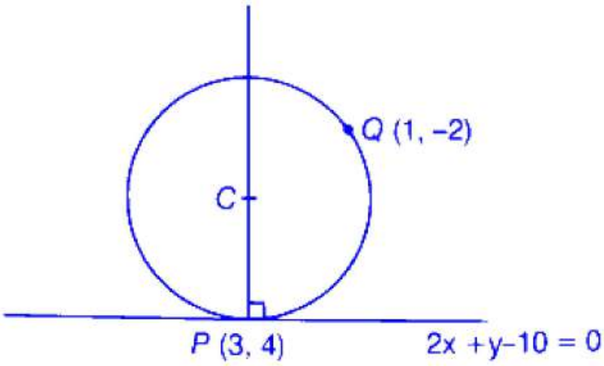
D.

$(4, -5)$

Answer: C

Solution:

Given, line $2x + y - 10 = 0$ touches the circle at the point $(3, 4)$.



$\because$ Perpendicular to this line passes through centre of circle.

Equation of line perpendicular to $2x + y - 10 = 0$ is

$$\begin{aligned} & x - 2y + \lambda = 0 \\ \Rightarrow \quad & 3 - 8 + \lambda = 0 \Rightarrow \lambda = 5 \\ \therefore \quad & x - 2y + 5 = 0 \end{aligned}$$

Centre of the circle be $C(2k - 5, k)$

Now radius $r^2 = CP^2 = CQ^2$

$r^2 = (2k - 8)^2 + (k - 4)^2 = (2k - 6)^2 + (k + 2)^2$

On solving this, we get $k = 2$

$\because$ Centre $(-1, 2)$ and radius $= \sqrt{20}$

Equation of circle be

$(x + 1)^2 + (y - 2)^2 = 20$

Clearly $(-5, 4)$ lies on the circle.

Question43

If (a, b) is the common point for the circles $x^2 + y^2 - 4x + 4y - 1 = 0$ and $x^2 + y^2 + 2x - 4y + 1 = 0$, then $a^2 + b^2 =$

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Options:

A.

$\frac{1}{5}$

B.

5

C.

25

D.

$\frac{1}{25}$

Answer: A

Solution:

(a, b) is the common point for the two given circle

$$\begin{array}{l} a^2 + b^2 - 4a + 4b - 1 = 0 \quad \ldots(i) \\ \quad a^2 + b^2 + 2a - 4b + 1 = 0 \quad \ldots(ii) \\ \underline{- \quad - \quad - \quad + \quad -} \\ \textit{Subtrating Eqs. (ii) form(i), we get} \end{array}$$

$$\begin{array}{l} -6a + 8b - 2 = 0 \\ -3a + 4b - 1 = 0 \\ 4b = 3a + 1 \quad \ldots(iii) \end{array}$$

From Eqs. (i) and (iii), we get

$$a^2 + \left(\frac{3a+1}{4}\right)^2 - 4a + 3a + 1 - 1 = 0$$

$$\Rightarrow 25a^2 - 10a + 1 = 0$$

$$(5a - 1)^2 = 0$$

$$a = \frac{1}{5} \text{ and } b = \frac{2}{5}$$

Now, $a^2 + b^2 = \frac{1}{25} + \frac{4}{25} = \frac{5}{25} = \frac{1}{5}$

Question44

The angle between the tangents drawn from the point $(2, 2)$ to the circle $x^2 + y^2 + 4x + 4y + c = 0$ is $\cos^{-1}\left(\frac{7}{16}\right)$. If two such circles exist, then sum of the values of c is

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Options:

A.

16

B.

20

C.

-20

D.

-16

Answer: D

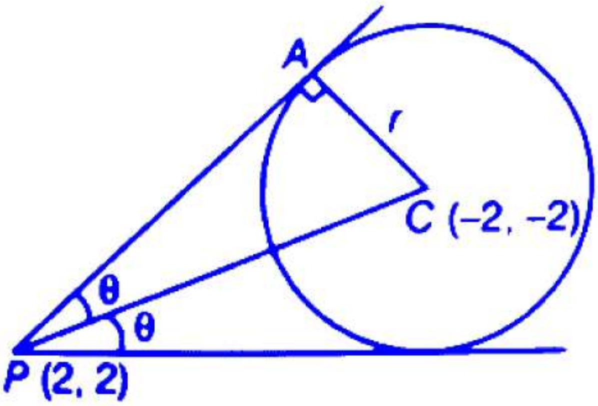
Solution:

We have the circle

$x^2 + y^2 + 4x + 4y + c = 0$

$C =$ Centre $= (-2, -2)$

$AC =$ radius $= \sqrt{8 - C}$



Angle between tangents

$\cos 2\theta = \frac{7}{16}, CP = \sqrt{16+16} = 4\sqrt{2}$

$\because \sin\theta = \frac{3}{4\sqrt{2}}$

Using $\sin\theta = \frac{r}{CP} = \frac{\sqrt{8-C}}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$

$\because \quad 8 - C = 9$

$C = -1$

Now, another circle is possible when angles be supplementary

$\because \theta$ is changed by $\pi - 2\theta$

$\because \quad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\because \quad \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \frac{\sqrt{23}}{4\sqrt{2}}$

$\because \quad \cos\theta = \frac{\sqrt{8-C}}{4\sqrt{2}} = \frac{\sqrt{23}}{4\sqrt{2}}$

$\because \quad 8 - c = 23$

$-c = -15$

$\because$ Now, sum of all possible values of C.

$= -1 - 15 = -16$

Question45

If the circle $S = x^2 + y^2 + 2gx + 4y + 1 = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x - 3 = 0$, then the radius of circle $S = 0$ is

Options:

A.

5

B.

$\sqrt{12}$

C.

25

D.

12

Answer: B

Solution:

We have the circle S

$S = x^2 + y^2 + 2gx + 4y + 1 = 0$

and the second circle be S_1,

$S_1 = x^2 + y^2 - 2x - 3 = 0$

$\because$ Centre $(1, 0)$ and radius $= 2$

Now, common chord of circles S and S_1 is

$S - S_1 = 0$
$(2g + 2)x + 4y + 4 = 0$

If a circle bisects the circumference of another circle, then common chord is a diameter of the second circle.

$$2g + 2 + 4 = 0 \Rightarrow g = -3$$
$$\because \quad S = x^2 + y^2 - 6x + 4y + 1 = 0$$

Centre $(3, -2)$

Radius $= \sqrt{9 + 4 - 1} = \sqrt{12}$

Question46

From a point P on the circle $x^2+y^2=4$, two tangents are drawn to the circle $x^2+y^2-6x-6y+14=0$. If A and B are the points of contact of those lines, then the locus of the centre of the circle passing through the points P, A and B is

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Options:

A.

$x^2+y^2-3x-3y+4=0$

B.

$2x^2+2y^2+6x+6y-7=0$

C.

$x^2+y^2+3x+3y-4=0$

D.

$2x^2+2y^2-6x-6y+7=0$

Answer: D

Solution:

Circle 1 given $x^2+y^2=4$

Circle 2 is given,

$x^2+y^2-6x-6y+14=0$

Centre $(3,3)$ radius $=2$

Let $P(x_1, y_1)$ be a point on circle 1 .

$\because$ Equation of chord of contact AB is

$$xx_1+yy_1-3(x+x_1)-3(y+y_1)+14=0$$
$$(x_1-3)x+(y_1-3)y-3x_1-3y_1+14=0$$

Now, equation of circle passing through P, A and B is

$$x^2+y^2-6x-6y+14+\lambda[(x_1-3)x+(y_1-3)y-3x_1-3y_1+14]=0$$

We have this circle passes through $P(x_1, y_1)$

$$\Rightarrow x_1^2+y_1^2-6x_1-6y_1+14+\lambda\left[(x_1-3)x_1 + (y_1-3)y_1-3x_1-3y_1+14\right]=0$$

$$\Rightarrow x_1^2+y_1^2-6x_1-6y_1+14+\lambda\left[x_1^2+y_1^2 -3x_1-3y_1-3x_1-3y_1+14\right]=x$$

Put $x_1^2+y_1^2=4$

$$\Rightarrow 4-6x_1-6y_1+14 +\lambda(4-6x_1-6y_1+14)=0$$

$$\Rightarrow 18-6x_1-6y_1+14+\lambda(18-6x_1 -6y_1)=0$$

$\because 1+\lambda=0$
$\lambda=-1$

$\Rightarrow$ Equation of circle be

$$\left(x^2+y^2-6x-6y+14\right) -\left[(x_1-3)x+(y_1-3)y -3x_1-3y_1+14\right]=0$$

$$\Rightarrow x^2+y^2-(3+x_1)x-(3+y_1)y +3x_1+3y_1=0$$

Centre $\left(\frac{3+x_1}{2}, \frac{3+y_1}{2}\right)$

$h=\frac{3+x_1}{2}$ and $k=\frac{3+y_1}{2}$

$x_1=2h-3 \quad y_1=2k-3$
$x_1^2+y_1^2=4$
$(2h-3)^2+(2k-3)^2=4$
$4h^2+4k^2-12h-12k+14=0$
$h^2+k^2-3h-3k+\frac{7}{2}=0$

$\because$ Taking locus of (h, k), we get

$2x^2+2y^2-6x-6y+7=0$

Question47

If the product of the lengths of the perpendicular drawn from the ends of a diameter of the circle $x^2+y^2=4$ on the line $x+y+1=0$ is maximum, then the two ends of that diameter are

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Options:

A.

$(-2, 0), (2, 0)$

B.

$(\sqrt{3}, 1), (-\sqrt{3}, -1)$

C.

$(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})$

D.

$(0, 2), (0, -2)$

Answer: C

Solution:

Let the ends of diameter of the given circle $x^2 + y^2 = 4$ is

$(2\cos\theta, 2\sin\theta)$, then other must be $(-2\cos\theta, -2\sin\theta)$

Then, length of perpendicular from the extremities of diameter to the line $x + y + 1 = 0$ is

$$\frac{2(\cos\theta + \sin\theta) + 1}{\sqrt{2}}$$

$$\times \frac{-2(\cos\theta + \sin\theta) + 1}{\sqrt{2}} = P(\text{ say })$$

$$\Rightarrow P = \frac{1 - 4(\cos\theta + \sin\theta)^2}{2}$$

$$= \frac{1 - 4(1 + \sin 2\theta)}{2}$$

$$\Rightarrow P = \frac{-3 - 4\sin 2\theta}{2}$$

$$\Rightarrow \quad \frac{dP}{d\theta} = \frac{1}{2}(-4)\cos 2\theta \cdot 2 = 0$$

$$\Rightarrow \quad \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

Hence, the points are $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$

Question 48

If the intercept made by a variable circle on the X -axis and Y-axis are 8 and 6 units respectively, then the locus of the centre of the circle is

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Options:

A.

$x^2 - y^2 + 28 = 0$

B.

$y^2 - x^2 - 7 = 0$

C.

$x^2 - y^2 - 28 = 0$

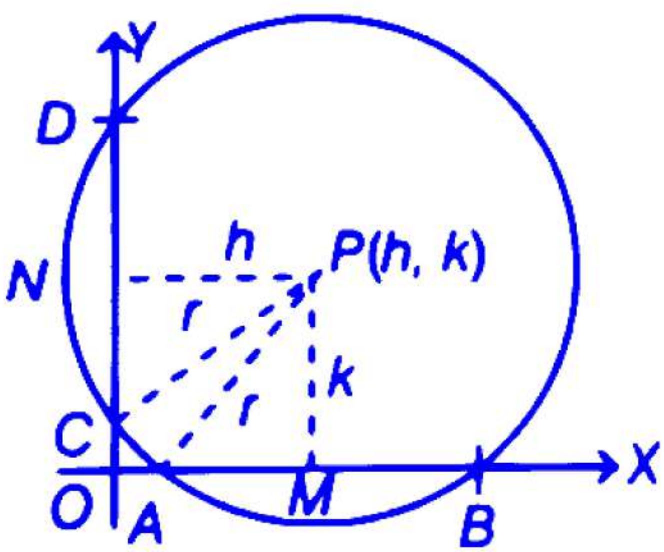
D.

$x^2 - y^2 - 7 = 0$

Answer: D

Solution:

Let the centre of circle be (h, k) and radius be r.



$$\begin{aligned} Clearly,\ PM &= k \\ PN &= h \\ AM &= 4, CN = 3 \\ r^2 &= 4^2 + k^2 \\ r^2 &= 3^2 + h^2 \end{aligned}$$

$$\begin{aligned} \because 16 + k^2 &= 9 + h^2 \\ h^2 - k^2 - 7 &= 0 \end{aligned}$$

Taking locus of $P(h, k)$, we get

$x^2 - y^2 - 7 = 0$

Question49

The slope of the non-vertical tangent drawn from the point $(3, 4)$ to the circle $x^2 + y^2 = 9$ is

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Options:

A.

$\frac{2}{3}$

B.

$\frac{3}{2}$

C.

$\frac{7}{24}$

D.

$\frac{24}{7}$

Answer: C

Solution:

Equation of tangent to the circle,

$x^2 + y^2 = 9$ is $y = mx + 3\sqrt{1+m^2}$

Tangent passes through the point $(3, 4)$,

$$(4-3m)^2 = 9\left(1+m^2\right)$$
$$\Rightarrow \quad 16 + 9m^2 - 24m = 9 + 9m^2$$
$$\Rightarrow \quad 24m = 7$$
$$m = \frac{7}{24}$$

Question50

If the acute angle between the circles $S \equiv x^2 + y^2 + 2kx + 4y - 3 = 0$ and $S' \equiv x^2 + y^2 - 4x + 2ky + 9 = 0$ is $\cos^{-1}\left(\frac{3}{8}\right)$ and the centre of $S' = 0$ lies in the first quadrant, then the radical axis of $S = 0$ and $S' = 0$ is

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Options:

A.

$x - 5y + 6 = 0$

B.

$x - 5y - 4 = 0$

C.

$5x - y - 6 = 0$

D.

$5x - y - 4 = 0$

Answer: A

Solution:

$$S = x^2 + y^2 + 2kx + 4y - 3 = 0$$
$$C_1 = (-k, -2) \text{ radius } r_1 = \sqrt{k^2 + 4 + 3}$$
$$S' = x^2 + y^2 - 4x + 2ky + 9 = 0$$
$$C_2 = (2, -k) \text{ radius } = \sqrt{4 + k^2 - 9}$$
$$= \sqrt{k^2 - 5}$$

Now, angle between two circles $S = 0$ and $S' = 0$ is

$\cos(180^\circ - \theta) = \frac{(r_1^2 + r_2^2 - (d^2)}{2r_1r_2}$

where r_1, r_2 are the radii and d is the distance between their centres.

$$= \frac{k^2 + 7 + k^2 - 5 - (k+2)^2 - (k-2)^2}{2\sqrt{k^2+7}\sqrt{k^2-5}}$$
$$\Rightarrow -\cos\theta = \frac{2k^2 + 2 - (2k^2 + 8)}{2\sqrt{k^2+7}\sqrt{k^2-5}}$$
$$= \frac{-6}{2\sqrt{k^2+7}\sqrt{k^2-5}}$$
$$\Rightarrow \quad \cos\theta = \frac{3}{\sqrt{k^2+7}\sqrt{k^2-5}} = \frac{3}{8}$$
$$\Rightarrow \quad (k^2+7)(k^2-9) = 64$$
$$\Rightarrow \quad k^4 + 2k^2 - 99 = 0$$
$$\Rightarrow \quad (k^2+11)(k^2-9) = 0 \Rightarrow k^2 = 9$$
$$\Rightarrow \quad k = \pm 3$$

Centre of $S' = 0$ lies in I quadrant

$[\therefore k = -3]$

Now, radical axis of $S = 0$ and $S' = 0$ is

$$x(2k+4) + y(4-2k) - 12 = 0$$
$$\Rightarrow \quad -2x + 10y - 12 = 0$$
$$x - 5y + 6 = 0$$

Question51

The circumference of a circle passing through the point $(4, 6)$ with two normals represented by $2x - 3y + 4 = 0$ and $x + y - 3 = 0$ is

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Options:

A. 5π

B. 10π

C. 25π

D. 8π

Answer: B

Solution:

The two given normal equations pass through the center of the circle. By solving the equations $2x - 3y + 4 = 0$ and $x + y - 3 = 0$, we find:

$x = 1, \quad y = 2$

Thus, the coordinates of the center of the circle are $(1, 2)$. Given that the point $(4, 6)$ lies on the circumference of the circle, we can calculate the radius as follows:

Radius $= \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9 + 16} = 5$

The circumference of the circle is then given by:

$2\pi r = 10\pi$

Question52

If the line through the point $P(5, 3)$ meets the circle $x^2 + y^2 - 2x - 4y + \alpha = 0$ at $A(4, 2)$ and $B(x_1, y_1)$, then $PA \cdot PB$ is equal to

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Options:

A. 6

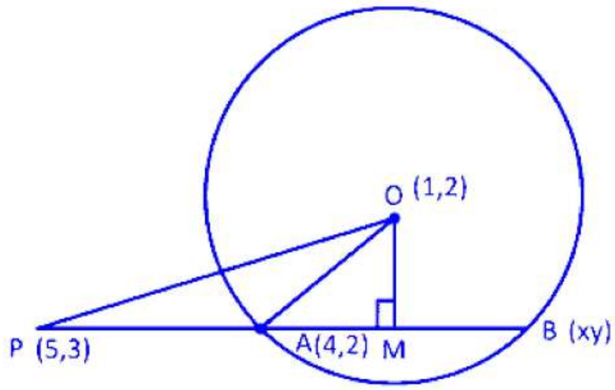
B. 12

C. 9

D. 8

Answer: D

Solution:



Coordinate of center of circle $x^2+y^2-2x-4y=0$ is $(1,2)$

$OM \perp AB$ and M, also M is mid-point of AB.

Radius of circle $=\sqrt{(4-1)^2+(2-2)^2}=3$

Equation of line PB is

$$y-3=\left(\frac{2-3}{4-5}\right)(x-5)$$
$$x-y=2$$
$$OM=\frac{|1-2-2|}{\sqrt{1^2+1^2}}=\frac{3}{\sqrt{2}}$$
$$AM=\sqrt{OA^2-OM^2}$$
$$=\sqrt{9-\frac{9}{2}}=\frac{3}{\sqrt{2}}$$
$$PB=PA+2AM$$
$$=\sqrt{(5-4)^2+(3-2)^2}+2\times\frac{3}{\sqrt{2}}$$
$$=\sqrt{2}+\frac{6}{\sqrt{2}}=\frac{8}{\sqrt{2}}$$

Now, $PA\times PB=\sqrt{2}\times\frac{8}{\sqrt{2}}=8$.

Question53

Consider the point $P(\alpha,\beta)$ on the line $2x+y=1$. If the P and $(3,2)$ are conjugate points with respect to the circle $x^2+y^2=4$, then $\alpha+\beta$ is equal to

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Options:

A. 3

B. -1

C. -5

D. 7

Answer: A

Solution:

P and $A(3, 2)$ are conjugates points of the circle $x^2 + y^2 = 4$.

Polar of A is $3x + 2y = 4$

Polar of A panes through P

$3\alpha + 2\beta = 4 \qquad \ldots(i)$

Also, point P is on the line $2x + y = 1$

$\Rightarrow \quad 2\alpha + \beta = 1 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$\alpha = -2$ and $\beta = 5$

Hence, $\alpha + \beta = -2 + 5 = 3$.

Question54

If $(1, 3)$ is the mid-point of a chord of the circle $x^2 + y^2 - 4x - 8y + 16 = 0$, then the area of the triangle formed by that chord with the coordinate axes is

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Options:

A. 16

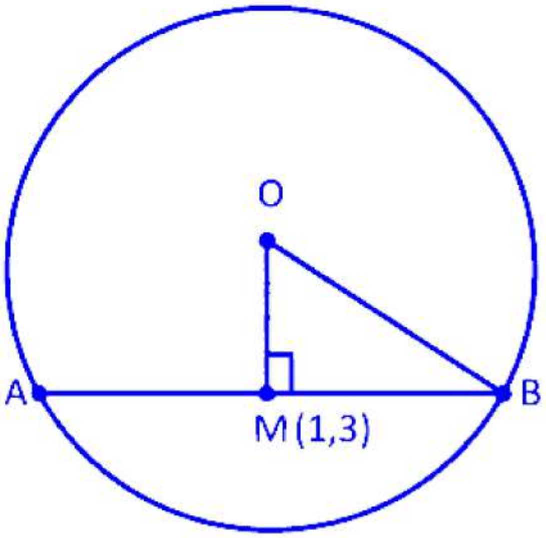
B. 8

C. 4

D. $8\sqrt{2}$

Answer: B

Solution:



Center of the given circle is $(2,4)$ and radius of the given circle

$$= \sqrt{4+16-16}$$
$$= 2$$

As the perpendicular line from center bisects the chord,

$AM = MB$

Slope of line $OM = \frac{4-3}{2-1} = 1$

Slope of chord $AB = -1 [\because AM \perp OM]$

Equation of chord AB is

$$y-3 = -1(x-1)$$
$$x+y = 4$$
$$\frac{x}{4}+\frac{y}{4} = 1$$

$\therefore x$ intercept $= y$ intercept $= 4$

$\therefore$ Required area of triangle

$= \frac{1}{2} \times 4 \times 4 = 8$ sq unit

Question55

If the circles $x^2+y^2+2\alpha x+2y-8=0$ and $x^2+y^2-2x+ay-14=0$ intersect orthogonally, then the distance between their centres is

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Options:

A. $\sqrt{242}$

B. $\sqrt{970}$

C. $\sqrt{629}$

D. $\sqrt{541}$

Answer: C

Solution:

If the two circle meets orthogonal, then

$$\begin{aligned} 2g_1g_2+2f_1f_2 &= c_1+c_2 \\ 2a(-1)+a &= -8-14 \\ a &= 22 \end{aligned}$$

Center of two given circle are $(-22,-1)$ and $(1,-11)$

$\therefore$ Required distance

$$\begin{aligned} &= \sqrt{(1+22)^2+(-11+1)^2} \\ &= \sqrt{529+100} \\ &= \sqrt{629} \end{aligned}$$

Question56

If the axes are rotated through angle ' α ', then the number of values of a such that the transformed equation of $x^2+y^2+2x+2y-5=0$ contains no liner terms is

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Options:

A. 0

B. 1

C. 2

D. Infinite

Answer: A

Solution:

The given equation is:

$x^2 + y^2 + 2x + 2y - 5 = 0$

We need to consider the transformation when the axes are rotated by an angle α. The equations for the transformed coordinates are:

$x = X \cos \alpha - Y \sin \alpha$

$y = X \sin \alpha + Y \cos \alpha$

Substituting these into the given equation, we get:

$(X \cos \alpha - Y \sin \alpha)^2 + (X \sin \alpha + Y \cos \alpha)^2 + 2(X \cos \alpha - Y \sin \alpha) + 2(X \sin \alpha + Y \cos \alpha) - 5 = 0$

Expanding and simplifying, the equation becomes:

$X^2 + Y^2 + 2X \cos \alpha - 2Y \sin \alpha + 2X \sin \alpha + 2Y \cos \alpha - 5 = 0$

Further simplifying, we have:

$X^2 + Y^2 - 2X(\cos \alpha + \sin \alpha) + 2Y(\cos \alpha - \sin \alpha) - 5 = 0$

For the transformed equation to have no linear terms, the coefficients of X and Y must be zero. Therefore, we set:

$\cos \alpha + \sin \alpha = 0$

$\cos \alpha - \sin \alpha = 0$

However, both equations cannot be simultaneously satisfied. Since these conditions are not possible together, there is no value of α that can make the transformed equation free of linear terms.

Question57

The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If $Q = (3, 4)$ and $R = (-4, 3)$, then $\angle QPR$ is equal to

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Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

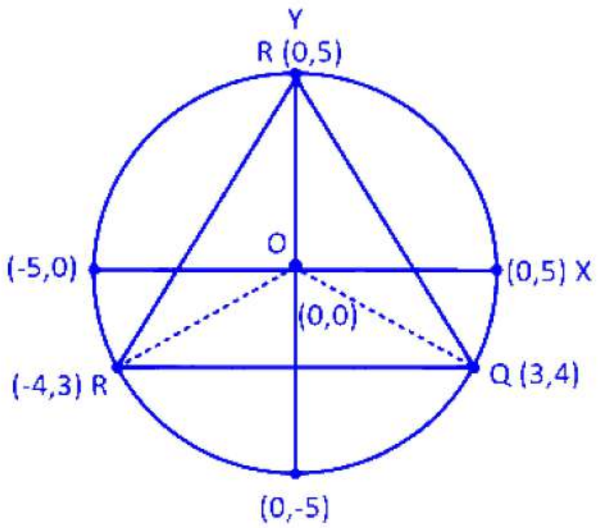
C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

$x^2 + y^2 = 25$



$$\begin{aligned} RQ &= \sqrt{(3+4)^2 + (4-3)^2} \\ &= \sqrt{49+1} = 5\sqrt{2} \\ OQ &= OR = 5 \end{aligned}$$

$$\cos \angle ROQ = \frac{OR^2 + OQ^2 - RQ^2}{2OR \cdot OQ}$$

$$= \frac{25 + 25 - 50}{2 \times 5 \times 5} = 0$$

$\Rightarrow \angle ROQ = 90^\circ$

Now,

$$\angle RPQ = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 90^\circ = 45^\circ = \frac{\pi}{4}$$

Question58

The locus of the point of intersection of perpendicular tangents drawn to the circle $x^2 + y^2 = 10$ is

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Options:

A. $x^2 + y^2 = 5$

B. $x^2 + y^2 = 20$

C. $x^2 + y^2 = 25$

D. $x^2 + y^2 = 100$

Answer: B

Solution:

To determine the locus of the point of intersection of perpendicular tangents to the circle given by $x^2 + y^2 = 10$, we need to consider the concept of a director circle.

The director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the original circle.

For the circle $x^2 + y^2 = 10$, the radius is $\sqrt{10}$. Thus, the radius of the director circle is:

$\sqrt{2} \times \sqrt{10} = \sqrt{20}$

Thus, the equation of the director circle is:

$x^2 + y^2 = 20$

Question59

The normal drawn at $(1, 1)$ to the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ is

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Options:

A. $4x + 3y = 7$

B. $4x + y = 5$

C. $x + y = 2$

D. $4x - y = 3$

Answer: B

Solution:

To find the equation of the normal to the circle $x^2 + y^2 - 4x + 6y - 4 = 0$ at the point $(1, 1)$, we first determine the circle's center.

The given equation can be rewritten as:

$(x^2 - 4x) + (y^2 + 6y) = 4$

Completing the square for both x and y:

$(x - 2)^2 - 4 + (y + 3)^2 - 9 = 4$

$(x - 2)^2 + (y + 3)^2 = 17$

Thus, the center of the circle is $(2, -3)$.

The normal line from $(1, 1)$ passes through the center $(2, -3)$. The slope of this normal line is calculated as follows:

slope $= \frac{-3-1}{2-1} = -4$

Using the point-slope form of a line with point $(1, 1)$:

$y - 1 = -4(x - 1)$

Simplifying this equation:

$y - 1 = -4x + 4$

$y = -4x + 5$

In standard form, this can be written as:

$4x + y - 5 = 0$

Thus, the equation of the normal is $4x + y - 5 = 0$.

Question60

Parametric equations of the circle $2x^2 + 2y^2 = 9$ are

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Options:

A. $x = \frac{3}{2}\cos\theta, \quad y = \frac{3}{2}\sin\theta$

B. $x=\frac{3}{\sqrt{2}}\cos\theta, \quad y=3\sin\theta$

C. $x=\frac{3}{\sqrt{2}}\sin\theta, \quad y=\frac{3}{\sqrt{2}}\cos\theta$

D. $x=3\sin\theta, \quad y=\frac{3}{2}\cos\theta$

Answer: C

Solution:

Given equation of circle is

$$\begin{aligned} & 2x^2+2y^2=9 \\ \Rightarrow \quad & 2\left(x^2+y^2\right)=9 \Rightarrow x^2+y^2=9/2 \\ \Rightarrow \quad & x^2+y^2=\left(\frac{3}{\sqrt{2}}\right)^2 \end{aligned}$$

Now, the parametric equation of circle is

$x=r\cos\theta, y=r\sin\theta$

$$\therefore \quad x=\frac{3}{\sqrt{2}}\cos\theta \quad \begin{bmatrix} \because r^2=x^2+y^2 \\ r=\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\text{and} \quad y=\frac{3}{\sqrt{2}}\sin\theta$$

Question61

Angle between the circles $x^2+y^2-4x-6y-3=0$ and $x^2+y^2+8x-4y+11=0$ is

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Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

Solution:

To find the angle between the circles c_1 and c_2, where:

$c_1 : x^2 + y^2 - 4x - 6y - 3 = 0$

$c_2 : x^2 + y^2 + 8x - 4y + 11 = 0,$

we can use the formula:

$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$

where r_1 and r_2 are the radii of the circles, and d is the distance between their centers.

First, calculate the radii:

$r_1 = \sqrt{2^2 + 3^2 + 3} = \sqrt{16} = 4$

$r_2 = \sqrt{4^2 + 2^2 - 11} = \sqrt{9} = 3$

Next, determine the centers of the circles:

The center of c_1 is $(2, 3)$.

The center of c_2 is $(-4, 2)$.

Now, calculate the distance d between the centers:

$d = \sqrt{(-4-2)^2 + (2-3)^2} = \sqrt{36+1} = \sqrt{37}$

Substitute all values into the formula for the angle:

$\cos\theta = \left|\frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right| = \left|\frac{16+9-37}{2\times4\times3}\right| = \frac{1}{2}$

Thus, the angle θ is $\frac{\pi}{3}$.

Question62

From a point $(1, 0)$ on the circle $x^2 + y^2 - 2x + 2y + 1 = 0$ if chords are drawn to this circle, then locus of the poles of these chords with respect the circle $x^2 + y^2 = 4$ is

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Options:

A. $x = 4$

B. $x + 2y = 5$

C. $x^2+y^2-x-y=0$

D. $2y^2=(x+1)$

Answer: A

Solution:

$x^2+y^2-2x+2y+1=0$

is a circle with centre $(1,-1)$ and radius 1 When, chord are drawn from of the point $(1,0)$ on the circle the locus will be $x^2+y^2=4$.

For a given point (x_1,y_1) on circle $x^2+y^2=4$ the equation of chord joining the point of tangent any of the tangent from (x_1,y_1) to the circle is $xx_1+yy_1=4$ given original point

(1,0).

$\Rightarrow x\cdot 1+y\cdot 0=4\Rightarrow x=4$

Thus, locus of poles of these chords is a vertical line.

Question63

If A and B are the centres of similitude with respect to the circles $x^2+y^2-14x+6y+33=0$ and $x^2+y^2+30x-2y+1=0$, then mid-point of AB is

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Options:

A. $\left(\frac{7}{3},\frac{4}{5}\right)$

B. $\left(\frac{3}{2},\frac{1}{5}\right)$

C. $\left(\frac{39}{2},\frac{-7}{4}\right)$

D. $\left(\frac{39}{4},\frac{-7}{2}\right)$

Answer: D

Solution:

$x^2+y^2-14x+6y+33=0 \qquad \ldots(i)$

$x^2+y^2+30x-2y+1=0 \qquad \ldots(ii)$

using Eq. (i), we can write it as

$$(x-7)^2-49+(y+3)^2-9+33=0$$
$$(x-7)^2+(y+3)^2-25=0$$

So, centre $(7,-3)$ and radius $=5$ using Eq. (ii) we can write it as

$$(x+15)^2-225+(y-1)^2-1+1=0$$
$$(x+15)^2+(y-1)^2=225$$

So, centre $(-15,1)$ and radius $=15$

Now, A(External centre similitude)

$=\frac{R_2C_1-R_1C_2}{R_2-R_1}$

And B (Internal centre similitude) $=\frac{R_2C_1+R_1C_2}{R_2+R_1}$, where C_1 and C_2 are

centers and R_1 and R_2 are their radii.

$$\begin{aligned} A &= \frac{15(7,-3)-5(-15,1)}{15-5} \\ &= \frac{(105,-45)+(75,-5)}{10} \\ &= \frac{(180,-50)}{10}=(18,-5) \\ B &= \frac{15(7,-3)+(5)(-15,1)}{15+5} \\ &= \frac{(105,-45)+(-75,5)}{20} \\ &= \frac{(30,-40)}{20}=(1.5,-2) \end{aligned}$$

Finally mid-point M of line segment

$$\begin{aligned} M &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &= \left(\frac{18+1.5}{2}, \frac{-5+(-2)}{2}\right) \\ M &= \left(\frac{19.5}{2}, \frac{-7}{2}\right)=\left(\frac{39}{4}, \frac{-7}{2}\right) \end{aligned}$$

Question64

C_1 is the circle with centre at $O(0,0)$ and radius $4, C_2$ is a variable circle with centre at (α, β) and radius 5 . If the common chord of C_1 and C_2 has slope $\frac{3}{4}$ and of maximum length, then one of the possible values of $\alpha+\beta$ is

Options:

A. $\frac{21}{5}$

B. $\frac{3}{5}$

C. $\frac{1}{5}$

D. $\frac{19}{5}$

Answer: B

Solution:

Given that C_1 is a circle centered at $O(0,0)$ with a radius of 4, and C_2 is a variable circle with a center at (α, β) and a radius of 5.

The equations of the circles are:

$C_1 : x^2 + y^2 = 16$

$C_2 : (x-\alpha)^2 + (y-\beta)^2 = 25$

For the common chord to exist and have maximum length, the distance between the centers $(\sqrt{\alpha^2+\beta^2})$ is:

$\sqrt{5^2 - 4^2} = 3$

The slope of the common chord is given as $\frac{3}{4}$. Therefore, the equation of the chord can be written as:

$3x - 4y = 0 \quad \ldots (i)$

Since the distance between the points $(0,0)$ and (α, β) should be 3, we find:

$\left|\frac{3\alpha-4\beta}{\sqrt{3^2+4^2}}\right| = 3$

Simplifying further, we have:

$3\alpha - 4\beta \pm 15 = 0 \quad \ldots (ii)$

Given that the product of the slopes of perpendicular lines is -1, and the line connecting the centers O and (α, β) is perpendicular to the chord, we have:

$\frac{3}{4} \times \frac{\beta}{\alpha} = -1$

Which simplifies to:

$3\beta + 4\alpha = 0 \quad \ldots (iii)$

By solving equations (ii) and (iii), we find:

$\alpha = -\frac{9}{5}$

$\beta = \frac{12}{5}$

Therefore, $\alpha + \beta = \frac{3}{5}$.

Question65

If the pair of tangents drawn to the circle $x^2 + y^2 = a^2$ from the point $(10, 4)$ are perpendicular. then $a =$

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Options:

A. $\sqrt{58}$

B. 58

C. $2\sqrt{63}$

D. $2\sqrt{45}$

Answer: A

Solution:

To determine the radius a for which the tangents from the point $(10, 4)$ to the circle $x^2 + y^2 = a^2$ are perpendicular, we use the property that tangents from a given point (x_1, y_1) to a circle are perpendicular if:

$x_1^2 + y_1^2 = 2r^2$

Given that the point is $(10, 4)$ and the circle is defined by $x^2 + y^2 = a^2$, we set $x_1 = 10$, $y_1 = 4$, and $r = a$. Plugging these into the formula gives:

$10^2 + 4^2 = 2a^2$

Calculating further:

$100 + 16 = 2a^2$

$116 = 2a^2$

Dividing by 2:

$a^2 = \frac{116}{2} = 58$

Thus, the radius a is:

$a=\sqrt{58}$

Question66

If $x-4=0$ is the radical axis of two orthogonal cirlces out of which one is $x^2+y^2=36$, then the centre of the other circle is

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Options:

A. $(8,0)$

B. $(9,0)$

C. $(6,0)$

D. $(12,0)$

Answer: B

Solution:

Given the circle $x^2+y^2=36$, we have a circle S_2 centered at $(0,0)$ with a radius of 6. The radical axis of two orthogonal circles is given by $x-4=0$.

For two circles to be orthogonal, the condition is:

$2(g_1g_2+f_1f_2)=c_1+c_2$

where (g_1,f_1) and (g_2,f_2) are the centers of the circles, and c_1 and c_2 are the constants in their respective equation forms $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$.

Let's denote the other circle as S_1. According to the given problem, the radical axis is:

$x-4=0$

The equation of the other circle S_1 can be expressed as:

$S_1-S_2=k(x-4)$

So:

$S_1=S_2+k(x-4)$

This translates to:

$S_1 = x^2 + y^2 - 36 + kx - 4k = 0$

Substituting into the orthogonality condition:

$c_1 + c_2 = 0$

Hence:

$-4k - 36 - 36 = 0$

Solving for k:

$-4k - 72 = 0 \Rightarrow k = -18$

Therefore, the equation of the other circle S_1 becomes:

$x^2 + y^2 - 18x + 36 = 0$

The center of this circle is $(9, 0)$.

Question67

The perimeter of the locus of the point P which divides the line segment QA internally in the ratio $1 : 2$, where $A = (4, 4)$ and Q lies on the circle $x^2 + y^2 = 9$, is

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Options:

A. 8π

B. 4π

C. π

D. 9π

Answer: B

Solution:

Let $Q = (x_1, y_1)$ and $A = (4, 4)$.

The circle is defined by the equation $x^2 + y^2 = 9$.

The coordinates of the point P, which divides the line segment QA in the ratio $m : n = 1 : 2$, are determined by the formula:

$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Substituting the given ratio and coordinates:

$P\left(\frac{1\cdot4+2\cdot x_1}{1+2}, \frac{1\cdot4+2\cdot y_1}{1+2}\right) \Rightarrow P\left(\frac{4+2x_1}{3}, \frac{4+2y_1}{3}\right)$

Since Q is on the circle $x^2+y^2=9$, we have:

$x_1^2+y_1^2=9$

Using parametric equations for the circle, let $x_1=3\cos\theta$ and $y_1=3\sin\theta$. Substituting into the expression for P from Eq. (i):

$P\left(\frac{4+6\cos\theta}{3}, \frac{4+6\sin\theta}{3}\right) \Rightarrow P\left(\frac{4}{3}+2\cos\theta, \frac{4}{3}+2\sin\theta\right)$

Let:

$x=\frac{4}{3}+2\cos\theta \quad \Rightarrow \quad \cos\theta=\frac{x-\frac{4}{3}}{2}$

$y=\frac{4}{3}+2\sin\theta \quad \Rightarrow \quad \sin\theta=\frac{y-\frac{4}{3}}{2}$

Substitute into the trigonometric identity $\sin^2\theta+\cos^2\theta=1$:

$\left(\frac{x-\frac{4}{3}}{2}\right)^2+\left(\frac{y-\frac{4}{3}}{2}\right)^2=1$

Simplifying further:

$\left(x-\frac{4}{3}\right)^2+\left(y-\frac{4}{3}\right)^2=4$

This equation represents a circle with center $\left(\frac{4}{3}, \frac{4}{3}\right)$ and radius 2. Thus, the perimeter of this circle is:

$2\pi \times \text{radius}=4\pi$

Question68

If the equation of the circle whose radius is 3 units and which touches internally the circle $x^2+y^2-4x-6y-12=0$ at the point $(-1,-1)$ is $x^2+y^2+px+qy+r=0$, then $p+q-r=$

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Options:

A. 2

B. $\frac{5}{2}$

C. $\frac{26}{5}$

D. 3

Answer: A

Solution:

Radius of circle is 3 units and which touches internally the circle $\Rightarrow C \equiv x^2 + y^2 - 4x - 6x - 12 = 0$ at points $(-1, 1)$

Centre of circle C is $(2, 3)$ and radius is 5 units

If $C_2(h, K)$ is the centre of circle of radius 3. Which touches circle C internally at $(1, -1)$, then

$\Rightarrow C_2 = 3$ and $\Rightarrow C_1, C_2 = R - r = 5 - 3 = 2$

Thus, $C_2(h, K)$ divides C in the ratio $2 : 3$ internally therefore,

$$\Rightarrow h = \frac{2(-1) + 3 \times 2}{2 + 3} = \frac{4}{5}$$
$$\Rightarrow k = \frac{2(-1) + 3 \times 3}{2 + 3} = \frac{7}{5}$$

Hence, equations of circle C_2 is

$$\Rightarrow \left(x - \frac{4}{5}\right)^2 + \left(y - \frac{7}{5}\right)^2 = (3)^2$$
$$\Rightarrow x^2 + \frac{16}{25} - \frac{8x}{5} + y^2 + \frac{49}{25} - \frac{14y}{5} = 9$$
$$\Rightarrow x^2 + y^2 - \frac{8x}{5} - \frac{14y}{5} - \frac{32}{5} = 0$$

On comparing with given equation of circle

$x^2 + y^2 + px + qy + r = 0$

We get, $p = \frac{-8}{5}, q = \frac{-14}{5}, r = \frac{-32}{5}$

$$\Rightarrow p + q - r = \frac{-8}{5} - \frac{14}{5} + \frac{32}{5} = 2$$
$$\Rightarrow p + q - r = 2$$

Question69

The equation of the circle touching the circle $x^2 + y^2 - 6x + 6y + 17 = 0$ externally and to which the lines $x^2 - 3xy - 3x + 9y = 0$ are normal is

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Options:

A. $x^2 + y^2 - 3x + 2y - 2 = 0$

B. $x^2+y^2-6x-2y+1=0$

C. $x^2+y^2+6x-2y-1=0$

D. $x^2+y^2-9x-3y+2=0$

Answer: B

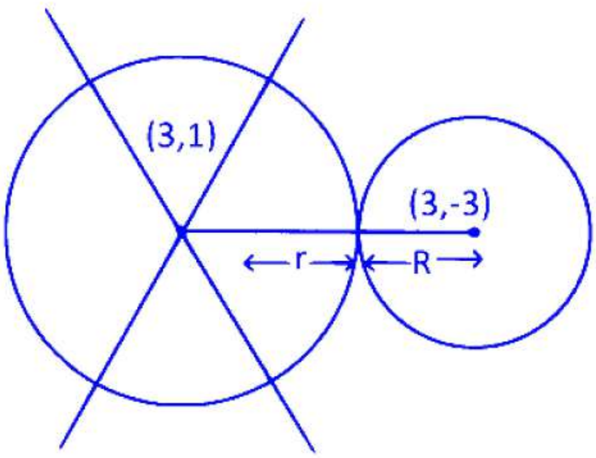
Solution:

Equation of circle

$x^2+y^2-6x-6y+17=0$ $\quad\quad\ldots(i)$

and equation of line

$x^2-3xy-3x+9y=0$ $\quad\quad\ldots(ii)$



From Eq. (ii), we get

$$\begin{aligned}
\Rightarrow\quad & x^2-3xy-3x+9y=0\\
\Rightarrow\quad & x^2-3x-3xy+9y=0\\
\Rightarrow\quad & x(x-3)-3y(x-3)=0\\
\Rightarrow\quad & (x-3)(x-3y)=0\\
\Rightarrow\quad & x=3, x=3y\\
\Rightarrow\quad & x=3, y=1\\
& R=\sqrt{9+9-17}\\
& R=\sqrt{1}=1
\end{aligned}$$

According to the given condition,

$$\begin{aligned}
\Rightarrow\quad & C_1C_2=R+r\\
\Rightarrow\quad & \sqrt{(3-3)^2+(1+3)^2}=1+r\\
\Rightarrow\quad & 4=1+r\\
\Rightarrow\quad & r=3
\end{aligned}$$

Equation of circle having coordinates $(3,1)$ and radius 3 is

$$\Rightarrow (x-3)^2+(y-1)^2=9$$
$$\Rightarrow x^2+9-6x+y^2+1-2y=9$$
$$\Rightarrow x^2+y^2-6x-2y+1=0$$

Above equation is equation of circle which touches the circle.

Question70

The pole of the straight line $9x+y-28=0$ with respect to the circle $2x^2+2y^2-3x+5y-7=0$ is

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Options:

A. $(-1,3)$

B. $(2,-3)$

C. $(3,-1)$

D. $(3,-3)$

Answer: C

Solution:

We have, equation of straight line

$9x+y-28=0 \quad \ldots(i)$

and equation of circle

$C \equiv 2x^2+2y^2-3x+5y-7=0 \quad \ldots(ii)$

Let (x_1, y_1) be the required point, line (i) must coincide with the polar of (x_1, y_1) whose equation is

$$\Rightarrow 2xx_1+2yy_1-\frac{3}{2}(x+x_1)+\frac{5}{2}(y+y_1)-7$$
$$=0$$
$$\Rightarrow x(4x_1-3)+y(4y_1+5)-3x_1+5y_1-14=0 \quad \ldots(iii)$$

Since, Eqs. (i) and (iii) are same, we have

$\frac{4x_1-3}{9}=\frac{y_1+5}{1}=\frac{-3x_1+5y_1-14}{-28}$

$\Rightarrow \quad x_1 = 9y_1 + 12$
$\Rightarrow 3x_1 - 117y_1 = 126$

On solving, we get

$x = 3 \Rightarrow y = -1$

So, the required point is $(3, -1)$.

Question71

The equation of a circle which touches the straight lines $x + y = 2, x - y = 2$ and also touches the circle $x^2 + y^2 = 1$ is

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Options:

A. $(x + \sqrt{2})^2 + y^2 = 3 - \sqrt{2}$

B. $(x + \sqrt{2})^2 + y^2 = 1 - 2\sqrt{2}$

C. $(x - \sqrt{2})^2 + y^2 = 2(1 - \sqrt{2})$

D. $(x - \sqrt{2})^2 + y^2 = 3 - 2\sqrt{2}$

Answer: D

Solution:

We have, equations of lines

$$\Rightarrow \quad L_1 \equiv x + y = 2 \quad \ldots (i)$$
$$\Rightarrow \quad L_2 \equiv x - y = 2 \quad \ldots (ii)$$

Let the cordinate of centre of circle be (g, f) which touches the line i.e. it will satisfy the equations of lines L_1 and L_2.

$$\Rightarrow \quad g + f = 2 \quad \ldots (iii)$$
$$\Rightarrow \quad g - f = 2 \quad \ldots (iv)$$

On solving Eqs. (iii) and (iv), we get (g. f) $= (2, 0)$

The center of the required circle will lie on bisector of acute angle between these tangents i.e. on.

$\frac{x+y-2}{\sqrt{2}} = \frac{x-y-2}{\sqrt{2}}$ i.e.

Let it be ($h, 0$) where h is (positive). If r be the radius, then it touches the circle $x^2 + y^2 = 1$ externally

According to the condition,

$$C_1C_2 = r_1 + r_2$$
$$h = 1 + r \qquad \ldots.(v)$$
$$\therefore \quad h = 2 \pm \sqrt{2r} = 1 + r$$

On solving, we get, $r = \sqrt{2} - 1$

Hence, the equation of circle with center $(\sqrt{2}, 0)$ and radius $(\sqrt{2} - 1)$ is

$$\Rightarrow (x - \sqrt{2})^2 + y^2 = (\sqrt{2} - 1)^2$$
$$\Rightarrow (x - \sqrt{2})^2 + y^2 = 3 - 2\sqrt{2}$$

Question72

The radical axis of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x + 2y + 1 = 0$. Then,

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Options:

A. $g = \frac{3}{8}$ or $f = 1$

B. $g = \frac{2}{3}$ or $t = 3$

C. $g = \frac{1}{2}$ or $f = 1$

D. $g = \frac{3}{4}$ or $f = 2$

Answer: D

Solution:

We have two given circles:

$$S_1 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$
$$S_2 \equiv 2x^2 + 2y^2 + 3x + 8y + 2c = 0$$

The equation of the second circle can be simplified by dividing by 2:

$$S_2 \equiv x^2 + y^2 + \frac{3}{2}x + 4y + c = 0$$

The radical axis of the circles $S_1 = 0$ and $S_2 = 0$ is determined by subtracting one equation from the other:

$S_1 - S_2 = 0$

$\Rightarrow (x^2 + y^2 + 2gx + 2fy + c) - \left(x^2 + y^2 + \frac{3}{2}x + 4y + c\right) = 0$

$\Rightarrow 2gx + 2fy - \frac{3}{2}x - 4y = 0$

$\Rightarrow x(g - \frac{3}{4}) + y(f - 2) = 0$

This radical axis touches the circle:

$S_3 \equiv x^2 + y^2 + 2x + 2y + 1 = 0$

The center of this circle is $(-1, -1)$ and it has a radius of 1. For tangency, the distance from the center to the radical axis must equal the radius, i.e., $p = r$:

$\frac{-1(g-\frac{3}{4})-(f-2)}{\sqrt{(g-\frac{3}{4})^2+(f-2)^2}} = 1$

Squaring both sides, we arrive at:

$(g - \frac{3}{4})^2 + (f - 2)^2 + 2(g - \frac{3}{4})(f - 2)$

$= (g - \frac{3}{4})^2 + (f - 2)^2$

$\therefore 2(g - \frac{3}{4})(f - 2) = 0$

This implies either $g - \frac{3}{4} = 0$ or $f - 2 = 0$:

$g = \frac{3}{4} \quad \text{or} \quad f = 2$

Question73

$2x - 3y + 1 = 0$ and $4x - 5y - 1 = 0$ are the equations of two diameters of the circle $S \equiv x^2 + y^2 + 2gx + 2fy - 11 = 0$. Q and R are the points of contact of the tangents drawn from the point $P(-2, -2)$ to this circle. If C is the centre of the circle $S = 0$, then the area (in square units) of the quadrilateral $PQCR$ is

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Options:

A. 25

B. 30

C. 24

D. 36

Answer: B

Solution:

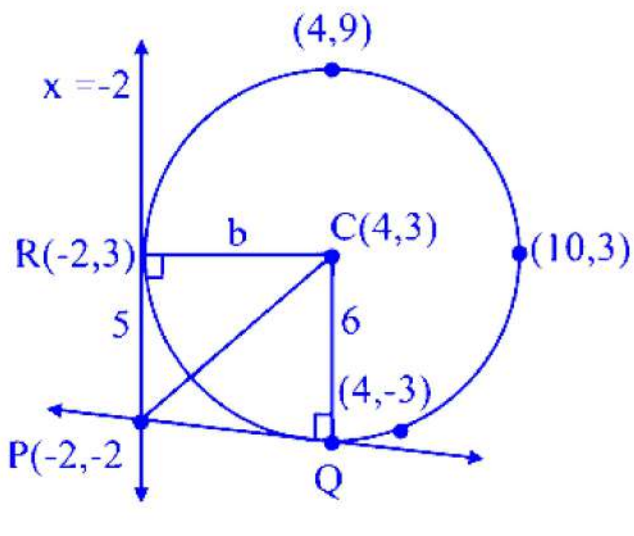
The point of intersection of the given linear equation is $(4,3)$.

So, center of the circle will be $(4,3)$.

$$\begin{aligned} &\therefore S \equiv x^2+y^2-8x-6y-11=0 \\ &\text{Radius } =\sqrt{(-4)^2+(-3)^2-(-11)} \\ &\qquad =\sqrt{36}=6 \end{aligned}$$

The most rightward point of circle will be $(-2,3)$. The x-coordinate of this point is same as the x-coordinate of the point $P(-2,-2)$.

So, the line $x=-2$ will be a tangent from P and it will touch the circle at $(-2;3)$.



Clearly, $\triangle CPR \cong \triangle CPQ$ and $\triangle CPR$ is a right-angled triangle.

$\therefore$ Area of quadrilateral $PQCR = 2\times$ Area of $\triangle CPR$

$$\begin{aligned} &=2\times\left(\frac{1}{2}\times 5\times 6\right) \\ &=30 \end{aligned}$$

Question74

If the inverse point of the point $(-1,1)$ with respect to the circle $x^2+y^2-2x+2y-1=0$ is (p,q), then $p^2+q^2=$

Options:

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer: B

Solution:

Given, if the inverse point of the point $(-1, +1)$ w.r.t. the

Circle $x^2 + y^2 - 2x + 2y - 1 = 0$ is (p, q) then, $p^2 + q^2 = ?$

Now, $x^2 - 2x + y^2 + 2y = 1$

1 Add or subtract

$$\begin{aligned}&\left(x^2 - 2x + 1 - 1\right) + \left(y^2 + 2y + 1 - 1\right) = 1\\&\left(x^2 - 2x + 1\right) - 1 + \left(y^2 + 2y + 1\right) - 1 = 1\\&(x-1)^2 - 1 + (y+1)^2 - 1 = 1\\&(x-1)^2 + (y+1)^2 = 3\end{aligned}$$

So, center of circle $(1, -1)$ and Radius $= \sqrt{3}$

Now, find the distance from the point $(-1, 1)$ to the center of the circle.

$$\begin{aligned}&\sqrt{(-1-1)^2 + (1-(-1))^2}\\&= \sqrt{(-2)^2 + (2)^2} = 2\sqrt{2}\end{aligned}$$

Now, determine the inverse point (p, q) of $(-1, 1)$ with respect to the circle lies on the same line passing through the point.

Here, $(x_1, y_1) = (-1, 1)$

$$\begin{aligned}(a, b) &= (1, -1),\\ r^2 &= 3\end{aligned}$$

Formula for inverse point

$$p=\left(a+\left\{r^2+\left(x_1-a\right)/\left(\left(x_1-a\right)^2+\left(y_1-b\right)^2\right)\right\}\right)$$
$$q=\left(b+\left\{r^2\left(y_1-b\right)/\left(\left(x_1-a\right)^2+\left(y_1-b\right)^2\right)\right\}\right)$$
$$p=1+\frac{3x-2}{8}=\frac{8-6}{8}=\frac{2}{8}=\frac{1}{4}$$
$$q=-1+\frac{3(1+1)}{4+4}$$
$$=-1+\frac{6}{8}=-\frac{1}{4}$$
$$p^2+q^2=\left(\frac{1}{4}\right)^2+\left(-\frac{1}{4}\right)^2=\frac{1}{8}$$

Question75

If (a, b) is the mid-point of the chord $2x-y+3=0$ of the circle $x^2+y^2+6x-4y+4=0$, then $2a+3b=$

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Options:

A. -1

B. 0

C. 1

D. 3

Answer: C

Solution:

Mid-point (a, b) of the chord. $(2x-y+3=0)$ of the circle. $x^2+y^2+6x-4y+4=0$

$$\left(x^2+6x\right)+\left(y^2-4y\right)+4=0$$
$$\left(x^2+9-9+6x\right)+\left(y^2+4-4-4y\right)+4=0$$
$$\left(x^2+9+9+6x\right)-9+\left(y^2+4-4y\right)=0$$
$$(x+3)^2+(y-2)^2=9$$

Center $(-3,2)$ and Radius $=3$

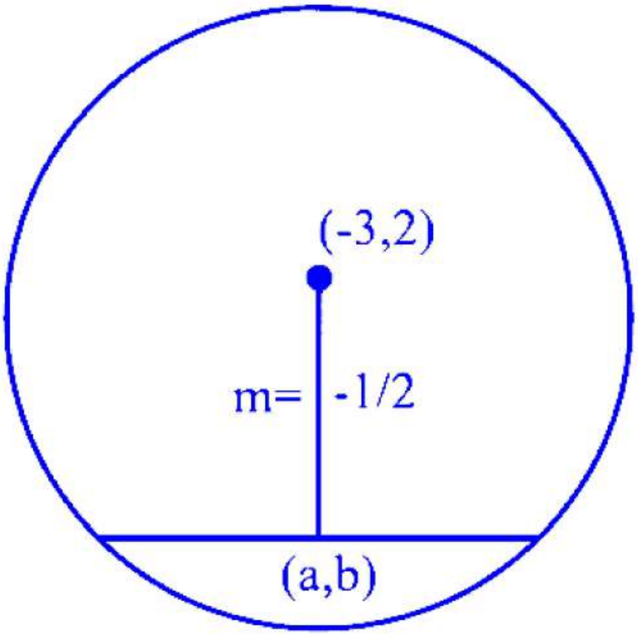
The chord equation, $2x-y+3=0$

The mid-point (a, b) is perpendicular bisector of the chord which passes thought center of the circle $(-3,2)$. The equation of the perpendicular bisectord the chord can be found using the given chord equation and the center of the circle rewriting chord equation.

$y=2x+3$

Slope of this line is 2 . Therefore, the slope of perpendicular bisector is $-\frac{1}{2}$

Now, center $(-3,2) m=\frac{-1}{2}$



$$
\begin{aligned}
y-2 & =-\frac{1}{2}(x+3) \\
y & =-\frac{1}{2} x+\frac{1}{2}
\end{aligned}
$$

The mid-point (a, b) of the chord is the intersection. Point of the original chord $2x-y \neq 3=0$ and perpendicular bisettr $y=\frac{-1}{2} x+\frac{1}{2}$.

On substitute the value,

$2x-\left(-\frac{1}{2} x+\frac{1}{2}\right)+3=0$

On solving $x=11$ and $y=1$

So, the mid-point (a, b) is $(-1,1)$.

Calculate $2a+3b$

$\Rightarrow 2(-1)+3(1) \Rightarrow 1$

Therefore, $2a+3b=1$.

Question76

If a direct common tangent drawn to the circle $x^2+y^2-6x+4y+9=0$ and $x^2+y^2+2x-2y+1=0$ touches the circles at A and B, then $AB=$

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Options:

A. 9

B. 16

C. $4\sqrt{6}$

D. $2\sqrt{6}$

Answer: D

Solution:

Given, first circle,

$x^2+y^2-6x+4y+9=0$

By complete the square,

$(x-3)^2+(y+2)^2=4$

Center $C_1(3,-2)$, Radius $(R_1)=2$

Similarly, for second circle.

$x^2+y^2+2x-2y+1=0$

By complete the square,

$(x+1)^2+(y-1)^2=1$

Center $C_2(-1,1)$, Radius $R_2=1$

Distance between C_1 and C_2

$d=\sqrt{(3-(-1))^2+(-2-1)^2}=\sqrt{4^2+(-3)^2}=5$

$$\begin{aligned}\text{Length of tangent } &= \sqrt{d^2-(R_1-R_2)^2} \\ &= \sqrt{5^2-(2-1)^2} \\ &= \sqrt{25-1} \\ &= \sqrt{24} \\ &= 2\sqrt{6}\end{aligned}$$

Therefore, the length of the common tangent is $2\sqrt{6}$ unit.

Question77

The radius of the circle which cuts the circles $x^2 + y^2 - 4x - 4y + 7 = 0, x^2 + y^2 + 4x - 4y + 6 = 0$ and $x^2 + y^2 + 4x + 4y + 5 = 0$ orthogonally is

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Options:

A. $\frac{\sqrt{193}}{4\sqrt{2}}$

B. $\frac{\sqrt{193}}{8}$

C. $\frac{\sqrt{193}}{4}$

D. $\frac{\sqrt{193}}{2\sqrt{2}}$

Answer: A

Solution:

Given, 1st circle

$= x^2 + y^2 - 4x - 4y + 7 = 0$

2nd circle $= x^2 + y^2 + 4x - 4y + 6 = 0$

3rd circle $= x^2 + y^2 + 4x + 4y + 5 = 0$

Now, find the center and radii of the given circles.

1st circle $x^2 + y^2 - 4x - 4y + 7 = 0$

By complete the square

$$(x - 2)^2 + (y - 2)^2 = 2^2 + 2^2 - 7$$
$$(x - 2)^2 + (y - 2)^2 = 1$$

Center $C_1(2, 2)$ Radius $R_1 = 1$

Similarly, 2nd Circle $(x + 2)^2 + (y - 2)^2 = 2$

Center $C_2(-2, 2)$ and Radius $R_2 = \sqrt{2}$

3rd circle $(x+2)^2+(y+2)^2=3$

$C_3(-2,-2), R_3=\sqrt{3}$

Now, let the Radius R and center (a, b) be the circle which cut each of given circles orthogonally, then the below condition must hold.

$$(a-2)^2+(b-2)^2=R^2+1$$
$$(a+2)^2+(b-2)^2=R^2+2$$
$$(a+2)^2+(b+2)^2=R^2+3$$

On subtract Eq. (i) from Eq. (ii), we get

$a=\frac{1}{8}$

On subtracting Eq. (i) from Eq. (iii), we get

$b=\frac{1}{8}$

On substitute a and b in Eq. (i)

$$\left(\frac{1}{8}-2\right)^2+\left(\frac{1}{8}-2\right)^2=R^2+1$$
$$\left(\frac{-15}{8}\right)^2+\left(\frac{-15}{8}\right)^2=R^2+1$$
$$\frac{450}{64}=R^2+1$$
$$R^2=\frac{193}{32}$$
$$R=\sqrt{\frac{193}{32}}$$
$$R=\frac{\sqrt{193}}{4\sqrt{2}}$$

Therefore, the radius of the circle that cuts all the given circle orthogonally is $\frac{\sqrt{193}}{4\sqrt{2}}$ unit.

Question78

$A(2,3), B(-1,1)$ are two points. If P is a variable point such that $\angle APB=90^{\circ}$, then locus of P is

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Options:

A. $x^2+y^2-x-4y+1=0$

B. $x^2+y^2+x+4y-1=0$

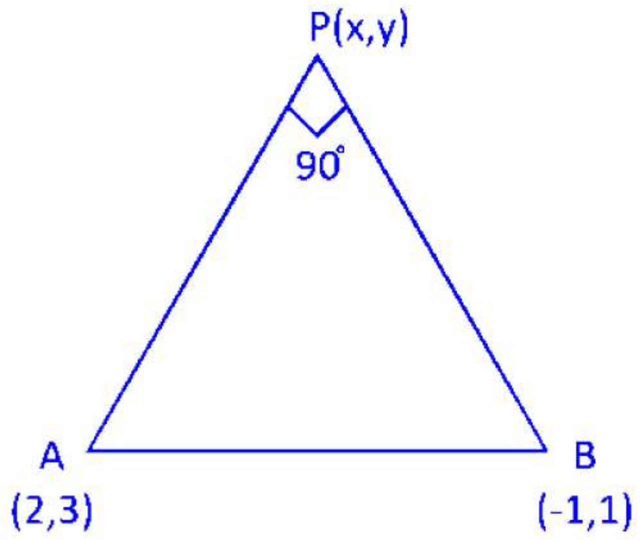
C. $x^2 + y^2 - x + 4y - 1 = 0$

D. $x^2 + y^2 + x - 4y + 1 = 0$

Answer: A

Solution:

In $\triangle APB$, we have



$$m_1 = \text{ Slope of } AP = \frac{y-3}{x-2}$$
$$m_2 = \text{ Slope of } BP = \frac{y-1}{x+1}$$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\tan 90^\circ = \left|\frac{\frac{(y-3)}{(x-2)} - \frac{(y-1)}{(x+1)}}{1 + \frac{(y-3)}{(x-2)}\frac{(y-1)}{(x+1)}}\right|$$
$$= \left|\frac{(y-3)(x+1)-(y-1)(x-2)}{(x-2)(x+1)+(y-3)(y-1)}\right|$$
$$\frac{1}{\infty} = 0 = \frac{(x-2)(x+1)+(y-3)(y-1)}{(y-3)(x+1)-(y-1)(x-2)}$$
$$\Rightarrow x^2 - x - 2 + y^2 - 4y + 3 = 0$$
$$\Rightarrow x^2 - x + y^2 - 4y + 1 = 0$$

Question79

The largest among the distances from the point $P(15,9)$ to the points on the circle $x^2+y^2-6x-8y-11=0$ is

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Options:

A. 12

B. 13

C. 19

D. 7

Answer: C

Solution:

Given the equation of the circle:

$x^2+y^2-6x-8y-11=0$

we can determine the center of the circle, C, which is $(3,4)$, and the radius, r, as follows:

$r=\sqrt{9+16+11}=\sqrt{36}=6$

Now, let's consider the point $P(15,9)$.

The distance between the point P and the center C is calculated as:

$CP=\sqrt{(15-3)^2+(9-4)^2}=\sqrt{144+25}=\sqrt{169}=13$

To find the largest possible distance from the point P to any point on the circle, we add the radius to CP:

Greatest distance from P to the circle $=CP+r=13+6=19$

Question80

The circle $x^2+y^2-8x-12y+\alpha=0$ lies in the first quadrant without touching the coordinate axes. If $(6,6)$ is an interior point to the circle, then

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Options:

A. $4 < \alpha < 6$

B. $6 < \alpha < 16$

C. $16 < \alpha < 48$

D. $36 < \alpha < 48$

Answer: D

Solution:

Given the circle equation:

$$x^2 + y^2 - 8x - 12y + \alpha = 0$$

And the point $P(6, 6)$ is an interior point of the circle. To determine the range of α, we perform the following steps:

Point $P(6, 6)$ lies inside the circle, thus:

$$6^2 + 6^2 - 8 \cdot 6 - 12 \cdot 6 + \alpha < 0$$

$$36 + 36 - 48 - 72 + \alpha < 0$$

Simplifying:

$$72 - 48 - 72 + \alpha < 0$$

$$\alpha < 48$$

To ensure the circle does not touch the coordinate axes and remains in the first quadrant, the radius must be less than both the x-coordinate and y-coordinate of the center. The center of the circle (h, k) is found by completing the square:

$$h = \frac{8}{2} = 4, \quad k = \frac{12}{2} = 6$$

Thus, the center is at $(4, 6)$ and the radius $\sqrt{4^2 + 6^2 - \alpha} < 4$ and also < 6:

$$\sqrt{52 - \alpha} < 4 \quad \text{and} \quad \sqrt{52 - \alpha} < 6$$

Solving these gives:

$$52 - \alpha < 16 \quad \text{and} \quad 52 - \alpha < 36$$

Which simplifies to:

$$\alpha > 36$$

Combining the inequalities from steps 1 and 2, we have:

$$36 < \alpha < 48$$

Question81

The equation of the circle whose diameter is the common chord of the circles $x^2+y^2-6x-7=0$ and $x^2+y^2-10x+16=0$ is

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Options:

A. $8x^2+8y^2-92x+197=0$

B. $x^2+y^2-23x+197=0$

C. $x^2+y^2-\frac{23}{2}x+\frac{197}{4}=0$

D. $4x^2+4y^2-46x+197=0$

Answer: A

Solution:

To find the equation of the circle whose diameter is the common chord of the circles given by:

$x^2+y^2-6x-7=0 \quad (i)$

$x^2+y^2-10x+16=0 \quad (ii)$

First, we find the common chord by subtracting equation (i) from equation (ii):

$$\begin{aligned} & x^2+y^2-10x+16-(x^2+y^2-6x-7)=0, \\ \Rightarrow & -4x+23=0, \\ \Rightarrow & x=\frac{23}{4}. \end{aligned}$$

Substitute $x=\frac{23}{4}$ back into either equation (let's use equation (ii)) to find the y-coordinates of the intersection points:

$$x^2+y^2-10x=-16,$$
$$\left(\frac{23}{4}\right)^2+y^2-10\cdot\frac{23}{4}=-16,$$
$$y=\frac{\pm 3\sqrt{15}}{4}.$$

The points of intersection are $\left(\frac{23}{4},\frac{-3\sqrt{15}}{4}\right)$ and $\left(\frac{23}{4},\frac{3\sqrt{15}}{4}\right)$.

The midpoint of the chord (diameter) is:

$\left(\frac{23}{4},0\right)$

The radius is the distance from this midpoint to either endpoint:

$$d=\sqrt{\left(\frac{23}{4}-\frac{23}{4}\right)^2+\left(0-\frac{3\sqrt{15}}{4}\right)^2}$$
$$=\frac{3\sqrt{15}}{4}$$

Thus, the equation of the circle is:

$$\left(x-\frac{23}{4}\right)^2+y^2=\left(\frac{3\sqrt{15}}{4}\right)^2,$$
$$\Rightarrow\left(x-\frac{23}{4}\right)^2+y^2=\frac{135}{16},$$
$$\Rightarrow 16\left(x^2-\frac{23}{2}x\right)+16y^2=135,$$
$$\Rightarrow 8x^2+8y^2-92x+197=0.$$

This is the equation of the circle whose diameter is the common chord of the given circles.

Question82

If the locus of the mid-point of the chords of the circle $x^2+y^2=25$, which subtend a right angle at the origin is given by $\frac{x^2}{\alpha^2}+\frac{y^2}{\alpha^2}=1$, then $|\alpha|=$

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Options:

A. $\frac{2}{5}$

B. $\frac{5}{\sqrt{2}}$

C. $\frac{2}{25}$

D. $5\sqrt{2}$

Answer: B

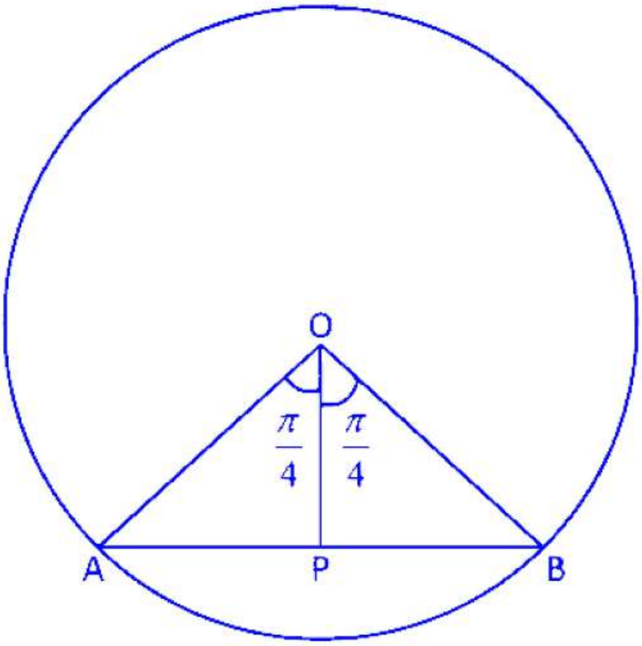
Solution:

Let mid-point be $P(h,k)$

and origin $O(0,0)$ is also the centre of the circle.

Since, chord making right angle at origin and P is mid-point, OP will bisect the right angle.

$OP = r \cos 45^\circ$, where r is radius of given circle.



$\Rightarrow \quad \sqrt{(h-0)^2 + (k-0)^2} = 5 \times \frac{1}{\sqrt{2}}$

$\Rightarrow \quad h^2 + k^2 = \left(\frac{5}{\sqrt{2}}\right)^2 = \frac{25}{2}$

$\therefore \qquad OP = \frac{5}{\sqrt{2}}$

Let the coordinates of P be (x_1, y_1)

$OP = \sqrt{x_1^2 + y_1^2} = \frac{5}{\sqrt{2}}$

Question83

The radical centre of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$, $x^2 + y^2 + x - y + 3 = 0, x^2 + y^2 - 3x + 2y + 5 = 0$

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Options:

A. $\left(-\frac{7}{38}, \frac{6}{19}\right)$

B. $\left(\frac{6}{19}, \frac{14}{19}\right)$

C. $\left(\frac{14}{19}, \frac{6}{19}\right)$

D. $\left(\frac{2}{19}, \frac{3}{19}\right)$

Answer: C

Solution:

Let $S_1 : x^2 + y^2 + 2x + 3y + 1 = 0$

$S_2 : x^2 + y^2 + x - y + 3 = 0$
$S_3 : x^2 + y^2 - 3x + 2y + 5 = 0$

The radical axis of the circles will be

$$\begin{aligned} & S_1 - S_2 = 0 \\ \Rightarrow \quad & x + 4y - 2 = 0 \quad \ldots \text{(i)} \\ \text{and} \quad & S_2 - S_3 = 0 \\ & 4x - 3y - 2 = 0 \quad \ldots \text{(ii)} \\ \text{and} \quad & S_3 - S_1 = 0 \\ \Rightarrow \quad & -5x - y + 4 = 0 \quad \ldots \text{(iii)} \end{aligned}$$

On multiply Eqs. (i) by 4 and then subtract from (ii), we get

$y = \frac{6}{19}$

On substitute this value in Eq. (i), we get

$x = \frac{14}{19}$

Since, x and y values also satisfies Eqs. (iii).

Thus, the radical centre of three circles is $\left(\frac{14}{19}, \frac{6}{19}\right)$.

Question84

If a circle is inscribed in an equilateral triangle of side a, then the area of any square (in sq units) inscribed in this circle is

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Options:

A. $\frac{2a^2}{3}$

B. $\sqrt{3}\frac{a^2}{2}$

C. $\frac{a^2}{2\sqrt{3}}$

D. $\frac{a^2}{6}$

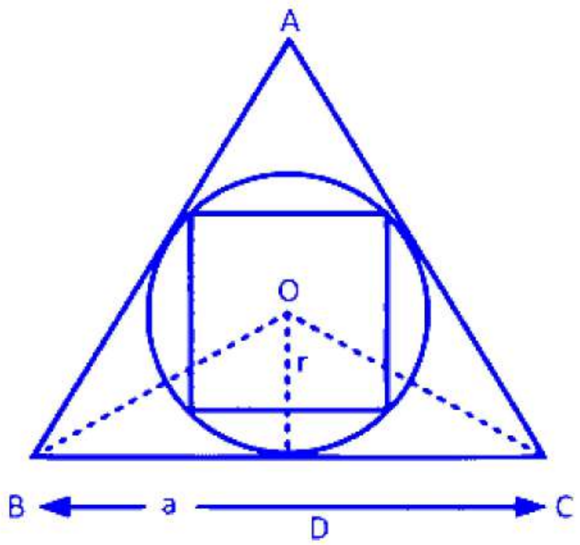
Answer: D

Solution:

In, figure BA and BC are tangents y the circle. So, OB is the angle bisecton of $\angle B$.

Then, $\angle B = 60^\circ$

$$\angle OBD = \frac{60}{2} = 30^\circ$$



In $\triangle OBD, \angle ODB = 90^\circ$

$$\tan 30^\circ = \frac{OD}{BD}$$

$$\frac{1}{3} = \frac{r}{\frac{a}{2}} = \frac{2r}{a}$$

$$r = \frac{a}{2\sqrt{3}}$$

Now, diagonal of square $=$ diameter of circle

$= 2r = 2 \times \frac{a}{2\sqrt{3}} = \frac{a}{\sqrt{3}}$

Now, area of square $= \frac{1}{2}(2r)^2$

$= \frac{1}{2} \times \left(\frac{a}{\sqrt{3}}\right)^2 = \frac{a^2}{6}$

Question85

If the line segment joining the points $(1,0)$ and $(0,1)$ subtends an angle of 45° at a variable point P, then the equation of the locus of P

is

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Options:

A. $\left(x^2+y^2-1\right)\left(x^2+y^2-2x-2y+1\right)=0, x \neq 0,1$

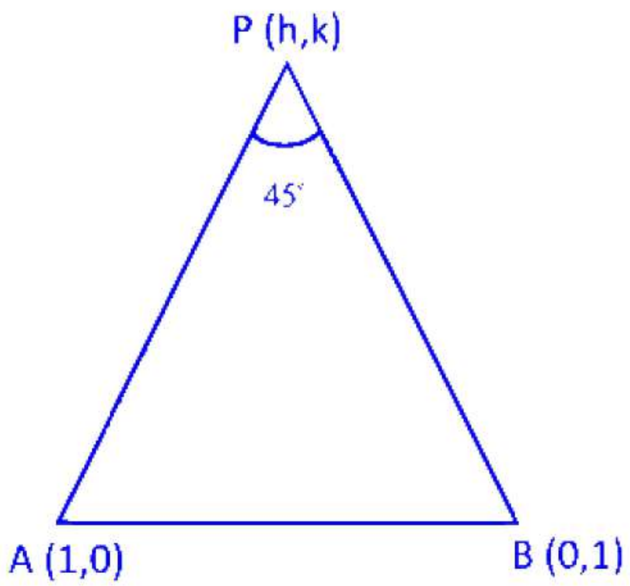
B. $\left(x^2+y^2-1\right)\left(x^2+y^2+2x+2y+1\right)=0, x \neq 0,1$

C. $x^2+y^2+2x+2y+1=0$

D. $x^2+y^2=4$

Answer: A

Solution:



Slope of $AP=\frac{k}{h-1}$

Slope of $BP=\frac{k-1}{h}$

$$\tan 45^{\circ}=\left|\frac{\frac{k}{h-1}-\frac{k-1}{h}}{1+\frac{k(k-1)}{h(h-1)}}\right|$$
$$\pm 1=\frac{kh-(k-1)(h-1)}{h(h-1)+k(k-1)}$$
$$\pm 1=\frac{kh-kh+h+k-1}{h^2-h+k^2-k}$$
$$\Rightarrow \pm\left(h^2-h+k^2-k\right)=h+k-1$$

If, $h+k-1=-\left(h^2-h+k^2-k\right)$,

$\Rightarrow h+k-1=-h^2+h-k^2+k\,h^2+k^2-1$

If $h+k-1=h^2-h+k^2-12$

$\Rightarrow h^2+k^2-2h-2k+1=0$

Equation of locus of P is $\left(x^2+y^2-1\right)=0$

or $x^2+y^2-2x-2y+1=0$

Hence, equation of locus of p is

$\left(x^2+y^2-1\right)\left(x^2+y^2-2x-2y+1\right)=0$
$x \neq 0,1$

Question86

Equation of the circle having its centre on the line $2x+y+3=0$ and having the lines $3x+4y-18=0, 3x+4y+2=0$ as tangents is

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Options:

A. $x^2+y^2+6x+8y+4=0$

B. $x^2+y^2-6x-8y+18=0$

C. $x^2+y^2-8x+10y+37=0$

D. $x^2+y^2+8x-10y+37=0$

Answer: D

Solution:

Let centre of circle be (h, k).

Centre of circle lies on $2x+y+3=0$

$2h+k+3=0$

The two tangents $3x+4y-18=0$ and $3x+4y+2=0$ are parallel as their slopes are equal

$2r=\frac{|2+18|}{\sqrt{3^2+4^2}}=\frac{20}{5}=4$

$\therefore \quad r=2$ units

Now, $r=\frac{|3h+4k-18|}{\sqrt{3^2+4^2}}=2$

$$
\begin{aligned}
\Rightarrow \quad & 3h+4k-18=\pm 10 \\
& 3h+4k-18+10=0 \\
\text{or} \quad & 3h+4k-18-10=0 \\
\Rightarrow \quad & 3h+4k-8=0 \\
\text{Or} \quad & 3h+4k-28=0
\end{aligned}
$$

From Eqs. (i) and (ii), we get $h=-4, k=5$

From Eqs. (i) and (iii), we get $h=-8, k=13$

Neglecting $h=-8$ and $k=13$

because distance from centre to tangent $\neq 2$

$\therefore$ Centre of circle is $(-4,5)$

$\therefore$ Equation of circle is

$$
\begin{aligned}
(x+4)^2+(y-5)^2 &= 2^2 \\
x^2+y^2+8x-10y+16+25 &= 4 \\
x^2+y^2+8x-10y+37 &= 0
\end{aligned}
$$

Question87

If power of a point $(4,2)$ with respect to the circle $x^2+y^2-2\alpha x+6y+\alpha^2-16=0$ is 9 , then the sum of the lengths of all possible intercepts made by such circles on the coordinate axes is

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Options:

A. $16+4\sqrt{6}$

B. $16+4\sqrt{6}-6\sqrt{2}$

C. $16+4\sqrt{6}+6\sqrt{2}$

D. $16+6\sqrt{2}$

Answer: A

Solution:

The power of the point $(4, 2)$ with respect to the given circle is 9.

To find the circle's equation, use the power of the point:

$16 + 4 - 8\alpha + 12 + \alpha^2 - 16 = 9$

Solving this equation gives:

$\alpha^2 - 8\alpha + 7 = 0$

which results in:

$\alpha = 1 \quad \text{or} \quad \alpha = 7$

Thus, the possible equations of the circles are:

$x^2 + y^2 - 2x + 6y - 15 = 0$

$x^2 + y^2 - 14x + 6y + 33 = 0$

For the equation $x^2 + y^2 - 2x + 6y - 15 = 0$:

Length of intercepts on the X-axis:

$2\sqrt{g^2 - c} = 2\sqrt{1 + 15} = 2\sqrt{16} = 8$

Length of intercepts on the Y-axis:

$2\sqrt{f^2 - c} = 2\sqrt{9 + 15} = 2\sqrt{24} = 4\sqrt{6}$

For the equation $x^2 + y^2 - 14x + 6y + 33 = 0$:

Length of intercept on the X-axis:

$2\sqrt{g^2 - c} = 2\sqrt{49 - 33} = 2\sqrt{16} = 8$

Length of intercept on the Y-axis is not possible (as the term under the square root is negative).

Therefore, the sum of the lengths of all possible intercepts is:

$8 + 8 + 4\sqrt{6} = 16 + 4\sqrt{6}$

Question88

Let α be an integer multiple of 8 . If S is the set of all possible values of α such that the line $6x + 8y + \alpha = 0$ intersects the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ at two distinct points, then the number of elements in S is

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Options:

A. 4

B. 6

C. 2

D. 1

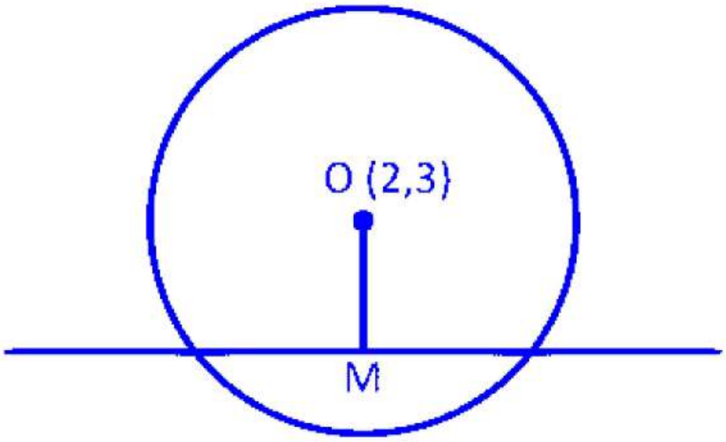
Answer: A

Solution:

We have,

Equation of circle is

$x^2 + y^2 - 4x - 6y + 9 = 0$
$\text{Centre of circle } = (2, 3), r = 2$



As the line intersects the circle at 2 points, perpendicular distance from centre to line in less than radius

$$0 < \frac{|12 + 24 + \alpha|}{\sqrt{6^2 + 8^2}} < 2$$
$0 < |36 + \alpha| < 20$
$\Rightarrow \text{ Either } 0 < 36 + \alpha < -16$
$\text{or} \quad 0 < -(36 + \alpha) < 20$

$\Rightarrow$ Either $-36 < \alpha < -16$, the possible values of α are $-32 - 24[\because \alpha$ is a multiple of 8]

If $-36 > \alpha > -56$, then possible values of α are $-40, -48$.

Hence, the number of elements is set $s = 4$.

Question89

If the circle $x^2 + y^2 - 8x - 8y + 28 = 0$ and $x^2 + y^2 - 8x - 6y + 25 - \alpha^2 = 0$ have only one common tangent, then $\alpha =$

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Options:

A. $\alpha = 4$

B. $\alpha = 2$

C. $\alpha = 1$

D. $\alpha = 5$

Answer: C

Solution:

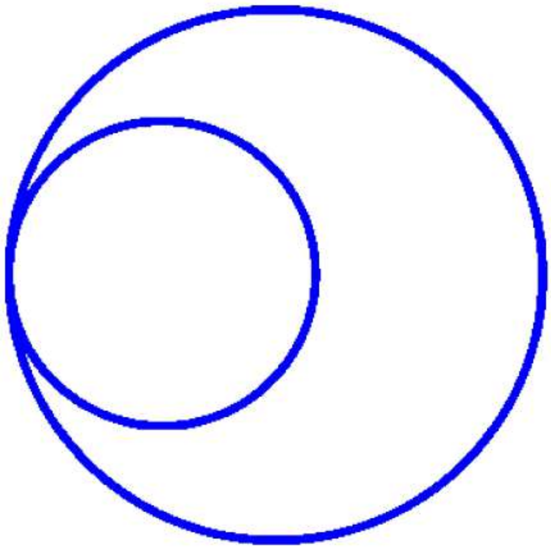
C_1 : Centre of circle

$$x^2 + y^2 - 8x - 8y + 28 = 0 \text{ is } (4, 4)$$
$$r_1 = \sqrt{16 + 16 - 28} = 2$$

C_2 : Centre of circle

$$x^2 + y^2 - 8x - 6y + 25 - \alpha^2 = 0 \text{ is } (4, 3)$$
$$r_2 = \sqrt{16 + 9 - 25 + \alpha^2} = \alpha$$

As the two circles have one common tangent, then the two circle coincides internally at point



$C_1C_2 = |r_1 - r_2| \sqrt{(4-4)^2 + (4-3)^2} = |2 - \alpha|$

$1 = |2 - \alpha| \Rightarrow 2 - \alpha = \pm 1$

$\Rightarrow \quad 2 - \alpha = 1 \text{ or } 2 - \alpha = -1$

$\Rightarrow \quad \alpha = 1 \text{ or } \alpha = 3$

Question90

If the equation of the circle passing through the points of intersection of the circles $x^2 - 2x + y^2 - 4y - 4 = 0$, $x^2 + 2x + y^2 + 4y - 4 = 0$ and the point $(3, 3)$ is given by $x^2 + y^2 + \alpha x + \beta y + \gamma = 0$, then $3(\alpha + \beta + \gamma) =$

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Options:

A. 32

B. -32

C. -26

D. 26

Answer: C

Solution:

The equation of the circle passing through the intersection of two given circles can be described by:

$x^2 - 2x + y^2 - 4y - 4 + \lambda(x^2 + 2x + y^2 + 4y - 4) = 0$

This simplifies to:

$x^2(1 + \lambda) + y^2(1 + \lambda) + x(-2 + 2\lambda) + y(-4 + 4\lambda) - 4 - 4\lambda = 0$

The circle in equation (i) must pass through the point $(3, 3)$, so we substitute these coordinates into the equation:

$9(1 + \lambda) + 9(1 + \lambda) + 3(-2 + 2\lambda) + 3(-4 + 4\lambda) - 4 - 4\lambda = 0$

Simplifying the above expression yields:

$18 + 18\lambda - 6 + 6\lambda - 12 + 12\lambda - 4 - 4\lambda = 0$

This further simplifies to:

$32\lambda - 4 = 0 \implies \lambda = \frac{1}{8}$

Substituting $\lambda = \frac{1}{8}$ back into equation (i), the equation of the circle becomes:

$$\frac{9}{8}x^2 + \frac{9}{8}y^2 - \frac{14}{8}x - \frac{28}{8}y - \frac{36}{8} = 0$$
$$\Rightarrow x^2 + y^2 - \frac{14}{9}x - \frac{28}{9}y - 4 = 0$$

From this, we identify $\alpha = \frac{-14}{9}$, $\beta = \frac{-28}{9}$, and $\gamma = -4$.

Now, calculating $3(\alpha + \beta + \gamma)$:

$3\left(-\frac{14}{9} - \frac{28}{9} - 4\right) = -\frac{78}{3} = -26$

Question91

The angle subtended by the chord $x + y - 1 = 0$ of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$ at the origin is

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Options:

A. $\cos^{-1}\left(\frac{6}{\sqrt{34}}\right)$

B. $\frac{\pi}{2}$

C. $\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Given

Chord:

$$x + y - 1 = 0$$

Circle:

$$x^2 + y^2 - 2x + 4y + 4 = 0$$

We have to find the **angle subtended by this chord at the origin.**

Step 1: Find the points of intersection of chord and circle

From the chord:

$$y = 1 - x$$

Substitute in the circle:

$$x^2 + (1 - x)^2 - 2x + 4(1 - x) + 4 = 0$$

$$x^2 + 1 - 2x + x^2 - 2x + 4 - 4x + 4 = 0$$

$$2x^2 - 8x + 9 = 0$$

Let the roots be x_1, x_2. Then:

$$x_1 + x_2 = \frac{8}{2} = 4, \quad x_1x_2 = \frac{9}{2}$$

Corresponding points are:

$$A(x_1, 1 - x_1), \quad B(x_2, 1 - x_2)$$

Step 2: Slopes of lines OA and OB

Slope of OA:

$$m_1 = \frac{1 - x_1}{x_1}$$

Slope of OB:

$$m_2 = \frac{1 - x_2}{x_2}$$

Find $m_1 + m_2$

$$m_1 + m_2 = \left(\frac{1}{x_1} - 1\right) + \left(\frac{1}{x_2} - 1\right)$$

$$= \left(\frac{x_1 + x_2}{x_1 x_2}\right) - 2$$

$$= \frac{4}{9/2} - 2 = \frac{8}{9} - 2 = -\frac{10}{9}$$

Find $m_1 m_2$

$$m_1 m_2 = \frac{(1 - x_1)(1 - x_2)}{x_1 x_2}$$

$$= \frac{1 - (x_1 + x_2) + x_1 x_2}{x_1 x_2}$$

$$= \frac{1 - 4 + 9/2}{9/2} = \frac{3/2}{9/2} = \frac{1}{3}$$

Step 3: Use formula for angle between two lines

For slopes m_1, m_2:

$$\cos\theta = \frac{|1 + m_1 m_2|}{\sqrt{(1 + m_1^2)(1 + m_2^2)}}$$

First find:

$$m_1^2 + m_2^2 = (m_1 + m_2)^2 - 2m_1 m_2$$

$$= \left(-\frac{10}{9}\right)^2 - 2\left(\frac{1}{3}\right) = \frac{100}{81} - \frac{2}{3} = \frac{100 - 54}{81} = \frac{46}{81}$$

Now:

$$(1+m_1^2)(1+m_2^2) = 1 + (m_1^2+m_2^2) + m_1^2 m_2^2$$
$$= 1 + \frac{46}{81} + \frac{1}{9} = \frac{81+46+9}{81} = \frac{136}{81}$$

Also:

$$1 + m_1 m_2 = 1 + \frac{1}{3} = \frac{4}{3}$$

Step 4: Substitute in cosine formula

$$\cos\theta = \frac{4/3}{\sqrt{136/81}} = \frac{4/3}{\sqrt{136}/9} = \frac{4}{3} \times \frac{9}{\sqrt{136}} = \frac{12}{\sqrt{136}}$$

Since $136 = 4 \times 34$:

$$\cos\theta = \frac{12}{2\sqrt{34}} = \frac{6}{\sqrt{34}}$$

✅ Final Answer

$$\boxed{\theta = \cos^{-1}\left(\frac{6}{\sqrt{34}}\right)}$$

Correct option: (A)

Question92

Let P be any point on the circle $x^2+y^2=25$. Let L be the chord of contact of P with respect to the circle $x^2+y^2=9$. The locus of the poles of the lines L with respect to the circle $x^2+y^2=36$ is

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Options:

A. $y^2 = 20x$

B. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

C. $x^2 + y^2 = 400$

D. $\frac{x^2}{25}-\frac{y^2}{16}=1$

Answer: C

Solution:

Given a point P on the circle $x^2+y^2=25$, let the coordinates of P be (x_1, y_1). Thus, we have:

$x_1^2+y_1^2=25 \quad$ (Equation 1)

The chord of contact from point P to the circle $x^2+y^2=9$ is given by the equation:

$x x_1+y y_1=9$

Let (h, k) be the pole of this chord with respect to the circle $x^2+y^2=36$. The polar of (h, k) is given by:

$h x+k y=36$

Since the line L is the polar of P with respect to the circle $x^2+y^2=36$, the equation of the polar line, in terms of (x_1, y_1), is simply:

$x x_1+y y_1=0$

The ratio given by comparing the polar equations is:

$\frac{h}{x_1}=\frac{k}{y_1}=\frac{36}{9}=4$

This implies:

$x_1=\frac{h}{4} \quad$ and $\quad y_1=\frac{k}{4}$

Substituting x_1 and y_1 from above into Equation 1, we have:

$\left(\frac{h}{4}\right)^2+\left(\frac{k}{4}\right)^2=25$

Simplifying:

$\frac{h^2}{16}+\frac{k^2}{16}=25$

Multiplying through by 16:

$h^2+k^2=400$

Therefore, the locus of the poles (h, k) is given by:

$x^2+y^2=400$

Question93

If the circles $S \equiv x^2+y^2-14x+6y+33=0$ and $S^1 \equiv x^2+y^2-a^2=0 (a \in N)$ have 4 common tangents, then possible number of values of a is

Options:

A. 13

B. 5

C. 14

D. 2

Answer: D

Solution:

We are given the circle equation:

$S \equiv x^2 + y^2 - 14x + 6y + 33 = 0$

The center, C_1, of this circle is $(7, -3)$, and we can determine its radius, r_1, as follows:

$r_1 = \sqrt{7^2 + (-3)^2 - 33} = \sqrt{49 + 9 - 33} = \sqrt{25} = 5$

For the circle $S^1 \equiv x^2 + y^2 = a^2$, the center, C_2, is $(0, 0)$ and its radius, r_2, is a.

To have four common tangents between the two circles, the condition is:

$C_1C_2 > r_1 + r_2$

Calculating C_1C_2, we have:

$C_1C_2 = \sqrt{(7-0)^2 + (-3-0)^2} = \sqrt{49 + 9} = \sqrt{58}$

Thus, we require:

$\sqrt{58} > a + 5$

Approximating $\sqrt{58} \approx 7.6$, we get:

$a + 5 < 7.6$

$a < 7.6 - 5$

$a < 2.6$

Since a is a natural number ($a \in \mathbb{N}$), the possible values of a are 1 and 2.

Therefore, there are 2 possible values of a.

Question94

If the area of the circum-circle of triangle formed by the line $2x + 5y + \alpha = 0$ and the positive coordinate axes is $\frac{29\pi}{4} Sq$, units, then $|\alpha| =$

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Options:

A. 25

B. 10

C. 20

D. 400

Answer: B

Solution:

The line $2x + 5y + \alpha = 0$ intersects the x-axis and y-axis at points $A\left(-\frac{\alpha}{2}, 0\right)$ and $B\left(0, -\frac{\alpha}{5}\right)$ respectively. This forms a right-angled triangle with the axes.

The circumcenter of a right-angled triangle is at the midpoint of the hypotenuse AB. Thus, the center of the circumcircle is $\left(\frac{-\alpha}{4}, \frac{-\alpha}{10}\right)$.

To find the radius of the circumcircle, we calculate the distance from the center to one of the vertices, say A:

$$\begin{aligned}
\text{Radius of the circle} &= \sqrt{\left(\frac{-\alpha}{2} - \frac{-\alpha}{4}\right)^2 + \left(0 - \frac{-\alpha}{10}\right)^2} \\
&= \sqrt{\left(-\frac{\alpha}{4}\right)^2 + \left(\frac{\alpha}{10}\right)^2} \\
&= \sqrt{\frac{\alpha^2}{16} + \frac{\alpha^2}{100}} \\
&= \sqrt{\frac{25\alpha^2 + 4\alpha^2}{400}} \\
&= \frac{\sqrt{29}\alpha}{20}.
\end{aligned}$$

The area of the circumcircle is given by πr^2:

$$\begin{aligned}
\pi\left(\frac{29}{400}\alpha^2\right) &= \frac{29\pi}{4} \\
\Rightarrow \quad \frac{29\alpha^2}{400} &= \frac{29}{4} \\
\Rightarrow \quad \alpha^2 &= 100 \\
\Rightarrow \quad |\alpha| &= 10.
\end{aligned}$$

Question95

The circle $S \equiv x^2 + y^2 - 2x - 4y + 1 = 0$ cuts the Y-axis at $A, B(OA > OB)$. If the radical axis of $S \equiv 0$ and $S' \equiv x^2 + y^2 - 4x - 2y + 4 = 0$ cuts the Y-axis at C, then the ratio in which C divides AB is

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Options:

A. $7 + 2\sqrt{3} : -7 + 2\sqrt{3}$

B. $\sqrt{3} + 2 : \sqrt{3} - 2$

C. $6 - 2\sqrt{3} : 2\sqrt{3} - 6$

D. $-3 : \sqrt{3}$

Answer: A

Solution:

We are given a circle defined by the equation:

$S \equiv x^2 + y^2 - 2x - 4y + 1 = 0$

This can be rewritten by completing the square as:

$(x - 1)^2 + (y - 2)^2 = 4$

To find where this circle intersects the Y-axis, set $x = 0$:

$y^2 - 4y + 1 = 0$

Solving this quadratic equation:

$y = \frac{4 \pm \sqrt{16-4}}{2}$

$y = 2 \pm \sqrt{3}$

Thus, the points of intersection are $A(0, 2 + \sqrt{3})$ and $B(0, 2 - \sqrt{3})$. Since $OA > OB$, we have $OA = 2 + \sqrt{3}$ and $OB = 2 - \sqrt{3}$.

The radical axis of the circles $S \equiv x^2 + y^2 - 2x - 4y + 1 = 0$ and $S' \equiv x^2 + y^2 - 4x - 2y + 4 = 0$ is found by subtracting:

$S - S' = 0$

$(x^2 + y^2 - 2x - 4y + 1) - (x^2 + y^2 - 4x - 2y + 4) = 0$

$-2x + 4x - 4y + 2y + 1 - 4 = 0$

$2x - 2y - 3 = 0$

This simplifies to the line equation:

$2x - 2y = 3$

Now, solve for y when $x = 0$ (where it intersects the Y-axis):

$2(0) - 2y = 3$

$y = -\frac{3}{2}$

Thus, the point C where it intersects the Y-axis is $(0, -\frac{3}{2})$.

Assume C divides AB in the ratio $k : 1$. Using section formula for coordinates:

$-\frac{3}{2} = \frac{k(2-\sqrt{3})+(2+\sqrt{3})}{k+1}$

Solving for k:

$-3k - 3 = 4k - 2\sqrt{3}k + 4 + 2\sqrt{3}$

Simplifying:

$-7k + 2\sqrt{3}k = 7 + 2\sqrt{3}$

$k = \frac{7+2\sqrt{3}}{-7+2\sqrt{3}}$

Therefore, the ratio in which C divides the segment AB is:

$(7 + 2\sqrt{3}) : (-7 + 2\sqrt{3})$

Question96

If the circle $S = 0$ cuts the circles $x^2 + y^2 - 2x + 6y = 0$, $x^2 + y^2 - 4x - 2y + 6 = 0$ and $x^2 + y^2 - 12x + 2y + 3 = 0$ orthogonally, then equation of the tangent at $(0, 3)$ on $S = 0$ is

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Options:

A. $x + y - 3 = 0$

B. $y = 3$

C. $x = 0$

D. $x - y + 3 = 0$

Answer: B

Solution:

Let the equation of the desired circle be:

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

The circle S is orthogonal to three given circles. Using the property of orthogonal circles, we find the equations involving g, f, and c.

For the Circle $x^2 + y^2 - 2x + 6y = 0$:

Since S and this circle cut orthogonally, we have:

$2 \cdot g \cdot (-1) + 2 \cdot f \cdot 3 = c + 0$

Simplifying:

$-2g + 6f = c \quad$ ...(i)

For the Circle $x^2 + y^2 - 4x - 2y + 6 = 0$:

Orthogonality gives us:

$2g \cdot (-2) + 2f \cdot (-1) = c + 6$

Simplifying:

$-4g - 2f = c + 6$

For the Circle $x^2 + y^2 - 12x + 2y + 3 = 0$:

Orthogonality yields:

$2g \cdot (-6) + 2f \cdot 1 = c + 3$

Simplifying:

$-12g + 2f = c + 3$

From equation (i), substituting c from the orthogonality conditions, we proceed to simplify:

Solving the System of Equations:

Using equations:

$-4g - 2f = c + 6$

$-12g + 2f = c + 3$

Rearrange equation (i):

$c = -2g + 6f \quad$ (equation i)

From $-4g - 2f = -2g + 6f + 6$:

$-2g - 8f = 6 \quad \Rightarrow g + 4f = -3 \quad$ ...(ii)

From $-12g+2f=-2g+6f+3$:

$-10g-4f=3 \quad \Rightarrow 10g+4f=-3$

Solving equations (ii) and the above, we find:

$g=0, \quad f=-\frac{3}{4}$

Substituting into equation (i) for c:

$c=6\times\left(-\frac{3}{4}\right)=-\frac{9}{2}$

Equation of the Circle S:

$x^2+y^2-\frac{3}{2}y-\frac{9}{2}=0$

Finding the Equation of the Tangent at $(0,3)$:

The equation of the tangent to this circle at the point (x_1,y_1) is given by:

$xx_1+yy_1-\frac{3}{2}\left(\frac{y+y_1}{2}\right)-\frac{9}{2}=0$

Substituting point $(0,3)$:

$3y-\frac{3}{2}\left(\frac{y+3}{2}\right)-\frac{9}{2}=0$

Simplifying:

$3y-\frac{3}{4}y-\frac{9}{4}-\frac{9}{2}=0$

Convert all terms to common denominator:

$\frac{12y}{4}-\frac{3y}{4}-\frac{9}{4}-\frac{18}{4}=0$

Re-arrange the equation:

$\frac{9y}{4}=\frac{27}{4}$

Solve for y:

$y=3$

Thus, the equation of the tangent is:

$y=3$

Question97

If θ is the angle between the tangents drawn from the point $(2,3)$ to the circle $x^2+y^2-6x+4y+12=0$ then $\theta=$

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Options:

A. $\cos^{-1}\left(\frac{5}{13}\right)$

B. $\sin^{-1}\left(\frac{4}{5}\right)$

C. $2\tan^{-1}\left(\frac{5}{12}\right)$

D. $\tan^{-1}\left(\frac{5}{12}\right)$

Answer: D

Solution:

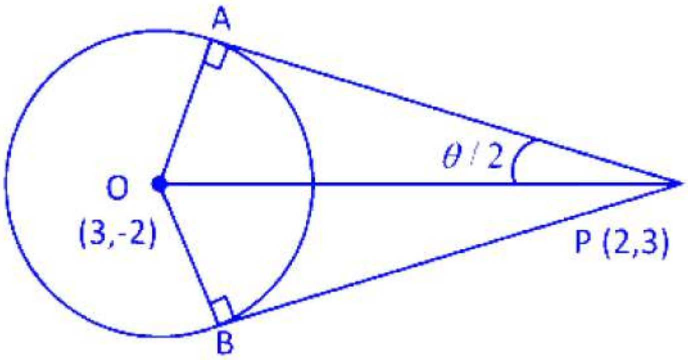
We have,

Equation, of circle is

$x^2 + y^2 - 6x + 4y + 12 = 0$

Centre of circle $= (\lambda - 2)$

radius $= \sqrt{9 + 4 - 12} = 1$



$$\begin{aligned} OP &= \sqrt{(3-2)^2 + (-2-3)^2} \\ &= \sqrt{1+25} = \sqrt{26} \\ AP &= \sqrt{OP^2 - AO^2} \\ &= \sqrt{26-1} = 5 \end{aligned}$$

In $\triangle MOP, \tan\frac{\theta}{2} = \frac{AO}{AP} = \frac{1}{5}$

$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2} = \frac{\frac{2}{5}}{\frac{24}{25}} = \frac{5}{12}$

Hence, $\theta = \tan^{-1}\frac{5}{12}$

Question98

If $2x - 3y + 3 = 0$ and $x + 2y + k = 0$ are conjugate lines with respect to the circle $S = x^2 + y^2 + 8x - 6y - 24 = 0$, then the length of the tangent drawn from the point $\left(\frac{k}{4}, \frac{k}{3}\right)$ to the circle $S = 0$, is

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Options:

A. 7

B. 1

C. 12

D. 24

Answer: B

Solution:

Let (a, b be the point on the line

$x + 2y + K = 0$

We have, equation of circle is

$x^2 + y^2 + 8x - 6y - 24 = 0$

Equation of polar for the given circle is obtained as

$$ax + by + 4(x + d) - 3(y + b) - 24 = 0$$
$$(a + 4x + (b - 3)y + 4a - 3 - 24$$

Now, comparing this line with

$2x - 3y + 3 = 0$ we get

$$\frac{a+4}{2} = \frac{b-3}{-3} = \frac{4a - y - 24}{3}$$
$$-3a - 12 = 2b - 6 \Rightarrow 3a + 2b = -6$$
$$\text{and } 3a + 12 = 8a - 6b - 4s$$
$$\Rightarrow 5a - 6b = 60 \ldots \text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$a = 3$ and $b = \frac{-15}{2}$

Now. $\left(3, \frac{15}{2}\right)$ lies on line $x + 2y + k = 0$

$3 - 15 + K = 0 \Rightarrow K = 12$

Required length of tangent from a point

$\left(\frac{12}{4}, \frac{12}{3}\right)$ i.e. $(3,4)$

$= \sqrt{3^3 + 4^2 + 24 - 24 - 24}$

$= \sqrt{9 + 16 - 24} = 1$

Question99

If $Q(h,k)$ is the inverse point of the point $P(1,2)$ with respect to the circle $x^2 + y^2 - 4x + 1 = 0$, then $2h + k =$

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Options:

A. 3

B. 4

C. 7

D. 11

Answer: B

Solution:

The equation of pole with respect ts the point $\beta, 2$ to the given circle is

$x + 2y - 2(x+1) + 1 = 0$

$\Rightarrow -x + 2y - 1 = 0$ or $x - 2y + 1 = 0$

Inverse of point $P(1,2)$ is the foot (h,k) of the perpendicalar from the point $(1,2)$

to the line $x - 2y + 1 = 0$

$\Rightarrow \quad \frac{h-1}{1} = \frac{k-2}{-2} = -\left[\frac{1-4+1}{1^2+(-2^3}\right]$

$\Rightarrow \quad h - 1 = \frac{K-2}{-2} = \frac{2}{5}$

$\Rightarrow \quad h - 1 = \frac{2}{5}$ and $\frac{K-2}{-2} = \frac{2}{5}$

$\Rightarrow \quad h = \frac{7}{5}$ and $K = \frac{-4}{5} + 2 = \frac{6}{5}$

Hence, $2h + K = \frac{14}{5} + \frac{6}{5} = \frac{20}{5} = 4$

Question100

If (a, b) and (c, d) are the internal and external centres of similitudes of the circles $x^2+y^2+4x-5=0$ and $x^2+y^2-6y+8=0$ respectively, then $(a+d)(b+q)=$

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Options:

A. 4

B. 9

C. 13

D. 22

Answer: C

Solution:

We have, equation of two circlet

$x^2+y^2+4x-5=0$

and $x^2+y^2-6y+8=0$

Centre and radius of both circles are

$c_1=\left(-2,0, r_1=\sqrt{4+5}=3\right.$

$c_2=(0,3), r_2=\sqrt{9-8}=1$

Let P and Q be the internal and external centre of similitude, it divides $C_1 C_2$ is ine ratio $3:1$.

$P=\frac{x_2+C_1}{3+1}=\left(\frac{-2}{4}, \frac{9}{4}\right)=\left(\frac{-1}{2} ; \frac{9}{4}\right)$

Here, $a=\frac{-1}{2}$ and $b=\frac{9}{4}$

$Q=\frac{3 C_2-C_1}{C_1-C_2}=\left(\frac{2}{2}, \frac{9}{2}\right)=\left(1, \frac{9}{2}\right)$

Here, $C=1$ and $d=\frac{9}{2}$

$$
\begin{aligned}
\text{Hence, } (a+\Delta)(b+c) & =\left(\frac{-1}{2}+\frac{9}{2}\right)\left(\frac{9}{4}+1\right) \\
& =\frac{4 \times 13}{4}=13
\end{aligned}
$$

Question101

A circle s passes through the points of intersection of the circles $x^2+y^2-2x+2y-2=0$ and $x^2+y^2+2x-2y+1=0$. If the centre of this circle S lies on the line $x-y+6=0$, then the radius of the circle S is

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Options:

A. $\sqrt{5}$

B. 5

C. $\sqrt{41}$

D. $\sqrt{14}$

Answer: D

Solution:

The equation of a circle that passes through the intersection points of the given circles is determined as follows:

First, start with the general form that includes both circles and a parameter λ:

$x^2+y^2-2x+2y-2+\lambda(x^2+y^2+2x-2y+1)=0$

Simplifying this equation leads to:

$(1+\lambda)x^2+(1+\lambda)y^2+(2\lambda-2)x+(2-2\lambda)y+(-2+\lambda)=0$

To rewrite in standard circle form, factor out the coefficients:

$x^2+y^2+2x\left(\frac{\lambda-1}{1+\lambda}\right)+2y\left(\frac{1-\lambda}{1+\lambda}\right)-\frac{2-\lambda}{1+\lambda}=0$

The center of the circle (h,k) from this equation is:

$\left(\frac{1-\lambda}{1+\lambda},\frac{\lambda-1}{1+\lambda}\right)$

Since this center lies on the line $x-y+6=0$, we substitute the center into the line's equation:

$1-\lambda-(\lambda-1)+6(1+\lambda)=0$

Solving:

$1-\lambda-\lambda+1+6\lambda+6=0 \quad \Rightarrow \quad 4\lambda+8=0 \quad \Rightarrow \quad \lambda=-2$

Substitute $\lambda=-2$ back into the center, we get:

$(h,k)=(-3,3)$

Finally, calculate the radius of the circle:

$\text{Radius} = \sqrt{(-3)^2 + 3^2 - \frac{2-(-2)}{1+(-2)}} = \sqrt{(-3)^2 + 3^2 - \frac{4}{-1}}$

Solving further:

$\text{Radius} = \sqrt{9+9+4} = \sqrt{14}$

Question102

The locus of mid-points of points of intersection of $x \cos \theta + y \sin \theta = 1$ with the coordinate axes is

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Options:

A. $x^2 + y^2 = 4$

B. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{4}$

C. $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$

D. $x^2 + y^2 = 2$

Answer: B

Solution:

Let the line cut the axes in A and B and if (h, k) be the mid-point of AB, then

$2h = \frac{1}{\cos \theta}, 2k = \frac{1}{\sin \theta}$

In order to find the locus, eliminate the variable θ by $\cos^2 \theta + \sin^2 \theta = 1$

$\frac{1}{4h^2} + \frac{1}{4k^2} = 1 \Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 4$

Question103

The radius of the circle having. $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ as its tangents is

Options:

A. $\frac{3}{2}$

B. 3

C. 6

D. $\frac{3}{4}$

Answer: D

Solution:

Equation of the given tangents are

$$E_1 = 3x - 4y + 4 = 0$$
$$E_2 = 6x - 8y - 7 = 0$$
$$\Rightarrow \quad 3x - 4y - \frac{7}{2} = 0$$

Here, $a = 3, b = -4c_1 = 4, c_2 = \frac{-7}{2}$

Since, slopes of the given tangents are equal, i.e. $\frac{3}{4}$.

$\therefore$ The given tangents are parallel,

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right| = \left|\frac{4 - \left(\frac{-7}{2}\right)}{\sqrt{3^2 + (-4)^2}}\right|$$
$$d = \frac{\frac{15}{2}}{\sqrt{9 + 16}} = \frac{15}{2 \times 5} = \frac{3}{2}$$

As we know that distance between two parallel tangents of a circle is equal to the diameter of that circle.

$$\therefore \text{ Diameter of given circle } = \frac{3}{2}$$
$$\therefore \text{ Radius of the given circle } = \frac{\text{Diameter}}{2}$$
$$= \left(\frac{3/2}{2}\right) = \frac{3}{4}$$

Question104

A circle is such that $(x-2)\cos\theta+(y-2)\sin\theta=1$ touches it for all values of θ. Then, the circle is

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Options:

A. $x^2+y^2-4x-4y+7=0$

B. $x^2+y^2+4x+4y+7=0$

C. $x^2+y^2-4x-4y-7=0$

D. $x^2+y^2+4x+4y-7=0$

Answer: A

Solution:

Since, the line $(x-2)\cos\theta+(y-2)\sin\theta=1$ touches a circle. So it is a tangent equation to a circle.

Equation of tangent to a circle at(x_1, y_1) is $(x-h)x_1+(y-k)y_1=a^2$ to a circle

$(x-h)^2+(y-k)^2=a^2$

Then, after comparing

$x-h=x-2y-k=y-2$ and $a^2=1$
$x_1=\cos\theta, y_1=\sin\theta$

$\therefore$ Required equation of circle

$$(x-2)^2+(y-2)^2=1$$
$$\Rightarrow \quad x^2+y^2-4x-4y+7=0$$

Question105

The least distance of the point $(10,7)$ from the circle $x^2+y^2-4x-2y-20=0$ is

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Options:

A. 6

B. 7

C. 4

D. 5

Answer: D

Solution:

Given equation $\left(x^2+y^2-4x-2y-20=0\right)$

So, $C=(2,1)$ and radius $=\sqrt{(g)^2+(f)^2-c}$

$=\sqrt{(2)^2+(1)^2+20}$

Radius $=5$

When substituted $x=10$ and $y=7$ in equation, then value becomes

$(10)^2+(7)^2-4(10)-2(7)-20=75$

which is greater than zero.

Thus, the point $(10,7)$ lies outside the circle.

Its distance from the centre of the circle $(2,1)$ is

$\sqrt{(10-2)^2+(7-1)^2}=10$ units

So, the minimum distance from the circle therefore becomes $10-5=5$ units

----

Question106

Suppose that the x-coordinates of the points A and B satisfy $x^2+2x-a^2=0$ and their y-coordinates satisfy $y^2+4y-b^2=0$. Then, the equation of the circle with AB as its diameter is

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Options:

A. $x^2+y^2+2x+4y-a^2-b^2=0$

B. $x^2+y^2+2x+4y+a^2+b^2=0$

C. $x^2+y^2-2x-4y-a^2-b^2=0$

D. $x^2+y^2-2x-4y+a^2+b^2=0$

Answer: A

Solution:

Let x_1 and x_2 are roots of equation $x^2+2x-a^2=0$ and y_1 and y_2 are roots of equation $y^2+4b-b^2=(x-x_1)(x-x_2)=0$ and $y^2+4b-b^2 \rightarrow y_1, y_2=(y-y_1)(y-y_2)$ (roots of equation)

Let $A(x_1, y_1)$ and $B(x_2, y_2)$.

Now, the equation of circle with diameter AB is given by $\Rightarrow (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow x^2+2x-a^2+y^2+4y-b^2=0$
$\Rightarrow x^2+y^2+2x+4y=a^2+b^2$
$\Rightarrow x^2+y^2+2x+4y-a^2-b^2=0$

Question107

The radical centre of the three circles $x^2+y^2-1=0, x^2+y^2-8x+15=0$ and $x^2+y^2+10y+24=0$ is

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Options:

A. $\left(2, \frac{-5}{2}\right)$

B. $\left(2, \frac{5}{2}\right)$

C. $\left(-2, \frac{5}{2}\right)$

D. $\left(-2, \frac{-5}{2}\right)$

Answer: A

Solution:

Let $S_1 : x^2 + y^2 - 1 = 0$ .... (i)

$S_2 : x^2 + y^2 - 8x + 15 = 0$ .... (ii)

and $S_3 : x^2 + y^2 + 10y + 24 = 0$ .... (iii)

From Eqs. (i) and (ii),

$$8x - 15 = 1$$
$$\Rightarrow \quad x = 2$$

Now, from Eqs. (i) and (iii),

$$-10y - 24 = 1$$
$$\Rightarrow \quad -10y = 25 \Rightarrow y = \frac{-5}{2}$$

Hence, the radical centre is $\left(2, \frac{-5}{2}\right)$.

Question108

For any real number t, the point $\left(\frac{8t}{1+t^2}, \frac{4(1-t^2)}{1+t^2}\right)$ lies on a / an

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Options:

A. circle of radius 2

B. circle of radius 4

C. ellipse with 4 as its major axis length

D. ellipse with 4 as its minor axis length

Answer: B

Solution:

$$\text{Let } x = \frac{8t}{1+t^2} \quad .... \text{(i)}$$
$$y = \frac{4\left(1-t^2\right)}{1+t^2} \quad .... \text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$\begin{aligned} x^2+y^2 &= \frac{64t^2}{(1+t^2)^2}+\frac{16\left(1-2t^2+t^4\right)}{(1+t^2)^2} \\ &= \frac{16\left(1+2t^2+t^4\right)}{(1+t^2)^2}=\frac{16\left(1+t^2\right)^2}{(1+t^2)^2}=16 \end{aligned}$$

$\Rightarrow x^2+y^2=(4)^2$

which is circle of radius 4 units.

Question109

The area of the circle passing through the points $(5, \pm 2),(1,2)$ is

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Options:

A. 8π

B. 4π

C. 2π

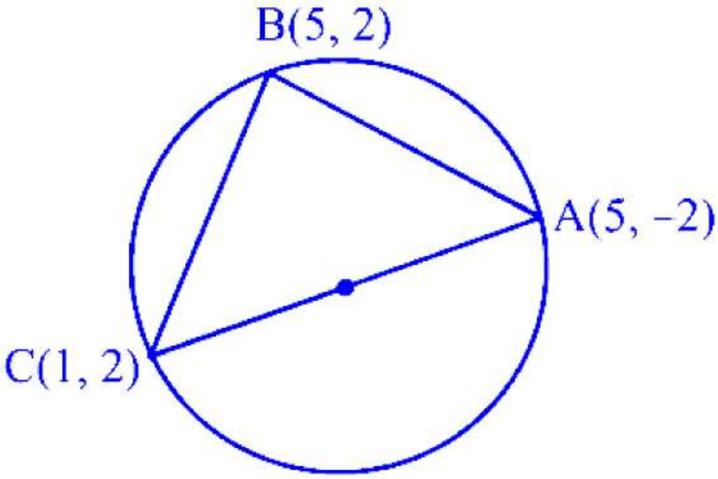
D. 16π

Answer: A

Solution:

Centre of circle passing through $(1,2),(5,2)$ and $(5,-2)$ is mid-point of $C(1,2)$ and $A(5,-2)$. We know that diameter always form a right angle on a circle.

$\therefore$ Centre $=\left(\frac{1+5}{2}, \frac{2-2}{2}\right)=(3,0)$



$$\text{and Radius } = \frac{\text{Diameter}}{2} = \frac{\sqrt{(5-1)^2+(-2-2)^2}}{2} = 2\sqrt{2}$$

Thus, area is $\pi(2\sqrt{2})^2 = 8\pi$ (units)^2

Question110

The ratio of the largest and shortest distances from the point $(2, -7)$ to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$ is

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Options:

A. 15 : 13

B. 7 : 1

C. 3 : 2

D. 14 : 1

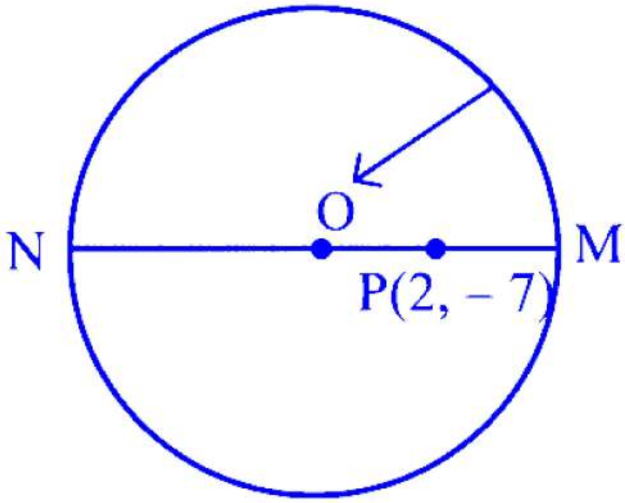
Answer: D

Solution:

To find the shortest and longest distances from the point $(2, -7)$ to the circle

$x^2 + y^2 - 14x - 10y - 151 = 0$

So, to find the solution we first draw the diagram,



So, we have to find the coordinates of the centre 0 and the radius of the circle, i.e $OM = ON =$ radius Thus, the shortest distance $PM = OM - OP$ and the longest distance $PN = MN - PM$ First we have to determine the point $P(2, -7)$ lies in which side of circle. So, we put the value $x = 2, y = -7$ in the left hand side of the equation, we get $x^2 + y^2 - 14x - 10y - 151 = (2)^2$

$= (2)^2 + (-7)^2 - 14 \times 2 - 10(-7) - 151 = -56 < 0$

Thus, the point lies inside of the circle. Thus, we know any equation of the circle is in the from $x^2 + y^2 + 2gx + 2fy + c = 0$.

Thus, $g = -7$, and $f = -5, c = -151$

So, centre is $(-g, -f) = (7, 5)$

$$\begin{aligned} \text{and radius } &= r = OM = ON \\ &= \sqrt{7^2 + 5^2 - (-151)} \\ &= \sqrt{49 + 25 + 151} \\ &= \sqrt{225} = 15 \end{aligned}$$

Now, distance between centre $O(7, 5)$ and $P(2, -7)$is $OP = \sqrt{(7-2)^2 + (5+7)^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$

Thus, the shortest distance from the point $P(2, -7)$ to the circle is $PM = OM - OP = 15 - 13 = 2$ units and the longest distance is

$$\begin{aligned} PN = MN - PM &= (OM + ON) - PM \\ &= (15 + 15) - 2 = 28 \text{ units} \end{aligned}$$

Thus, the required ratio is $28 : 2 = 14 : 1$

Question111

A circle has its centre in the first quadrant and passes through $(2, 3)$. If this circle makes intercepts of length 3 and 4 respectively on $x = 2$ and $y = 3$, its equation is

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Options:

A. $x^2 + y^2 + 3x - 5y + 8 = 0$

B. $x^2 + y^2 - 4x - 6y + 13 = 0$

C. $x^2 + y^2 - 6x - 8y + 23 = 0$

D. $x^2 + y^2 - 8x - 9y + 30 = 0$

Answer: D

Solution:

Let the centre of circle be (h, k). Equation of circle will be

$x^2 + y^2 - 2hx - 2ky + c = 0 \quad$ ... (i)

Circle passes through $(2, 3)$

$$\begin{aligned} & 2^2 + 3^2 - 4h - 6k + c &= 0 \\ \Rightarrow \quad & 4h + 6k - c = 13 \\ \text{or} \quad & c = 4h + 6k - 13 \end{aligned}$$

Equation of circle be

$x^2 + y^2 - 2hx - 2ky + 4h + 6k - 13$

This circle makes intercept of length 3 on the line $x = 2$

$|y_1 - y_2| = 3$, when $x = 2$ is substituted in the equation of circle, where y_1 and y_2 are the two roots

$4+y^2-4h-2ky+4h+6k-13=0$
$\Rightarrow y^2-2ky+6k-9=0$
$y_1+y_2=2k$ and $y_1y_2=6k-9$
$|y_1-y_2|=3$
$\sqrt{(y_1+y_2)^2-4y_1y_2}=3$
$\Rightarrow 4k^2-4(6k-9)=9$
$\Rightarrow 4k^2-24k+36-9=0 \Rightarrow 4k^2-24k+27=0$
$\Rightarrow k=\frac{9}{2}, k\neq\frac{3}{2}$

[$\because$ when substitute $k=\frac{3}{2}$ in y_1y_2, we get y_1y_2 as 0]

Now, similarly substitute $y=3$

$x^2+9-2hx-6k+4h+6k-13=0$
$\Rightarrow x^2-2hx+4h-4=0$
$\Rightarrow x_1+x_2=2h_1, x_1x_2=4h-4$
$\because \quad |x_1-x_2|=4$
$\sqrt{(x_1+x_2)^2-4x_1x_2}=4$
$\Rightarrow \quad 4h^2-16h+16=16 \Rightarrow 4h(h-4)=0$
$h=4$ and $h\neq 0$

$\therefore$ Equation of the required circle is

$x^2+y^2-8x-9y+30=0$

Question112

The image of the point $(3,4)$ with respect to the radical axis of the circles $x^2+y^2+8x+2y+10=0$ and $x^2+y^2+7x+3y+10=0$ is

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Options:

A. $(3,4)$

B. $(-4,-3)$

C. $(4,3)$

D. $(-3,-4)$

Answer: C

Solution:

The given equation of circles

$S_1 : x^2 + y^2 + 8x + 2y + 10 = 0$
$S_2 : x^2 + y^2 + 7x + 3y + 10 = 0$

As we know the equation of radical axis of two circles is given by

$$S_1 - S_2 = 0$$
$$\Rightarrow (x^2 + y^2 + 8x + 2y + 10) - (x^2 + y^2 + 7x + 3y + 10) = 0$$
$$\Rightarrow (8x - 7x) + (2y - 3y) = 0$$
$$\Rightarrow x - y = 0$$

Thus, equation of radical axis of the given circle is

$x - y = 0$

Hence, the image of the point $(3, 4)$ with respect to $y = x$ line is $(4, 3)$.

Question113

The locus of centers of the circles, possessing the same area and having $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ as their common tangent, is

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Options:

A. $12x - 16y - 15 = 0$

B. $3x - 4y + \frac{11}{2} = 0$

C. $12x - 16y + 15 = 0$

D. $3x - 4y - \frac{11}{2} = 0$

Answer: A

Solution:

1. **Same area ⇒ same radius**

   Circles having the same area must have the **same radius.**

2. **Given common tangents:**

$$L_1 : 3x - 4y + 4 = 0, \quad L_2 : 6x - 8y - 7 = 0$$

3. **Condition for same radius:**

   The distance of the center (x, y) from both tangent lines must be **equal.**

4. **Distance formula from a line:**

   Distance of (x, y) from $ax + by + c = 0$ is

$$\frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$

5. **Apply distance equality:**

$$\frac{|3x - 4y + 4|}{\sqrt{3^2 + (-4)^2}} = \frac{|6x - 8y - 7|}{\sqrt{6^2 + (-8)^2}}$$

6. **Simplify denominators:**

$$\sqrt{25} = 5, \quad \sqrt{100} = 10$$

$$\frac{|3x - 4y + 4|}{5} = \frac{|6x - 8y - 7|}{10}$$

7. **Cross-multiply (taking same sign case):**

$$2(3x - 4y + 4) = 6x - 8y - 7$$

8. **Simplify:**

$$6x - 8y + 8 = 6x - 8y - 7$$

$$\Rightarrow 15 = 0 \quad \text{(shift constants)}$$

   Rearranging gives:

$$12x - 16y - 15 = 0$$

✅ **The correct answer is: A** $\quad 12x - 16y - 15 = 0$

Question114

For any two non-zero real numbers a and b if this line $\frac{x}{a} + \frac{y}{b} = 1$ is a tangent to the circle $x^2 + y^2 = 1$, then which of the following is true?

Options:

A. $\left(\frac{1}{a}, \frac{1}{b}\right)$ lies inside the circle

B. (a, b) lies inside the circle

C. $\left(\frac{1}{a}, \frac{1}{b}\right)$ lies on the circle

D. (a, b) lies on the circle.

Answer: C

Solution:

Curve : $x^2 + y^2 = 1$

Let $\left(\frac{1}{a}, \frac{1}{b}\right) = (x_1, y_1)$ touches the circles, then equation of tangent at (x_1, y_1) is

$$xx_1 + yy_1 = 1$$

$$\Rightarrow \quad x \cdot \left(\frac{1}{a}\right) + y \cdot \left(\frac{1}{b}\right) = 1 \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

Question115

The length of the intercept on the line $4x - 3y - 10 = 0$ by the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ is

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Options:

A. 5

B. 2

C. 10

D. 6

Answer: C

Solution:

$$
\begin{aligned}
& 4x-3y-10=0 \\
& \Rightarrow \quad x=\frac{3y+10}{4} \\
& \text{and } x^2+y^2-2x+4y-20=0 \\
& \Rightarrow \quad \left(\frac{3y+10}{4}\right)^2+y^2-2\left(\frac{3y+10}{4}\right)+4y-20=0 \\
& \Rightarrow \quad 9y^2+100+60y+16y^2-24y-80+64y-320=0 \\
& \Rightarrow \quad 25y^2+100y-300=0 \Rightarrow y^2+4y-12=0 \\
& \Rightarrow \quad y^2+6y-2y-12=0 \Rightarrow y(y+6)-2(y+6)=0 \\
& \Rightarrow \quad (y+6)(y-2)=0 \\
& y=-6,2 \\
& y=-6 \Rightarrow x=\frac{-18+10}{4}=-2 \Rightarrow(-2,-6) \\
& y=2 \Rightarrow x=\frac{6+10}{4}=4 \Rightarrow(4,2)
\end{aligned}
$$

Required length

$$
\begin{aligned}
& =\sqrt{(4+2)^2+(2+6)^2} \\
& =\sqrt{36+64}=\sqrt{100}=10
\end{aligned}
$$

Question116

The pole of the line $\frac{x}{a}+\frac{y}{b}=1$ with respect to the circle $x^2+y^2=c^2$ is

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Options:

A. $\left(\frac{c^2}{a}, \frac{c^2}{b}\right)$

B. $\left(\frac{c^2}{b}, \frac{c^2}{a}\right)$

C. $\left(\frac{c}{a}, \frac{c}{b}\right)$

D. $\left(\frac{c}{b}, \frac{c}{a}\right)$

Answer: A

Solution:

Let the pole be (x_1, y_1), then the equation of the polar with respect to the circle $x^2 + y^2 = c^2$ is $xx_1 + yy_1 = c^2$

$\Rightarrow \quad \frac{xx_1}{c^2} + \frac{yy_1}{c^2} = 1 \quad \ldots$ (i)

Eq. (i) and $\frac{x}{a} + \frac{y}{b} = 1$ represents the same line.

$$\therefore \quad \frac{x_1}{c^2} = \frac{1}{a} \Rightarrow x_1 = \frac{c^2}{a}$$
$$\frac{y_1}{c^2} = \frac{1}{b} \Rightarrow y_1 = \frac{c^2}{b}$$

Pole : $(x_1, y_1) = \left(\frac{c^2}{a}, \frac{c^2}{b}\right)$

Question117

If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the Y-axis, then the length of PQ is

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Options:

A. 5

B. 6

C. 4

D. 3

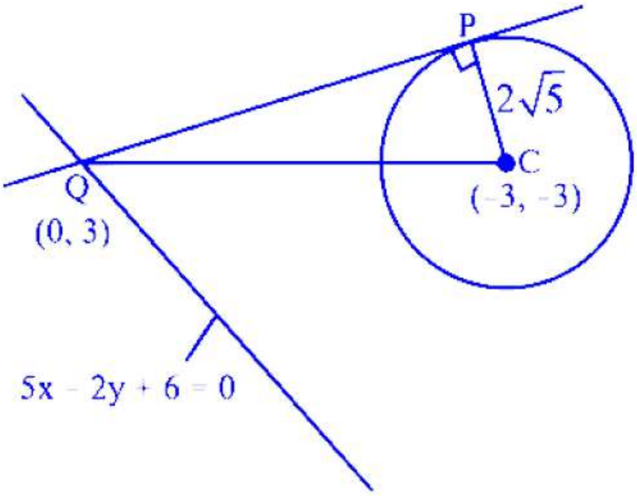
Answer: A

Solution:

Line: $5x - 2y + 6 = 0$

On Y-axis, $x = 0 \Rightarrow y = \frac{6}{2} = 3$

i.e. $Q \equiv (0, 3)$



Circle : $x^2+y^2+6x+6y-2=0$
Centre $(-g,-f) \equiv(-3,-3)$
Radius $=\sqrt{g^2+f^2-c}$
$=\sqrt{9+9+2}=\sqrt{20}$
$=2\sqrt{5}$
$CQ=\sqrt{(0+3)^2+(3+3)^2}=\sqrt{9+36}=\sqrt{45}$
In $\triangle PQC$,
$(CQ)^2=(CP)^2+(QP)^2$
$\Rightarrow \quad 45=20+(QP)^2$
$\Rightarrow \quad 25=(QP)^2 \Rightarrow QP=5$

Question118

The equation of the pair of straight lines parallel to x-axis and touching the circle $x^2+y^2-6x-4y-12=0$ is

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Options:

A. $y^2-4y-21=0$

B. $y^2+4y-21=0$

C. $y^2-4y+21=0$

D. $y^2+4y+21=0$

Answer: A

Solution:

Equation parallel to X-axis is given as

$y=a$

or $y-a=0$ .... (i)

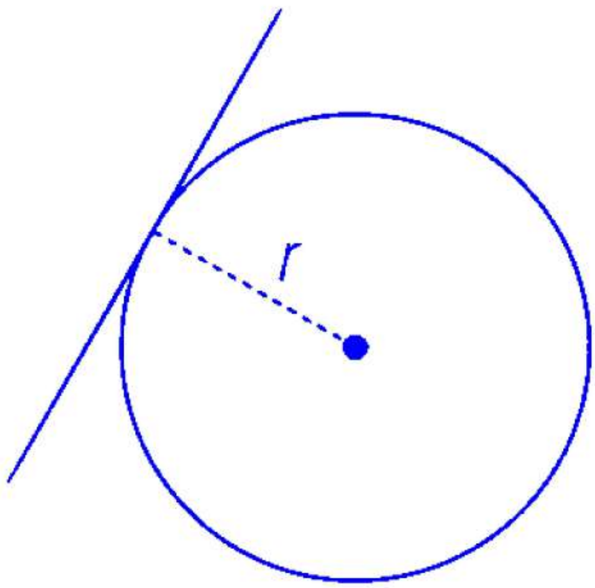
If Eq. (i) touches the circle

$x^2+y^2-6x-4y-12=0$ ..... (ii)

Then, Eq. (i) becomes tangent to circle.

Centre of Eq. (ii) is $(3,2)$

Radius of Eq. (ii) is $\sqrt{9+4+12}=5$



Then, perpendicular distance from centre to line (i) is radius i.e.

Perpendicular distance between $(3,2)$ and $y-a=0$ is equal to 5

$$\text{i.e.} \quad \left|\frac{2-a}{l}\right|=5$$
$$\Rightarrow \quad 2-a=\pm 5$$
$$\Rightarrow \quad a=7,-3$$

$\therefore$ Equation of pair of lines $=(y+3)(y-7)=0$

$$\text{i.e. } (y+3)(y-7)=0$$
$$\Rightarrow y^2-4y-21=0$$

Question119

The points where the circle $x^2+y^2-3x-4y+2=0$ cuts the X-axis are

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Options:

A. $(1,2)$ and $(2,0)$

B. $(2,0)$ and $(3,0)$

C. $(0,2)$ and $(0,1)$

D. $(1,0)$ and $(2,0)$

Answer: D

Solution:

Given circle,

$x^2+y^2-3x-4y+2=0$ ..... (i)

It cut the X-axis, when $y=0$

Put $y=0$ in Eq. (i), we obtain

$$\begin{aligned} & x^2-3x+2=0 \\ \Rightarrow \quad & (x-2)(x-1)=0 \\ \therefore \quad & x=2,1 \end{aligned}$$

Thus, circle (i) touches X-axis at $(2,0),(1,0)$.

Question120

The center and radius of the circle $x^2+y^2+8x+10y-8=0$ respectively are and units

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Options:

A. $(-4,-5), 7$

B. $(4,5), 49$

C. $(-8,-10), 8$

D. $(-4,5), 7$

Answer: A

Solution:

$$\begin{aligned} & x^2+y^2+8x+10y-8=0 \\ \Rightarrow\quad & (x+4)^2+(y+5)^2-16-25-8=0 \\ \Rightarrow\quad & (x+4)^2+(y+5)^2=49 \\ \Rightarrow\quad & (x+4)^2+(y+5)^2=(7)^2 \end{aligned}$$

which is standard form of circle.

Radius = 7 and Centre = $(-4,-5)$

Question121

The poles of the tangents to the circle $x^2+y^2=4$ with respect to the circle $(x+2)^2+y^2=8$, lie on

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Options:

A. $y^2+8x=0$

B. $x^2+8y=0$

C. $y^2-8x=0$

D. $x^2-8y=0$

Answer: A

Solution:

We know an important result:

If you draw tangents to the circle $x^2+y^2=a^2$, and then find their poles with respect to the circle $(x+a)^2+y^2=2a^2$, all those poles will be on the curve $y^2+4ax=0$.

In our question, $a=2$. So, we are looking at the circles $x^2+y^2=4$ and $(x+2)^2+y^2=8$.

Using the result, the poles of the tangents to $x^2+y^2=4$ with respect to $(x+2)^2+y^2=8$ will be on:

$y^2+4\times 2x=0$

which simplifies to:

$y^2+8x=0$

Question122

If the power of the point $(1,6)$ with respect to the circle $x^2+y^2+4x-6y-a=0$ is -16 then a equals

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Options:

A. 5

B. 11

C. 21

D. 6

Answer: C

Solution:

Given equation of circle is

$S: x^2+y^2+4x-6y-a=0$ ...... (i)

Power of point w.r.t. Eq. (i) is -16 , i.e.

$S(1,6) = -16$

or $(1)^2 + (6)^2 + 4 - 36 - a = -16$

$\Rightarrow 5 - a = -16$

$\Rightarrow a = 21$

Question123

The equation of radical axis of the circles $x^2 + y^2 + 4x + 6y + 7 = 0$ and $4x^2 + 4y^2 + 8x + 12y - 9 = 0$ is

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Options:

A. $x + y + 1 = 0$

B. $8x + 12y = 0$

C. $8x + 12y + 37 = 0$

D. $2x + 3y + 7 = 0$

Answer: C

Solution:

$$S_1 : x^2 + y^2 + 4x + 6y + 7 = 0$$
$$S_2 : x^2 + y^2 + 2x + 3y - \frac{9}{4} = 0$$

Equation of radical axis

$$S_1 - S_2 = 0$$
$$\left(x^2 + y^2 + 4x + 6y + 7\right) - \left(\left(x^2 + y^2 + 2x + 3y - \tfrac{9}{4}\right) = 0\right.$$
$$\Rightarrow 2x + 3y + \tfrac{37}{4} = 0$$
$$\text{or} \quad 8x + 12y + 37 = 0$$

Question124

The radical axis of the circles $S_1 : x^2 + y^2 - 4x + 6y - 10 = 0$ and $S_2 : x^2 + y^2 + 2x - 6y + 2 = 0$, cut the circle S_1 in

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Options:

A. two real and distinct points

B. one real point

C. imaginary points

D. can't be determined

Answer: A

Solution:

$S_1 : x^2 + y^2 - 4x + 6y - 10 = 0$ ...... (i)

$S_2 : x^2 + y^2 + 2x - 6y + 2 = 0$ ...... (ii)

Equation of radical axis

$S_1 - S_2 = 0$

$$\begin{aligned} \Rightarrow \quad & (x^2 + y^2 - 4x + 6y - 10) \\ & - (x^2 + y^2 + 2x - 6y + 2) = 0 \\ \Rightarrow \quad & 6x - 12y + 12 = 0 \\ \Rightarrow \quad & x - 2y + 2 = 0 \quad \text{......(iii)} \end{aligned}$$

Given that Eq. (iii) cut the circle (i),

Using Eq. (iii) in Eq. (i) as, $x = 2y - 2$

$$(2y - 2)^2 + y^2 - 4(2y - 2) + 6y - 10 = 0$$
$$\Rightarrow 4y^2 + 4 - 8y + y^2 - 8y + 8 + 6y - 10 = 0$$
$$\Rightarrow \quad 5y^2 - 10y + 2 = 0$$
$$\Rightarrow \quad y = \frac{10 \pm \sqrt{100 - 40}}{10} = \frac{5 \pm \sqrt{15}}{5}$$
$$\Rightarrow \quad x = 2\left(\frac{5 \pm \sqrt{15}}{5}\right)$$

Thus, Eq. (iii) cuts S_1 at two real and distinct points.

Question125

The locus of a point, which is at a distance of 4 units from $(3, -2)$ in xy-plane is

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Options:

A. $x^2 + y^2 + 6x - 4y + 16 = 0$

B. $x^2 + y^2 - 6x - 4y + 3 = 0$

C. $x^2 + y^2 - 6x + 4y - 16 = 0$

D. $x^2 + y^2 - 6x + 4y - 3 = 0$

Answer: D

Solution:

Let the point be (h, k).

So, we have

$$\begin{aligned} (h-3)^2 + (k+2)^2 &= 4^2 \\ \Rightarrow h^2 + 9 - 6h + k^2 + 4 + 4k &= 16 \\ \Rightarrow \quad h^2 + k^2 - 6h + 4k - 3 &= 0 \end{aligned}$$

Replace $h \rightarrow x$ and $k \rightarrow y$

$x^2 + y^2 - 6x + 4y - 3 = 0$

Question126

Find the equation of the circle which passes through origin and cuts off the intercepts -2 and 3 over the X and Y-axes respectively.

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Options:

A. $x^2+y^2-2x+8y=0$

B. $2\left(x^2+y^2\right)+2x-3y=0$

C. $x^2+y^2-2x-8y=0$

D. $x^2+y^2+2x-3y=0$

Answer: D

Solution:

$$\begin{aligned} 2\sqrt{g^2-c} &= 2; \\ 2\sqrt{f^2-c} &= 3 \text{ and } c=0 \\ g=\pm 1, f &= \pm\frac{3}{2} \end{aligned}$$

$x^2+y^2\pm 2x\pm 3y=0$

It passes through $(-2,0)$ and $(0,3)$.

So, centre must be in second quadrant or fourth guadrant.

The x-coordinate of centre is negative.

$\therefore \quad x^2+y^2+2x-3y=0$

Question127

The angle between the pair of tangents drawn from $(1,1)$ to the circle $x^2+y^2+4x+4y-1=0$ is

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Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

1. **Write the circle in standard form**

$$x^2 + y^2 + 4x + 4y - 1 = 0$$

$$(x+2)^2 + (y+2)^2 = 9$$

So,
Center $C(-2, -2)$, Radius $r = 3$.

2. **Distance of point $P(1, 1)$ from the center**

$$CP = \sqrt{(1+2)^2 + (1+2)^2} = \sqrt{9+9} = 3\sqrt{2}$$

3. **Use formula for angle between tangents**

   If tangents are drawn from an external point at distance d from the center,

$$\sin\frac{\theta}{2} = \frac{r}{d}$$

4. **Substitute values**

$$\sin\frac{\theta}{2} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

5. **Find angle**

$$\frac{\theta}{2} = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$$

✅ The correct answer is: A — $\frac{\pi}{2}$

Question128

If the circle $x^2 + y^2 - 4x - 8y - 5 = 0$ intersects the line $3x - 4y - m = 0$ in two distinct points, then the number of integral values of 'm' is

Options:

A. 52

B. 51

C. 50

D. 49

Answer: D

Solution:

$x^2 + y^2 - 4x - 8y - 5 = 0$

$$\begin{aligned} &\text{Centre } = (2,4) \\ &\text{Radius } = 2\sqrt{4+16+5} = 10 \end{aligned}$$

Line : $3x - 4y - m = 0$

Distance of L from Centre < Radius

$$\begin{aligned} &\frac{|3 \cdot 2 - 4 \cdot 4 - m|}{\sqrt{3^2 + 4^2}} < 10 \\ \Rightarrow \quad &\frac{|6 - 16 - m|}{5} < 10 \\ \Rightarrow \quad &|-10 - m| < 50 \\ \Rightarrow \quad &-50 < -10 - m < 50 \\ \Rightarrow \quad &-40 < -m < 60 \\ \Rightarrow \quad &-60 < m < 40 \end{aligned}$$

Only (d) satisfies the above inequality.

Question129

Let C be the circle center $(0,0)$ and radius 3 units. The equation of the locus of the mid-points of the chords of the circle c that subtends an angle of $\frac{2\pi}{3}$ at its centre is

Options:

A. $x^2 + y^2 = \frac{1}{4}$

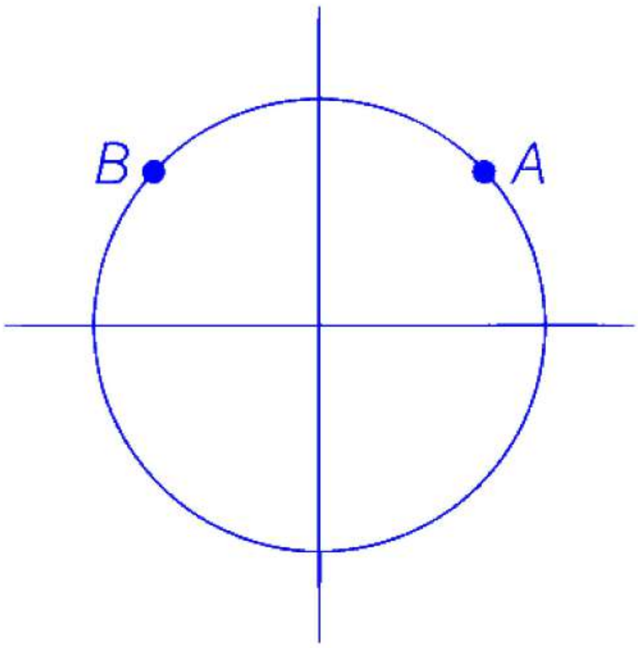
B. $x^2 + y^2 = \frac{27}{4}$

C. $x^2 + y^2 = \frac{9}{4}$

D. $x^2 + y^2 = \frac{5}{4}$

Answer: C

Solution:



Let $A = (3\cos\theta, 3\sin\theta)$

Then, $B = \left(3\cos\left(\theta + \frac{2\pi}{3}\right), 3\sin\left(\theta + \frac{2\pi}{3}\right)\right)$

Let (h, k) be the required point

$2h = 3\cos\theta + 3\cos\left(\theta + \frac{2\pi}{3}\right)$

$= 3\left[\cos\theta + \cos\theta\left(-\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{3}{2}(\cos\theta - \sqrt{3}\sin\theta)$

$2k = 3\sin\theta + 3\sin\left(\theta + \frac{2\pi}{3}\right)$

$= 3\left[\sin\theta + \sin\theta\left(\frac{-1}{3}\right) + \cos\theta\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{3}{2}(\sin\theta + \sqrt{3}\cos\theta)$

$\cos\theta - \sqrt{3}\sin\theta = \frac{4h}{3} \Rightarrow \sqrt{3}\cos\theta + \sin\theta = \frac{4k}{3}$

$\sqrt{3}\cos\theta - 3\sin\theta = \frac{4h}{\sqrt{3}}$

$4\sin\theta = \frac{4}{3}(k - \sqrt{3}h)$

$\sin\theta = \left(\frac{k-\sqrt{3}h}{3}\right)$

$\cos\theta = \left(\frac{3k+\sqrt{3}h}{3\sqrt{3}}\right) = \left(\frac{\sqrt{3}k+h}{3}\right)$

$\Rightarrow \left(\frac{k-\sqrt{3}h}{3}\right)^2 + \left(\frac{\sqrt{3}k+h}{3}\right)^2 = 1$

$\Rightarrow k^2 + 3h^2 - 2\sqrt{3}hk + 3k^2 + h^2 + 2\sqrt{3}kh = 9$

$\Rightarrow 4(k^2 + h^2) = 9 \Rightarrow x^2 + y^2 = \frac{9}{4}$

Question130

The length of the common chord of the circles $x^2 + y^2 + 3x + 5y + 4 = 0$ and $x^2 + y^2 + 5x + 3y + 4 = 0$ is ___________ units.

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Options:

A. 3

B. 2

C. 6

D. 4

Answer: D

Solution:

$$x^2+y^2+3x+5y+4=0$$
$$x^2+y^2+5x+3y+4=0gP-2x+2y=0$$
$$\Rightarrow \quad x=y \Rightarrow 2x^2+8x+4=0$$
$$\Rightarrow \quad x^2+4x+2=0 \Rightarrow (x+2)^2=2$$
$$\Rightarrow \quad x+2=\pm\sqrt{2} \Rightarrow x=-2\pm\sqrt{2}$$
$$\text{Length } =\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{2}\,(x_1-x_2)$$
$$=\sqrt{2}(-2+\sqrt{2})-(-2-\sqrt{2})=4$$

Question131

Find the equation of the circle which passes through the point $(1,2)$ and the points of intersection of the circles $x^2+y^2-8x-6y+21=0$ and $x^2+y^2-2x-15=0$

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Options:

A. $x^2+y^2-18x-12y+27=0$

B. $2\left(x^2+y^2\right)-18x-12y+27=0$

C. $3\left(x^2+y^2\right)-18x-12y+27=0$

D. $4\left(x^2+y^2\right)-18x-12y+27=0$

Answer: C

Solution:

$$\left(x^2+y^2-8x-6y+21\right)+\lambda\left(x^2+y^2-2x-15\right)=0$$
$$\Rightarrow \quad (1+4-8-12+21)+\lambda(1+4-2-15)=0$$
$$\Rightarrow \quad 6+(-12\lambda)=0 \Rightarrow \lambda=1/2$$
$$\Rightarrow \quad 3\left(x^2+y^2\right)-18x-12y+27=0$$

Question132

Given, two fixed points $A(-2,1)$ and $B(3,0)$.Find the locus of a point P which moves such that the angle $\angle APB$ is always a right angle.

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Options:

A. $x^2+y^2+x+y+6=0$

B. $x^2+y^2-x-y-6=0$

C. $x+y+6=0$

D. $2x^2+2y^2-2x-2y+1=0$

Answer: B

Solution:

$\angle APB = 90\Upsilon$

$\Rightarrow$ Locus of P is a circle with A and B as extremities of diameter.

$\therefore$ Locus of P is given by

$$\begin{aligned} (x+2)(x-3)+(y-1)(y-0)&=0 \\ x^2-x-6+y^2-y&=0 \\ \text{or} \quad x^2+y^2-x-y-6&=0 \end{aligned}$$

Question133

The equations of the tangents to the circle $x^2+y^2=4$ drawn from the point $(4,0)$ are

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Options:

A. $y=\pm\frac{1}{\sqrt{3}}(x-4)$

B. $y = \pm\frac{2}{\sqrt{3}}(x-4)$

C. $x = \pm\frac{1}{\sqrt{3}}(y-4)$

D. $x = \pm\frac{2}{\sqrt{3}}(y-4)$

Answer: A

Solution:

Let $S = x^2 + y^2 - 4 = 0$, then $T = xx_1 + yy_1 - 4$

Equation of tangent from $(4,0)$ is $x_1 = 4, y_1 = 0$ and $S_1 = x_1^2 + y_1^2 - 4$

$SS_1 = T^2$

$$\begin{aligned} &\Rightarrow && \left(x^2+y^2-4\right)(16-4) = (4x-4)^2 \\ \Rightarrow 3\left(x^2+y^2-4\right) && &= 4(x-1)^2 \\ &\Rightarrow && 3x^2+3y^2-12 && = 4\left(x^2-2x+1\right) \\ &\Rightarrow && 3y^2 = (x-4)^2 \\ &\Rightarrow && y = \pm\frac{1}{\sqrt{3}}(x-4) \end{aligned}$$

Question134

If $P(-9,-1)$ is a point on the circle $x^2 + y^2 + 4x + 8y - 38 = 0$, then find equation of the tangent drawn at the other end of the diameter drawn through P

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Options:

A. $7x - 3y = 60$

B. $7x - 3y = 56$

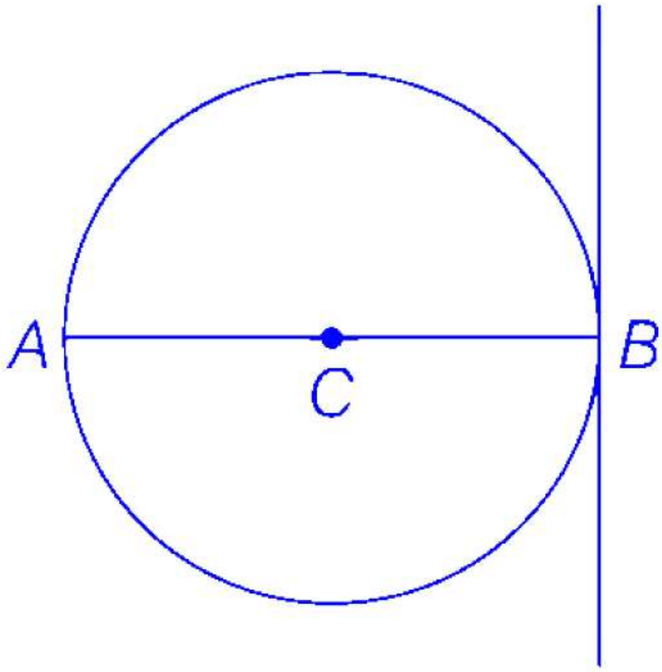
C. $7x + 3y = 56$

D. $7x + 3y = 60$

Answer: B

Solution:

Let $P(-9,-1)$



On the circle

$$x^2+y^2+4x+8y-38=0$$
$$\because 2g=4 \Rightarrow g=2$$
$$2f=8 \Rightarrow f=4$$
$$\text{Centre } =(-g,-f)$$

Whose centre is $C(-2,-4)$.

Let PB be a diameter. Then, C in mid-point of PB and Let $B(x,y)$.

Then, $(-2,-4)=\left(\frac{-9+x}{2},\frac{-1+y}{2}\right)$

$$\Rightarrow \quad \frac{-9+x}{2}=-2 \text{ and } \frac{-1+y}{2}=-4$$
$$\Rightarrow \quad x=5 \text{ and } y=-7$$
$$\therefore \quad B(5,-7)$$

Tangent at $(5,-7)$

$$5x-7y+2(x+5)+4(y-7)-38=0$$
$$5x-7y+2x+10+4y-28-38=0$$
$$\Rightarrow \quad 7x-3y=56$$

Question135

Find the equation of a circle whose radius is 5 units and passes through two points on the X-axis, which are at a distance of 4 units from the origin

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Options:

A. $x^2 + y^2 - 6x - 25 = 0$

B. $x^2 + y^2 - 6y - 25 = 0$

C. $x^2 + y^2 + 6y - 16 = 0$

D. $x^2 + y^2 + 6x - 16 = 0$

Answer: C

Solution:

Given, radius = 5

and points on X-axis at 4 units distance from origin be (−4, 0) and (4, 0).

Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (i)

Equation (i) passes through (−4, 0) and (4, 0)

$\therefore \quad 16 - 8g + c = 0$

and $\quad 16 + 8g + c = 0$

On solving, we have

$c = -16, g = 0$

$\therefore$ radius = 5

$\Rightarrow \sqrt{g^2 + f^2 - c} = 5$

or $f^2 + 16 = 25$

or $f = \pm 3$

$\therefore$ Required equation of circle

$x^2 + y^2 \pm 6y - 16 = 0$

Question136

If a foot of the normal from the point $(4, 3)$ to a circle is $(2, 1)$ and $2x - y - 2 = 0$, is a diameter of the circle, then the equation of circle is

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Options:

A. $x^2 + y^2 + 2x + 1 = 0$

B. $x^2 + y^2 + 2x - 1 = 0$

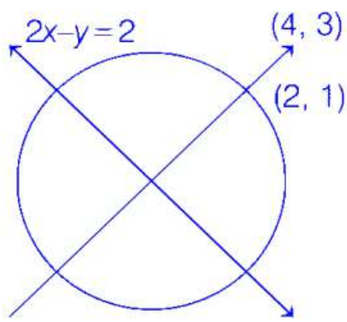
C. $x^2 + y^2 - 2x - 1 = 0$

D. $2\left(x^2 + y^2\right) - 2x - 1 = 0$

Answer: C

Solution:

Equation of normal passing through



(4, 3) and (2, 1)

$y - 3 = \frac{1-3}{2-4}(x - 4)$

$y - 3 = x - 4$

$x - y = 1$ ..... (i)

Equation of diameter

$2x - y = 2$ ...... (ii)

Solving Eqs. (i) and (ii), we have coordinate of centre of circle

$\therefore \quad x = 1, y = 0$

Centre of Circle $(1, 0)$

Radius of circle $=$ distance between $(1, 0)$ and $(2, 1)$

$$= \sqrt{(2-1)^2 + (1-0)^2} = \sqrt{2}$$

Radius $= \sqrt{2}$

$\therefore$ Equation of circle

$$(x-1)^2 + (y-0)^2 = (\sqrt{2})^2$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 1 = 0$$

Question137

The length of the tangent from any point on the circle $(x-3)^2+(y+2)^2=5r^2$ to the circle $(x-3)^2+(y+2)^2=r^2$ is 16 units, then the area between the two circles in square units is

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Options:

A. 32π

B. 4π

C. 8π

D. 256π

Answer: D

Solution:

Let $P(x, y)$ be any point on the circle, therefore it will satisfy the circle

$(x_1-3)^2+(y_1+2)^2=5r^2$ ..... (i)

The length of tangent drawn from point $P(x_1, y_1)$ to the circle $(x-3)^2+(y+2)^2=r^2$ is

$$\begin{aligned}\sqrt{(x_1-3)^2+(y_1+2)^2-r^2} &= \sqrt{5r^2-r^2} \text{ [from Eq. (i)]} \\ &= \sqrt{4r^2}=2r\end{aligned}$$

According to the question,

$$\begin{aligned} 16 &= 2r \\ \Rightarrow \quad r &= 8 \end{aligned}$$

$$\begin{aligned}\therefore \text{ Area between two circles } &= 5\pi r^2-\pi r^2 \\ &= 4\pi r^2 = 4\pi \cdot 8^2 \\ &= 256\pi \text{ squnits}\end{aligned}$$

Question138

The equation of the circle, which cuts orthogonally each of the three circles

$x^2 + y^2 - 2x + 3y - 7 = 0,$
$x^2 + y^2 + 5x - 5y + 9 = 0$ and
$x^2 + y^2 + 7x - 9y + 29 = 0$

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Options:

A. $x^2 + y^2 - 16x - 18y - 4 = 0$

B. $x^2 + y^2 = a^2$

C. $x^2 + y^2 - 16x = 0$

D. $y^2 - x^2 + 2x = 0$

Answer: A

Solution:

Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (i)

This circle is orthogonal to

$x^2 + y^2 - 2x + 3y - 7 = 0$ .... (ii)

$x^2 + y^2 + 5x - 5y + 9 = 0$ .... (iii)

$x^2 + y^2 + 7x - 9y + 29 = 0$ ... (iv)

By condition of orthogonality, we have

$-2g + 3f = c - 7$ .... (vi)

$5g - 5f = c + 9$ .... (vii)

$7g - 9f = c + 29$ .... (viii)

from Eq. (vii) $c = 5g - 5f - 9$

From Eqs. (vi) and (vii), we get

$-7g + 8f = -16$

and $g - 2f = 10$ {on subtracting Eq. (vii) from Eq. (viii)}

On solving, we have $g = -8, f = -9, c = -4$

Required equation of circle,

$\therefore x^2 + y^2 - 16x - 18y - 4 = 0$

Question139

Find the equations of the tangents drawn to the circle $x^2 + y^2 = 50$ at the points where the line $x + 7 = 0$ meets it.

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Options:

A. $7x + y + 50 = 0$ and $7x - y + 50 = 0$

B. $x + y = 0$ and $x - y = 0$

C. $x + 7y + 5 = 0$ and $y - 7x + 5 = 0$

D. $x + 7y + 50 = 0$ and $x - 7y + 50 = 0$

Answer: A

Solution:

Given circle $x^2 + y^2 = 50$ ..... (i)

and a line $x + 7 = 0$ ..... (ii)

Point of intersection of Eqs. (i) and (ii) is $x = -7, y = \pm 1$

$\therefore (-7, 1)$ and $(-7, -1)$ are points of contact of Eqs. (i) and (ii),

for circle $x^2 + y^2 = r^2$

Equation of tangent at (x_1, y_1) is

$xx_1 + yy_1 = r^2$

$\therefore$ Equation of tangent of Eq. (i) at $(-7, 1)$ and $(-7, -1)$ are

$$\begin{aligned} & -7x + y = 50 \text{ or } 7x - y + 50 = 0 \\ & \text{and } -7x - y = 50 \text{ or } 7x + y + 50 = 0 \end{aligned}$$

Question140

If the chord of contact of tangents from a point on the circle $x^2+y^2=r_1^2$ to the circle $x^2+y^2=r_2^2$ touches the circle $x^2+y^2=r_3^2$, then r_1, r_2 and r_3 are in

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Options:

A. AP

B. HP

C. GP

D. AGP

Answer: C

Solution:

(c) Let (x_1, y_1) be any point on the circle

$x^2+y^2=r_1^2$

Then, $x_1^2+y_1^2=r_1^2$ ..... (i)

Equation of chord of contact of tangents from (x_1, y_1) to the circle $x^2+y^2=r_2^2$ is

$x x_1+y y_1=r_2^2$ .... (ii)

$\because$ Eq. (ii) touches the circle

$x^2+y^2=r_3^2$

The length of perpendicular from centre $(0,0)$ on Eq. (ii) is radius $=r_3$

$$\Rightarrow \quad \frac{r_2^2}{\sqrt{x_1^2+y_1^2}}=r_3 \Rightarrow \frac{r_2^2}{\sqrt{r_1^2}}=r_3$$

$$\Rightarrow \quad r_2^2=r_1 r_3$$

$\Rightarrow \quad r_1, r_2, r_3$ are in GP.

Question141

Find the equation of the circle passing through $(1, -2)$ and touching the X-axis at $(3, 0)$.

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Options:

A. $x^2 + y^2 + 6x - 4y - 9 = 0$

B. $x^2 + y^2 - 6x - 4y + 9 = 0$

C. $x^2 + y^2 - 6x - 4y - 9 = 0$

D. $x^2 + y^2 - 6x + 4y + 9 = 0$

Answer: D

Solution:

Let (h, k) be centre and r be the radius

Then, $h = 3$

Equation of circle

$(x-3)^2 + (y-k)^2 = r^2$

It passes through $(3, 0)$ and $(1, -2)$

$$\Rightarrow k^2 = r^2$$
$$\text{and } 4 + k^2 + 4k + 4 = r^2$$

$$\Rightarrow \quad 8 + 4k = 0$$
$$\Rightarrow \quad k = -2$$

$\therefore$ Required equation of circle

$$(x-3)^2 + (y+2)^2 = 2^2$$
$$\text{or} \quad x^2 + y^2 - 6x + 4y + 9 = 0$$

Question142

Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle

$x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations represent L_1

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Options:

A. $x+y=0$ and $x+7y=0$

B. $x-y=0$ and $x+7y=0$

C. $x-7y=0$ and $x+y=0$

D. $x-7y=0$ and $x-y=0$

Answer: B

Solution:

Let $L_1 : y=mx$ (equation of line passes through origin)

Intercept made by L_1 and L_2 are equal $\Rightarrow L_1$ and L_2 are at the same distance from centre of circle $x^2+y^2-x+3y=0$

Now, centre $\left(\frac{1}{2}, \frac{-3}{2}\right)$

$[\because$ centre $=(-g,-f)]$

Distance of L_1 from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{\left|m \cdot \frac{1}{2}+\frac{3}{2}\right|}{\sqrt{m^2+1}}$

Distance of L_2 from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{\left|\frac{1}{2}-\frac{3}{2}-1\right|}{\sqrt{1+1}}$

$\because L_2 : x+y-1=0$

$[\because y=mx \Rightarrow mx-y=0]$

$\Rightarrow \quad \frac{\left|\frac{m+3}{2}\right|}{\sqrt{m^2+1}}=\frac{2}{\sqrt{2}}$

Squaring both sides, we get

$\frac{(m+3)^2}{4(m^2+1)}=\frac{4}{2}$

$\Rightarrow \quad m^2+6m+9=8\left(m^2+1\right)$

$\Rightarrow \quad 7m^2-6m-1=0$

$\Rightarrow \quad m=\frac{6 \pm \sqrt{36+28}}{14}=\frac{6 \pm 8}{14}$

$=1, \frac{-1}{7}$

$\therefore \quad L_1$ is $y=x$

and

$y=\frac{-1}{7} x$

or

$x-y=0$

or

$x+7y=0$

Question143

The radius of the circle whose center lies at $(1,2)$ while cutting the circle $x^2+y^2+4x+16y-30=0$ orthogonally, is units.

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Options:

A. $\sqrt{41}$

B. $\sqrt{31}$

C. $\sqrt{21}$

D. $\sqrt{11}$

Answer: D

Solution:

Given, equation of circle

$x^2+y^2+4x+16y-30=0$ ..... (i)

Comparing with $x^2+y^2+2g_1x+2f_1y+c_1=0$

We have, $g_1=2, f_1=8, c_1=-30$

Let equation of circle whose centre at $(1,2)$ is

$x^2+y^2-2x-4y+c=0$ ...... (ii)

Here, $g_2=-1, f_2=-2, c_2=c$

Circles Eqs. (i) and (ii) are orthogonal, then

$$\begin{aligned} \Rightarrow \quad & 2\left(g_1 g_2+f_1 f_2\right)=c_1+c_2 \\ \text{or} \quad & 2(-2-16)=-30+c \\ & c=-6 \end{aligned}$$

From Eqs. (ii), we get

$$\begin{aligned} & x^2+y^2-2x-4y-6=0 \\ \text{Radius } & =\sqrt{(-1)^2+(-2)^2-(-6)}=\sqrt{11} \end{aligned}$$

Question144

The point which has the same power with respect to each of the circles $x^2+y^2-8x+40=0, x^2+y^2-5x+16=0$ and $x^2+y^2-8x+16y+160=0$ is

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Options:

A. $\left(-8, \frac{-15}{2}\right)$

B. $\left(8, \frac{-15}{2}\right)$

C. $\left(8, \frac{15}{2}\right)$

D. $\left(-8, \frac{15}{2}\right)$

Answer: B

Solution:

$$\begin{aligned} \text{Let } S_1: x^2+y^2-8x+40=0 \\ S_2: x^2+y^2-5x+16=0 \\ S_3: x^2+y^2-8x+16y+160=0 \end{aligned}$$

The point which has same power is obtained by solving

$S_2 - S_1 = 0$

and $S_3 - S_1 = 0$

$\Rightarrow \quad 3x - 24 = 0$

and $\quad 16y + 120 = 0$

$\Rightarrow \quad x = 8$ and $y = \frac{-15}{2} \Rightarrow 2at = 2$

$\therefore$ Required point is $\left(8, \frac{-15}{2}\right)$.
