Parabola

Question1

If $x-y-3=0$ is a normal drawn through the point $(5,2)$ to the parabola $y^2=4x$, then the slope of the other normal that can be drawn through the same point to the parabola $y^2=4x$ is

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Options:

A.

0

B.

-1

C.

2

D.

-2

Answer: D

Solution:

$\because y^2=4x$

So, $\quad a=1$

$\therefore$ The equation of the normal

$$y=mx-2am-am^3$$
$$\Rightarrow \quad y=mx-2m-m^3$$

which passes through $(5, 2)$

$\therefore \quad 2 = 5m - 2m - m^3$

$\Rightarrow \quad m^3 - 3m + 2 = 0$

and the given normal is

$y = x - 3$

$\therefore \quad m = 1$, put the value

$(1)^3 - 3 \times 1 + 2 = 0$

Thus, $m = 1$ is a root.

$\therefore m^3 - 3m + 2 = (m-1)^2(m+2) = 0$

So, there are two normals with slopes 1 and -2

$\because$ The given normal has slope 1

So, the other normal has slope $= -2$

Question2

If the normal chord drawn at the point $\left(\frac{15}{2}, \frac{15}{\sqrt{2}}\right)$ to the parabola $y^2 = 15x$ subtends an angle θ at the vertex of the parabola, then $\sin\frac{\theta}{3} + \cos\frac{2\theta}{3} - \sec\frac{4\theta}{3} =$

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Options:

A.

0

B.

3

C.

1

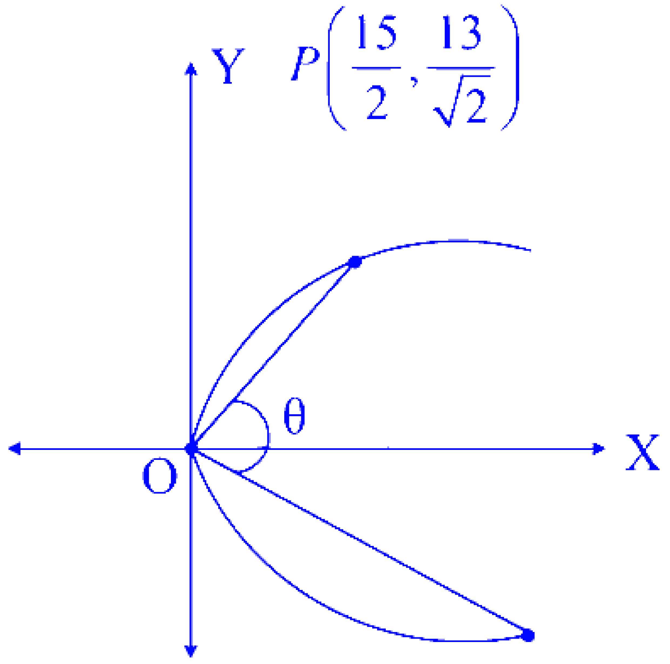
D.

Answer: B

Solution:

$y^2 = 15x$

Equation of normal at $\left(\frac{15}{2}, \frac{15}{\sqrt{2}}\right)$



$$2y\frac{dy}{dx} = 15$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{15}{2y}$$

$$\Rightarrow \quad m_N = \frac{-1}{\frac{dy}{dx}} = \frac{-2y}{15} \quad \left(\because y = \frac{15}{\sqrt{2}}\right)$$

$$\Rightarrow \quad m_N = \frac{-2 \times 15/\sqrt{2}}{15} = -\sqrt{2}$$

Equation of normal

$$y - \frac{15}{\sqrt{2}} = -\sqrt{2}\left(x - \frac{15}{2}\right)$$

$$\Rightarrow \sqrt{2}y - 15 = -2\left(x - \frac{15}{2}\right)$$

$$\Rightarrow \sqrt{2}y - 15 = -2x + 15$$

$$\Rightarrow 2x + \sqrt{2}y = 30$$

$$\Rightarrow \frac{2x + \sqrt{2}y}{30} = 1$$

Homogenisation of $y^2 = 15x$ w.r.t L

$$y^2 = 15x\left(\frac{2x+\sqrt{2}y}{30}\right)$$

$$\Rightarrow \quad 2y^2 = 2x^2 + 2\sqrt{2}xy$$

$$\Rightarrow \quad 2x^2 + 2\sqrt{2}xy - 2y^2 = 0$$

$$\because \quad a + b = 0$$

$$\therefore \quad \theta = \pi/2$$

Now, $\sin\frac{\theta}{3} + \cos\frac{2\theta}{3} - \sec\frac{4\theta}{3}$

$$= \sin\frac{\pi}{6} + \cos\frac{\pi}{3} - \sec\left(\frac{2\pi}{3}\right)$$

$$= 1/2 + 1/2 + 2 = 3$$

Question3

Tangents are drawn at three points $P(t_1), Q(t_2), R(t_3)$ on the parabola $y^2 = x$. Let these tangents intersect each other at the points L, M, N. If $t_1 = 2, t_2 = -4, t_3 = 6$, then the area of the $\triangle LMN$ is

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Options:

A.

24

B.

18.5

C.

7.5

D.

12

Answer: C

Solution:

Given, parabola is $y^2 = x$ and $t_1 = 2t_2 = -4$ and $t_3 = 6$.

So, intersection of tangents at t_1 and t_2, then tangents are $2y = x + 1$ and $-4y = x + 4$

Solving these two equations, we get $y = \left(\frac{-1}{2}\right)$ and $x = -2$

So, point $L = \left(-2, -\frac{1}{2}\right)$

Now, intersection of tangents at $t_2 = -4$ and $t_3 = 6$, then tangents are

$-4y = x + 4, 6y = x + 9$

Solving these two,

we get $x = -6, y = \frac{1}{2}$

So, print $M = \left(-6, \frac{1}{2}\right)$

Now, intersection of tangents at $t_1 = 2$ and $t_3 = 6$, so tangents are $2y = x + 1$ and $6y = x + 9$

Solving these two equation, we get $x = 3, y = 2$

So, point $N = (3, 2)$

Area of triangle with vertices $L\left(-2, -\frac{1}{2}\right), M\left(-6, \frac{1}{2}\right)$ and $N(3, 2)$ is

$$\text{Area } = \left|\frac{1}{2}\right| \begin{matrix} -2 & -\frac{1}{2} & 1 \\ -6 & \frac{1}{2} & 1|| \\ 3 & 2 & 1 \end{matrix}$$

$$= \left|\frac{-15}{2}\right| = 7.5$$

Question4

If the tangents of the parabola $y^2 = 8x$ passing through the point $P(1, 3)$ touches the parabola at A and B, then the area (in sq. units) of $\triangle PAB$ is

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Options:

A.

1

B.

$\frac{3}{4}$

C.

$\frac{1}{2}$

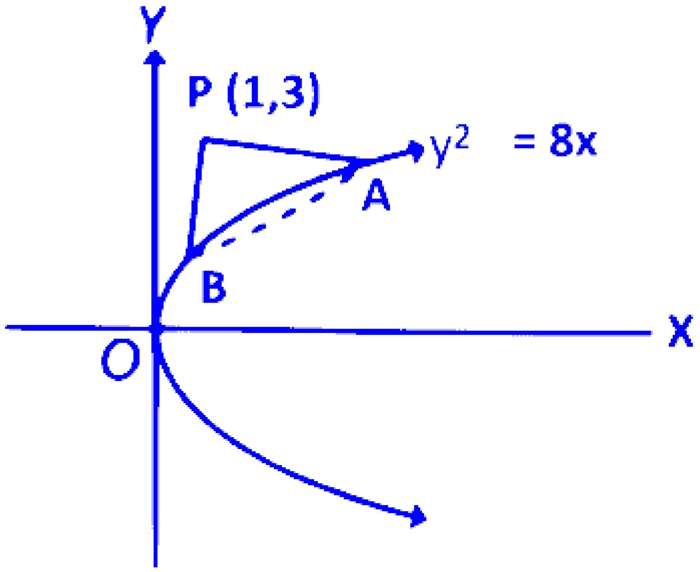
D.

$\frac{1}{4}$

Answer: D

Solution:

$y^2=8x, a=2$



Equation of tangent

$y=mx+\frac{2}{m}$

at (1,3)

$$3=m+\frac{2}{m} \Rightarrow m^2-3m+2=0$$
$$\Rightarrow \quad m=1,2$$

So, equation of tangent (put point $(1,3)$ on $y=mc+c$)

$$y=x+c, y=2x+c$$
$$\Rightarrow \quad y=x+2y=2x+1$$

to find coordinates of A and B, put the line on parabola.

$$(x+2)^2=8x \Rightarrow x^2-4x+4=0$$
$$\Rightarrow \quad x=2,-2$$
$$\text{at} \quad x=2, y=\sqrt{8\times 2}=4$$

So, $A=(2,4)$

and $(2x+1)^2=8x$

$\Rightarrow 4x^2-4x+1=0$

$\Rightarrow x=\frac{1}{2},-\frac{1}{2}$

at $x=\frac{1}{2}, y=2$

So, $B=\left(\frac{1}{2},2\right)$

Now, area of $\triangle PAB$

$$A=\frac{1}{2}\begin{vmatrix}1 & 3 & 1\\ 2 & 4 & 1\\ \frac{1}{2} & 2 & 1\end{vmatrix}$$

$$=\left|\frac{1}{2}\left(1(4-2)-3\left(2-\frac{1}{2}\right)+1(4-2)\right|$$

$$=\left|\frac{1}{2}\left(2-\frac{9}{2}+2\right)\right|=\left|\frac{1}{2}\left(4-\frac{9}{2}\right)\right|$$

$$=\left|\frac{1}{2}\left(-\frac{1}{2}\right)\right|=\frac{1}{4}$$

Question5

The lengths of the two focal chords of the parabola $y^2=16x$ is 25 units each. If these two chords cut the parabola at A, B, C and D, then the area (in sq. units) of the quadrilateral formed by A, B, C and D is

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Options:

A.

$\frac{625}{2}$

B.

180

C.

150

D.

300

Answer: D

Solution:

Given equation of parabola is

$y^2 = 16x \qquad \ldots(i)$

Comparing it with $y^2 = 4ax$ we get, $a = 4$

Let coordinates of end points of focal chord of a parabola (i) is, $\left(at^2, 2at\right)$ and $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

$$= \text{ its length } = \sqrt{\left(at^2 - \frac{a}{t^2}\right)^2 + \left(2at + \frac{2a}{t}\right)^2} \text{ (given)}$$
$$= 25$$
$$\Rightarrow \sqrt{\left(4\left(t^2 - \frac{1}{t^2}\right)\right)^2 + \left(8\left(t + \frac{1}{t}\right)\right)^2} = 25$$
$$\text{(putting } a = 4)$$
$$\Rightarrow \quad 16\left(t^2 - \frac{1}{t^2}\right)^2 + 64\left(t + \frac{1}{t}\right)^2 = 625$$
$$\Rightarrow 16\left(t^4 + \frac{1}{t^4} - 2\right) + 64\left(t^2 + \frac{1}{t^2} + 2\right) = 625 \qquad \ldots(ii)$$

$$\text{Let } t + \frac{1}{t} = u \text{ so that } \left(t + \frac{1}{t}\right)^2 = u^2$$
$$\Rightarrow \quad t^2 + \frac{1}{t^2} + 2 = u^2$$
$$\Rightarrow \quad t^2 + \frac{1}{t^2} = u^2 - 2$$
$$\Rightarrow \quad \left(t^2 + \frac{1}{t^2}\right)^2 = \left(u^2 - 2\right)^2$$
$$\Rightarrow \quad t^4 + \frac{1}{t^4} + 2 = u^4 + 4 - 2u^2$$

Putting these values in Eq. (ii) and solving then we get,

$t = 2, t = \frac{1}{2}, t = -2, t = \frac{-1}{2}$

So, Points are $A(16, 16), B(1, 4), C(1, -4), D(16, -16)$

$\therefore$ Area of quadrilateral $ABCD =$

$$\frac{1}{2}\left| x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1) \right|$$
$$= \frac{1}{2}\left| 16 \times 4 + 1 \times (-4) + 1 \times (-16) + 16 \times 16 - (16 \times 1 + 4 \times 1 + (-4) \times 16 + (-16)(16) \right|$$
$= 300$ sq. units

Question6

If the perpendicular distance from the focus of a parabola $y^2 = 4ax$ to its directrix is $\frac{3}{2}$, then the equation of the normal drawn at $(4a, -4a)$ is

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Options:

A.

$2x + y = 3$

B.

$2x - y = 9$

C.

$x - 2y = 9$

D.

$x + 2y + 3 = 0$

Answer: B

Solution:

$y^2 = 4ax$, focus $= (a, 0)$

Equation of directrix is $x = -a$.

The perpendicular distance from the focus $(a, 0)$ to the directrix $x + a = 0$

$$\Rightarrow \quad \frac{|a+a|}{\sqrt{1+0}} = |2a| = |2a|$$
$$\Rightarrow \quad |2a| = \frac{3}{2}$$
$$\Rightarrow \quad 2a = \frac{3}{2} \text{ or } 2a = \frac{-3}{2}$$
$$\Rightarrow \quad 2a = \frac{3}{2} \Rightarrow a = \frac{3}{4}$$
$$\Rightarrow \quad y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

Point $(4a, -4a)$

$$x_1 = 4a, y_1 = -4a$$
$$\Rightarrow \quad y - (-4a) = -\frac{(-4a)}{2a}(x - 4a)$$
$$\Rightarrow \quad y + 4a = -(-2)(x - 4a)$$
$$\Rightarrow \quad y + 4a = 2(x - 4a)$$
$$\Rightarrow \quad y + 4a = 2x - 8a$$
$$\Rightarrow \quad 2x - y - 12a = 0$$

Substituting $a = \frac{3}{4}$

$$\Rightarrow \quad 2x - y - 12\left(\frac{3}{4}\right) = 0$$
$$\Rightarrow \quad 2x - y - 9 = 0$$
$$2x - y = 9$$

Question7

PQ is a focal chord of the parabola $y^2 = 4x$ with focus S. If $P = (4, 4)$, then $SQ =$

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Options:

A.

2

B.

$\frac{5}{4}$

C.

5

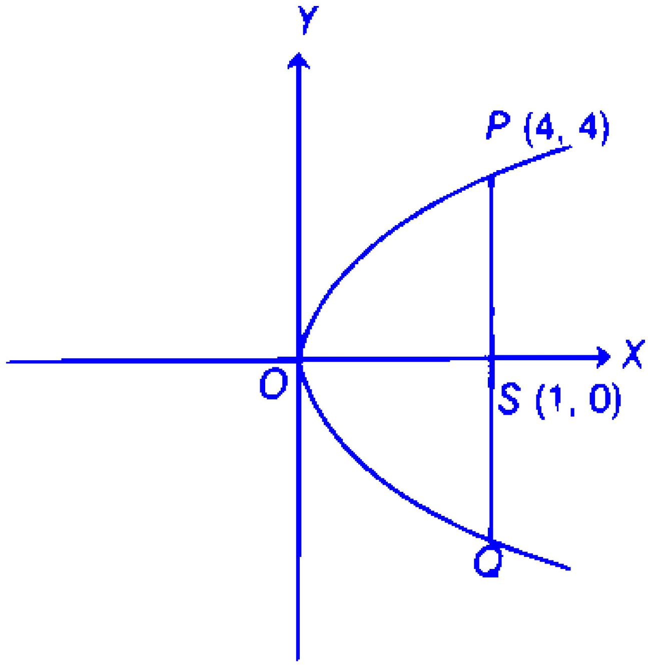
D.

$\frac{3}{2}$

Answer: B

Solution:

Given parabola $y^2 = 4x$



$S \equiv (1, 0), \quad a = 1$
Other end of focal chord Q

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a} \Rightarrow$$

$$\Rightarrow \quad \frac{1}{5} + \frac{1}{SQ} = \frac{4}{5} \Rightarrow SQ = \frac{5}{4}$$

Question8

The angle between the tangents drawn from the point $(1, 4)$ to the parabola $y^2 = 4x$ is

Options:

A.

$\frac{\pi}{6}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{3}$

D.

$\frac{\pi}{2}$

Answer: C

Solution:

$y^2 = 4x$

Equation of tangent to the parabola

$y^2 = 4ax$ is $y = mx + \frac{a}{m}$

$\because$ Equation of tangent is

$y = mx + \frac{1}{m}$

Given, tangents passes through $(1, 4)$

$$
\begin{aligned}
& 4 = m + \frac{1}{m} \\
\Rightarrow \quad & m^2 - 4m + 1 = 0 \\
\Rightarrow \quad & m_1 + m_2 = 4, \quad m_1 m_2 = 1
\end{aligned}
$$

$$
\begin{aligned}
\because \tan \alpha &= \frac{m_1 - m_2}{1 + m_1 m_2} \\
&= \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2} \\
&= \frac{\sqrt{16 - 4}}{1 + 1} = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3} \\
\therefore \quad \alpha &= \tan^{-1}\sqrt{3} = \frac{\pi}{3}
\end{aligned}
$$

Question9

If L is the normal drawn to the parabola $y^2 = 8x$ at the point $t = \frac{1}{\sqrt{2}}$, then the foot of the perpendicular drawn from the focus of the parabola on to the normal L is

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Options:

A.

$(3, 2)$

B.

$(5, \sqrt{2})$

C.

$(0, \sqrt{2})$

D.

$(3, \sqrt{2})$

Answer: D

Solution:

We have parabola, $y^2 = 8x$

Equation of normal at $\left(at^2, 2at\right)$ is

$y + xt = 2at + at^3$

But $t = \frac{1}{\sqrt{2}}$

$$y + x\frac{1}{\sqrt{2}} = 4\frac{1}{\sqrt{2}} + \frac{2}{2\sqrt{2}}$$
$$\sqrt{2}y + x = 5$$

It is equation of L.

Focus of parabola $y^2 = 8x$ is $(2, 0)$

Foot of perpendicular from focus to the line L is

$$\frac{x-2}{1}=\frac{y-0}{\sqrt{2}}=-\left(\frac{2-5}{3}\right)=1$$
$$x=3 \text{ and } y=\sqrt{2}$$

$\because$ Foot of perpendicular $=(3, \sqrt{2})$.

Question10

If P is a point which divides the line segment joining the focus of the parabola $y^2=12x$ and a point on the parabola in the ratio $1:2$. Then, the locus of p is

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Options:

A. $y^2=2(x-2)$

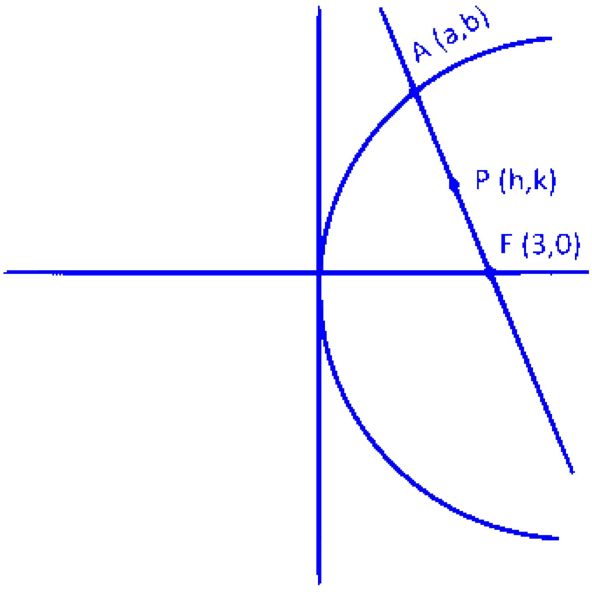
B. $y^2=4x$

C. $y^2=4(x-2)$

D. $y^2=9(x-3)$

Answer: C

Solution:



Let $A(a, b)$ be any point on the parabola and point $P(h, k)$ divides FA in ratio $1:2$

$$\Rightarrow \quad h=\frac{a+6}{3} \text{ and } k=\frac{b}{3}$$
$$\Rightarrow \quad a=3h-6 \text{ and } b=3k$$

$A(a, b)$ lies on parabola, so $b^2=12a$

$$9k^2=12(3h-6)$$
$$k^2=4(h-2)$$

Hence, locus of P is $y^2=4(x-2)$.

Question11

Equation of the line touching both parabolas $y^2=4x$ and $x^2=-32y$ is

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Options:

A. $x+2y+4=0$

B. $2x+y-4=0$

C. $x-2y-4=0$

D. $x - 2y + 4 = 0$

Answer: D

Solution:

To find the equation of the line that touches both parabolas $y^2 = 4x$ and $x^2 = -32y$, we start by considering the line in the form $y = mx + c$.

For the parabola $y^2 = 4x$, the condition for the line to be tangent is:

$c = \frac{1}{m}$

Thus, the line can be expressed as:

$y = mx + \frac{1}{m}$

For the parabola $x^2 = -32y$, the condition for tangency is:

$c = 8m^2$

Therefore, the line in this case is:

$y = mx + 8m^2$

By equating the expressions for c, we have:

$\frac{1}{m} = 8m^2$

Solving this equation for m, we get:

$m^3 = \frac{1}{8}$

$m = \frac{1}{2}$

Substituting $m = \frac{1}{2}$ back into the line equation, we obtain:

$y = \frac{x}{2} + 2$

Rewriting this equation in a standard form, we have:

$x - 2y + 4 = 0$

Thus, the equation of the tangent line is:

$x - 2y + 4 = 0$

Question12

If the normal chord drawn at $(2a, 2a\sqrt{2})$ on the parabola $y^2 = 4ax$ subtends an angle θ at its vertex, then $\theta =$

Options:

A. 45°

B. 90°

C. 135°

D. 60°

Answer: B

Solution:

The normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1) can be expressed as:

$y = \frac{-y_1}{2a}(x - x_1) + y_1$

For the point $(2a, 2a\sqrt{2})$, the equation becomes:

$$\begin{aligned} y &= \frac{-2a\sqrt{2}}{2a}(x - 2a) + 2a\sqrt{2} \\ &= -\sqrt{2}(x - 2a) + 2a\sqrt{2} \\ y &= -\sqrt{2}x + 4a\sqrt{2} \end{aligned}$$

Substituting this expression into $y^2 = 4ax$, we get:

$$\begin{aligned} (-\sqrt{2}x + 4a\sqrt{2})^2 &= 4ax \\ 2(x - 4a)^2 &= 4ax \\ x^2 + 16a^2 - 8ax &= 2ax \\ x^2 + 16a^2 - 10ax &= 0 \end{aligned}$$

This factors as:

$$\begin{aligned} x^2 - 8ax - 2ax + 16a^2 &= 0 \\ (x - 8a)(x - 2a) &= 0 \end{aligned}$$

Thus, $x = 8a$ or $x = 2a$.

Now, for $x = 8a$:

$$\begin{aligned} y &= -\sqrt{2} \times 8a + 4a\sqrt{2} \\ &= -4a\sqrt{2} \end{aligned}$$

For $x = 2a$:

$$y = -\sqrt{2} \cdot 2a + 4a\sqrt{2}$$
$$= 2a\sqrt{2}$$

The chord lies between points $(2a, 2a\sqrt{2})$ and $(8a, -4a\sqrt{2})$.

The slope m of this chord is:

$$m = \frac{-4a\sqrt{2} - 2a\sqrt{2}}{8a - 2a} = -\sqrt{2}$$

Considering the line from the vertex $(0, 0)$ to $(2a, 2a\sqrt{2})$, the slope is $\sqrt{2}$.

The angle θ between two lines with slopes m_1 and m_2 is calculated by:

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$
$$= \frac{\sqrt{2} + \sqrt{2}}{1 - \frac{2}{2}}$$
$$= \frac{2\sqrt{2}}{0}$$

Since $\tan\theta = \tan\frac{\pi}{2}$, this implies:

$$\theta = \frac{\pi}{2} = 90^\circ$$

Question13

If the ordinates of points P and Q on the parabola $y^2 = 12x$ are in the ratio $1 : 2$. Then, the locus of the point of intersection of the normals to the parabola at P and Q is

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Options:

A. $y + 18\left(\frac{x-6}{21}\right)^{\frac{3}{2}} = 0$

B. $y - 18\left(\frac{x-6}{12}\right)^{\frac{3}{2}} = 0$

C. $y + 12\left(\frac{x-6}{14}\right)^{\frac{1}{2}} = 0$

D. $y - 12\left(\frac{x-6}{18}\right)^{\frac{1}{2}} = 0$

Answer: A

Solution:

Given the equation of the parabola:

$y^2 = 12x$

By comparing this with the standard form $y^2 = 4ax$, we find:

$a = 3$

Consider the coordinates of the points P and Q on the parabola:

$P = (at_1^2, 2at_1) = (3t_1^2, 6t_1)$

$Q = (at_2^2, 2at_2) = (3t_2^2, 6t_2)$

Given that the ordinates of P and Q are in the ratio 1:2, we have:

$\frac{6t_2}{6t_1} = 2$

This simplifies to:

$t_2 = 2t_1$

Let $R(x, y)$ be the point of intersection of the normals at P and Q.

For a point t on the parabola $y^2 = 4ax$, the point of intersection of the normals at t_1 and t_2 is given by:

$\left(a(t_1^2 + t_2^2 + t_1t_2 + 2), -at_1t_2(t_1 + t_2)\right)$

Substituting the values, we have:

$x = 3(t_1^2 + t_2^2 + t_1t_2 + 2)$

Substitute $t_2 = 2t_1$:

$x = 3(t_1^2 + (2t_1)^2 + t_1 \cdot 2t_1 + 2)$

$x = 3(t_1^2 + 4t_1^2 + 2t_1^2 + 2) = 3(7t_1^2 + 2)$

$x = 21t_1^2 + 6$

From this, we can express:

$t_1^2 = \frac{x-6}{21}$

Now, calculate y:

$y = -3t_1 \cdot 2t_1(3t_1)$

$y = -18t_1^3$

Substituting $t_1^2 = \frac{x-6}{21}$:

$t_1 = \left(\frac{x-6}{21}\right)^{1/2}$

Therefore:

$y = -18\left(\frac{x-6}{21}\right)^{3/2}$

Hence, the locus of the point of intersection of the normals at P and Q is:

$y+18\left(\frac{x-6}{21}\right)^{3/2}=0$

Question14

A common tangent to the circle $x^2+y^2=9$ and parabola $y^2=8x$ is

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Options:

A. $3x-\sqrt{3y}+2=0$

B. $x-\sqrt{3}y+6=0$

C. $2x-\sqrt{3}y+3=0$

D. $x-3y+6=0$

Answer: B

Solution:

To find the common tangent to the circle $x^2+y^2=9$ and the parabola $y^2=8x$, we start with the equation of the tangent to the parabola. The general form for the tangent to $y^2=8x$ is given by:

$y=mx+\frac{2}{m}$

This tangent must also touch the circle $x^2+y^2=9$. The condition for this line to be tangent to the circle is:

$\left|\frac{2}{m\sqrt{1+m^2}}\right|=3$

Simplifying the equation, we get:

$\frac{4}{m^2(1+m^2)}=9$

Solving this, we arrive at the polynomial equation:

$9m^4+9m^2-4=0$

By factoring, we find:

$9m^4+12m^2-3m^2-4=0$

This can be factored further into:

$(3m^2+4)(3m^2-1)=0$

We obtain:

$m^2 = \frac{1}{3}$

Note that the solution $3m^2 + 4 = 0$ is rejected since it doesn't provide a valid, real number for m^2.

Thus, the tangent can be expressed as:

$y = \pm \frac{1}{\sqrt{3}} x \pm 2\sqrt{3}$

This translates to:

$\sqrt{3} y = \pm(x + 6)$

The resulting equations are:

$x \pm \sqrt{3} y + 6 = 0$

Thus, the common tangents are:

$x + \sqrt{3} y + 6 = 0 \quad \text{or} \quad x - \sqrt{3} y + 6 = 0$

Question15

The normal drawn at a point $(2, -4)$ on the parabola $y^2 \pm 8x$ cuts again the same parabola at (α, β), then $\alpha + \beta =$

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Options:

A. 8

B. 16

C. 24

D. 30

Answer: D

Solution:

To find where the normal at the point $(2, -4)$ on the parabola $y^2 = 8x$ intersects the same parabola again, we proceed as follows:

Equation of the Normal:

The equation of the normal to the parabola $y^2 = 8x$ at a point (x_1, y_1) is given by $y - y_1 = -\frac{2x_1}{y_1}(x - x_1)$.

Here, the point is $(2, -4)$, so $x_1 = 2$ and $y_1 = -4$.

Substituting into the normal equation:

$y + 4 = -\frac{4}{-4}(x - 2)$

$y + 4 = x - 2$

$x - y = 6$

Thus, $x = y + 6$.

Substitute Back into the Parabola's Equation:

Substituting $x = y + 6$ into the parabola equation $y^2 = 8x$:

$y^2 = 8(y + 6)$

$y^2 = 8y + 48$

$y^2 - 8y - 48 = 0$

Solve the Quadratic Equation:

Factor the quadratic:

$(y - 12)(y + 4) = 0$

The solutions are $y = 12$ and $y = -4$.

Find the Corresponding x Values:

For $y = 12$, $x = 12 + 6 = 18$.

We already know for $y = -4$, $x = 2$, which is the original point.

Conclusion:

The other intersection point is $(\alpha, \beta) = (18, 12)$.

Thus, $\alpha + \beta = 18 + 12 = 30$.

Question16

If the axes are rotated through an angle 45° about the origin in anticlockwise direction, then the transformed equation of $y^2 = 4ar$ is

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Options:

A. $(x + y)^2 = 4\sqrt{2}a(x - y)$

B. $(x-y)^2=4\sqrt{2}a(x+y)$

C. $(x-y)^2=\frac{43}{\sqrt{2}}(x-y)$

D. $(x+y)^2=\frac{4a}{\sqrt{2}}(x-y)$

Answer: A

Solution:

To solve the problem of transforming the equation $y^2=4ax$ when the coordinate axes are rotated by 45° anticlockwise about the origin, we start by expressing the new coordinates (x,y) in terms of the original coordinates (X,Y).

Given that the coordinates are rotated 45° anticlockwise, we can use the following transformation equations:

$x=X\cos 45^\circ-Y\sin 45^\circ=\frac{X}{\sqrt{2}}-\frac{Y}{\sqrt{2}}=\frac{X-Y}{\sqrt{2}}$

$y=X\sin 45^\circ+Y\cos 45^\circ=\frac{X}{\sqrt{2}}+\frac{Y}{\sqrt{2}}=\frac{X+Y}{\sqrt{2}}$

Substitute these expressions into the given equation $y^2=4ax$:

$\left(\frac{X+Y}{\sqrt{2}}\right)^2=4a\left(\frac{X-Y}{\sqrt{2}}\right)$

Simplifying the equation, we get:

$\frac{(X+Y)^2}{2}=4a\frac{(X-Y)}{\sqrt{2}}$

Multiply through by 2 to eliminate the denominator:

$(X+Y)^2=4\sqrt{2}a(X-Y)$

Thus, the transformed equation is:

$(x+y)^2=4\sqrt{2}a(x-y)$

This is the equation of the curve after rotating the axes by 45° anticlockwise.

Question17

The line $x-2y-3=0$ cuts the parabola $y^2=4\,\text{ar}$ at the points P and Q. If the focus of this parabola is $\left(\frac{1}{4},k\right)$. then $PQ=$

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Options:

A. $16a\sqrt{5}$

B. $8a\sqrt{5}$

C. $4a\sqrt{5}$

D. $2a\sqrt{5}$

Answer: A

Solution:

Focus of the parabola $y^2 = 42x$ is $(\alpha, 0)$

Here, $a = \frac{1}{4}$ and $K = 0$

So, equation of parabola is $y^2 = x$

As, the line $x = 2y + 3$ cuts the parabola

$$\begin{aligned} y^2 &= x \\ y^2 &= 2y + 3 \\ \Rightarrow y^2 - 2y - 3 &= 0 \\ y^2 - 3y + y - 3 &= 0 \\ y &= 3, -1 \end{aligned}$$

If $y = 3$, then $x = 9$ and

If $y = -1$, then $x = 1$

The two intersecting points

P and Q are (9.3) and $(1, -1)$

$$\begin{aligned} PQ &= \sqrt{(9-1)^2 + (3+1)^2} \\ &= \sqrt{64+16} = \sqrt{80} \\ &= 4\sqrt{5} \\ &= 16a\sqrt{5} \quad \left[\because a = \frac{1}{4}\right] \end{aligned}$$

Question18

Which of the following represents a parabola?

Options:

A. $x=4 \cos t, y=4 \sin t$

B. $x^2-2=-2 \cos t, y=\cos ^2\left(\frac{t}{2}\right)$

C. $\sqrt{x}=\tan t, \sqrt{y}=\sec t$

D. $x=\sqrt{1-\sin t}, y=\sin \left(\frac{t}{2}\right)+\cos \left(\frac{t}{2}\right)$

Answer: B

Solution:

From option (a).

$$x=4 \cos t, y=4 \sin t$$
$$x^2=16 \cos ^2 t, y^2=16 \sin ^2 t$$

So, $x^2+y^2=16$, so this equation of a circle.

From option (b),

$$x^2-2=-2 \cos t, y=\cos ^2\left(\frac{t}{2}\right)$$

$$x^2=2-2 \cos t \quad \text{... (i)}$$
$$y=\frac{1+\cos t}{2} \quad \text{... (ii)}$$

From Eq. (ii),

$$\cos t=2 y-1$$
$$\text{Now, } x^2=2-2 \cos t$$
$$x^2=2-2(2 y-1) \Rightarrow x^2=2-4 y+2$$
$$x^2=4-4 y$$

So, this is represent the parabola.

Question19

Suppose a parabola passes through $(0,4),(1,9)$ and $(4,5)$ and has its axis parallel to the Y-axis. Then, the equation of the parabola is

Options:

A. $19x^2 + 12y - 79x - 48 = 0$

B. $19x^2 + 12y - 79x + 48 = 0$

C. $19y^2 + 12x - 79y - 48 = 0$

D. $19y^2 + 12x - 79y + 48 = 0$

Answer: A

Solution:

Assume the vertex of the parabola is (h, k) and the length of its latus rectum is $4a$. Since its axis is parallel to the Y-axis, the equation of the parabola can be written as:

$(x - h)^2 = 4a(y - k)$ .... (i)

The parabola passes through the points $(0, 4), (1, 9)$, and $(4, 5)$. Substituting these points into equation (i), we get:

$\therefore (0 - h)^2 = 4a(4 - k)$

$\Rightarrow h^2 = 4a(4 - k)$ ... (ii)

$(1 - h)^2 = 4a(9 - k)$

$\Rightarrow 1 - 2h + h^2 = 4a(9 - k)$ ... (iii)

$(4 - h)^2 = 4a(5 - k)$

$\Rightarrow 16 - 8h + h^2 = 4a(5 - k)$ ... (iv)

Next, we subtract Eqs. (ii) and (iii), and Eqs. (iii) and (iv), giving us:

$1 - 2h = 20a$ ... (v)

$15 - 6h = -16a$ ... (vi)

By solving Eqs. (v) and (vi), we obtain $a = -\frac{3}{19}$ and $h = \frac{79}{38}$.

Substituting these values back, we find $k = \frac{9889}{912}$.

Therefore, the equation of the parabola becomes:

$$\left(x-\frac{79}{38}\right)^2=\frac{-12}{19}(y-\frac{9889}{912})$$

$\Rightarrow 19x^2+12y-79x-48=0$

Question20

Suppose a parabola with focus at $(0,0)$ has $x-y+1=0$ as its tangent at the vertex. Then, the equation of its directrix is

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Options:

A. $x-y+2=0$

B. $x-y-2=0$

C. $x-y+3=0$

D. $x-y+4=0$

Answer: A

Solution:

Focus $\equiv(0,0)$

Tangent at vertex of parabola : $x-y+1=0$

The distance between the focus and tangent at the

vertex $=\left|\frac{0-0+1}{\sqrt{1^2+1^2}}\right|=\frac{1}{\sqrt{2}}$

The directrix is the line parallel to the tangent at vertex and at a distance $2\times\frac{1}{\sqrt{2}}$ from focus.

Let the equation of directrix be

$x-y+\lambda=0$

Here, $\frac{\lambda}{\sqrt{1^2+1^2}}=\frac{2}{\sqrt{2}}$

$\Rightarrow \quad \lambda=2$

$\therefore$ Equation of directrix is $x-y+2=0$

Question21

If $ax + by = 1$ is a normal to the parabola $y^2 = 4px$, then the condition is

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Options:

A. $4ab = a^2 + b^2$

B. $4pab + ab^3 = a^2b^2$

C. $pa^3 = b^2 - 2pab^2$

D. $pa^2 + 4pa = a + b$

Answer: C

Solution:

Parabola : $y^2 = 4px$ ... (i)
Normal : $ax + by = 1$ ... (ii)

$$\Rightarrow \quad y = -\frac{a}{b}x + \frac{1}{b}$$

Slope, $m = -\frac{a}{b}$ and constant, $c = \frac{1}{b}$

Condition of normal says that

$$c = -2m\left(\frac{\text{Coefficient of } x}{4}\right) - \left(\frac{\text{Coefficient of } x}{4}\right)m^3$$

$$\Rightarrow \frac{1}{b} = -2\left(\frac{-a}{b}\right)\left(\frac{4p}{4}\right) - \left(\frac{4p}{4}\right)\left(\frac{-a}{b}\right)^3$$

$$\Rightarrow \frac{1}{b} = \frac{2pa}{b} + \frac{pa^3}{b^3}$$

$$\Rightarrow b^2 = 2pab^2 + pa^3$$

$$\Rightarrow pa^3 = b^2 - 2pab^2$$

Question22

The point of intersection of the latus rectum and axis of the parabola $y^2 + 4x + 2y - 8 = 0$ is

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Options:

A. $\left(\frac{9}{4}, -1\right)$

B. $\left(\frac{5}{4}, -1\right)$

C. $\left(\frac{7}{2}, \frac{5}{2}\right)$

D. $\left(\frac{-5}{4}, 1\right)$

Answer: B

Solution:

Given parabola,

$y^2 + 4x + 2y - 8 = 0$ ..... (i)

Point of intersection of latusrectum and axis of parabola is foci.

Now, from Eq. (i), we get

$$\begin{aligned} & (y+1)^2 - 1 + 4x - 8 = 0 \\ \Rightarrow \quad & (y+1)^2 = -4x + 9 \\ \Rightarrow \quad & (y+1)^2 = -4\left(x - \frac{9}{4}\right) \end{aligned}$$

Compare with general standard form of parabola

$(y-k)^2 = -4a(x-h)$

where, foci $= (-a, 0)$

Here, $4a = +4$

$\Rightarrow \quad a = +1$

Then, $\left(x - \frac{9}{4}\right) = -1 \Rightarrow x = -1 + \frac{9}{4} = \frac{5}{4}$

and $(y+1) = 0 \Rightarrow y = -1$

$\therefore$ Focus $= \left(\frac{5}{4}, -1\right) =$ Point of intersection

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Question23

The coordinates of the focus of the parabola described parametrically by $x = 5t^2 + 2$ and $y = 10t + 4$ (where t is a parameter) are

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Options:

A. (7, 4)

B. (3, 4)

C. (3, −4)

D. (−7, 4)

Answer: A

Solution:

$x = 5t^2 + 2$ .... (i)

$y = 10t + 4 \Rightarrow t = \frac{y-4}{10}$

From Eq. (i), $(x - 2) = 5\left(\frac{y-4}{10}\right)^2$

$$\begin{aligned} &\Rightarrow \quad (y-4)^2 = 20(x-2) \quad \begin{cases} x - 2 = 5 \\ y - 4 = 0 \end{cases} \\ &\Rightarrow \quad (7, 4) \text{ focus} \end{aligned}$$

Question24

Find the equation of the parabola whichpasses through (6, −2), has its vertex at theorigin and its axis along the Y-axis.

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Options:

A. $y^2 = 18x$

B. $x^2 = 18y$

C. $y^2 = -18x$

D. $x^2 = -18y$

Answer: D

Solution:

Equation of parabola whose vertex is at origin and axis is Y-axis is

$x^2 = 4ay$

This equation passes through $(6, -2)$

$$\therefore \quad 6^2 = 4a(-2)$$
$$\Rightarrow \quad \frac{-36}{8} = a \Rightarrow a = \frac{-9}{2}$$

$\therefore$ Required parabola $x^2 = -18y$

Question25

If one end of focal chord of the parabola $y^2 = 8x$ is $\left(\frac{1}{2}, 2\right)$, then the length of the focal chord is .... units.

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Options:

A. $\frac{625}{4}$

B. $\frac{5}{\sqrt{2}}$

C. $\frac{25}{2}$

D. 25

Answer: C

Solution:

Given parabola, $y^2 = 8x$

Comparing with $y^2 = 4ax$

We have, $a = 2$

One end of focal chord $\left(\frac{1}{2}, 2\right)$

Parametric form of ends of focal chord $= \left(at_1^2, 2at\right)$

$$\Rightarrow 2at = 2$$
$$\therefore 2 \cdot 2 \cdot t = 2 \Rightarrow t = \frac{1}{2}$$

$$\begin{aligned}\text{Length of focal chord } &= a\left(t + \frac{1}{t}\right)^2 \\ &= 2\left(\frac{1}{2} + 2\right)^2 = 2 \cdot \left(\frac{5}{2}\right)^2 = \frac{25}{2}\end{aligned}$$
