Hyperbola

Question1

If θ is the angle subtended by a latus rectum at the centre of the hyperbola having eccentricity $\frac{2}{\sqrt{7}-\sqrt{3}}$, then $\sin\theta =$

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Options:

A.

$\frac{1}{2}\tan\frac{\theta}{2}$

B.

$2\cos\frac{\theta}{2}$

C.

$\frac{1}{\sin\frac{\theta}{2}+\cos\frac{\theta}{2}}$

D.

$1-\cos\frac{\theta}{2}$

Answer: A

Solution:

The latus rectum of a hyperbola is a line segment that is perpendicular to the main axis (transverse axis) and passes through a focus.

The angle θ is the angle at the center of the hyperbola made by one latus rectum.

For a hyperbola, the angle θ subtended at the center by the latus rectum always follows this rule:

$\sin\theta = \frac{1}{2}\tan\left(\frac{\theta}{2}\right)$

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Question2

The tangent drawn at an extremity (in the first quadrant) of latus rectum of the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$ meets the X-axis and Y-axis at A and B respectively. If O is the origin, then $(OA)^2-(OB)^2=$

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Options:

A.

$-\frac{20}{9}$

B.

$\frac{16}{9}$

C.

$-\frac{4}{9}$

D.

$-\frac{4}{3}$

Answer: A

Solution:

$\frac{x^2}{4}-\frac{y^2}{5}=1$

So, $a=2, b=\sqrt{5}$ and $e=\sqrt{1+\frac{5}{4}}=3/2$

foci are $(\pm ae, 0)=\left(\pm 2\times\frac{3}{2}, 0\right)=(\pm 3.0)$

The latus rectum for the focus in the positive x-direction is at $x=ac=3$

$\therefore \quad \frac{3^2}{4}-\frac{y^2}{5}=1 \Rightarrow y=\pm 5/2$

in first quadrant, the extremity is ($3, 5/2$) and the equation of tangental ($3, 5/2$) is

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$$\frac{x \cdot 3}{4} - \frac{y \cdot \frac{5}{2}}{5} = 1 \Rightarrow \frac{3x}{4} - \frac{y}{2} = 1$$
$\Rightarrow 3x - 2y = 4$

the tangent meet X-axis at $y = 0$

$\Rightarrow 3x = 4 \Rightarrow x = 4/3$

$\therefore (OA)^2 = \left(\frac{4}{3} - 0\right)^2 + (0-0)^2 = 16/9$

and when $x = 0 \Rightarrow -2y = 4 \Rightarrow y = -3$

So, $OB^2 = (0-0)^2 + (0+2)^2 = 4$

Hence, $(OA)^2 - (OB)^2 = \frac{16}{9} - 4 = -20/9$

Question3

Let $P(a \sec \theta, b \tan \theta)$ and $Q(a \sec \phi, b \tan \phi)$, where $\theta + \phi = \frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ If (h, k) is the point of intersection of the normals drawn at P and Q then $K =$

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Options:

A.

$\frac{a^2+b^2}{a}$

B.

$-\left(\frac{a^2+b^2}{b}\right)$

C.

$-\left(\frac{a^2+b^2}{a}\right)$

D.

$\frac{a^2+b^2}{b}$

Answer: B

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Solution:

$P(a\sec\theta, b\tan\theta)$ and

$Q(a\sec\phi, b\tan\phi)$

$\because \theta+\phi=\pi/2$

Equation of normal at $(a\sec\theta, b\tan\theta)$ to hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ is}$$
$$ax\cos\theta+by\cot\theta=a^2+b^2 \quad \ldots(i)$$
$$ax\cos\phi+by\cot\phi=a^2+b^2$$
$$\Rightarrow \phi=\frac{\pi}{2}-\theta$$
$$\therefore ax\sin\theta+by\tan\theta=a^2+b^2 \quad \ldots(ii)$$

Multiply Eq. (i) by $\sin\theta$ and Eq. (ii) $\cos\theta$ and subtract

$$\text{by } \cot\theta\cdot\sin\theta-\text{ by } \tan\theta\cos\theta$$
$$=\left(a^2+b^2\right)(\sin\theta-\cos\theta)$$
$$\text{by }(\cos\theta-\sin\theta)=-\left(a^2+b^2\right)$$
$$\Rightarrow \quad y=-\frac{a^2+b^2}{b}$$

Question4

If the angle between the asymptotes of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\tan^{-1}\left(\frac{2}{3}\right)$ and $a^2-b^2=45$, then $ab=$

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Options:

A.

20

B.

24

C.

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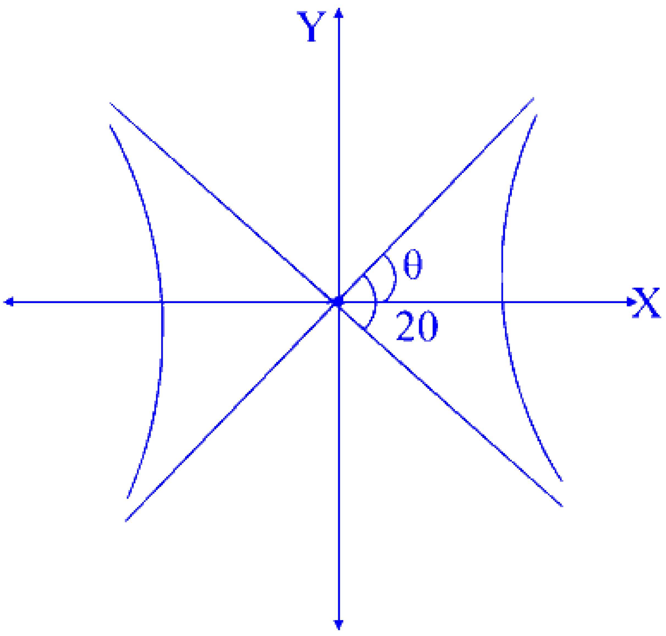
45

D.

54

Answer: D

Solution:



$$\because \quad 2\theta = 2\tan^{-1} 2/3$$
$$\Rightarrow \quad \theta = \tan^{-1} 2/3$$
$$\Rightarrow \tan\theta = 2/3$$

$$\because \quad e = \sec\theta$$
$$\Rightarrow \quad e^2 = \sec^2\theta = 1 + \frac{4}{9} = \frac{13}{9}$$
$$\therefore \quad e^2 = \frac{13}{9}$$
$$\Rightarrow \quad 1 + \frac{b^2}{a^2} = \frac{13}{9} \Rightarrow \frac{b^2}{a^2} = \frac{4}{9}$$
$$\Rightarrow \quad \frac{b}{a} = \frac{2}{3} \qquad \ldots(i)$$
$$\Rightarrow \quad a^2 - b^2 = 45 \Rightarrow a^2 - \frac{4a^2}{9} = 45$$
$$\Rightarrow \quad 5a^2 = 9 \times 45$$
$$\Rightarrow \quad a^2 = 81$$
$$\Rightarrow \quad a = 9 \Rightarrow b = \frac{2}{3} \times 9 = 6$$
$$\therefore \quad ab = 9 \times 6 = 54$$

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Question5

If $3\sqrt{2}x - 4y = 12$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{5}{4}$ is its eccentricity, then $a^2 - b^2 =$

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Options:

A.

5

B.

7

C.

9

D.

11

Answer: B

Solution:

Given, equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Now, the equation of the tangent to be hyperbola is

$3\sqrt{2}x - 4y = 12$

$\Rightarrow \quad y = \frac{3\sqrt{2}}{4}x - 3$

So, slope of the tangent, $m_t = \frac{3\sqrt{2}}{4}$ and y-intercept is $c = -3$

Now, the line $y = mx + c$ tangent to the hyperbola is given by

$$\begin{aligned} c^2 &= a^2m^2 - b^2 \\ \Rightarrow \quad (-3)^2 &= a^2\left(\frac{3\sqrt{2}}{4}\right)^2 - b^2 \\ \Rightarrow \quad 9 &= \frac{9}{8}a^2 - b^2 \\ \Rightarrow \quad 9a^2 - 8b^2 &= 72 \qquad \ldots(i) \end{aligned}$$

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Given, eccentricity, $e = \frac{5}{4}$

$$\begin{aligned} So,\ b^2 &= a^2\left(e^2-1\right) \\ &= a^2\left(\left(\frac{5}{4}\right)^2-1\right) \\ &= a^2\left(\frac{25}{16}-1\right)=\frac{9}{16}a^2 \end{aligned}$$

$$\Rightarrow \quad 16b^2 = 9a^2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$b^2 = 9 \text{ and } a^2 = 16$$

$$\therefore a^2 - b^2 = 16 - 9 = 7$$

Question6

If the normal drawn to the hyperbola $xy = 16$ at $(8, 2)$ meets the hyperbola again at a point (α, β), then $|\beta| + \frac{1}{|\alpha|} =$

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Options:

A.

40

B.

34

C.

28

D.

54

Answer: B

Solution:

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Given hyperbola equation is $xy = 16$

Differentiate it w.r.t x, we get

$$y + x \cdot \frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$$

Now, slope of tangent at $(8, 2)$ is

$m_1 = \frac{-2}{8} = \frac{-1}{4}$

And slope of normal is

$m_n = \frac{-1}{-1/4} = 4$

Now, equation of normal at $(8, 2)$ is

$$y - 2 = 4(x - 8)$$
$$\Rightarrow \quad y = 4x - 30$$

Normal intersects the hyperbola again at a point (α, β),

So, $\alpha\beta = 16$ and $\beta = 4\alpha - 30$

Solving these two equations, we get

$$\alpha(4\alpha - 30) = 16$$
$$\Rightarrow \quad 2\alpha^2 - 15\alpha - 8 = 0$$

$$So, \alpha = \frac{-(-15) \pm \sqrt{(-15)^2 - 4 \cdot 2 \cdot (-8)}}{2 \cdot 2}$$
$$= \frac{15 \pm \sqrt{289}}{4} = \frac{15 \pm 17}{4}$$

So, $\alpha_1 = \frac{15+17}{4} = \frac{32}{4} = 8$ and

$\alpha_2 = \frac{15-17}{4} = \frac{-1}{2}$

Since $(8, 2)$ is one point, the other point is where $\alpha = \frac{-1}{2}$

$$\text{So, } \beta = 4\alpha - 30$$
$$= 4\left(\frac{-1}{2}\right) - 30 = -32$$
$$\therefore |\beta| + \frac{1}{|\alpha|} = |-32| + \frac{1}{|-1/2|}$$
$$= 32 + 2 = 34$$

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Question7

If $3x + 2\sqrt{2}y + k = 0$ is a normal to the hyperbola $4x^2 - 9y^2 - 36 = 0$ making positive intercepts on both the axes, then $k =$

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Options:

A.

$13\sqrt{2}$

B.

$-5\sqrt{2}$

C.

$-2\sqrt{2}$

D.

$-13\sqrt{2}$

Answer: D

Solution:

$4x^2 - 9y^2 = 36 \Rightarrow \frac{x^2}{9} - \frac{y^2}{4} = 0$

So, $a^2 = 9, b^2 = 4$

Thus, equation of normal

$\frac{9x}{x_1} + \frac{4y}{y_1} = (9 + 4) = 13 \quad \ldots (i)$

and the given equation is

$$\begin{aligned} & 3x + 2\sqrt{2}y + k = 0 \\ \Rightarrow \quad & 3x + 2\sqrt{2}y = -k \quad \ldots (ii) \end{aligned}$$

$\because$ Both equation are proportional

Thus, $\frac{\frac{9}{x_1}}{3} = \frac{\frac{4}{y_1}}{2\sqrt{2}} = \frac{13}{-k}$

$\Rightarrow \quad \frac{3}{x_1} = \frac{\sqrt{2}}{y_1} \Rightarrow x_1 = \frac{3}{\sqrt{2}} y_1$

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Put in equation of hyperbola

$\frac{\left(\frac{3}{\sqrt{2}}y_1\right)^2}{9} - \frac{y^2}{4} = 1 \Rightarrow y_1^2 = 4 \Rightarrow y_1 = \pm 2$

at $y_1 = 2 \Rightarrow x_1 = 3\sqrt{2}$ and at $y_1 = -2$

$\Rightarrow x_1 = -3\sqrt{2}$

Also, $\frac{3}{x_1} = -\frac{13}{k} \Rightarrow -k = \frac{13}{3} \times 3\sqrt{2} = 13\sqrt{2}$

Hence, $k = -13\sqrt{2}$

Question8

If a hyperbola has asymptotes $3x - 4y - 1 = 0$ and $4x - 3y - 6 = 0$, then the transverse and conjugate axes of that hyperbola are

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Options:

A.

$x + y - 5 = 0, x - y - 1 = 0$

B.

$4x - 3y = 0, 3x + 4y = 0$

C.

$3x - 4y = 0, 4x + 3y = 0$

D.

$x + 2y - 1 = 0, 2x - y + 1 = 0$

Answer: A

Solution:

Asymptotes are

$$3x - 4y - 1 = 0 \qquad \ldots(i)$$
$$4x - 3y - 6 = 0 \qquad \ldots(ii)$$

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By Eqs. (i) and (ii), we get

$x = 3, y = 2$

Now, the direction vectors of asymptotes for $3x - 4y = 1 \Rightarrow \mathrm{d}_1 = \langle 4, 3\rangle$ and for $4x - 3y = 6 \Rightarrow \mathbf{d}_2 = \langle 3, 4\rangle$ Thus, transverse and conjugate axis directions.

$$\frac{L_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{L_2}{\sqrt{a_2^2+b_2^2}}$$

$\Rightarrow \quad \frac{3x-4y-1}{\sqrt{3^2+(-4)^2}} = \pm\frac{4x-3y-6}{\sqrt{4^2+(-3)^2}}$

$\Rightarrow \quad 3x - 4y - 1 = \pm 4x - 3y - 6$

So, $\quad 3x - 4y - 1 = 4x - 3y - 6$

or $\quad 3x - 4y - 1 = -4x + 3y + 6$

$\Rightarrow \quad x + y - 5 = 0$

or $\quad 7x - 7y - 7 = 0$

$\Rightarrow \quad x + y - 5 = 0$ or $x - y - 1 = 0$

Hence, transverse and conjugate axis are

$x + y = 5, x - y = 1$

Question9

$x + y + 3 = 0, 2x - y + 1 = 0$ are the equations of the asymptotes of a hyperbola.

If $(1, -2)$ is a point on this hyperbola, then the equation of its conjugate hyperbola is

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Options:

A.

$2x^2 + xy - y^2 + 7x - 2y - 1 = 0$

B.

$2x^2 + xy - y^2 + 7x - 2y + 13 = 0$

C.

$2x^2 + xy + y^2 - 7x - 2y - 1 = 0$

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D.

$2x^2 + xy + y^2 - 7x - 2y + 13 = 0$

Answer: B

Solution:

Given equation of asymptotes

$x + y = -3 \qquad \ldots (i)$

and $\quad 2x - y = -1 \qquad \ldots (ii)$

On solving Eqs. (i) and (ii), we get centre of hyperbola i.e. $x = \frac{-4}{3}, y = \frac{-5}{3}$

i.e. Centre $\equiv \left(\frac{-4}{3}, \frac{-5}{3}\right)$

Also combined equation of asymptotes is $(x + y + 3)(2x - y + 1) = 0$

$\Rightarrow \quad 2x^2 - y^2 + xy + 7x - 2y + 3 = 0$

Now, equation of hyperbola is (when equation of asymptotes are known) is

$(x + y + 3)(2x - y + 1) + c = 0 \qquad \ldots (iii)$

where c is constant.

$\because$ (iii) passes through $(1, -2)$.

So, $(1 - 2 + 3)(2 + 2 + 1) + c = 0$

$\Rightarrow \quad (2 \times 5) + c = 0 \Rightarrow c = -10$

$\therefore$ Equation of hyperbola is

$(x + y + 3)(2x - y + 1) - 10 = 0$

Also, equation of conjugate hyperbola is (as we know that, equation of hyperbola, conjugate hyperbola, and pair of asymptotes are same but only difference in constant term).

$= 2x^2 + xy - y^2 + 7x - 2y + 13 = 0$

Question10

If θ is the acute angle between the tangents drawn from the point $(1, 1)$ to the hyperbola $4x^2 - 5y^2 - 20 = 0$, then $\tan\theta =$

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Options:

A.

$2\sqrt{21}$

B.

$\frac{4}{5}$

C.

$\frac{\sqrt{7}}{2}$

D.

$\frac{2}{\sqrt{7}}$

Answer: A

Solution:

Given equation of hyperbola can be written as

$\frac{x^2}{5} - \frac{y^2}{4} = 1$ which is of the form

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \ldots (i)$

where, $a = \sqrt{5}, b = 2$

As we know that, equation of any tangent to the hyperbola Eq. (i) having slope m is

$y = mx \pm \sqrt{a^2m^2 - b^2} = mx + \sqrt{5m^2 - 4} \qquad \ldots (ii)$

According to question, Eq. (ii) passes through $(1, 1)$.

$$\begin{aligned} &\text{So, } 1 = m \times 1 \pm \sqrt{5m^2 - 4} \\ \Rightarrow \quad & 1 - m = \pm\sqrt{5m^2 - 4} \\ \Rightarrow \quad & (1 - m)^2 = 5m^2 - 4 \\ \Rightarrow \quad & 4m^2 + 2m - 5 = 0 \qquad \ldots (iii) \end{aligned}$$

Let m_1, m_2 slopes of these two tangents be the roots of Eq. (iii).

So, $m_1 + m_2 = \frac{-2}{4} = \frac{-1}{2}$

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$$m_1 m_2 = \frac{-5}{4}$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\sqrt{\left(\frac{-1}{2}\right)^2 - 4 \times \left(\frac{-5}{4}\right)}}{1 + \left(\frac{-5}{4}\right)}\right|$$

$$= \left|\frac{\sqrt{\frac{1}{4} + \frac{20}{4}}}{1 - \frac{5}{4}}\right| = \left|\frac{\sqrt{21}}{2} \times \frac{4}{-1}\right|$$

$$= |-2\sqrt{21}| = 2\sqrt{21}$$

Question11

If the equation of the tangent of the hyperbola $5x^2 - 9y^2 - 20x - 18y - 34 = 0$ which makes an angle 45° with the positive X-axis in positive direction is $x + by + c = 0$, then $b^2 + c^2 =$

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Options:

A.

2 or 13

B.

5 or 26

C.

2 or 26

D.

26 or 28

Answer: C

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Solution:

The slope of tangent $m = \tan 45^\circ = 1$

Now, $5x^2 - 9y^2 - 20x - 18y - 34 = 10$

$$\Rightarrow 5\left(x^2 - 4x\right) - 9\left(y^2 + 2y\right) = 34$$
$$\Rightarrow 5\left(x^2 - 4x + 4\right) - 9\left(y^2 + 2y + 1\right) = 34 + 20 - 9$$

$$\Rightarrow 5(x-2)^2 - 9(y+1)^2 = 45$$
$$\Rightarrow \frac{(x-2)^2}{9} - \frac{(y+1)^2}{5} = 1$$

The equation of the tangent with slope m is

$y + 1 = m(x-2) \pm \sqrt{9m^2 - 5}$

Since, $m = 1$

$$\Rightarrow \quad y + 1 = (x-2) \pm \sqrt{9-5}$$
$$\Rightarrow \quad y = x - 1 \text{ or } y = x - 5$$

$\Rightarrow$ The equation $x - y - 1 = 0$

or $x - y - 5 = 0$

Comparing with $x + by + c = 0$

$$\Rightarrow b = -1 \text{ and } c = -1, -5$$
$$\Rightarrow b^2 + c^2 = (-1)^2 + (-1)^2 = 1 + 1 = 2$$
$$\text{and } b^2 + c^2 = (-1)^2 + (-5)^2 = 1 + 25 = 26$$
$$\Rightarrow b^2 + c^2 = 2 \text{ or } 26$$

Question12

If the distance between the foci of a hyperbola H is 26 and distance between its directrices is $\frac{50}{13}$, then the eccentricity of the conjugate hyperbola of the hyperbola H is

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Options:

A.

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$\frac{13}{12}$

B.

$\frac{25}{17}$

C.

$\frac{13}{7}$

D.

$\frac{25}{13}$

Answer: A

Solution:

Distance between the foci $= 2ae$

$\Rightarrow \quad 2ae = 26$

distance between the directrices $= \frac{2a}{e}$

$$\begin{aligned}
&\Rightarrow \quad \frac{2a}{e} = \frac{50}{13} \\
&\Rightarrow \quad \frac{2ae}{\frac{2a}{e}} = \frac{26}{\frac{50}{13}} \\
&\Rightarrow \quad e^2 = \frac{13 \cdot 13}{25} \\
&\Rightarrow \quad e = \frac{13}{5} \\
&\Rightarrow \quad \frac{1}{e_H^2} + \frac{1}{e_{CH}^2} = 1 \\
&\Rightarrow \quad \frac{25}{169} + \frac{1}{e_{CH}^2} = 1 \\
&\Rightarrow \quad e_{CH}^2 = \frac{169}{144} \Rightarrow e_{CH} = \frac{13}{12}
\end{aligned}$$

Question13

By rotating the axes about the origin in anti-clockwise direction with certain angle, if the equation $x^2 + 4xy + y^2 = 1$ is transformed to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = l$, then $\sqrt{\frac{a^2+b^2}{a^2}} =$

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Options:

A.

2

B.

$\frac{\sqrt{13}}{3}$

C.

$\frac{3}{2}$

D.

$\sqrt{10}$

Answer: A

Solution:

Given equation

$$\begin{aligned}
&x^2+4xy+y^2=1 \qquad \ldots(i)\\
&ax^2+2hxy+by^2=1\\
&x=X\cos\theta-Y\sin\theta\\
&y=X\sin\theta+Y\cos\theta\\
&a=1,h=2,b=1\\
&h^2-ab=3>0 \Rightarrow \text{ Given}
\end{aligned}$$

curve and hyperbola,

$$\begin{aligned}
\tan 2\theta &= \frac{2h}{a-b}=\frac{4}{0}=\frac{\pi}{2}\\
\theta &= \frac{\pi}{4}\\
x &= \frac{X-Y}{\sqrt{2}}, y=\frac{X+Y}{\sqrt{2}}
\end{aligned}$$

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From (i), we get

$$\frac{X^2+Y^2-2XY}{2}+\frac{4\left(X^2-Y^2\right)}{2}+\frac{X^2+Y^2+2XY}{2}=1$$

$$\frac{1}{2}\left[6X^2-2Y^2\right]=1 \Rightarrow 3X^2-Y^2=1$$

$$\frac{X^2}{1/3}-\frac{Y^2}{1}=1 \Rightarrow a^2=\frac{1}{3}, b^2=1$$

$$\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{\frac{1}{3}+1}{\frac{1}{3}}}=2$$

Question14

If a tangent to the hyperbola $xy=-1$ is also a tangent to the parabola $y^2=8x$, then the equation of that tangent is

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Options:

A.

$3y+x=2$

B.

$y=3x+4$

C.

$y=x+2$

D.

$y=2x+1$

Answer: C

Solution:

Given equation of hyperbola

$xy=-1$

Let equation of tangent to $xy=-1$ be $y=mx+c$

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$$\Rightarrow \quad x(mx+c)+1=0$$
$$mx^2+cx+1=0 \text{ for tangent } Q=0$$
$$c^2-4m=0 \Rightarrow c^2=4m \qquad \ldots(i)$$

Equation of tangent to $y^2=8x$

$$y=mx+\frac{2}{m}$$
$$\frac{2}{m}=c \Rightarrow \frac{4}{m^2}=4m$$
$$m=1, c=2$$
$$y=x+2$$

Question15

The distance between the tangents of the hyperbola $2x^2-3y^2=6$ which are perpendicular to the line $x-2y+5=0$ is

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Options:

A.

$2\sqrt{2}$

B.

4

C.

$\sqrt{2}$

D.

$3\sqrt{2}$

Answer: A

Solution:

Given equation of hyperbola

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$2x^2 - 3y^2 = 6$

$\frac{x^2}{3} - \frac{y^2}{2} = 1 \qquad \ldots(i)$

Equation of tangent, $y =$ Slope of give line $m_1 = \frac{1}{2}$

Slope of perpendicular line $= -2$

$y = mx + c$ is tangent to hyperbola

$c^2 = a^2m^2 - b^2 \Rightarrow c = \pm\sqrt{10}$

Equation of tangents $2x + y + \sqrt{10} = 0$,

$2x + y - \sqrt{10}$ distance $= \frac{2\sqrt{10}}{\sqrt{5}} = 2\sqrt{2}$

Question16

The tangents drawn to the hyperbola $5x^2 - 9y^2 = 90$ through a variable point P make the angles α and β with its transverse axis. If α, β are the complementary angles then the locus of P is

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Options:

A.

$x^2 + y^2 = 8$

B.

$x^2 - y^2 = 8$

C.

$x^2 - y^2 = 28$

D.

$x^2 + y^2 = 28$

Answer: C

Solution:

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Given, hyperbola $5x^2 - 9y^2 = 90$

$\frac{x^2}{18} - \frac{y^2}{10} = 1$

Let equation of tangent to the ellipse be

$y = mx \pm \sqrt{18m^2 - 10}$

or $(y - mx)^2 = 18m^2 - 10$

$$\begin{aligned} \therefore \quad & y^2 + m^2x^2 - 2mxy - 18m^2 + 10 = 0 \\ & m^2\left(x^2 - 18\right) - 2mxy + y^2 + 10 = 0 \end{aligned}$$

Given tangent makes α and β angles with transverse axis are complementary

$$\begin{aligned} & \because \quad \alpha + \beta = \frac{\pi}{2} \\ & \tan(\alpha + \beta) = \text{ not defined} \\ & \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \text{ not defined} \\ & \text{And } m_1 = \tan\alpha \quad m_2 = \tan\beta \\ & m_1 + m_2 = \frac{2xy}{x^2 - 18} \text{ and } m_1m_2 = \frac{y^2 + 10}{x^2 - 18} \\ & \because \quad 1 - m_1m_2 = 0 \Rightarrow m_1m_2 = +1 \\ & \quad \frac{y^2 + 10}{x^2 - 18} = +1 \\ & \Rightarrow \quad y^2 + 10 = +x^2 - 18 \\ & \Rightarrow \quad y^2 = x^2 - 28 \text{ or } x^2 - y^2 = 28 \end{aligned}$$

Question17

If θ is the acute angle between the asymptotes of a hyperbola $7x^2 - 9y^2 = 63$, then $\cos\theta =$

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Options:

A.

$\frac{1}{4}$

B.

$\frac{3}{4}$

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C.

$\frac{1}{8}$

D.

$\frac{4}{3}$

Answer: C

Solution:

We have hyperbola

$$7x^2 - 9y^2 = 63$$
$$\frac{x^2}{9} - \frac{y^2}{7} = 1$$

Now, its asymptotes is

$$\frac{x}{3} \pm \frac{y}{\sqrt{7}} = 0$$
$$r = \frac{x^2}{9} + 0 \cdot xy - \frac{y^2}{7} = 0$$
$$a = \frac{1}{9}, b = -\frac{1}{7}$$

Angle between pair of asymptotes

$$\tan\theta = \frac{2\sqrt{0+\frac{1}{63}}}{\frac{1}{9}-\frac{1}{7}} = \left|\frac{2\frac{1}{\sqrt{63}}}{\frac{-2}{63}}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{2}{\sqrt{63}} \times \frac{63}{2} = \sqrt{63}$$

$$\therefore \quad \cos\theta = \frac{1}{8}$$

Question18

One of the latus recta of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ subtends an angle $2\tan^{-1}\left(\frac{3}{2}\right)$ at the centre of the hyperbola. If $b^2 = 36$ and e is the eccentricity of the given hyperbola, then $\sqrt{a^2 + e^2} =$

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Options:

A.

4

B.

$\sqrt{14}$

C.

6

D.

$\sqrt{21}$

Answer: A

Solution:

Given, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $b^2 = 36$ Coordinate of one of $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{36}{a^2}$ The latus rectum is $L\left(ae, +\frac{b^2}{a}\right)$ and $L'\left(ae, -\frac{b^2}{a}\right)$.

Now, slope of $OL = \frac{b^2}{a^2 e}$ Slope of $OL' = -\frac{b^2}{a^2 e}$ Now, angle between OL and OL'

$$2\tan^{-1}\frac{b^2}{a^2 e} = 2\tan^{-1}\frac{3}{2}$$
$$\therefore \quad \frac{b^2}{a^2+e} = \frac{3}{2} \Rightarrow \frac{3e}{2} = \frac{b^2}{a^2}$$
$$\frac{3e}{2} = e^2 - 1 \quad \left[\because e^2 = 1 + \frac{b^2}{a^2}\right]$$
$$2e^2 - 3e - 2 = 0$$
$$(e-2)(2e+1) = 10 \Rightarrow e = 2, e \neq -\frac{1}{2}$$
$$a^2 = \frac{2b^2}{3e} = \frac{2 \times 36}{3 \times 2} = 12 \quad [\because b^2 = 36]$$
$$\therefore \sqrt{a^2+e^2} = \sqrt{12+4} = 4$$

Question19

If the equation of the hyperbola having $(8,3),(0,3)$ as foci and $\frac{4}{3}$ as eccentricity is $\frac{(x-\alpha)^2}{p} - \frac{(y-\beta)^2}{q} = 1$, then $p+q=$

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Options:

A.

β^2

B.

$\alpha + \beta$

C.

α^2

D.

$\alpha\beta$

Answer: C

Solution:

Given, foci $(8,3)$ and $(0,3)$ and $e = \frac{4}{3}$ centre $= (4,3)$

Distance between foci $= 2ae = 8$

$$ae = 4$$
$$a \cdot \frac{4}{3} = 4$$
$$a = 3 \Rightarrow a^2 = 9 \text{ and } b^2 = 7$$

$\because$ Equation of hyperbola is

$\frac{(x-4)^2}{9} - \frac{(y-3)^2}{7} = 1$

Clearly $\alpha = 4, p = 9$ and $q = 7$

$\therefore p + q = 16 = \alpha^2$

Question20

If $y = x + \sqrt{2}$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2} = 1$, then equations of its directrices are

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Options:

A. $x = \pm\sqrt{3}$

B. $x = \pm\sqrt{\frac{8}{3}}$

C. $x = \pm\sqrt{\frac{2}{3}}$

D. $x = \pm\sqrt{\frac{4}{3}}$

Answer: B

Solution:

To find the directrices of the hyperbola given by the equation $\frac{x^2}{a^2} - \frac{y^2}{2} = 1$, where the tangent line is $y = x + \sqrt{2}$, we proceed with the following steps:

The equation for the tangent line of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ can be written as:

$y = mx \pm \sqrt{a^2m^2 - b^2}$

For our hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2} = 1$, $b^2 = 2$, and the given tangent line is $y = x + \sqrt{2}$, implying that the slope $m = 1$. Substituting these values, we find:

$a^2m^2 - 2 = 2$

$a^2 \times 1^2 - 2 = 2 \implies a^2 - 2 = 2 \implies a^2 = 4$

Next, we determine the eccentricity e of the hyperbola, given by:

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{2}{4}} = \sqrt{\frac{3}{2}}$

The equation of the directrices is given by:

$x = \pm\frac{a}{e}$

Substituting the known values:

$x = \pm\frac{2}{\sqrt{3/2}} = \pm\frac{2\cdot\sqrt{2}}{\sqrt{3}} = \pm\sqrt{\frac{8}{3}}$

Thus, the directrices of the hyperbola are $x = \pm\sqrt{\frac{8}{3}}$.

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Question21

The area of the quadrilateral formed with the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and its conjugate hyperbola is (in sq units)

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Options:

A. 24

B. 25

C. 16

D. 50

Answer: D

Solution:

We have the hyperbola given by $\frac{x^2}{16} - \frac{y^2}{9} = 1$. In this equation, $a^2 = 16$ and $b^2 = 9$.

To find the area of the quadrilateral formed with the foci of this hyperbola and its conjugate hyperbola, we use the formula:

Area $= 2(a^2 + b^2)$

Substituting the known values:

Area $= 2(16 + 9) = 50$ square units.

Question22

The line $21x + 5y = k$ touches the hyperbola $7x^2 - 5y^2 = 232$, then k is equal to

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Options:

A. 116

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B. 232

C. 58′

D. 110

Answer: A

Solution:

To determine the value of k for which the line $21x + 5y = k$ is tangent to the hyperbola $7x^2 - 5y^2 = 232$, we apply the condition for tangency of a line to a hyperbola.

Tangency Condition:

For a line $y = mx + c$ to be tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the condition is $c^2 = a^2m^2 - b^2$.

Given:

The line equation is:

$$y = \frac{-21}{5}x + \frac{k}{5}$$

Hence, $m = \frac{-21}{5}$ and $c = \frac{k}{5}$.

The hyperbola $7x^2 - 5y^2 = 232$ can be written as:

$$\frac{x^2}{\frac{232}{7}} - \frac{y^2}{\frac{232}{5}} = 1$$

Here,

$$a^2 = \frac{232}{7}, \quad b^2 = \frac{232}{5}$$

Applying the Tangency Condition:

$$\left(\frac{k}{5}\right)^2 = \frac{232}{7} \times \left(\frac{-21}{5}\right)^2 - \frac{232}{5}$$

Simplifying:

$$\frac{k^2}{25} = \frac{232}{7} \times \frac{441}{25} - \frac{232}{5}$$

$$\frac{k^2}{25} = \frac{232 \times 441}{175} - \frac{232 \times 5}{25}$$

Factor out 232:

$$\frac{k^2}{25} = \frac{232 \times (441 - 5)}{175}$$

Calculate:

$$= \frac{232 \times 436}{175}$$

Now solving for k:

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$k^2 = \frac{13456}{25}$

Therefore:

$k = 116$

Thus, the value of k is $\boxed{116}$.

Question23

If the equation $\frac{x^2}{7-k} + \frac{y^2}{5-k} = 1$ represents a hyperbola, then

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Options:

A.

$5 < k < 7$

B. $k < 5$ or $k > 7$

C. $k < 5$

D.

$k \neq 5, \quad k \neq 7, -\infty < k < \infty$

Answer: A

Solution:

Given, equation $\frac{x^2}{7-k} + \frac{y^2}{5-k} = 1$ represent at hyperbola.

We know that equation of hyperbola,

$$\begin{aligned} & \frac{x^2}{a} - \frac{y^2}{b} = 1 \\ \therefore \quad & a^2 > 0 \text{ and } b^2 > 0 \\ \Rightarrow \quad & 7 - k > 0 \text{ and } k - 5 > 0 \\ \Rightarrow \quad & k < 7 \text{ and } k > 5 \\ \Rightarrow \quad & 5 < k < 7 \end{aligned}$$

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Question24

The transformed equation of $x^2 - y^2 + 2x + 4y = 0$ when the origin is shifted to the point $(-1, 2)$ is

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Options:

A. $x^2 + y^2 = 1$

B. $x^2 + 3y^2 = 1$

C. $x^2 - y^2 + 3 = 0$

D. $4x^2 + 9y^2 = 36$

Answer: C

Solution:

To transform the equation $x^2 - y^2 + 2x + 4y = 0$ when the origin is shifted to the point $(-1, 2)$, we proceed as follows:

Given equation:

$x^2 - y^2 + 2x + 4y = 0$

New origin:

$(h, k) = (-1, 2)$

Transform the variables:

Let $X = x - h$ and $Y = y - k$.

So,

$X = x - (-1) = x + 1$

$Y = y - 2$

Solving for x and y:

$x = X - 1$

$y = Y + 2$

Now, substituting these into the given equation:

$(x - 1)^2 - (y + 2)^2 + 2(x - 1) + 4(y + 2) = 0$

Expanding each term:

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$(x-1)^2 = x^2 - 2x + 1$

$(y+2)^2 = y^2 + 4y + 4$

Substitute and simplify:

$[x^2 - 2x + 1] - [y^2 + 4y + 4] + 2(x-1) + 4(y+2) = 0$
$x^2 - y^2 - 2x + 1 - 4y - 4 + 2x - 2 + 4y + 8 = 0$

Combine and simplify:

$x^2 - y^2 + 3 = 0$

Thus, the transformed equation is:

$x^2 - y^2 + 3 = 0$

Question25

If the ellipse $4x^2 + 9y^2 = 36$ is confocal with a hyperbola whose length of the transverse axis is 2 , then the points of intersection of the ellipse and hyperbola lie on the circle

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Options:

A. $x^2 + y^2 = 81$

B. $x^2 + y^2 = 16$

C. $x^2 + y^2 = 25$

D. $x^2 + y^2 = 5$

Answer: D

Solution:

Given the ellipse equation:

$4x^2 + 9y^2 = 36$

or equivalently,

$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad \text{... (i)}$

This ellipse is confocal with a hyperbola whose transverse axis has a length of 2. For the hyperbola, the transverse axis is $2a = 2$, giving us $a = 1$.

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For confocal conics, the foci are at the same distance from the origin. The distance of the focus for the ellipse is:

$$c = \sqrt{a^2 - b^2} = \sqrt{\sqrt{9^2} - \sqrt{4^2}} = \sqrt{9 - 4} = \sqrt{5}$$

For the hyperbola, we have:

$$c = \sqrt{a^2 + b^2}$$

Thus,

$$c = \sqrt{1 + b^2} = \sqrt{5}$$

Solving for b^2, we find $b^2 = 4$. Therefore, the equation of the hyperbola is:

$$\frac{x^2}{1} - \frac{y^2}{4} = 1 \quad \Rightarrow \quad x^2 - \frac{y^2}{4} = 1 \quad \text{... (ii)}$$

This implies:

$$y^2 = 4(x^2 - 1) \quad \text{... (iii)}$$

From equations (i) and (iii), we get:

$$\begin{aligned} 4x^2 + 9\left(4(x^2 - 1)\right) &= 36 \\ 4x^2 + 36x^2 - 36 &= 36 \\ 40x^2 &= 72 \\ x^2 &= \frac{9}{5} \\ x &= \pm\frac{3\sqrt{5}}{5} \end{aligned}$$

Thus, using equation (iii):

$$y^2 = 4 \times \left(\frac{9}{5} - 1\right) = 4 \times \frac{4}{5} = \frac{16}{5}$$

$$y = \pm\frac{4\sqrt{5}}{5}$$

The points of intersection are $\left(\pm\frac{3\sqrt{5}}{5}, \frac{4\sqrt{5}}{5}\right)$.

The equation of a circle passing through these points is:

$$\begin{aligned} x^2 + y^2 &= \left(\frac{3\sqrt{5}}{5}\right)^2 + \left(\frac{4\sqrt{5}}{5}\right)^2 \\ &= \frac{45 + 80}{25} = \frac{125}{25} \\ \therefore \quad x^2 + y^2 &= 5 \end{aligned}$$

Question26

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If the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\sec\alpha$, then area of the triangle formed by the asymptotes of the hyperbola with any of its tangent is

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Options:

A. $a^2b^2\sec^2\alpha$

B. $\frac{b^2}{|\tan\alpha|}$

C. $a^2\tan^2\alpha$

D. $\left(a^2+b^2\right)\tan^2\alpha$

Answer: B

Solution:

For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the eccentricity e is given by the formula:

$e=\sqrt{1+\frac{b^2}{a^2}}$

Since the eccentricity is provided as $\sec\alpha$, we have:

$\sec\alpha=\sqrt{1+\frac{b^2}{a^2}}$

Squaring both sides of the equation, we obtain:

$\sec^2\alpha=1+\frac{b^2}{a^2}$

Subtracting 1 from both sides gives:

$\sec^2\alpha-1=\frac{b^2}{a^2}=\tan^2\alpha$

or

$\frac{b^2}{a^2}=\tan^2\alpha$

The equations of the asymptotes of the hyperbola are:

$y=\pm\frac{b}{a}x=\pm\tan\alpha\cdot x$

A general form for the tangent to the hyperbola is:

$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

Given $x_1=a\sec\theta$ and $y_1=b\tan\theta$, when $x=0$, the equation transforms to:

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$\frac{-y\tan\theta}{a\tan\alpha}=1\Rightarrow y=-\frac{a\tan\alpha}{\tan\theta}$

Thus, the vertices of the triangle formed by asymptotes and the tangent will be at $(\pm a,0)$ and $\left(0,-\frac{a\tan\alpha}{\tan\theta}\right)$.

The area of the triangle, with the base as $2a$ and height $-\frac{a\tan\alpha}{\tan\theta}$, is calculated by:

Area $=\frac{1}{2}\times$ Base $\times$ Height $=\frac{1}{2}\times 2a\times\left(\frac{a\tan\alpha}{\tan\theta}\right)$

Simplifying this, we get:

$=\left|\frac{a^2\tan\alpha}{\tan\theta}\right|=\left|\frac{a^2b^2}{a^2\tan\theta}\right|$

From our earlier identity, this equals:

Area $=\frac{b^2}{\tan\theta}=\frac{b^2}{|\tan\alpha|}$

Question27

If e_1 and e_2 are respectively the eccentricities of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its conjugate hyperbola, then the line $\frac{x}{2e_1}+\frac{y}{2e_2}=1$ touches the circle having centre at the origin, then its radius is

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Options:

A. 2

B. e_1+e_2

C. e_1e_2

D. 4

Answer: A

Solution:

Given the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, its eccentricity e_1 is defined by:

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$e_1 = \sqrt{1 + \frac{b^2}{a^2}}$

For the conjugate hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, the eccentricity e_2 is:

$e_2 = \sqrt{1 + \frac{a^2}{b^2}}$

We are given the line:

$\frac{x}{2e_1} + \frac{y}{2e_2} = 1$

Rewriting it as:

$\frac{x}{2e_1} + \frac{y}{2e_2} - 1 = 0$

Our task is to find the radius of the circle, centered at the origin, which this line touches.

The distance from the origin $(0, 0)$ to a line $Ax + By + C = 0$ is:

$d = \frac{|C|}{\sqrt{A^2+B^2}}$

For equation (ii), the distance becomes:

$d = \frac{|-1|}{\sqrt{\left(\frac{1}{2e_1}\right)^2+\left(\frac{1}{2e_2}\right)^2}}$

Substituting, we find:

$d = \frac{2}{\sqrt{\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}}}$

$d = \frac{2}{\sqrt{\frac{a^2+b^2}{a^2+b^2}}} = \frac{2}{\sqrt{1}}$

So, the distance d is 2. Therefore, the radius of the circle is 2.

Question28

The descending order of magnitude of the eccentricities of the following hyperbolas isA. A hyperbola whose distance between foci is three times the distance between its directrices.B. Hyperbola in which the transverse axis is twice the conjugate axis.C. Hyperbola with asymptotes $x + y + 1 = 0, x - y + 3 = 0$

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Options:

A. C, A, B

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B. B, C, A

C. C, B, A

D. A, C, B

Answer: D

Solution:

Explanation

Hyperbola A

Given: $2c = 3 \times \frac{2a}{e}$

This implies: $c = \frac{3a}{e}$

Using the relation $c^2 = a^2 + b^2$, we derive:

$\left(\frac{3a}{e}\right)^2 = a^2 + b^2$

$\Rightarrow \frac{9a^2}{e^2} = a^2 + b^2 \Rightarrow 9a^2 = e^2(a^2 + b^2)$

$\Rightarrow 9 = e^2(1 + \frac{b^2}{a^2}) \Rightarrow 9 = e^2(1 + e^2 - 1)$

$\Rightarrow e^4 = 9 \Rightarrow e = \sqrt{3}$

Hyperbola B

Let $2a$ be the transverse axis and $2b$ be the conjugate axis, given:

$2a = 2 \times 2b \Rightarrow a = 2b$

Using the relation $c^2 = a^2 + b^2$, we derive:

$c^2 = (2b)^2 + b^2 \Rightarrow c^2 = 5b^2 \Rightarrow c = \sqrt{5}b$

Therefore:

$e = \frac{\sqrt{5}b}{2b} = \frac{\sqrt{5}}{2} \quad [\because e = \frac{c}{a}]$

Hyperbola C

Given the asymptotes equations:

$x + y + 1 = 0 \Rightarrow y = -x - 1$

and $x - y + 3 = 0 \Rightarrow y = x - 3$

The standard form of asymptotes for a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by:

$y = \pm\frac{b}{a}x$

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The slopes of asymptotes are -1 and 1, meaning:

$\frac{b}{a} = \pm 1 \Rightarrow b = a$

Using the relation $c^2 = a^2 + b^2$, we derive:

$c^2 = a^2 + a^2 \Rightarrow c = \sqrt{2}a$

Therefore:

$e = \frac{\sqrt{2}a}{a} \Rightarrow e = \sqrt{2}$

Conclusion

In descending order of the magnitudes of the eccentricities, we have $\sqrt{3} > \sqrt{2} > \frac{\sqrt{5}}{2}$. Therefore, the order is A, C, B.

Question29

The equation of the pair of asymptotes of the hyperbola $4x^2 - 9y^2 - 24x - 36y - 36 = 0$ is

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Options:

A. $2x^2 - xy - 3y^2 - 14x - 9y - 12 = 0$

B. $2x^2 - xy - 3y^2 - 2x + 3y = 0$

C. $2x^2 - 5xy + 3y^2 - 22x + 27y + 60 = 0$

D. $4x^2 - 9y^2 - 24x - 36y = 0$

Answer: D

Solution:

To determine the equation of the asymptotes for the hyperbola given by:

$4x^2 - 9y^2 - 24x - 36y - 36 = 0$

we first rearrange and complete the square:

Reorganize the terms:

$(4x^2 - 24x) - (9y^2 + 36y) - 36 = 0$

Complete the square for each set of terms:

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$4(x^2 - 6x) - 9(y^2 + 4y) - 36 = 0$

Completing the square:

$4(x^2 - 6x + 9 - 9) - 9(y^2 + 4y + 4 - 4) - 36 = 0$

Simplifying:

$4(x-3)^2 - 9(y+2)^2 - 36 = 0$

Set the equation in standard hyperbola form:

$4(x-3)^2 - 9(y+2)^2 = 36$

$\frac{(x-3)^2}{9} - \frac{(y+2)^2}{4} = 1$

The asymptotes of a hyperbola in the standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ are given by:

$(y-k) = \pm\frac{b}{a}(x-h)$

For our hyperbola:

$h = 3, k = -2$

$a = 3, b = 2$

Thus, the equations of the asymptotes are:

$(y+2) = \pm\frac{2}{3}(x-3)$

Expanding this gives:

First Asymptote:

$y + 2 = \frac{2}{3}(x-3)$

$3(y+2) = 2(x-3)$

Simplifying:

$3y + 6 = 2x - 6 \quad \Longrightarrow \quad 2x - 3y = 12$

Second Asymptote:

$y + 2 = -\frac{2}{3}(x-3)$

$3(y+2) = -2(x-3)$

Simplifying:

$3y + 6 = -2x + 6 \quad \Longrightarrow \quad 2x + 3y = 0$

After multiplying and expanding these, we recover the simplified asymptote equation:

$4x^2 - 9y^2 - 24x - 36y = 0$

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Question30

The equation of one of the tangents drawn from the point $(0,1)$ to the hyperbola $45x^2 - 4y^2 = 5$ is

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Options:

A. $4y + 5 = 0$

B. $3x + 4y - 4 = 0$

C. $5x - 6y + 6 = 0$

D. $9x - 2y + 2 = 0$

Answer: D

Solution:

The equation of one of the tangent drawn from the point $(0,1)$ to the hyperbola $45x^2 - 4y^2 = 5$.

We can use the fact that the equation of the tangent to a hyperbola.

$Ax^2 + By^2 = C$ at point (x_1, y_1) is given

$Ax_1x + By_1y = C$

Here, $A = 45, B = -4, C = 5$

Let (x_1, y_1) be the point of tangency, the equation of the tangent line is

$45x_1x - 4y_1y = 5$

The line must, also pass through the point (0,1) substituting ($x = 0$) and ($y = 1$) into the tangent equation gives

$$45x_1(0) - 4y_1(1) = 5$$
$$\Rightarrow \quad y_1 = \frac{-5}{4}$$
$$45x_1x - 4\left(\frac{-5}{4}\right)y = 5$$
$$9x_1x_1 + y = 1$$

Using the hyperbola equation

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$$45x_1^2 - 4\left(\frac{25}{16}\right) = 5$$

$$\Rightarrow 45x_1^2 - \frac{25}{4} = 5$$

$$\Rightarrow 45x_1^2 = 5 + \frac{25}{4} = \frac{20+25}{4} = \frac{45}{4}$$

$$\Rightarrow \quad 45x_1^2 = \frac{45}{4}$$

$$\Rightarrow \quad x_1^2 = \frac{1}{4} \Rightarrow x_1 = \pm\frac{1}{2}$$

$$\Rightarrow \quad x_1 = \frac{1}{2}$$

$$\Rightarrow \quad 9\left(\frac{1}{2}\right)x + y = 1$$

$$\Rightarrow \quad 9x + 2y = 2$$

$$\Rightarrow \quad x_1 = \frac{-1}{2} \text{ and } 9\left(\frac{-1}{2}\right)x + y = 1$$

$$\Rightarrow \quad -9x + 2y = 2$$

$$\Rightarrow 9x - 2y + 2 = 0$$

Question31

If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $\sqrt{5}x = 4$ and e is its eccentricity, then $e^2 =$

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Options:

A. $\frac{\sqrt{7}}{2}$

B. $\frac{7}{2}$

C. $\frac{35}{4}$

D. $2\sqrt{3}$

Answer: B

Solution:

To solve for e^2 where the hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the hyperbola passes through the point $(4, -2\sqrt{3})$, we can proceed as follows:

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Plug the point into the hyperbola equation:

$\frac{16}{a^2} - \frac{12}{b^2} = 1 \quad \ldots \text{(i)}$

Relate b^2 and a^2 with eccentricity e:

$b^2 = a^2(e^2 - 1)$

Given the directrix $\sqrt{5}x = 4$:

$x = \frac{4}{\sqrt{5}} = \frac{a}{e}$

Therefore,

$a^2 = \frac{16}{5}e^2 \quad \ldots \text{(ii)}$

Substitute the expression for a^2 from (ii) into (i):

$\frac{16}{\frac{16}{5}e^2} - \frac{12}{b^2} = 1$

Substitute $b^2 = a^2(e^2 - 1)$:

$\frac{5}{e^2} - \frac{12}{a^2(e^2-1)} = 1$

Replace a^2 with $\frac{16}{5}e^2$:

$\frac{5}{e^2} - \frac{12}{\frac{16}{5}e^2(e^2-1)} = 1$

Simplify the equation:

$\frac{5}{e^2} - \frac{60}{16e^2(e^2-1)} = 1$

On further simplification:

$\frac{5}{e^2}\left[1 - \frac{12}{16(e^2-1)}\right] = 1$

Solve for e^2:

Upon solving, we find that:

$e^2 = \frac{7}{2}$

This process allows us to determine that the value of e^2 is $\frac{7}{2}$.

Question32

If l_1 and l_2 are the lengths of the perpendiculars drawn from a point on the hyperbola $5x^2 - 4y^2 - 20 = 0$ to its asymptotes, then $\frac{{l_1}^2{l_2}^2}{100} =$

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Options:

A. $\frac{20}{9}$

B. $\frac{16}{81}$

C. $\frac{4}{81}$

D. $\frac{2}{9}$

Answer: C

Solution:

Given hyperbola $5x^2 - 4y^2 - 20 = 0$

$\Rightarrow \quad 5x^2 - 4y^2 = 20$
$\Rightarrow \quad \frac{5x^2}{20} - \frac{4y^2}{20} = \frac{20}{20}$
$\Rightarrow \quad \frac{x^2}{4} - \frac{y^2}{5} = 1$

Asymptotes are $x \pm \frac{2}{\sqrt{5}}y = 0$

Let $P(x_0, y_0)$ be any point on $S = 0$, Length of the perpendicular from P to $x + \frac{2}{\sqrt{5}}y = 0$ is

$$l_1 = \frac{\left|x_0 + \frac{2}{\sqrt{5}}y_0\right|}{\sqrt{1 + \frac{4}{5}}} = \frac{\left|x_0 + \frac{2}{\sqrt{5}}y_0\right|}{\sqrt{\frac{9}{5}}} = \frac{\left|\sqrt{5}x_0 + 2y_0\right|}{3}$$

Length of the perpendicular from P to $x - \frac{2}{\sqrt{5}}y = 0$ is

$$l_2 = \frac{\left|x_0 - \frac{2}{\sqrt{5}}y_0\right|}{\sqrt{1+\frac{4}{5}}} = \frac{\left|x_0 - \frac{2}{\sqrt{5}}y_0\right|}{\sqrt{\frac{9}{5}}} = \frac{\left|\sqrt{5}x_0 - 2y_0\right|}{3}$$

$$\therefore l_1^2 \cdot l_2^2 = \frac{\left[\left(\sqrt{5}x_0+2y_0\right)^2\left(\sqrt{5}x_0-2y_0\right)^2\right]}{(3)^2\times(3)^2}$$

$$= \frac{\left(5x_0-4y_0\right)^2}{81} = \frac{400}{81}$$

$$\therefore \quad \frac{l_1^2 l_2^2}{100} = \frac{400}{81\times100} = \frac{4}{81}$$

Question33

If a circle of radius 4 cm passes through the foci of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{4} = 1$ and concentric with the hyperbola, then the eccentricity of the conjugate hyperbola of that hyperbola is

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Options:

A. 2

B. $2\sqrt{3}$

C. $2/\sqrt{3}$

D. $\sqrt{3}$

Answer: A

Solution:

The eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{4} = 1$ is given by the formula:

$\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{a^2}}$

The center of this hyperbola is at the origin, $(0, 0)$.

The equation of the circle is:

$x^2 + y^2 = 16$

This circle passes through the foci of the hyperbola, which are located at $(ae, 0)$. Thus, we can write:

$a^2e^2 = 16$

Since $e = \sqrt{1 + \frac{4}{a^2}}$, we substitute:

$a^2\left(1 + \frac{4}{a^2}\right) = 16$

Expanding and simplifying gives:

$a^2 + 4 = 16 \quad \Rightarrow \quad a^2 = 12$

Further simplifying the eccentricity:

$e = \sqrt{1 + \frac{4}{12}} = \frac{2}{\sqrt{3}}$

Now, let e' be the eccentricity of the conjugate hyperbola. We have:

$\frac{1}{e'^2} + \frac{3}{4} = 1$

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Solving for e':

$\frac{1}{e'^2} = 1 - \frac{3}{4} \quad \Rightarrow \quad e'^2 = 4 \quad \Rightarrow \quad e' = 2$

Question34

If a tangent to the hyperbola $x^2 - \frac{y^2}{3} = 1$ is also a tangent to the parabola $y^2 = 8x$, then equation of such tangent with the positive slope is

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Options:

A. $y - x - \frac{1}{2} = 0$

B. $y - 2x - 1 = 0$

C. $2y - 4x - 1 = 0$

D. $y - x - 1 = 0$

Answer: B

Solution:

To determine the equation of the tangent line that is tangent to both the hyperbola $x^2 - \frac{y^2}{3} = 1$ and the parabola $y^2 = 8x$, we start with the tangent equation for the parabola. The general equation for a tangent to the parabola $y^2 = 8x$ is:

$y = mx + \frac{2}{m}$

Since this line is also tangent to the hyperbola, it must satisfy the condition for tangency to the hyperbola as well. The condition for tangency to a hyperbola of the form $x^2 - \frac{y^2}{a^2} = 1$ is given by:

$c = \pm\sqrt{m^2 - 3}$

Given $c = \frac{2}{m}$, equating this to the tangency condition gives:

$\frac{2}{m} = \pm\sqrt{m^2 - 3}$

Squaring both sides, we get:

$\frac{4}{m^2} = m^2 - 3$

Rearranging terms gives:

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$m^4 - 3m^2 - 4 = 0$

We can factor this as:

$(m^2 - 4)(m^2 + 1) = 0$

Solving for m^2, we find:

$m^2 = 4 \quad (\text{since } m^2 \neq -1)$

Thus, $m = \pm 2$. For the tangent with a positive slope:

$m = 2$

Substituting $m = 2$ back into the tangent equation of the parabola, we have:

$y = 2x + \frac{2}{2} = 2x + 1$

So, the equation of the tangent line with a positive slope is:

$y - 2x - 1 = 0$

Question35

The locus of the mid-points of the chords of the hyperbola $x^2 - y^2 = a^2$ which touch the parabola $y^2 = 4ax$ is

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Options:

A. $x\left(y^2 - x^2\right) = ay^2$

B. $x\left(x^2 + y^2\right) = y^2 + x$

C. $ax^3 + y^3 = 3x$

D. $x\left(x^2 - y^2\right) = a^2$

Answer: A

Solution:

To find the locus of the midpoints of the chords of the hyperbola $x^2 - y^2 = a^2$ that touch the parabola $y^2 = 4ax$, we begin by considering a point (h, k), which is the midpoint of a chord of the hyperbola.

The equation of this chord in the format $T = S_1$ is:

$hx - ky = h^2 - k^2$

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Rewriting it in slope-intercept form gives:

$y = \frac{h}{k}x + \frac{k^2-h^2}{k}$

Since this line is tangent to the parabola $y^2 = 4ax$, we use the point of tangency conditions:

For a line $y = mx + c$ tangent to the parabola, the condition is:

$c = \frac{a}{m}$

Substituting $m = \frac{h}{k}$ and $c = \frac{k^2-h^2}{k}$, we equate and simplify:

$\frac{k^2-h^2}{k} = \frac{ak}{h}$

Solving this equation gives:

$hk^2 - h^3 = ak^2$

Thus, the required locus of the midpoint (h, k) satisfies:

$x(y^2 - x^2) = ay^2$

Question36

If the product of eccentricities of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = -1$ is 1 , then $b^2 =$

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Options:

A. $\frac{12}{25}$

B. 144

C. 25

D. $\frac{144}{25}$

Answer: D

Solution:

To find the value of b^2, we need to consider the properties of the given ellipse and hyperbola. Let's denote e_1 and e_2 as the eccentricities of the ellipse and hyperbola, respectively.

For the ellipse given by $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$, the eccentricity e_1 is calculated as:

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$e_1 = \sqrt{1 - \frac{b^2}{16}}$

For the hyperbola given by $\frac{x^2}{9} - \frac{y^2}{16} = -1$, the eccentricity e_2 is calculated as:

$e_2 = \sqrt{1 + \frac{9}{16}}$

We are given that the product of these eccentricities is 1:

$e_1 \times e_2 = 1$

Substituting the expressions for e_1 and e_2:

$\sqrt{1 - \frac{b^2}{16}} \times \sqrt{1 + \frac{9}{16}} = 1$

Squaring both sides to eliminate the square roots, we get:

$(1 - \frac{b^2}{16})(1 + \frac{9}{16}) = 1$

Simplifying the equation:

$\frac{(16-b^2)(9+16)}{16 \times 16} = 1$

This further simplifies to:

$16 - b^2 = \frac{256}{25}$

Solving for b^2:

$b^2 = 16 - \frac{256}{25}$

Putting the expression under a common denominator and simplifying gives us:

$b^2 = \frac{400}{25} - \frac{256}{25} = \frac{144}{25}$

Therefore, $b^2 = \frac{144}{25}$.

Question37

If the line $5x - 2y - 6 = 0$ is a tangent to the hyperbola $5x^2 - ky^2 = 12$, then the equation of the normal to this hyperbola at the point $(\sqrt{6}, p)(p < 0)$ is

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Options:

A. $\sqrt{6}x + 2y = 0$

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B. $2\sqrt{6}x + 3y = 3$

C. $\sqrt{6}x - 5y = 21$

D. $3\sqrt{6}x - y = 21$

Answer: C

Solution:

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Step 1: Find the value of k

The condition for the line $lx + my + n = 0$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{a^2l^2}{n^2} - \frac{b^2m^2}{n^2} = 1$.

The given line is $5x - 2y - 6 = 0$, so $l = 5$, $m = -2$, $n = -6$.

The given hyperbola is $5x^2 - ky^2 = 12$, which can be written as $\frac{x^2}{12/5} - \frac{y^2}{12/k} = 1$. So $a^2 = \frac{12}{5}$ and $b^2 = \frac{12}{k}$.

Substitute these into the tangency condition:

$$\frac{(12/5)(5^2)}{(-6)^2} - \frac{(12/k)(-2^2)}{(-6)^2} = 1$$

$$\frac{12 \times 25}{5 \times 36} - \frac{12 \times 4}{k \times 36} = 1$$

$$\frac{60}{36} - \frac{48}{36k} = 1$$

$$\frac{5}{3} - \frac{4}{3k} = 1$$

$$\frac{4}{3k} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$k = \frac{4 \times 3}{2 \times 3} = 2$$

Step 2: Find the point of tangency

The hyperbola equation is $5x^2 - 2y^2 = 12$. The point $(\sqrt{6}, p)$ lies on the hyperbola, so:

$$5(\sqrt{6})^2 - 2p^2 = 12$$

$$5(6) - 2p^2 = 12$$

$$30 - 2p^2 = 12$$

$$2p^2 = 18$$

$$p^2 = 9$$

$$p = \pm 3$$

Given $p < 0$, we take $p = -3$. The point is $(\sqrt{6}, -3)$.

Step 3: Find the equation of the normal

The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at point (x_1, y_1) is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$.

Using $a^2 = \frac{12}{5}$, $b^2 = \frac{12}{k} = \frac{12}{2} = 6$, $x_1 = \sqrt{6}$, $y_1 = -3$:

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$$\frac{(12/5)x}{\sqrt{6}} + \frac{6y}{-3} = \frac{12}{5} + 6$$

$$\frac{12x}{5\sqrt{6}} - 2y = \frac{12+30}{5}$$

$$\frac{12x}{5\sqrt{6}} - 2y = \frac{42}{5}$$

Multiply by $5\sqrt{6}$ to clear denominators:

$$12x - 10\sqrt{6}\,y = 42\sqrt{6}$$

Divide by 2:

$$6x - 5\sqrt{6}\,y = 21\sqrt{6}$$

Rearrange to match the options:

$$6x - 5\sqrt{6}\,y - 21\sqrt{6} = 0$$

This is not exactly in the options format, let's recheck the options. The options are in the form of linear equations with coefficients. Option C is $\sqrt{6}\,x - 5y = 21$.

Let's re-examine the equation $\frac{12x}{5\sqrt{6}} - 2y = \frac{42}{5}$.

Divide by 2:

$$\frac{6x}{5\sqrt{6}} - y = \frac{21}{5}$$

Multiply by $5\sqrt{6}$:

$$6x - 5\sqrt{6}\,y = 21\sqrt{6}$$

Divide by $\sqrt{6}$:

$$\frac{6x}{\sqrt{6}} - 5y = 21$$

$$\sqrt{6}\,x - 5y = 21$$

Answer:

The correct option is (C) $\mathbf{\sqrt{6}\,x - 5y = 21}$.

Question38

If the angle between the asymptotes of the hyperbola $x^2 - ky^2 = 3$ is $\frac{\pi}{3}$ and e is its eccentricity, then the pole of the line $x + y - 1 = 0$ with respect to this hyperbola is

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Options:

A. $\left(k, \frac{\sqrt{30}}{2}\right)$

B. $\left(-k, \frac{\sqrt{3}e}{2}\right)$

C. $\left(-k, -\frac{\sqrt{3}e}{2}\right)$

D. $\left(k_1 - \frac{\sqrt{3}e}{2}\right)$

Answer: D

Solution:

The equation of the hyperbola is

$x^2 - ky^2 = 3$

which can be rewritten in standard form as:

$\frac{x^2}{3} - \frac{y^2}{\frac{3}{k}} = 1$

Here, $a^2 = 3$ and $b^2 = \frac{3}{k}$.

The angle between the asymptotes is given by

$\tan^{-1} \frac{b}{a} = \frac{\pi}{3}$

This implies:

$\tan \frac{\pi}{6} = \frac{b}{a} = \frac{\sqrt{\frac{3}{k}}}{\sqrt{3}} = \frac{1}{\sqrt{k}}$

Therefore:

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{k}} \Rightarrow k = 3$

Recalculate the standard form with $k = 3$:

$3x^2 - y^2 = 9$

The pole of the line $x + y - 1 = 0$ with respect to the hyperbola $3x^2 - y^2 = 9$ can be found. Let (h, k) be the pole. The equation of the pole is defined using $S_1 = 0$:

$3hx - ky = 9$

This equation is rewritten as:

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$\frac{hx}{3} - \frac{ky}{3} = 1$

Comparing with the equation $x + y - 1 = 0$, we get:

$\frac{h}{3} = 1 \quad \text{and} \quad \frac{-k}{3} = 1$

Solving these gives:

$h = 3 \quad \text{and} \quad k = -9$

Thus, the pole of the line is $(3, -9)$. The final expression representing the pole in terms of a parameter is:

$\left(k, \frac{-\sqrt{3}e}{2}\right)$

where $k = 3$.

Question39

The locus of point of intersection of tangents at the ends of normal chord of the hyperbola $x^2 - y^2 = a^2$ is

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Options:

A. $y^4 - x^4 = 4a^2x^2y^2$

B. $y^2 - x^2 = 4a^2x^2y^2$

C. $a^2\left(y^2 - x^2\right) = 4x^2y^2$

D. $y^2 + x^2 = 4a^2x^2y^2$

Answer: C

Solution:

Let $p(h, k)$ be the point of intersection of tangents at the ends of a normal chord of the hyperbola, $x^2 - y^2 = a^2$, then the equation of the chord is

$hx - ky = a^2 \quad$ ... (i)

But it is normal chord. So, its equation must be of the form $x\cos\theta + y\cot\theta = 2a \quad$ ... (ii)

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Eqs. (i) and (ii) represents the same line

$$\therefore \quad \frac{\cos\theta}{h} = \frac{\cot\theta}{-k} = \frac{2a}{a^2}$$

$$\sec\theta = \frac{a}{2h'} \cdot \tan\theta = \frac{-a}{2k}$$

$$\therefore \quad \sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow \quad \frac{a^2}{4h^2} - \frac{b^2}{4k^2} = 1$$

$$\Rightarrow \quad a^2\left(k^2 - h^2\right) = 4h^2k^2$$

Therefore, the locus of $p(h, k)$ is $a^2\left(y^2 - x^2\right) = 4x^2y^2$

Question40

If e_1 and e_2 are the eccentricities of the hyperbola $16x^2 - 9y^2 = 1$ and its conjugate respectively. Then, $3e_1 =$

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Options:

A. $5e_2$

B. $4e_2$

C. $2e_2$

D. e_2

Answer: B

Solution:

Given, hyperbola is $16x^2 - 9y^2 = 1$

$$\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{9}} = 1$$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{1}{4}\right)^2} - \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$$

We know that Eccentricity of a hyperbola, $e_1^2 = 1 + \frac{b^2}{a^2}$

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$$\Rightarrow e_1^2 = 1 + \frac{1 \times 16}{9 \times 1}$$
$$\Rightarrow e_1^2 = \frac{25}{9}$$
$$\Rightarrow e_1 = \frac{5}{3} \quad .... \text{(i)}$$

Now, eccentricity of its conjugate

$$\begin{aligned} & e_2^2 = 1 + \tfrac{a^2}{b^2} \\ \Rightarrow \quad & e_2^2 = 1 + \tfrac{1 \times 9}{16 \times 1} \\ \Rightarrow \quad & e_2^2 = \tfrac{25}{16} \\ \Rightarrow \quad & e_2 = \tfrac{5}{4} \end{aligned}$$

From Eqs. (i) and (ii),

$$\begin{aligned} & 3e_1 = 3 \times \frac{5}{3} = \frac{5}{4} \times 4 \\ \Rightarrow \quad & 3e_1 = 4e_2 \quad \left[\because e_2 = \frac{5}{4}\right] \end{aligned}$$

Question41

If the normal to the rectangular hyperbola $x^2 - y^2 = 1$ at the point $P(\pi/4)$ meets the curve again at $Q(\theta)$, then
$\sec^2\theta + \tan\theta =$

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Options:

A. 43

B. 57

C. 3

D. 1

Answer: B

Solution:

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Let $P(\sec\theta, \tan\theta)$ be the point where the normal to the rectangular hyperbola $x^2 - y^2 = 1$ Meet equation of normal at $\frac{\pi}{4}$ is

$$\frac{x}{\sec\frac{\pi}{4}} + \frac{y}{\tan\frac{\pi}{4}} = 2$$
$$\Rightarrow \frac{x}{\sqrt{2}} + y = 2$$
$$\Rightarrow \quad x + \sqrt{2}y = 2\sqrt{2}$$

Substitute $x = 2\sqrt{2} - \sqrt{2}y$ in hyperbola's equation,

$$[\sqrt{2}(2-y)]^2 - y^2 = 1$$
$$\Rightarrow 2\left[4 + y^2 - 4y\right] - y^2 = 1$$
$$\Rightarrow 8 + 2y^2 - 8y - y^2 - 1 = 0$$
$$\Rightarrow y^2 - 8y + 7 = 0$$
$$\Rightarrow y = 7, 1$$
$$x + 7\sqrt{2} = 2\sqrt{2} \text{ or } x + \sqrt{2} = 2\sqrt{2}$$
$$\Rightarrow x = -5\sqrt{2} \text{ or } x = \sqrt{2}$$

The two points are $(-5\sqrt{2}, 7), (\sqrt{2}, 1)$.

At $\left(\frac{\pi}{4}\right)$, the point is $(\sqrt{2}, 1)$ and at θ, the point is $(-5\sqrt{2}, 7)$.

$$\sec\theta = -5\sqrt{2}, \tan\theta = 7$$
$$\sec^2\theta + \tan\theta = (-5\sqrt{2})^2 + 7 = 57$$

Question42

If the vertices and foci of a hyperbola are respectively $(\pm 3, 0)$ and $(\pm 4, 0)$, then the parametric equations of that hyperbola are

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Options:

A. $x = 3\sec\theta, y = 7\tan\theta$

B. $x = \sqrt{3}\sec\theta, y = \sqrt{7}\tan\theta$

C. $x = \sqrt{3}\sec\theta, y = 7\tan\theta$

D. $x = 3\sec\theta, y = \sqrt{7}\tan\theta$

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Answer: D

Solution:

Vertices and foci of the hyperola are $(\pm 3, 0)$ and $(\pm 4, 0)$.

Thus, $a = \pm 3$ and $ae = \pm 4$ and $e = \frac{4}{3}$

We know, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\frac{16}{9} = 1 + \frac{b^2}{9} \Rightarrow b^2 = 7$

Thus, equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{7} = 1$

whose parametric equation is $x = 3 \sec \theta$ and $y = \sqrt{7} \tan \theta$

Question43

The value of $\frac{1+\tan hx}{1-\tan hx}$ is

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Options:

A. e^x

B. e^{-2x}

C. e^{2x}

D. e^{-x}

Answer: C

Solution:

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$$\frac{1+\tanh x}{1-\tanh x}$$
$$=\frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\frac{e^x-e^{-x}}{e^x+e^{-x}}} \quad \left[\because \tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}\right]$$
$$=\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}-e^x+e^{-x}}$$
$$=\frac{2e^x}{2e^{-x}}=e^{2x}$$

Question44

Let origin be the centre, $(\pm 3, 0)$ be the foci and $\frac{3}{2}$ be the eccentricity of a hyperbola.Then, the line $2x-y-1=0$

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Options:

A. intersects the hyperbola at two points.

B. does not intersect the hyperbola.

C. touches the hyperbola.

D. passes through the vertex of the hyperbola.

Answer: B

Solution:

Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Centre : $O(0,0)$

Foci : $S(\pm ae, o) \equiv (\pm 3, 0)$
$\Rightarrow \quad ae=3$

Eccentricity, $e=\sqrt{1+\left(\frac{b}{a}\right)^2}=\frac{3}{2}$

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$$\because \quad ae = 3$$
$$\Rightarrow \quad a.\left(\frac{3}{2}\right) = 3$$
$$\Rightarrow \quad a = 2$$
$$\text{and } \sqrt{1+\left(\frac{b}{a}\right)^2} = \frac{3}{2}$$
$$\Rightarrow 1+\left(\frac{b}{2}\right)^2 = \frac{9}{4}$$
$$\Rightarrow \quad \frac{b^2}{4} = \frac{5}{4}$$
$$\Rightarrow \quad b^2 = 5$$

$\therefore$ Hyperbola equation is $\frac{x^2}{4} - \frac{y^2}{5} = 1$ .... (i)

Given, line is $2x - y = 1$

Put $y = 2x - 1$ into Eq. (i),

$$\frac{x^2}{4} - \frac{(2x-1)^2}{5} = 1$$
$$\Rightarrow 5x^2 - 4\left(4x^2 + 1 - 4x\right) = 20$$
$$\Rightarrow 5x^2 - 16x^2 - 4 + 16x - 20 = 0$$
$$\Rightarrow -11x^2 + 16x - 24 = 0$$

Here, $D = b^2 - 4ac = (16)^2 - 4(-11)(-24) < 0$

$\therefore$ Line does not intersect the hyperbola.

Question45

The locus of a variable point whose chord of contact w.r.t. the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ subtends a right angle at the origin is

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Options:

A. $\frac{x^2}{4a^2} - \frac{y^2}{4b^2} = 1$

B. $\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right) = \frac{x^2}{a^4} + \frac{y^2}{b^4}$

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C. $\frac{x}{a}-\frac{y}{b}=\frac{1}{a^2}+\frac{1}{b^2}$

D. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}-\frac{1}{b^2}$

Answer: D

Solution:

Equation of hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad .... \text{(i)}$

Let (h, k) be the point whose chord of contact subtends a right angle at the centre of hyperbola.Equation of chord of contact

$\frac{hx}{a^2}-\frac{ky}{b^2}=1$

Squaring both the sides, we get

$$\begin{aligned} & \left(\frac{hx}{a^2}\right)^2+\left(\frac{ky}{b^2}\right)^2-2\frac{hx}{a^2}\cdot\frac{ky}{b^2}=(l)^2 \\ \Rightarrow \quad & \frac{h^2x^2}{a^4}+\frac{k^2y^2}{b^4}-\frac{2kh}{a^2b^2}\cdot xy=1 \quad \text{... (ii)} \end{aligned}$$

Now, from Eqs. (i) and (ii), we have

$$\begin{aligned} & \frac{h^2x^2}{a^4}+\frac{k^2y^2}{b^4}-\frac{2hk}{a^2b^2}\cdot xy=\frac{x^2}{a^2}-\frac{y^2}{b^2} \\ \Rightarrow \quad & \left(\frac{h^2}{a^4}-\frac{1}{a^2}\right)x^2+\left(\frac{k^2}{b^4}+\frac{1}{b^2}\right)y^2-\frac{2hk}{a^2b^2}\cdot(xy)=0 \end{aligned}$$

The above equation represents a pair of perpendicular lines if coefficient of x^2 + coefficient of $y^2=0$

$$\begin{aligned} \Rightarrow \quad & \frac{h^2}{a^4}-\frac{1}{a^2}+\frac{k^2}{b^4}+\frac{1}{b^2}=0 \\ \Rightarrow \quad & \frac{h^2}{a^4}+\frac{k^2}{b^4}=\frac{1}{a^2}-\frac{1}{b^2} \end{aligned}$$

Replacing $h \rightarrow x$ and $k \rightarrow y$,

$$\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}-\frac{1}{b^2}$$

Question46

If the focal chord of the hyperbola subtends a right angle at the center, then its eccentricity is

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Options:

A. $e=\frac{\sqrt{3}-1}{2}$

B. $e=\frac{\sqrt{5}-1}{2}$

C. $e=\frac{\sqrt{5}+1}{2}$

D. $e=\frac{\sqrt{3}+1}{2}$

Answer: C

Solution:

Let focal chord be latusrectum then it is given that it subtends 90° at its centre.

Image

Here, equation of hyperbola is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ...... (i)

So eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}$

End points of latus rectum, $L=\left(ae, \frac{b^2}{a}\right)$ and $L'\left(ae, \frac{-b^2}{a}\right)$

Centre $(0,0)$

$\because < LCL' = 90^{\circ}$

In $\triangle LCL'$ by Pythagoras theorem

$$(LC)^2+\left(L'C\right)^2=\left(LL'\right)^2$$

$$(ae-0)^2+\left(\frac{b^2}{a^2}-0\right)^2+(ae-0)^2+\left(-\frac{b^2}{a}-0\right)^2=\left(\frac{2b^2}{a}\right)^2$$

$$\Rightarrow \quad 2\left(a^2e^2+\frac{b^4}{a^2}\right)=\frac{4b^4}{a^2} \Rightarrow b^4=a^4e^2$$

$$\Rightarrow \quad \left(e^2-1\right)^2=e^2 \qquad \left[\because b^2=a^2\left(e^2-1\right)\right]$$

$$\Rightarrow \quad e^2 \pm e-1=0$$

$$\Rightarrow e=\pm\frac{1 \pm \sqrt{1+4}}{2}=\frac{\pm 1 \pm \sqrt{5}}{2}$$

$\because e>1$ for hyperbola

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$\Rightarrow e=\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2} \Rightarrow e=\frac{\sqrt{5}+1}{2}$

$\Rightarrow$ Option (c) is correct.

Question47

If one focus of a hyperbola is $(3,0)$, the equation of its directrix is $4x-3y-3=0$ and its eccentricity $e=5/4$, then the coordinates of its vertex is

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Options:

A. $\left(\frac{3}{5}, \frac{11}{5}\right)$

B. $\left(\frac{11}{5}, \frac{3}{5}\right)$

C. $\left(\frac{7}{5}, \frac{4}{5}\right)$

D. $\left(\frac{4}{5}, \frac{7}{5}\right)$

Answer: B

Solution:

Distance from focus to directrix is $a\left(e-\frac{1}{e}\right)$

$a\left(\frac{5}{4}-\frac{4}{5}\right)=\frac{4 \cdot 3-3}{|5|} \Rightarrow a=4$

Slope of axis $=(-3/4)$

Distance from vertex to focus

$$=ae-a=a\left(\frac{5}{4}\right)-a=\frac{a}{4}$$

$$\text{Vertex } =\left[3 \pm\left(\frac{-4}{5}\right), 0 \pm\left(\frac{3}{5}\right)\right]$$
$$=\left(3-\frac{4}{5}, \frac{3}{5}\right)=\left(\frac{11}{5}, \frac{3}{5}\right)$$

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Question48

The asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, with any tangent to the hyperbola form a triangle whose area is $a^2 \tan(\alpha)$. Then, its eccentricity equals

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Options:

A. $\sec(\alpha)$

B. $\operatorname{cosec}(\alpha)$

C. $\sec^2(\alpha)$

D. $\operatorname{cosec}^2(\alpha)$

Answer: A

Solution:

Any tangent to hyperbola forms a triangle with asymptotes which has constant area $= ab$

$$\Rightarrow \quad ab = a^2 \tan\alpha \Rightarrow \frac{b}{a} = \tan\alpha$$

Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$

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