Functions

Question1

The domain of the real valued function $f(x)=\frac{3}{4-x^2}+\log_{10}\left(x^3-x\right)$ is

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Options:

A.

$(1,2)\cup(2,\infty)$

B.

$(-1,0)\cup(1,2)$

C.

$(-1,0)\cup(1,2)\cup(2,\infty)$

D.

$(-\infty,-1)\cup(1,2)\cup(2,\infty)$

Answer: C

Solution:

$$f(x)=\frac{3}{4-x^2}+\log_{10}\left(x^3-x\right)$$

for $\frac{3}{4-x^2}, 4-x^2\neq 0$

$\Rightarrow \quad x\neq\pm 2$

and for $\log_{10}\left(x^3-x\right)$,

$x^3-x>0\Rightarrow x\left(x^2-1\right)>0$

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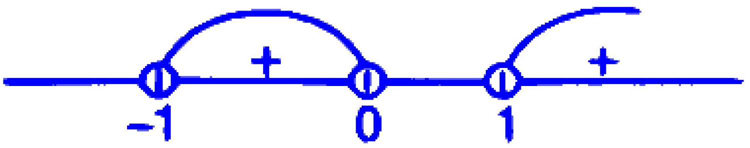

and $x \neq 2$

Here, $x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

Question2

A real valued function $f : A \to B$ defined by $f(x) = \frac{4-x^2}{4+x^2} \forall x \in A$ is a bijection. If $-4 \in A$, then $A \cap B =$

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Options:

A.

$(-1, 1]$

B.

$[0, 1]$

C.

$[0, \infty)$

D.

$(-1, 0]$

Answer: D

Solution:

$\because f(x) = \frac{4-x^2}{4+x^2}, \forall x \in A$ and $f(x)$ is bijective So, Domain of $f(x) 4 + x^2 \neq 0$ which is always true. therefore, Domain of $f(x) \in R$ Lets check if $f(x)$ be injective

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$f(x_1) = f(x_2)$

$$\Rightarrow \frac{4 - x_1^2}{4 + x_1^2} = \frac{4 - x_2^2}{4 + x_2^2}$$

$$\Rightarrow 16 + 4x_2^2 - 4x_1^2 - x_1^2 x_2^2$$
$$= 16 + 4x_1^2 - 4x_2^2 - x_1^2 x_2^2$$
$$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_2 = \pm x_1$$

for $f(x)$ to be injective.

We must have $x_1 = x_2$ and $-4 \in A$ therefore, domain must be $(-\infty, 0]$ Now, determine the range

Let, $y = \frac{4-x^2}{4+x^2}$

$$\Rightarrow y(4 + x^2) = 4 - x^2 \Rightarrow 4y + yx^2 = 4 - x^2$$
$$\Rightarrow x^2 = \frac{4(1-y)}{1+y}$$
$$\because x^2 \geq 0$$

So, $\frac{4(1-y)}{1+y} \geq 0$

So, $1 - y \geq 0$ and $1 + y > 0$

therefore, range of $f(x) \in (-1, 1]$

that means, $A \subseteq (-\infty, 0]$

and $B = (-1, 1]$

Hence, $A \cap B = (-1, 0]$

Question3

Let $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2 . x \in R$. If b and c are non-zero real numbers such that min $f(x) > \max g(x)$, then $\left|\frac{c}{b}\right|$ lies in the interval

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Options:

A.

$\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

B.

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$\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$

C.

$(\sqrt{2}, \infty)$

D.

$(0, 1)$

Answer: C

Solution:

$f(x) = x^2 + 2bx + 2c^2$ is a quadratic, opening upward

So, minmum at $(x) = \frac{-2b}{2} = -b$

then, $f(-b) = b^2 - 2b^2 + 2c^2 - b^2 + 2c^2$
and $g(x) = -x^2 - 2cx + b^2$ is quadratic opening downward.

So, maximum of $g(x)$ at $x = \frac{-(-2c)}{2(-1)} = -c$

then, $g(-c) = -c^2 + 2c^2 + b^2 = c^2 + b^2$

and given that

$\min f(x) > \max g(x)$

$\Rightarrow \quad -b^2 + 2c^2 > c^2 + b^2$

$\Rightarrow \quad c^2 > 2b^2 \Rightarrow \left(\frac{c}{b}\right)^2 > 2$

$\Rightarrow \quad \left|\frac{c}{b}\right| > \sqrt{2}$

Hence, $\left|\frac{c}{b}\right| \in (\sqrt{2}, \infty)$

Question4

Let [x] represent the greatest integer less than or equal to x, $\{x\} = x - [x]\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$. If $f(x) = \left\{x + \left[\frac{x}{1+x^2}\right]\right\}$ is a real valued function, then $f(\sqrt{2}) + f(-\sqrt{3}) =$

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Options:

A.

0.682

B.

0.318

C.

0.146

D.

1.146

Answer: A

Solution:

We have,

$$f(x)=\left\{x+\left[\frac{x}{1+x^2}\right]\right\}$$

$$f(\sqrt{2})=\left\{\sqrt{2}+\left[\frac{\sqrt{2}}{5}\right]\right\}=\{\sqrt{2}\}=\sqrt{2}-1$$

$$\text{and } f(-\sqrt{3})=\left\{-\sqrt{3}+\left[\frac{-\sqrt{3}}{4}\right]\right\}$$

$$=\{-\sqrt{3}-1\}$$

$$\Rightarrow \quad f(-\sqrt{3})=2-\sqrt{3}$$

$$\therefore f(\sqrt{2})+f(-\sqrt{3})=\sqrt{2}-1+2-\sqrt{3}$$

$$=1+\sqrt{2}-\sqrt{3}=1+1.414-1.732$$

$$=2.414-1.732=0.682$$

Question5

If the range of the function $f(x)=-3x-3$ is $\{3,-6,-9,-18\}$, then which one of the following is not in the domain of f ?

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Options:

A.

-1

B.

-2

C.

2

D.

5

Answer: A

Solution:

The function given is $f(x) = -3x - 3$.

The range of the function is given as $\{3, -6, -9, -18\}$.

To find the elements in the domain corresponding to this range, we need to find the x values for which $f(x)$ equals each value in the range.

We set $f(x)$ equal to each value in the range and solve for x:

1. Set $f(x) = 3$:

   $-3x - 3 = 3$

   $-3x = 3 + 3$

   $-3x = 6$

   $x = \frac{6}{-3}$

   $x = -2$

2. Set $f(x) = -6$:

   $-3x - 3 = -6$

   $-3x = -6 + 3$

   $-3x = -3$

   $x = \frac{-3}{-3}$

   $x = 1$

3. Set $f(x) = -9$:

   $-3x - 3 = -9$

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$-3x = -9 + 3$

$-3x = -6$

$x = \frac{-6}{-3}$

$x = 2$

4. Set $f(x) = -18$:

$-3x - 3 = -18$

$-3x = -18 + 3$

$-3x = -15$

$x = \frac{-15}{-3}$

$x = 5$

So, the domain of f corresponding to the given range $\{3, -6, -9, -18\}$ is the set of x values $\{-2, 1, 2, 5\}$.

Now we need to check the given options to see which one is not in this domain:

A) -1: Is -1 in $\{-2, 1, 2, 5\}$? No.

B) -2: Is -2 in $\{-2, 1, 2, 5\}$? Yes. ($f(-2) = 3$)

C) 2: Is 2 in $\{-2, 1, 2, 5\}$? Yes. ($f(2) = -9$)

D) 5: Is 5 in $\{-2, 1, 2, 5\}$? Yes. ($f(5) = -18$)

The value -1 is not in the calculated domain.

The final answer is $\boxed{-1}$.

Question6

$[t]$ denotes the greatest integer function and $[t - m] = [t] - m$ when $m \in Z$.If $k = 2[2x - 1] - 1$ and $3[2x - 2] + 1 = 2[2x - 1] - 1$, then the range of $f(x) = [k + 5x]$ is

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Options:

A.

$\{7, 8, 9\}$

B.

$\{4, 5, 6\}$

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C.

$\{5,6,7\}$

D.

$\{6,7,8\}$

Answer: D

Solution:

$$\begin{aligned} \because [2x-2] &= [(2x-1)-1] \\ &= [2x-1]-1 \\ \text{let, } \quad [2x-1] &= a \\ \Rightarrow \quad [2x-2] &= a-1 \end{aligned}$$

$$\begin{aligned} &\text{so, } 3(a-1)+1 = 2a-1 \\ &\Rightarrow \quad 3a-3+1 = 2a-1 \Rightarrow a=1 \end{aligned}$$

thus $[2x-1]=1$

$$\begin{aligned} &\Rightarrow \quad 1 \leq (2x-1) < 2 \Rightarrow 2 \leq 2x < 3 \\ &\Rightarrow \quad 1 \leq x < 1.5 \end{aligned}$$

so, $x \in [1, 1.5)$

Now, $k = 2[2x-1]-1 = 2(1)-1 = 1$

for $x \in [1, 1.5)$

$$\begin{aligned} f(x) \quad & = [k+5x] = [1+5x] \\ \text{at} \quad & x=1, f(x) = [6] = 6 \quad (\text{ minimum }) \\ \text{at} \quad & x=1.5, f(x) = [1+5 \times 1.5] \\ & = [8.5] = 8 \quad (\text{maximum}) \end{aligned}$$

so, $f(x)$ can be $6,7,8$

Hence, the range of $f(x) = \{6,7,8\}$

Question7

If $f(x)=(x+1)^2-1, x \geq -1$, then $\left\{x \mid f(x)=f^{-1}(x)\right\}$ is

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Options:

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A.

$\{0,-1\}$

B.

$\{-1,0,1\}$

C.

$\left\{-1,0,\frac{-3+\sqrt{3}i}{2},\frac{-3-\sqrt{3}i}{2}\right\}$

D.

an empty set

Answer: A

Solution:

$\because \quad y=f(x)=(x+1)^2-1$

$\Rightarrow y+1=(x+1)^2$

$\Rightarrow \quad x=\sqrt{y+1}-1$

thus, $f^{-1}(x)=\sqrt{x+1}-1$, defined for

$x\geq -1$

Now, $f(x)=f^{-1}(x)$

$\Rightarrow (x+1)^2=\sqrt{x+1}$

let $x+1=t$

$\Rightarrow \quad t^4=t\Rightarrow t\left(t^3-1\right)=0$

therefore, $t=0$

$\Rightarrow \quad x+1=0\Rightarrow x=-1$

or $\quad t=1$

$\Rightarrow \quad x+1=1\Rightarrow x=0$

Hence, $\{x\mid f(x)=f^{-1}(x)\}$

$=\{0,-1\}$

Question8

Consider the following statements.

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Statement I	A function $f : A \to B$ is said to be one-one if and only if $f(x) \neq f(y) \Rightarrow x \neq y$
Statement II	A relation $f : A \to B$ is said to be a function if $x \neq y \Rightarrow f(x) \neq f(y)$

Then, which one of the following is true?

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Options:

A.

Only statement I is true.

B.

Only statement II is true.

C.

Both Statement I and Statement II are true.

D.

Neither Statement I nor Statement II is true.

Answer: A

Solution:

It is obvious. Since, f is said to be one-one if $f(x) = f(y) \Rightarrow x = y$ or $f(x) \neq f(y) \Rightarrow x \neq y$. Also, in a function, one image may have more than one pre-image.

Question9

The set of all real values of x for which $f(x) = \sqrt{\frac{|x|-2}{|x|-3}}$ is a well defined function is

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Options:

A.

$(-3,-2] \cup (2,3]$

B.

$R-[-3,-2) \cup (2,3]$

C.

$R-[-3,3]$

D.

$(-3,3)$

Answer: B

Solution:

$f(x)=\sqrt{\frac{|x|-2}{|x|-3}}$

$\Rightarrow \frac{|x|-2}{|x|-3} \geq 0$

Case I $|x|-2 \geq 0$ and $|x|-3>0$

$\Rightarrow \quad |x| \geq 2$ and $|x|>3$
$\Rightarrow \quad |x|>3$
$\Rightarrow \quad x>3$ or $x<-3$

Case II $|x|-2 \leq 0$ and $|x|-3<0$

$|x| \leq 2$ and $|x|<3$
$\Rightarrow \quad |x| \leq 2$
$-2 \leq x \leq 2$
$|x|-3 \neq 0 \Rightarrow |x| \neq 3$
$\Rightarrow x \neq 3$ and $x \neq -3$

Combining these, we get

$(-\infty,-3) \cup [-2,2] \cup (3, \infty)$

Finally we write

$R-[-3,-2) \cup (2,3]$

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Question10

Let $f: N \rightarrow N$ be a function such that $f(x+y)=f(x)+f(y)+xy$ for every $x, y \in N$. If $f(\mathbb{1})=2$, then $\sum_{k=0}^{10} f(k)=$

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Options:

A.

1650

B.

275

C.

550

D.

1025

Answer: B

Solution:

$f(x+y)=f(x)+f(y)+xy \qquad \ldots(i)$

$x, y \in N$

From Eq. (i),

$$f(1)=2=\frac{1(1+3)}{2}$$
$$x=1, y=1$$
$$f(2)=f(1)+f(1)+1$$
$$f(2)=5=\frac{2(2+3)}{2}$$

From Eq. (i), $x=1, y=2$

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$$f(3) = f(1) + f(2) + 2$$
$$= 2 + 5 + 2 = 9 = \frac{3(3+3)}{2}$$
$$f(4) = f(2) + f(2) + 4$$
$$= 14 = \frac{4(4+3)}{2}$$
$$\Rightarrow f(x) = \frac{n(n+3)}{2} = \frac{n^2}{2} + \frac{3n}{2}$$
$$\sum_{k=0}^{10} f(k) = \sum_{k=1}^{10} \frac{k(k+3)}{2} = \frac{1}{2}\sum_{k=1}^{10} k^2 + \frac{3}{2}\sum_{k=1}^{10} k$$
$$= \frac{1}{2}(385 + 165) = 275$$

Question11

If a real valued function $f : [-1, 2] \to B$ defined by

$$f(x) = \begin{cases} 1 - x, & \text{when } -1 \leq x \leq 1 \\ x - 1, & \text{when } 1 < x \leq 2 \end{cases}$$

is a surjection, then $B =$

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Options:

A.

$[-1, 2]$

B.

$[-1, 1]$

C.

$[0, 2]$

D.

$[0, 1]$

Answer: C

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Solution:

$$f(x)=\begin{cases}1-x, & x\in[-1,1]\\ x-1, & x\in(1,2]\end{cases}$$

$$\begin{aligned}-1&\leq x\leq 1\\ +1&\geq -x\geq -1\\ +2&\geq 1-x\geq +0\\ x&\in[-1,1]\\ f(x)&\in[0,2] \qquad \ldots(i)\end{aligned}$$

Now $x\in(1,2], f(x)\in(0,1]$ $\ldots(ii)$

From Eqs. (i) and (ii) range $\equiv[0,2]$

Question12

The sum of the least positive integer and the greatest negative integer in the range of the function $f(x)=\frac{x^2-5x+7}{x^2-5x-7}$ is

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Options:

A.

0

B.

1

C.

2

D.

-1

Answer: B

Solution:

Given, $f(x)=\frac{x^2-5x+7}{x^2-5x-7}=y$

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$\Rightarrow \quad (y-1)x^2 - 5(y-1)x - (7y+7) = 0$

for $x \in R$

$D \geq 0$

$\Rightarrow \quad 25(y-1)^2 + 4(y-1)(7y+\pi \geq 0$

$\Rightarrow \quad (y-1)[25y - 25 + 28y + 28] \geq 0$

$(y-1)(53y+3) \geq 0$

$y \in \left(-\infty, \frac{-3}{53}\right] \cup (1, \infty)$ at $y = 1, x$ is not

possible

$\therefore$ Required sum $= 2 + (-1) = 1$

Question13

The interval in which the curve represented by $f(x) = 2x + \log\left(\frac{x}{2+x}\right)$ is

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Options:

A.

$(-\infty, 0)$

B.

$(-2, \infty)$

C.

$(-\infty, -2) \cup (0, \infty)$

D.

$(-2, 0)$

Answer: C

Solution:

$f(x) = 2x + \log\left(\frac{x}{2+x}\right), \frac{x}{2+x} > 0$ and $\frac{x}{2+x} \neq 0 \Rightarrow x \neq 0$

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$x \in (-\infty, -2) \cup (0, \infty)$

$$f'(x) = 2 + \left(\frac{2+x}{x}\right) \cdot \left(\frac{2}{(2+x)^2}\right)$$

$$= 2 + \frac{2}{x(x+2)}, x \neq -2$$

$$f'(x) = 0, 2 = \frac{-2}{x(x+2)}$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0 \Rightarrow x = -1$$

$$x = -1 \notin (-\infty, -2) \cup (0, \infty)$$

$$f'(x) > 0 \text{ for } x \in (-\infty, -2) \cup (0, \infty)$$

So, required interval,

$(-\infty, -2) \cup (0, \infty)$

Question14

The set of real values of x such that $f(x) = \sqrt{\frac{[x]-1}{[x]^2-[x]-6]}}$ is a real valued function is

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Options:

A.

$[1, \infty)$

B.

$(-\infty, -2) \cup [4, \infty)$

C.

$[-1, 3)$

D.

$[-1, 2) \cup [4, \infty)$

Answer: D

Solution:

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$$f(x)=\sqrt{\frac{[x]-1}{[x]^2-[x]-6}}$$

$f(x)$ is defined when $\frac{[x]-1}{[x]^2-[x]-6} \geq 0$

$$\frac{[x]-1}{([x]-3)([x]+2)} \geq 0$$

$\Rightarrow \quad -2<[x] \leq 1$ or $[x] \geq 3$

$\Rightarrow \quad -1 \leq x<2$ or $x \geq 4$ or $x \in[-1,2) \cup[4, \infty)$

Question15

If a function $f: Z \rightarrow Z$ is defined by $f(x)=x-(-1)^x$, then $f(x)$ is

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Options:

A.

one-one, but not onto

B.

onto but not one-one

C.

both one-one and onto

D.

neither one-one nor onto

Answer: C

Solution:

$$f(x)=x-(-1)^x=\begin{cases}x-1, & x \in \text{ even} \\ x+1, & x \in \text{ odd}\end{cases}$$

$f(x)$ is a linear function. Hence, it is one-one and its range is integer for integral value of x so it is onto.

$\because$ It is both one-one and onto.

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Question16

Domain of the real valued function $f(x) = \log(x^2 - 1) + x \coth^{-1} x$ is

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Options:

A.

R

B.

$(-1, 1)$

C.

$R - [-1, 1]$

D.

$R - [0, 1]$

Answer: C

Solution:

$f(x) = \log(x^2 - 1) + x \cot h^{-1} x \log(x^2 - 1)$ is defined when $x^2 - 1 > 0 x^2 > 1$

$-\infty < x < -1$ or $1 < x < \infty$ And $\cot h^{-1} x$ is defined when $|x| > 1 \Rightarrow -\infty < x < -1$ or $1 < x < \infty$

$\because$ Domain of $f(x)$ is $x \in (-\infty, -1) \cup (1, \infty)$ or $R - [-1, 1]$

Question17

The domain and range of a real valued function $f(x) = \cos x - 3$ are respectively

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Options:

A.

$R \backslash \{0\}$ and $[-1, 1]$

B.

R and $[-1, 1]$

C.

$R \backslash \{0\}$ and $[-4, -2]$

D.

R and $[-4, -2]$

Answer: D

Solution:

We have $f(x) = \cos x - 3$
Domain $= R$
Range $-1 \leq \cos x \leq 1$
$-1 - 3 \leq \cos x - 3 \leq 1 - 3$
$-4 \leq f(x) \leq -2$
Range $= [-4, -2]$

Question18

If $f : R \to R$ and $g : R \to R$ are two functions defined by $f(x) = 2x - 3$ and $g(x) = 5x^2 - 2$, then the least value of the function $(g \circ f)(x)$ is

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Options:

A.

-2

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B.

2

C.

-4

D.

4

Answer: A

Solution:

Given, $f(x)=2x-3$ and $g(x)=5x^2-2$
$g \circ f(x)=5[f(x)]^2-2=5(2x-3)^2-2$
$\because$ Least value of $g \circ f(x)=-2$

Question19

If $A \subseteq Z$ and the function $f: A \rightarrow R$ is defined by $f(x)=\frac{1}{\sqrt{64-(0.5)^{24+x-x^2}}}$, then the sum of all absolute value of elements of A is

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Options:

A. 36

B. 5

C. 25

D. 11

Answer: C

Solution:

We have, $f(x)=\frac{1}{\sqrt{64-(0.5)^{24+x-x^2}}}$

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$f(x)$ will be defined, if $64-(0.5)^{24+x-x^2}>0$

$$\begin{aligned} &= 64 > (0.5)^{24+x-x^2} \\ &= 64 > 2^{x^2-x-24} \\ &= 2^6 > 2^{x^2-x-24} \\ &= x^2-x-24 < 6 \\ &= x^2-x-30 < 0 \\ &= (x+5)(x-6) < 0 \\ &= -5 < x < 6 \end{aligned}$$

$\because x=-4,-3,-2-1,0,1,2,3,4,5$

Then, $A=\{-4,-3,-2,-1,0,1,2,3,4,5\}$

Sum of all absolute values of A is

$$\begin{aligned} &= 4+3+2+1+0+1+2+3+4+5 \\ &= 25 \end{aligned}$$

Question20

Which of the following function are odd?

I. $f(x)=x\left(\frac{e^x-1}{e^x+1}\right)$

II. $f(x)=k^x+k^{-x}+\cos x$

III. $f(x)=\log\left(x+\sqrt{x^2+1}\right)$

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Options:

A. II

B. I II

C. III

D. I

Answer: C

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Solution:

We have, $f(x)=\log\left(x+\sqrt{x^2+1}\right)$

$$\begin{aligned} \Rightarrow \quad f(-x) &= \log\left(-x+\sqrt{x^2+1}\right) \\ &= \log\left[\sqrt{x^2+1}-x\times\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right] \\ &= \log\left[\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}\right] \\ &= \log\left(\frac{1}{\sqrt{x^2+1}+x}\right) \\ &= -\log\left(x+\sqrt{x^2+1}\right) \\ &= -f(x) \end{aligned}$$

$\therefore f(x)$ is an odd function.

Question21

The range of the real valued function $f(x)=\frac{15}{3\sin x+4\cos x+10}$ is

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Options:

A. $[0,3]$

B. $[-1,3]$

C. $[1,3]$

D. $[-1,1]$

Answer: C

Solution:

We need to determine the range of the function:

$f(x)=\frac{15}{3\sin x+4\cos x+10}$

Start with the expression inside the denominator:

$3\sin x+4\cos x$

The range of this expression is given by:

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$-\sqrt{3^2+4^2} \leq 3\sin x + 4\cos x \leq \sqrt{3^2+4^2}$

Calculating the bounds:

$-\sqrt{9+16} \leq 3\sin x + 4\cos x \leq \sqrt{9+16}$

$-5 \leq 3\sin x + 4\cos x \leq 5$

Adding 10 to the entire inequality, we have:

$5 \leq 3\sin x + 4\cos x + 10 \leq 15$

The function $f(x)$ can then be rewritten as:

$f(x) = \frac{15}{3\sin x + 4\cos x + 10}$

Therefore, the minimum value of the denominator is 5, and the maximum is 15.

The range of $f(x)$ can be calculated as follows:

Minimum value of $f(x)$:

$f(x)_{\min} = \frac{15}{15} = 1$

Maximum value of $f(x)$:

$f(x)_{\max} = \frac{15}{5} = 3$

Hence, the range of the function $f(x)$ is $[1,3]$.

Question22

Define the function, f, g and h from R to R such that $f(x) = x^2 - 1, g(x) = \sqrt{x^2+1}$ and $h(x) = \begin{cases} 0, \text{ if } & x \leq 0 \\ x, \text{ if } & x \geq 0 \end{cases}$ consider the following statements

(i) fog is invertible

(ii) h is an identify function

(iii) $f \circ g$ is not invertible

(iv) $(h \circ f \circ g)x = x^2$

Then, which one of the following is true ?

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Options:

A. II, IV

B. II, III

C. III, IV

D. I, II

Answer: C

Solution:

Given the functions:

$f(x) = x^2 - 1$

$g(x) = \sqrt{x^2 + 1}$

$$h(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Let's analyze the statements:

Function Composition:

$f \circ g = f(g(x)) = f(\sqrt{x^2 + 1}) = (\sqrt{x^2 + 1})^2 - 1 = x^2 + 1 - 1 = x^2$.

Since $f \circ g = x^2$, it is clear that this function is not one-to-one because different values of x (such as x and $-x$) yield the same x^2. Therefore, $f \circ g$ is not invertible.

Composition involving h:

$h \circ f \circ g = h(f(g(x))) = h(x^2)$.

The function $h(x^2)$ has the rule:

$$h(x^2) = \begin{cases} 0, & \text{if } x \leq 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$$

Since x^2 is always non-negative and $h(x^2) = x^2$ for $x \geq 0$, this simplifies to $h(x^2) = x^2$ across non-negative ranges, reinforcing that $h(x^2)$ is equivalent to x^2.

Considering these observations, statements (III) and (IV) are true:

Statement (III): $f \circ g$ is not invertible.

Statement (IV): $h \circ f \circ g = x^2$.

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Thus, statements (III) and (IV) are correct.

Question23

The domain of the real valued function $f(x)=\sqrt{9-\sqrt{x^2-144}}$ is

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Options:

A. $[-15,-12] \cup [12,15]$

B. $(-\infty,-12] \cup [12,\infty)$

C. $[-15,15]$

D. $[-12,-12]$

Answer: A

Solution:

For real values of $f(x)$

$\sqrt{9-\sqrt{x^2-144}} \geq 0$

On squaring both sides, we get

$$9-\sqrt{x^2-144} \geq 0$$
$$9 \geq \sqrt{x^2-144}$$

Again, squaring on both sides, we get

$$9^2 \geq x^2-144$$
$$81+144 \geq x^2$$
$$225 \geq x^2$$
$$x^2 \leq 225$$

On taking principal square root

$|x| \leq 15$

i.e. $x \leq 15$ or $x \geq -15$

$$Now, \quad x^2-144 \geq 0$$
$$x^2 \geq 144$$

On taking principal square root

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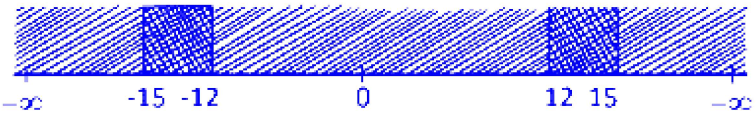
$$\sqrt{x^2} \geq \sqrt{12^2}$$
$$|x| \geq 12$$

i.e. $x \leq -12$ or $x \geq 12$



We can see on number line the domain of the given function will be cross shadow area.

i.e. $[-15, -12] \cup [12, 15]$

Question24

The real valued function $f : R \rightarrow \left[\frac{5}{2}, \infty\right)$ defined by $f(x) = |2x + 1| + |x - 2|$ is

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Options:

A. One - one function but not onto

B. Onto function but not one - one

C. Bijection

D. Neither one - one function not onto

Answer: B

Solution:

Given function $f : R \rightarrow \left[\frac{5}{2}, \infty\right]$ is defined by

$f(x) = |2x + 1| + |x - 2|$

Consider the graph

There are two points, where the expressions inside the absolute values change signs.

$x = -\frac{1}{2}$ or -0.5 and $x = 2$

If $x < -0.5$

$$\begin{aligned} f(x) &= -2x - 1 - x + 2 \\ &= -3x + 1 \end{aligned}$$

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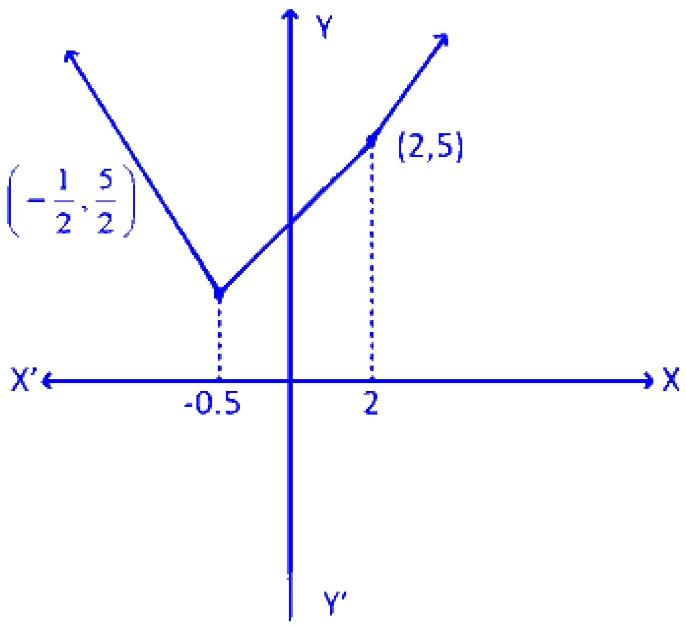
If -0.5 < x < 2

$f(x) = 2x + 1 - x + 2 = x + 3$

If $x > 2$

$f(x) = 2x + 1 + x - 2 = 3x - 1$

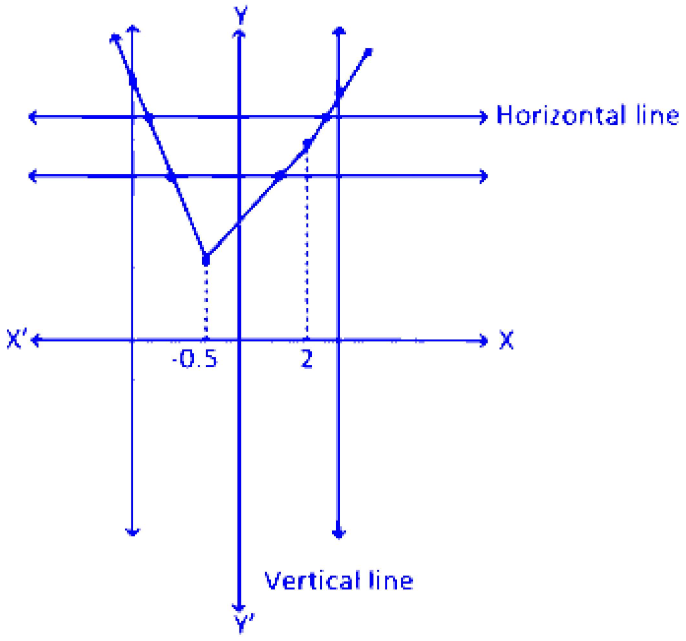


Each vertical line should intersect the graph at exactly one point. If not then function is not onto.

If a horizontal line intersects the graph more than once, the function is not one-one.

Clearly, from graph, through horizontal line two cut points implies that not one-one and through vertical line one cut point implies that onto.

Hence, function is onto but not one-one.



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Question25

If $3f(x) - 2f(1/x) = x$, then $f(2) =$

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Options:

A. 1

B. $1/2$

C. 2

D. $7/2$

Answer: B

Solution:

We start with the equation:

$3f(x) - 2f\left(\frac{1}{x}\right) = x.$

Differentiating both sides with respect to x, we have:

$3f'(x) - 2 \cdot f'\left(\frac{1}{x}\right) \times \left(-\frac{1}{x^2}\right) = 1.$

This simplifies to:

$3f'(x) + \frac{2}{x^2} f'\left(\frac{1}{x}\right) = 1.$

First, set $x = 2$:

$3f'(2) + \frac{2}{(2)^2} f'\left(\frac{1}{2}\right) = 1.$

This results in:

$3f'(2) + \frac{1}{2} f'\left(\frac{1}{2}\right) = 1.$

Multiply throughout by 2 to clear the fraction:

$6f'(2) + f'\left(\frac{1}{2}\right) = 2. \quad (i)$

Next, set $x = \frac{1}{2}$:

$3f'\left(\frac{1}{2}\right) + \frac{2}{\left(\frac{1}{2}\right)^2} f'(2) = 1.$

This becomes:

$3f'\left(\frac{1}{2}\right) + 8f'(2) = 1.$

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Solve for $f'\left(\frac{1}{2}\right)$:

$f'\left(\frac{1}{2}\right)=\frac{1-8f'(2)}{3}. \quad (ii)$

Substitute the expression for $f'\left(\frac{1}{2}\right)$ from equation (ii) into equation (i):

$6f'(2)+\frac{1-8f'(2)}{3}=2.$

Multiply through by 3 to eliminate the fraction:

$18f'(2)+1-8f'(2)=6.$

Simplify:

$10f'(2)=5.$

So, we find:

$f'(2)=\frac{5}{10}=\frac{1}{2}.$

Question26

The domain of the real valued function $f(x)$ $=\log_2\log_3\log_5\left(x^2-5x+11\right)$ is

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Options:

A. $(2,\infty)$

B. $(-\infty,3)$

C. $(2,3)$

D. $(-\infty,2)\cup(3,\infty)$

Answer: D

Solution:

Mathematics

(d) We have,

$f(x)=\log_2\log_3\log_5\left(x^2-5x+11\right)$

$\log_5$ must be positive.

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$\log_5\left(x^2-5x+11\right)$ is defined if

$x^2-5x+11>0$

Now, $D=b^2-4ac=(-5)^2-4(1)(11)$

$=-19$

$\because$ If the discriminant is negative and $a>0, x^2-5x+11>0, \forall$ real x.

Now, $\log_3$ must be positive.

$\log_3\left(\log_5\left(x^2-5x+11\right)\right)$ is defined if

$\log_5\left(x^2-5x+11\right)>0$

Since, $\log_5\left(x^2-5x+11\right)>0$

$$\Rightarrow \quad x^2-5x+11>5^{\circ}$$
$$\left[\because \log_a x=b, \text{ then } x=a^b\right]$$
$$\Rightarrow \quad x^2-5x+11>1$$
$$\Rightarrow \quad x^2-5x+10>0$$
$$D=b^2-4ac=(-5)^2-4(1)(10)=-15$$

The discriminate is negative, $x^2-5x+10$ has no real roots and is always positive. $\therefore x^2-5x+10>0, \forall$ real x

Now, $\log_2$ must be positive.

$\log_2\left(\log_3\left(\log_5\left(x^2-5x+11\right)\right)\right.$ is defined

If

$$\log_3\left(\log_5\left(x^5-5x+11\right)\right)>0$$
$$\log_3\left(\log_5\left(x^2-5x+11\right)\right)>0$$
$$\log_5\left(x^2-5x+11\right)>3^{\circ}$$

$$\Rightarrow \quad \log_5\left(x^2-5x+11\right)>1$$
$$\Rightarrow \quad x^2-5x+11>5$$
$$\Rightarrow \quad x^2-5x+6>0$$
$$(x-2)(x-3)>0$$

The quadratic x^2-5x+6 changes sign at $x=2$ and $x=3$.

Thus, $x^2-5x+6>0$ for $x\in(-\infty, 2)\cup(3, \infty)$.

Thus, the domain of $f(x)$ is $(-\infty, 2)\cup(3, \infty)$.

Question27

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The range of the real valued function $f(x) = \left(\frac{x^2+2x-15}{2x^2+13x+15}\right)$ is

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Options:

A. $R - \left\{-5, -\frac{3}{2}\right\}$

B. $R - \left\{-5, \frac{1}{2}\right\}$

C. $R - \left\{\frac{1}{2}, \frac{8}{7}\right\}$

D. $R - \left\{-\frac{3}{2}, \frac{8}{7}\right\}$

Answer: C

Solution:

To find the range of the function $f(x) = \frac{x^2+2x-15}{2x^2+13x+15}$, we begin by factorizing the numerator and denominator.

Denominator Factorization:

$2x^2 + 13x + 15 = 0$

$2x^2 + 10x + 3x + 15 = 0$

$2x(x+5) + 3(x+5) = 0$

$(x+5)(2x+3) = 0 \quad \Rightarrow \quad x = -5, \frac{-3}{2}$

Numerator Factorization:

$x^2 + 2x - 15 = 0$

$x^2 + (5x - 3x) - 15 = 0$

$x(x+5) - 3(x+5) = 0$

$(x+5)(x-3) = 0 \quad \Rightarrow \quad x = -5, 3$

Domain and Range:

The domain is $(-\infty, -5) \cup (-5, \frac{-3}{2}) \cup (\frac{-3}{2}, \infty)$.

The range is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{8}{7}\right) \cup \left(\frac{8}{7}, \infty\right)$, excluding $\frac{1}{2}$ and $\frac{8}{7}$, which do not correspond to any real x values for $f(x)$.

Thus, the range of the function is:

$R - \left\{\frac{1}{2}, \frac{8}{7}\right\}$

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Question28

$f: R \rightarrow R$ is defined by $f(x+y)=f(x)+12y, \forall x, y \in R$. If $f(1)=6$, then $\sum_{r=1}^{n} f(r)=$

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Options:

A. n^2

B. $5n^2$

C. $6n^2$

D. $\frac{3n(n+1)}{2}$

Answer: C

Solution:

Given, $f(\Omega)=6$

and $f(x+y)=f(x)+12y$

Now,

$$\begin{aligned} f(2) &= f(1+1)=f(1)+12(1) \\ &= 6+12=18 \\ f(3) &= f(1+2)=f(1)+12(2) \\ &= 6+24=30 \\ f(4) &= f(1+3)=f(1)+12(3) \\ &= 6+36=42 \end{aligned}$$

and, so on

$$\begin{aligned} \therefore \quad f(n) &= 6+(n-1)12 \\ &= 6+12n-12=12n-6 \end{aligned}$$

Now, $\sum_{r=1}^{n} f(r)=f(1)+f(2)+f(3)+\ldots+f(n)$

$$\begin{aligned} &= 6+18+30+42+\ldots+(12n-6) \\ &= 6[1+3+5+7+\ldots+(2n-1)] \\ &= 6n^2 \end{aligned}$$

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Question29

The domain of the real valued function $f(x) = \sqrt{2+x} + \sqrt{3-x}$ is

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Options:

A. $(-2, 3)$

B. $[-2, 3)$

C. $(-2, 3]$

D. $[-2, 3]$

Answer: D

Solution:

To determine the domain of the function $f(x) = \sqrt{2+x} + \sqrt{3-x}$, we need to ensure that all expressions under the square roots are non-negative.

For $\sqrt{2+x}$ to be defined, the expression inside the square root must be non-negative:

$2 + x \geq 0 \quad \Rightarrow \quad x \geq -2$

For $\sqrt{3-x}$ to be defined, the expression inside the square root must also be non-negative:

$3 - x \geq 0 \quad \Rightarrow \quad x \leq 3$

Combining both conditions, the value of x must satisfy:

$-2 \leq x \leq 3$

Therefore, the domain of the function $f(x)$ is the closed interval $[-2, 3]$.

Question30

Let $f(x) = 3 + 2x$ and $g_n(x) = (f \circ f \circ fo \ldots$ in times $)(x), \forall n \in N$ if all the lines $y = g_n(x)$ pass through a fixed point (α, β), then $\alpha + \beta =$

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Options:

A. -5

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B. -4

C. -3

D. -6

Answer: D

Solution:

Let $f(x) = 3 + 2x$ and $g_n(x)$ be the composition of n functions f with itself. If all the lines $y = g_n(x)$ pass through a fixed point (α, β), we need to find $\alpha + \beta$.

The function $f : \mathbb{R} \to \mathbb{R}$ is defined as:

$f(x) = 2x + 3 = g_1(x)$

Now, find $f \circ f(x) = g_2(x)$:

$$\begin{aligned} g_2(x) &= 2(2x + 3) + 3 \\ &= 4x + 6 + 3 \\ &= 4x + 9 \end{aligned}$$

Next, find $f \circ f \circ f(x) = g_3(x)$:

$$\begin{aligned} g_3(x) &= 2(4x + 9) + 3 \\ &= 8x + 18 + 3 \\ &= 8x + 21 \end{aligned}$$

Observing a pattern, we have:

$$\begin{aligned} g_1(x) &= 2x + 3 = 2^1(x + 3) - 3 \\ g_2(x) &= 4x + 9 = 2^2(x + 3) - 3 \\ g_3(x) &= 8x + 21 = 2^3(x + 3) - 3 \\ &\vdots \\ g_n(x) &= 2^n(x + 3) - 3 \end{aligned}$$

Then, the expression becomes:

$$\begin{aligned} y = g_n(x) &= 2^n(x + 3) - 3 \\ y + 3 &= 2^n(x + 3) \end{aligned}$$

Now, consider the equation of a line passing through (α, β):

$y - \beta = m(x - \alpha)$

Comparing the equations,

$$\begin{aligned} \alpha &= -3 \\ \beta &= -3 \end{aligned}$$

Thus, $\alpha + \beta = -3 + (-3) = -6$.

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Question31

Let $a > 1$ and $0 < \mathrm{b} < 1$. If $f : R \rightarrow [0, 1]$ is defined by
$f(x) = \begin{cases} a^x, & -\infty < x < 0 \\ b^x, & 0 \leq x < \infty \end{cases}$, then $f(x)$ is

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Options:

A. a bijection

B. one-one but not onto

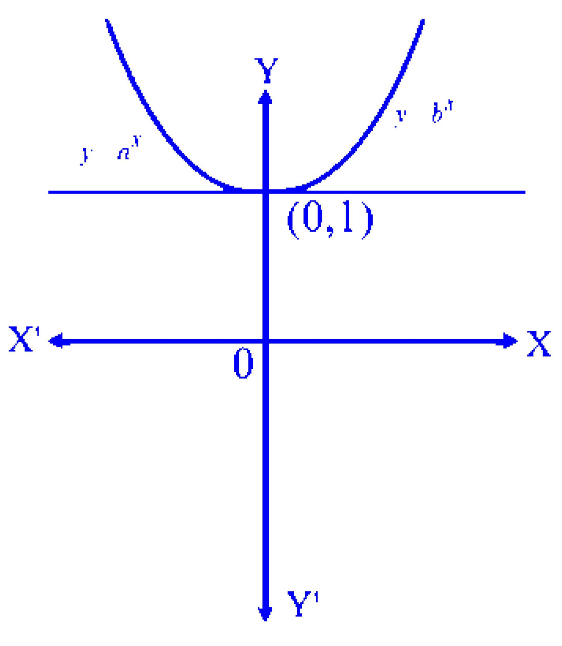
C. onto but not one-one

D. neither one-one nor onto

Answer: D

Solution:

The graph of $f(x)$ is shown below.



$\therefore$ The function is not onto.

The function is many one as image of $f(x)$ is for every value of a and b.

Hence, the function is neither one-one nor onto.

Question32

If $P(x)=x^5+ax^4+bx^3+cx^2+dx+e$ is a polynomial such that $P(0)=1, P(1)=2, P(2)=5, P(3)=10$ and $P(4)=17$, then $P(5)=$

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Options:

A. 26

B. 146

C. 126

D. 76

Answer: B

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Solution:

We have,

$$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

$\because \quad P(0) = 1$

$\Rightarrow \quad 0+0+0+0+0+e = 1$

$\Rightarrow \quad e = 1$

$\because \quad P(1) = 2$

$\Rightarrow \quad 1+a+b+c+d+1 = 2$

$\Rightarrow \quad a+b+c+d = 0 \ldots\ldots \text{(i)}$

$\Rightarrow \quad 32+16a+8b+4c+2d+1 = 5$

$\Rightarrow \quad 16a+8b+4c+2d = -28$

$\Rightarrow \quad 8a+4b+2c+d = -14 \ldots\ldots \text{(ii)}$

$\because P(3) = 10$

$\Rightarrow \quad 243+81a+27b+9c+3d+1 = 10$

$\Rightarrow 81a+27b+9c+3d = -234$

$\Rightarrow \quad 27a+9b+3c+d = -78 \ldots\ldots \text{(iii)}$

$P(4) = 17$

$\Rightarrow 1024+256a+64b+16c+4d+1 = 17$

$\Rightarrow 256a+64b+16c+4d = -1008$

$\Rightarrow 64a+16b+4c+d = -252 \ldots\ldots \text{(iv)}$

From Eqs. (i), (ii), (iii) and (iv), we get

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -14 \\ -78 \\ -252 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -78 \\ -252 \end{bmatrix}$$

$R_2 \to R_2 - 8R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -78 \\ -252 \end{bmatrix}$$

$R_3 \to R_3 - 27R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 0 & -18 & -24 & -26 \\ 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -78 \\ -252 \end{bmatrix}$$

$R_4 \to R_4 - 64R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 0 & -18 & -24 & -26 \\ 0 & -48 & -60 & -63 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -78 \\ -252 \end{bmatrix}$$

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$R_3 \to R_3 - \frac{9}{2}R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 0 & 0 & 3 & \frac{11}{2} \\ 0 & -48 & -60 & -63 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -15 \\ -252 \end{bmatrix}$$

$R_4 \to R_4 - 12R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 0 & 0 & 3 & \frac{11}{2} \\ 0 & 0 & 12 & 21 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -15 \\ -84 \end{bmatrix}$$

$R_4 \to R_4 - 4R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -7 \\ 0 & 0 & 3 & \frac{11}{2} \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -14 \\ -15 \\ -24 \end{bmatrix}$$

Thus,

$$\begin{aligned} a+b+c+d &= 0 \\ -4b-6c-7d &= -14 \\ 3c+\frac{11}{2}d &= -15 \\ -d &= -24 \end{aligned}$$

On solving, we get

$a=-10, b=35, c=-49, d=24$

Now, $P(5) = 3125 + 625 \times (-10)$
$+125 \times 35 + 25 \times (-49) + 24 \times 5 + 1$
$= 3125 - 6250 + 4375 - 1225 + 120 + 1$
$= 146$

Question33

If a real valued function $f : [a, \infty) \to [b, \infty)$ defined by $f(x) = 2x^2 - 3x + 5$ is a bijection. Then, $3a + 2b =$

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Options:

A. 20

B. 10

C. 12

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D. 6

Answer: B

Solution:

We are given a real-valued function $f:[a,\infty)\rightarrow[b,\infty)$ defined by $f(x)=2x^2-3x+5$. This function is a quadratic and is stated to be a bijection.

First, let's find the vertex of the quadratic function, as it will help determine the minimum value of the function, which is crucial for it to be a bijection.

The vertex formula for a quadratic function ax^2+bx+c is given by:

$\left(\frac{-b}{2a},\frac{-D}{4a}\right)$

where $D=b^2-4ac$. For our function, $a=2$, $b=-3$, and $c=5$.

Calculating the vertex:

Calculate x-coordinate of the vertex:

$\frac{-(-3)}{2\times2}=\frac{3}{4}$

Calculate the discriminant D:

$D=(-3)^2-4\times2\times5=9-40=-31$

Calculate y-coordinate of the vertex:

$\frac{-(-31)}{8}=\frac{31}{8}$

Thus, the vertex of the quadratic function is $\left(\frac{3}{4},\frac{31}{8}\right)$.

Since $f(x)=2x^2-3x+5$ is a parabola opening upwards (as the coefficient of x^2 is positive), the minimum value of $f(x)$ occurs at the vertex. Therefore, to achieve a bijective function, the domain should start from the x-coordinate of the vertex, and the range should start from the y-coordinate of the vertex.

This implies:

$a=\frac{3}{4}$

$b=\frac{31}{8}$

To find the value of $3a+2b$:

$3\times\frac{3}{4}+2\times\frac{31}{8}=\frac{9}{4}+\frac{62}{8}=\frac{9}{4}+\frac{31}{4}=\frac{40}{4}=10$

Thus, $3a+2b=10$.

Question34

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The domain of the real valued function $f(x) = \frac{1}{\sqrt{\log_{0.5}(2x-3)}} + \sqrt{4-9x^2}$ **is**

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Options:

A. $\left[\frac{2}{3}, \frac{3}{2}\right)$

B. Null Set

C. $\left[\frac{2}{3}, 2\right)$

D. $\left[-\frac{2}{3}, \frac{2}{3}\right]$

Answer: B

Solution:

To find the domain of the real-valued function $f(x) = \frac{1}{\sqrt{\log_{0.5}(2x-3)}} + \sqrt{4-9x^2}$, we analyze the conditions under which each part of the function is defined.

First Part: $\frac{1}{\sqrt{\log_{0.5}(2x-3)}}$

For the square root in the denominator to be defined and non-zero, the expression inside must be positive: $\log_{0.5}(2x-3) > 0$.

The base of the logarithm is less than 1. This means:

$2x - 3 < 1 \quad \Rightarrow \quad x < 2$

Additionally, the expression inside the logarithm must be positive:

$2x - 3 > 0 \quad \Rightarrow \quad x > \frac{3}{2}$

Second Part: $\sqrt{4-9x^2}$

The expression under the square root must be non-negative:

$4 - 9x^2 > 0 \quad \Rightarrow \quad x^2 < \frac{4}{9}$

Solving for x, we derive:

$-\frac{2}{3} < x < \frac{2}{3}$

Intersection of Conditions:

From the first part, we need x to satisfy both conditions: $\frac{3}{2} < x < 2$.

From the second part, x must be in the interval $-\frac{2}{3} < x < \frac{2}{3}$.

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The intersection of these conditions is empty, as there is no number that simultaneously satisfies $\frac{3}{2} < x < 2$ and $-\frac{2}{3} < x < \frac{2}{3}$.

Hence, the domain of the function is a null set.

Question35

If a function $f : R \rightarrow R$ is defined by $f(x) = x^3 - x$, then f is

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Options:

A. one-one and onto.

B. one-one but not onto.

C. onto but not one-one.

D. Neither one-one nor onto.

Answer: C

Solution:

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We are given the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = x^3 - x$$

We need to determine whether this function is one-to-one (injective) and onto (surjective).

Step 1: Check if the function is one-to-one (injective)

A function is one-to-one if each element in the domain corresponds to a unique element in the codomain. To check whether $f(x) = x^3 - x$ is one-to-one, we look at its derivative:

$$f'(x) = 3x^2 - 1$$

We find the critical points by setting $f'(x) = 0$:

$$3x^2 - 1 = 0 \quad \Rightarrow \quad x^2 = \frac{1}{3} \quad \Rightarrow \quad x = \pm\frac{1}{\sqrt{3}}$$

These critical points suggest that the function has a local maximum and minimum, which means the function is not strictly increasing or decreasing. Therefore, $f(x)$ is **not one-to-one.**

Step 2: Check if the function is onto (surjective)

A function is onto if every element of the codomain has a corresponding element in the domain. The function $f(x) = x^3 - x$ is a cubic function. As $x \to \infty$, $f(x) \to \infty$, and as $x \to -\infty$, $f(x) \to -\infty$. Thus, the function covers all real values, meaning it is **onto.**

Step 3: Conclusion

Since the function is onto but not one-to-one, the correct answer is:

C: onto but not one-to-one.

Question36

If $f(x) = \sqrt{x} - 1$ and $g\{f(x)\} = x + 2\sqrt{x} + 1$ then $g(x) =$

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Options:

A.

$(x + 2)^2$

B.

$(x - 2)^2$

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C.

$(\sqrt{x}+2)^2$

D.

$(\sqrt{x}-2)^2$

Answer: A

Solution:

To determine $g(x)$, let's first understand the given functions:

We have $f(x)=\sqrt{x-1}$. This means $f(x)$ is a function that transforms x into $\sqrt{x-1}$.

Next, the composition $g(f(x))=x+2x^2+1$ can be rewritten using substitution and simplification steps:

Begin by rewriting the expression for $g(f(x))$:

$g(f(x))=x+2\sqrt{x-1}+1$

Recognize a perfect square identity in:

$x+2\sqrt{x-1}+1=(\sqrt{x-1}+1)^2$

Here, $(\sqrt{x-1}+1)^2=\sqrt{x-1}^2+2\sqrt{x-1}+1$, which simplifies to $x-1+2\sqrt{x-1}+1=x+2\sqrt{x-1}$.

Notice then that:

$g(f(x))=(\sqrt{x-1}+1)^2=((\sqrt{x-1}-1)+2)^2=(f(x)+2)^2$

Thus, we can deduce that $g(x)$ must match the transformed output when $x=f(x)$ plus an added constant:

$g(x)=(x+2)^2$

Therefore, $g(x)$ is the function that squares the input, transformed by adding 2.

Question37

For real values of x and a, if the expression $\frac{x^3-3x^2-3x+1}{2x^2-3x+1}$ assumes all real values, then

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Options:

A. $a>-1$ or $a<-1/2$

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B. $-1 < a < a < -1/2$

C. $1/2 < a < 1$

D. $a < 1/2$ or $a > 1$

Answer: B

Solution:

Let's solve the problem such that the given expression assumes all real values.

We're looking at the expression:

$y = \frac{x+a}{2x^2-3x+1}$

To ensure that y can take all real values, the denominator must not be zero:

$2x^2 - 3x + 1 \neq 0$

This can be factored as:

$(2x-1)(x-1) \neq 0$

Thus, $x \neq 1$ and $x \neq \frac{1}{2}$.

For y to take all real values for real x, the quadratic expression $(3y+1)^2 - 4y(y-2) \geq 0$ must hold:

$(3y+1)^2 - 4[y(y-2) - a] \geq 0$

Simplifying the terms:

$9y^2 + 6y + 1 - (4y^2 - 8y - 4a) \geq 0$

This simplifies to:

$5y^2 + 14y + 1 + 4a \geq 0$

As a quadratic in terms of y, for it to be non-positive (invertible for any x), the discriminant must be negative:

The discriminant:

$6^2 - 4 \times 5 \times (1+4a) < 0$

$36 - 20(1+4a) < 0$

$36 - 20 - 80a < 0$

$16 < 80a$

Thus:

$a > \frac{1}{2}$

Therefore, the condition for a is:

$-1 < a < -\frac{1}{2}$

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The correct understanding implies the feasible range for a.

Question38

$f(x+h)=0$ represents the transformed equation of the equation $f(x)=x^4+2x^3-19x^2-8x+60=0$. If this transformation removes the term containing x^3 from $f(x)=0$, then $h=$

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Options:

A. $-1/2$

B. 1

C. 2

D. -1

Answer: A

Solution:

To find the value of h that removes the x^3 term from the polynomial $f(x)=x^4+2x^3-19x^2-8x+60=0$ when transformed to $f(x+h)=0$, follow these steps:

Polynomial Transformation: The transformation affects each term in the original polynomial. For the x^3 term in the polynomial, apply the transformation:

$f(x+h)=(x+h)^4+2(x+h)^3-19(x+h)^2-8(x+h)+60$

Focus on the x^3 Terms: When expanding, consider only the terms involving x^3:

From $(x+h)^4$:

${}^4C_1 \cdot x^3 \cdot h=4hx^3$

From $2(x+h)^3$:

$2 \times 1 \times x^3=2x^3$

Collect x^3 Terms: Adding these terms together gives:

$4hx^3+2x^3=(4h+2)x^3$

Set the Coefficient to Zero: To eliminate the x^3 term from the polynomial, the coefficient must be zero:

$4h+2=0$

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Solve for h:

$4h = -2 \quad \Longrightarrow \quad h = \frac{-2}{4} = \frac{-1}{2}$

Thus, the value of h that removes the x^3 term is $h = -\frac{1}{2}$.

Question39

$f(x) = \log\left(\left(\frac{2x^2-3}{x}\right) + \sqrt{\frac{4x^4-11x^2+9}{|x|}}\right)$ is

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Options:

A. an odd function

B. an even function

C. a polynomial function

D. not a function

Answer: A

Solution:

$f(x) = \log\left(\left(\frac{2x^2-3}{x}\right) + \sqrt{\frac{4x^4-11x^2+9}{|x|}}\right)$

$$f(-x) = \log\left(\frac{2(-x)^2-3}{(-x)} + \sqrt{\frac{4(-x)^4-11(-x)^2+9}{|-x|}}\right)$$

$$f(-x) = \log\left[\frac{2x^2-3}{-x} + \sqrt{\frac{4x^4-11x^2+9}{|x|}}\right]$$

We know that, if function $f(x)$ is an odd function. Then,

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$$f(x)+f(-x)=0$$

$$\log\left[\left(\frac{2x^2-3}{x}\right)+\sqrt{\frac{4x^4-11x^2+9}{|x|}}\right]$$

$$+\log\left[\sqrt{\frac{4x^4-11x^2+9}{|x|}}-\frac{(2x^2-3)}{x}\right]$$

$$=\log\left[\frac{4x^4-11x^2+9}{x^2}-\frac{(2x^2-3)^2}{x^2}\right]$$

$$=\log\left[\frac{4x^4-11x^2+9-4x^4-9+12x^2}{x^2}\right]$$

$$=\log 1=0$$

Question40

Let $f : R-\left\{\frac{-1}{2}\right\} \rightarrow R$ be defined by $f(x)=\frac{x-2}{2x+1}$. If α and β satisfy the equation $f(f(x))=-x$, then $4\left(\alpha^2+\beta^2\right)=$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. 17

B. 12

C. 24

D. 34

Answer: A

Solution:

$f(x)=\frac{x-2}{2x+1}$

$f(f(x))=-x$ [Given]

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$$\Rightarrow \quad \frac{f(x)-2}{2(f(x))+1}=-x$$

$$\Rightarrow \quad \frac{\frac{x-2}{2x+1}-2}{\frac{2x-4}{2x+1}+1}=-x$$

$$\Rightarrow \frac{x-2-4x-2}{2x-4+2x+1}=-x$$

$$\Rightarrow \quad \frac{-3x-4}{4x-3}=-x \Rightarrow \frac{3x+4}{4x-3}=x$$

$$\Rightarrow \quad 3x+4=4x^2-3x$$

$$\Rightarrow \quad 4x^2-6x-4=0$$

$$\Rightarrow \quad 2x^2-3x-2=0$$

Now, $\alpha+\beta=\frac{3}{2}, \alpha\beta=-1$

Now, $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

$\Rightarrow \quad \frac{9}{4}=\alpha^2+\beta^2-2 \quad [\because \alpha\beta=-1]$

$$\Rightarrow \alpha^2+\beta^2=\frac{17}{4}$$

$$\Rightarrow 4\left(\alpha^2+\beta^2\right)=17$$

Question41

The domain of the real valued function $f(x)=\sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is

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Options:

A. $(1,4)$

B. $(-1,1)$

C. $(-2,1)$

D. $(-2,4)$

Answer: C

Solution:

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Given, function $f(x)=\sin\left(\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right)$ is defined for $x \neq 1$ and $x > -2$ and also $x < 1$. Thus, the domain of above function is $D(f)=\{x \in R; x \in (-2,1)\}$

Question42

The range of the real valued function $f(x)=\sqrt{\frac{x^2+2x+8}{x^2+2x+4}}$ is

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\left[\sqrt{\frac{7}{3}}, \infty\right)$

B. $(0, \infty)$

C. $(1, \infty)$

D. $\left(1, \sqrt{\frac{7}{3}}\right]$

Answer: D

Solution:

$f(x)=\sqrt{\frac{x^2+2x+8}{x^2+2x+4}}$

Let $y=\frac{x^2+2x+8}{x^2+2x+4} \quad [\because f(x)=\sqrt{y}]$

$$\Rightarrow yx^2+2xy+4y=x^2+2x+8$$
$$\Rightarrow (y-1)x^2+2x(y-1)+4y-8=0$$

For x to be real, discriminant of equation must be greater than equals to zero.

$$\begin{aligned} & [2(y-1)]^2-4(y-1)(4y-8) && \geq 0 \\ \Rightarrow\ & 4(y-1)^2-4(y-1)4(y-2) && \geq 0 \\ \Rightarrow\ & (y-1)^2-(y-1)4(y-2) && \geq 0 \\ \Rightarrow\ & (y-1)[y-1-4y+8] && \geq 0 \\ \Rightarrow\ & (y-1)(-3y+7) && \geq 0 \\ & y=1, \frac{7}{3} \end{aligned}$$

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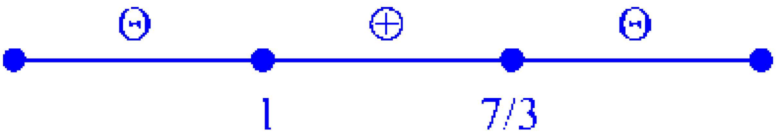

$$\therefore \quad (y-1)(-3y+7) \geq 0 \Rightarrow y \in \left(1, \frac{7}{3}\right]$$

$[\because$ At $y=1, x^2+2x+4 \neq x^2+2x+8]$

$$\because \quad y \in \left(1, \frac{7}{3}\right]$$

$$\therefore \quad f(x) \in \left(1, \sqrt{\frac{7}{3}}\right]$$

Question43

If $f(x)=\sqrt{2-x^2}$ and $g(x)=\log(1-x)$ are two real valued functions, then the domain of the function $(f+g)(x)$ is

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Options:

A. $[-2, 2]$

B. $[-2, 1)$

C. $(-\infty, 1)$

D. $(1, 2]$

Answer: B

Solution:

$f(x)=\sqrt{2-x^2}$ and $g(x)=\log(1-x)$

$$\begin{aligned} (f+g)(x) &= f(x)+g(x) \\ &= \sqrt{2-x^2}+\log(1-x) \end{aligned}$$

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$$2 - x^2 \geq 0$$
$$\Rightarrow x^2 \leq 2$$
$$\Rightarrow |x| \leq \sqrt{2}$$
$$\Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$$

Also, $1 - x > 0 \Rightarrow x < 1$

Required domain : $-\sqrt{2} \leq x \leq \sqrt{2} \cap x < 1$

$\Rightarrow \quad -\sqrt{2} \leq x < 1$

Question44

Let $f(x) = (x+2)^2 - 2, x \geq -2$. Then, $f^{-1}(x)$ is equal to

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Options:

A. $-\sqrt{2+x} - 2$

B. $\sqrt{2+x} + 2$

C. $\sqrt{2+x} - 2$

D. $-\sqrt{2+x} + 2$

Answer: C

Solution:

Let $f(x) = y$, then $x = f^{-1}(y)$, here

$y = (x+2)^2 - 2$

or

$$y + 2 = (x+2)^2$$
$$x + 2 = \sqrt{y+2}$$

$$\Rightarrow \quad x = \sqrt{y+2} - 2$$
$$\therefore \quad f^{-1}(y) = \sqrt{y+2} - 2$$
$$\therefore \quad f^{-1}(x) = \sqrt{x+2} - 2$$

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Question45

If f is the greatest integers function defined on R as $f(x)=[x]$ and g is the modulus function defined on R as $g(x)=|x|$, then the value of $(g \circ f)\left(\frac{-5}{3}\right)$ is

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

$f(x)=[x], g(x)=|x|$

Now, $f\left(-\frac{5}{3}\right)=\left[-\frac{5}{3}\right]$

$=[-1.667]=-2$

Then, $\quad g\left(f\left(-\frac{5}{3}\right)\right)=g(-2)$

$(g \circ f)\left(-\frac{5}{3}\right)=|-2|=2$

$\therefore \quad (g \circ f)\left(-\frac{5}{3}\right)=2$

Question46

If $f: R \rightarrow R$ and $g: R \rightarrow R$ are two functions defined by $f(x)=a x+b(a \neq 0), \forall x \in R$ and $g(x)=c x^3+d(c \neq 0), \forall x \in R$, then $(f \circ g)^{-1}(x)$ is equal to

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Options:

A. $\left(\frac{x-ad+b}{ac}\right)^{\frac{1}{2}}$

B. $\left(\frac{x+ad-b}{ac}\right)^{\frac{1}{3}}$

C. $\left(\frac{x-ad-b}{ac}\right)^{\frac{1}{3}}$

D. $\left(\frac{x+ad+b}{ac}\right)^{\frac{1}{3}}$

Answer: C

Solution:

Given, $f(x)=ax+b$ and $g(x)=cx^3+d$

$$\begin{aligned} f(g(x)) &= f\left(cx^3+d\right) \\ &= a\left(cx^3+d\right)+b \end{aligned}$$
$$(f \circ g)(x)=acx^3+ad+b \quad \ldots\ldots(\mathrm{i})$$

Let $(f \circ g)(x)=y$, then

$x=(f \circ g)^{-1} y$

From Eq. (i), we get

$y=acx^3+ad+b$

i.e.

$$\begin{aligned} acx^3 &= y-ad-b \\ x^3 &= (y-ad-b)/ac \\ x &= \left[\frac{(y-ad-b)}{ac}\right]^{1/3} \end{aligned}$$

$$\therefore \quad (f \circ g)^{-1}(y)=\left[\frac{(y-ad-b)}{ac}\right]^{1/3}$$
$$\text{or} \quad (f \circ g)^{-1}(x)=\left[\frac{(x-ad-b)}{ac}\right]^{1/3}$$

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Question47

If $f(10-x)=3x^2+4x-5$ and $f(x)=px^2+qx+r$, then $p+q+r$ is equal to

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Options:

A. 272

B. 274

C. 275

D. 273

Answer: B

Solution:

$f(10-x)=3x^2+4x-5$ and $f(x)=px^2+qx+r$

Let $10-x=y$, then $x=10-y$

$$\begin{aligned} f(y) &= 3(10-y)^2+4(10-y)-5 \\ &= 3\left(100+y^2-20y\right)+40-4y-5 \\ &= 340+3y^2-60y-4y-5 \\ &= 3y^2-64y+335 \end{aligned}$$

Replace y by $x \Rightarrow f(x)=3x^2-64x+335$

compare with $f(x)=px^2+qx+r$, we obtain

$$\begin{aligned} & p=3, q=-64, r=335 \\ \Rightarrow \quad & p+q+r=274 \end{aligned}$$

Question48

$f(x)=\sin x+\cos x \cdot g(x)=x^2-1$**, then** $g(f(x))$ **is invertible if**

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Options:

A. $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$

B. $\frac{-\pi}{2} \leq x \leq 0$

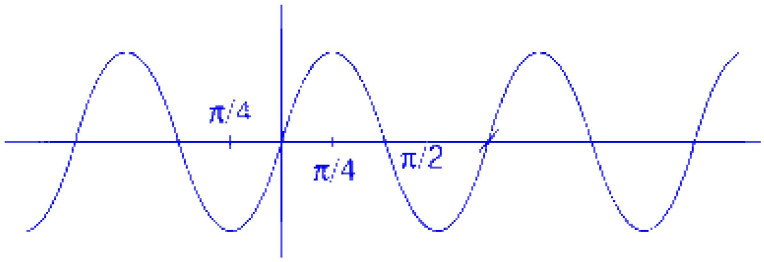
C. $\frac{-\pi}{2} \leq x \leq \pi$

D. $0 \leq x \leq \frac{\pi}{2}$

Answer: A

Solution:

$$\begin{aligned} f(x) &= \sin x + \cos x \\ g(x) &= x^2 - 1 \\ g[f(x)] &= (\sin x + \cos x)^2 - 1 \\ &= 1 + \sin 2x - 1 = \sin 2x \end{aligned}$$



Among the given options, $\sin 2x$ is monotonous (here strictly increasing) in $-\frac{\pi}{4} \leq 2 \leq \frac{\pi}{4}$.

Question49

If $f : z \to z$ is defined by $f(x) = x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 - 12x^3 + 11x^2 + x - 3, \forall x \in z$, then $f(11)$ is equal to

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Options:

A. 7

B. 8

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C. 6

D. 9

Answer: B

Solution:

$$\begin{aligned} f(x) &= x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 \\ &\quad - 12x^3 + 11x^2 + x - 3, \quad \forall x \in Z \\ &= x^8(x-11) - 2x^6(x-11) + x^3(x-11) \\ &\quad - x^2(x-11) + (x-11) + 8 \\ &= (x-11)\left[x^8 - 2x^6 + x^3 - x^2 + 1\right] + 8 \end{aligned}$$

So, $f(11) = 0 + 8 = 8$

Question50

Let $f(x) = x^3$ and $g(x) = 3^x$, then the quadratic equation whose roots are solutions of the equation $(f \circ g)(x) = (g \circ f)(x)$ (for $x \neq 0$) is

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Options:

A. $x^2 - 6x + 3 = 0$

B. $x^2 - 6x + 9 = 0$

C. $x^2 - x + 3 = 0$

D. $x^2 - 3 = 0$

Answer: D

Solution:

$f(x) = x^3$ and $g(x) = 3^x$

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$\Rightarrow \quad f[g(x)] = g[f(x)]$

$\Rightarrow \quad (3^x)^3 = 3^{x^3} \Rightarrow 3^{3x} = 3^{x^3}$

$\Rightarrow \quad 3x = x^3 \Rightarrow x\left(x^2 - 3\right) = 0$

As, $\quad x \neq 0$, So $x^2 - 3 = 0$

Question51

The real valued function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ defined on $R/\{0\}$ is

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Options:

A. an odd function

B. an even function

C. Both even and odd function

D. Neither even nor odd function

Answer: B

Solution:

$f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$

Now, $f(-x) = \frac{-x}{\frac{1}{e^x} - 1} - \frac{x}{2} + 1 = \frac{-xe^x}{1 - e^x} - \frac{x}{2} + 1$

$$\begin{aligned}
&= \frac{-2xe^x - x\left(1 - e^x\right)}{2\left(1 - e^x\right)} + 1 \\
&= \frac{-2xe^x - x + xe^x}{2\left(1 - e^x\right)} + 1 \\
&= \frac{-xe^x - 2x + x}{2\left(1 - e^x\right)} + 1 \\
&= \frac{-2x + x\left(1 - e^x\right)}{2\left(1 - e^x\right)} + 1 \\
&= \frac{-x}{1 - e^x} + \frac{x}{2} + 1 \\
&= \frac{x}{e^x - 1} + \frac{x}{2} + 1 = f(x)
\end{aligned}$$

$\therefore f(x)$ is an even function.

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Question52

The domain of the function $f(x)=\frac{1}{[x]-1}$, where $[x]$ is greatest integer function of x is

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Options:

A. $R-(1,2)$

B. $R-\{1\}$

C. $R-\{0,1\}$

D. $R-[1,2)$

Answer: D

Solution:

$f(x)=\frac{1}{[x]-1}$

$f(x)$ is not defined when

$$\begin{aligned} & [x]-1=0 \\ \Rightarrow \quad & [x]=1 \Rightarrow 1 \leq x<2 \\ \Rightarrow \quad & x \in[1,2) \end{aligned}$$

Domain $f=R-[1,2)$

Question53

Let $f: R \rightarrow R$ be a function defined by $f(x)=\frac{4^x}{4^x+2}$, what is the value of $f\left(\frac{1}{4}\right)+2 f\left(\frac{1}{2}\right)+f\left(\frac{3}{4}\right)$ is equal to

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Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

$f(x)=\frac{4^x}{4^x+2}$

$f(1-x)=\frac{4^{1-x}}{4^{1-x}+2}=\frac{\frac{4}{4^x}}{\frac{4}{4^x}+2}$

$=\frac{4}{4+2\,.\,4^x}$

$f(1-x)=\frac{2}{4^x+2}$ ...... (i)

and $1-f(x)=1-\frac{4^x}{4^x+2}$

$=\frac{4^x+2-4^x}{4^x+2}=\frac{2}{4^x+2}=f(1-x)$

$\Rightarrow f(1-x)+f(x)=1$ ..... (ii)

Put $x=\frac{1}{4}$ in Eq. (ii), we get

$f\left(\frac{3}{4}\right)+f\left(\frac{1}{4}\right)=1$ ...... (iii)

Now, put $x=\frac{1}{2}$ in Eq. (ii), we get

$f\left(\frac{1}{2}\right)+f\left(\frac{1}{2}\right)=1$

or $2f\left(\frac{1}{2}\right)=1$ ..... (iv)

Adding Eqs. (iii) and (iv), we have

$f\left(\frac{1}{4}\right)+2f\left(\frac{1}{2}\right)+f\left(\frac{3}{4}\right)=2$

Question54

Let $f: R\rightarrow R$ and $g: R\rightarrow R$ be defined by $f(x)=2x+1$ and $g(x)=x^2-2$ determine $(g\circ f)(x)$ is equal to

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Options:

A. $2x^2 - 3$

B. $4x^2 + 4x - 1$

C. $4x^2 + 4x + 1$

D. $2x^2 - 4$

Answer: B

Solution:

Given,

$$f(x) = 2x + 1$$
$$g(x) = x^2 - 2$$

Then,

$$\begin{aligned} g \circ f(x) &= g[f(x)] \\ &= g(2x+1) = (2x+1)^2 - 2 \\ &= 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1 \end{aligned}$$

Question55

Given, the function $f(x) = \frac{a^x + a^{-x}}{2}$, $(a > 2)$, then $f(x+y) + f(x-y)$ is equal to

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Options:

A. $f(x) - f(y)$

B. $f(y)$

C. $2f(x)f(y)$

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D. $f(x) f(y)$

Answer: C

Solution:

Given, $f(x)=\frac{a^x+a^{-x}}{2}$

Then,

$$f(x+y)+f(x-y)=\frac{a^{x+y}+a^{-(x+y)}}{2}$$
$$+\frac{a^{x-y}+a^{-(x-y)}}{2}$$
$$=\frac{a^x \cdot a^y+a^{-x} \cdot a^{-y}+a^x \cdot a^{-y}+a^{-x} a^y}{2}$$
$$=\frac{a^x\left(a^y+a^{-y}\right)+a^{-x}\left(a^y+a^{-y}\right)}{2}$$
$$=\frac{\left(a^x+a^{-x}\right)\left(a^y+a^{-y}\right)}{2}$$
$$=2 \cdot \frac{\left(a^x+a^{-x}\right)}{2} \cdot \frac{\left(a^y+a^{-y}\right)}{2}=2 f(x) f(y)$$

Question56

If f is a function defined on $(0,1)$ by $f(x)=\min \{x-[x],-x-[x]\}$, then $(f \circ f o f o f)(x)$ is equal to $\rightarrow$ ($[\cdot]$ greatest integer function)

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Options:

A. x

B. $-x$

C. $4x$

D. $2x$

Answer: A

Solution:

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$\because \{x\} = x - [x]$

$f : (0, 1) \rightarrow (0, 1)$

Defined by $f(x) = \min\{x - [x], -x - [x]\}$ which is an identity function, as $x \in (0, 1)$ and $\{x\} \in (0, 1)$
Where, $\{\cdot\}$ is fractional part function.

$\Rightarrow (f \circ f \circ f \circ f)(x) = x$

Question57

If $(x^2 + 5x + 5)^{x+5} = 1$, then the number of integers satisfying this equation is

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Options:

A. 2

B. 3

C. 4

D. 5

Answer: B

Solution:

$\because (x^2 + 5x + 5)^{x+5} = 1$

Case I $x + 5 = 0$

$\Rightarrow x = -5$

Case II $x^2 + 5x + 5 = 1$

or, $x^2 + 5x + 4 = 0$

or, $(x + 1)(x + 4) = 0$

$\Rightarrow x = -1, -4$

$x = -1, -4$ and -5 are the three integers satisfying given equation.

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Question58

If $\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$, then

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Options:

A. $f(x) = x^2 - 3x + 7$

B. $f(x) = x^2 + 3x + 7$

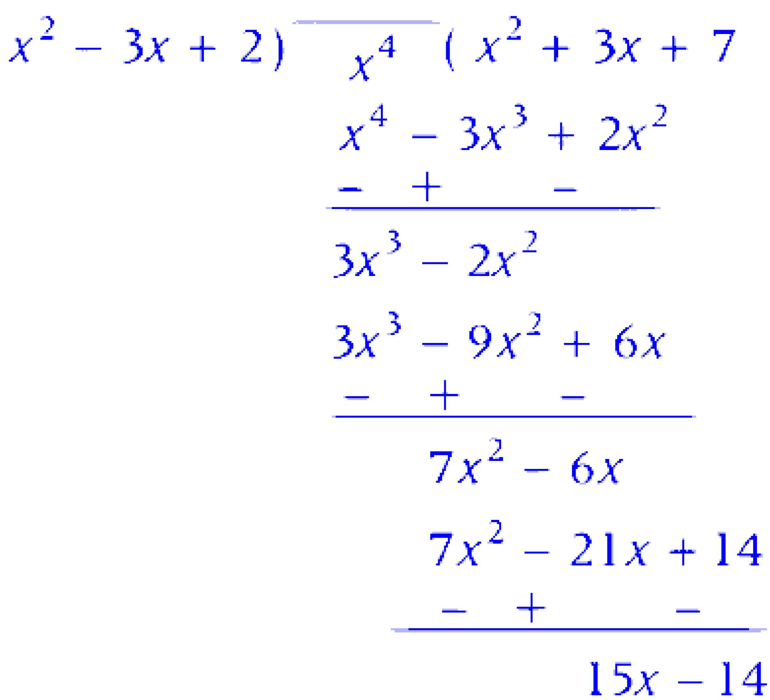
C. $A + B = 17$

D. $A - B = -18$

Answer: B

Solution:

$\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2-3x+2}$ is an improper fraction.



$\therefore \frac{x^4}{(x-1)(x-2)} = x^3 + 3x + 7 + \frac{15x-14}{(x-1)(x-2)}$

$= x^2 + 3x + 7 + \frac{A}{x-1} + \frac{B}{x-2}$

On comparing with given equation,

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$\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$, we have $f(x) = x^2 + 3x + 7$

Question59

Which statement among the following is true?

(i) the function $f(x) = x|x|$ is strictly increasing on $R - \{0\}$.

(ii) the function $f(x) = \log_{(1/4)} x$ is strictly increasing on $(0, \infty)$.

(iii) a one-one function is always an increasing function.

(iv) $f(x) = x^{1/3}$ is strictly decreasing on R

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Options:

A. (i)

B. (ii)

C. (iii)

D. (iv)

Answer: A

Solution:

(i) $f(x) = x|x| = \begin{cases} x(-x), & x < 0 \\ x(x), & x \geq 0 \end{cases}$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -2x, x < 0 \\ 2x, x \geq 0 \end{cases}$$

$f'(x) > 0, \forall x \in R - \{0\}$

$\Rightarrow f(x)$ is strictly increasing on $R - \{0\}$.

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