Ellipse

Question1

If the normal at the point $P\left(\frac{\pi}{4}\right)$ on the ellipse $x^2+4y^2-4=0$ meets the ellipse again at $Q(\alpha,\beta)$, then $\alpha=$

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$\sqrt{2}$

B.

$\frac{-23}{17\sqrt{2}}$

C.

$\frac{7\sqrt{2}}{17}$

D.

$\frac{1}{\sqrt{2}}$

Answer: C

Solution:

$x^2+4y^2-4=0 \Rightarrow \frac{x^2}{4}+y^2=1$

$\text{Center}=(0,0), a=2, b=1$

The parametric equation of point on the ellipse

$x=a\cos\theta, y=b\sin\theta$
at $\theta=\pi/4$

$x_p=2\times\frac{1}{\sqrt{2}}=\sqrt{2}, y_p=2\times\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

So, $p=\left(\sqrt{2},\frac{1}{\sqrt{2}}\right)$

Slope $=\frac{a^2y_1}{b^2x_1}=\frac{4\times\frac{1}{\sqrt{2}}}{1\times\sqrt{2}}=2$

$\therefore$ The equation of the normal at p_{at}

$y-\frac{1}{\sqrt{2}}=2(x-\sqrt{2})\Rightarrow y=2x-\frac{3}{\sqrt{2}}$

and the normal intersects at point $Q(\alpha,\beta)$

$$\therefore x^2+4\left(2x-\frac{3}{\sqrt{2}}\right)^2=4$$
$$\Rightarrow x^2+16x^2-24\sqrt{2}x+18=4$$
$$\Rightarrow 17x^2-24\sqrt{2}x+14=0$$
$$D=(-24\sqrt{2})^2-4(17)(14)=200$$
$$\therefore \quad x=\frac{24\sqrt{2}\pm\sqrt{200}}{34}=\frac{14\sqrt{2}}{34},\sqrt{2}$$
$\because \quad x=\sqrt{2}$ corresponds to point p
$$\therefore \quad x=\frac{14\sqrt{2}}{34}=\frac{7\sqrt{2}}{17}$$ corresponds top
$$\text{So, } \alpha=\frac{7\sqrt{2}}{17}$$

Question2

If a tangent having slope $\frac{1}{3}$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is a normal to the circle $(x+1)^2+(y+1)^2=1$, then a^2 lies in the interval

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$\left(\frac{\sqrt{2}}{\sqrt{5}},2\right)$

B.

$\left(\frac{2}{5}, 4\right)$

C.

$\left(1, \frac{10}{9}\right)$

D.

$(3, 5)$

Answer: B

Solution:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$

Equation of tangent having slope $\frac{1}{3}$ is

$y = mn \pm \sqrt{a^2m^2 + bc}$

$\Rightarrow \quad y = \frac{x}{3} \pm \sqrt{\frac{a^2}{9} + b^2} \qquad \ldots(i)$

$\therefore$ Equation (i) is normal of circle

$(x+1)^2 + (y+1)^2 = 1$

$\therefore$ It passes through $(-1, -1)$

$$-1 = \frac{-1}{3} \pm \sqrt{\frac{a^2}{9} + b^2}$$
$$\Rightarrow \quad \left(\frac{2}{3}\right)^2 = \frac{a^2}{9} + b^2 \Rightarrow 9b^2 = 4 - a^2$$
$$\because \quad a > b \Rightarrow a^2 > b^2$$
$$\Rightarrow \quad a^2 > \frac{4-a^2}{9} \Rightarrow 9a^2 > 4 - a^2$$
$$\Rightarrow \quad 10a^2 > 4 \Rightarrow a^2 > \frac{4}{10}$$
$$\Rightarrow \quad a^2 > \frac{2}{5} \text{ and } 4 - a^2 > 0$$
$$a^2 < 4$$
$$\therefore \quad a^2 \in \left(\frac{2}{5}, 4\right)$$

Question3

If $P(\alpha, \beta)$ is a point on the curve $9x^2 + 4y^2 = 144$ in the first quadrant and the minimum area of the triangle formed by the tangent of the curve at P with the coordinate axis is S, then

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$S=\sqrt{\alpha\beta}$

B.

$S=\alpha\beta$

C.

$S=2\sqrt{\alpha\beta}$

D.

$S=2\alpha\beta$

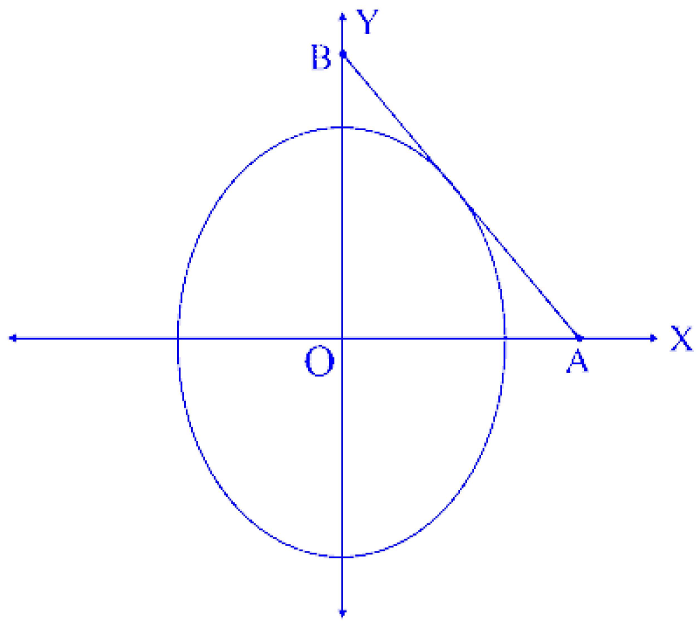
Answer: D

Solution:

$9x^2+4y^2=144 \Rightarrow \frac{x^2}{16}+\frac{y^2}{36}=1$

Equation of tangent at $(4\cos\theta, 6\sin\theta)$

$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{6}=1$



$\therefore A\left(\frac{4}{\cos\theta}, 0\right), B\left(0, \frac{6}{\sin\theta}\right)$

Area of $\triangle OAD = \frac{1}{2} \times \frac{4}{\cos\theta} \cdot \frac{6}{\sin\theta}$

$= \frac{24}{\sin 2\theta}$

$\therefore$ Minimum area $= 24$

$\because \quad \theta = \frac{\pi}{4}$
$\therefore \quad \alpha = \frac{4}{\sqrt{2}}, \beta = \frac{6}{\sqrt{2}} \Rightarrow \alpha\beta = 12$

$\therefore$ Minimum area $= 2\alpha\beta$

Question4

The area (in sq. units) of the triangle formed by the tangent and normal to the ellipse $9x^2 + 4y^2 = 72$ at the point $(2, 3)$ with the X-axis is

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$\frac{25}{2}$

B.

$\frac{39}{4}$

C.

$\frac{35}{4}$

D.

$\frac{45}{4}$

Answer: B

Solution:

Given, equation: $9x^2+4y^2=72$

Differentiate w.r.t x, we get

$$18x+8y\cdot\frac{dy}{dx}=0$$
$$\Rightarrow \quad \frac{dy}{dx}=\frac{-18x}{8y}=\frac{-9x}{4y}$$

Slope of tangent at point $(2,3)$ is

$m_t=\frac{-9(2)}{4(3)}=\frac{-18}{12}=\frac{-3}{2}$

So, equation of tangent using point-slope form is

$$(y-3)=-\frac{3}{2}(x-2)$$
$$\Rightarrow \quad 2y-6=-3x+6$$
$$\Rightarrow \quad 3x+2y=12$$

Now, slope of normal, $m_n=\frac{-1}{m_t}=\frac{2}{3}$

Now, slope of normal, $m_n=\frac{-1}{m_t}=\frac{2}{3}$

So, equation of normal is

$$(y-3)=\frac{2}{3}(x-2)$$
$$\Rightarrow \quad 3y-9=2x-4$$
$$\Rightarrow \quad 2x-3y=-5$$

Now, for the x-intercept of the tangent line, put $y=0$, then

$$3x+2(0)=12$$
$$\Rightarrow \quad x=4$$

so $(4,0)$

For the x-intercept of the normal line, put $y=0$, then

$$2x-3(0)=-5$$
$$\Rightarrow \quad x=-\frac{5}{2}, \text{ so, } \left(\frac{-5}{2},0\right)$$

Now, base of triangle $=$ Distance $\mathrm{b/w}\,x$-intercepts of the tangent and normal lines

$b=4-\left(\frac{-5}{2}\right)=\frac{13}{2}$

And, height of triangles is the y-coordinate of the point $(2,3)$ is $h=3$

So, area $=\frac{1}{2}bh=\frac{1}{2}\left(\frac{13}{2}\right)\times 3=\frac{39}{4}$

$\therefore$ Area $=\frac{39}{4}$ sq. units

Question5

The equation of the normal drawn at the point $(\sqrt{2}+1,-1)$ to the ellipse $x^2+2y^2-2x+8y+5=0$ is

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$x+y=\sqrt{2}$

B.

$x-2y=3+\sqrt{2}$

C.

$\sqrt{2}x-y=3+\sqrt{2}$

D.

$2x+y=2\sqrt{2}+1$

Answer: C

Solution:

$S(x,y)\equiv x^2+2y^2-2x+8y+5$

Thus, $\frac{\partial s}{\partial x}=2x-2\frac{\partial s}{\partial y}=4y+8$

at $P\equiv(\sqrt{2}+1,-1)$

$2x-2=2(\sqrt{2}+1)-2=2\sqrt{2}$

and $4y+8=-4+8=4$

So, $\mathbf{n}=\langle 2\sqrt{2},4\rangle$

So, slope $=\frac{4}{2\sqrt{2}}=\sqrt{2}$

Then, equation of normal (at $\sqrt{2}+1,-1$)

$$
\begin{aligned}
& y-(-1)=\sqrt{2}(x-(\sqrt{2}+1)) \\
\Rightarrow & y+1=\sqrt{2}x-2-\sqrt{2} \\
\Rightarrow & y-\sqrt{2}x+\sqrt{2}+3=0 \\
\Rightarrow & \sqrt{2}x-y-3-\sqrt{2}=0 \\
\Rightarrow & \sqrt{2}x-y=3+\sqrt{2}
\end{aligned}
$$

Question6

If the tangents drawn from a point P to the ellipse $4x^2+9y^2-16x+54y+61=0$ are perpendicular, then the locus of P is

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$x^2+y^2-4x+6y+4=0$

B.

$x^2+y^2-4x+6y=0$

C.

$x^2+y^2-6x+4y+9=0$

D.

$x^2+y^2-6x+4y=0$

Answer: B

Solution:

$$
\begin{aligned}
& 4x^2+9y^2-16x+54y+61=0 \\
\Rightarrow \quad & (2x-4)^2+(3y+9)^2=36 \\
\Rightarrow \quad & \frac{(x-2)^2}{9}+\frac{(y+3)^2}{4}=1
\end{aligned}
$$

Which is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

where $X=x-2Y=y+3, a=3$ and $b=2$

As we know equation of director circle (because locus of two perpendicular pair of tangents shows its director circle) is $x^2 + y^2 = a^2 + b^2$

$$\Rightarrow \quad (x-2)^2 + (y+3)^2 = (3)^2 + (2)^2$$
$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 9 + 6y = 13$$
$$\Rightarrow \quad x^2 + y^2 - 4x + 6y = 0$$

Question7

Let A_1 be the area of the given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let A_2 be the area of the region bounded by the curve which is the locus of mid-point of the line segment joining the focus of the ellipse and a point P on the given ellipse, then $A_1 : A_2 =$

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

$3 : 2$

B.

$a : b$

C.

$4 : 1$

D.

$2a : 3b$

Answer: C

Solution:

Area of ellipse $A_1 = \pi ab$

Let (x_1, y_1) be a point on the ellipse Let $(ae, 0)$ be the focus Let (h, k) be the mid-point.

$$h=\frac{x_1+ae}{2}, k=\frac{y_1}{2}$$
$$x_1=2h-ae, y_1=2k$$
$$\Rightarrow \quad \frac{(2h-ae)^2}{a^2}+\frac{(2k)^2}{b^2}=1$$
$$\Rightarrow \quad \frac{\left(h-\frac{ae}{2}\right)^2}{\left(\frac{a}{2}\right)^2}+\frac{k^2}{\left(\frac{b}{2}\right)^2}=1$$

$\Rightarrow$ Semi-major axis $=\frac{a}{2}$

Semi-minor axis $=\frac{b}{2}$

The area of the locus ellipse

$$A_2=\pi\cdot\frac{a}{2}\cdot\frac{b}{2}=\frac{ab\pi}{4}$$
$$\Rightarrow \quad \frac{A_1}{A_2}=\frac{\pi ab}{\frac{\pi ab}{4}}=\frac{4}{1}$$
$$\Rightarrow \quad A_1 : A_2=4:1$$

Question8

The angle between the tangents drawn from a point $(-3, 2)$ to the ellipse $4x^2+9y^2-36=0$ is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

45°

B.

$\tan^{-1}\left(\frac{2}{3}\right)$

C.

$\tan^{-1}\left(\frac{3}{2}\right)$

D.

90°

Answer: D

Solution:

Given equation of ellipse

$$4x^2 + 9y^2 - 36 = 0$$
$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 1 \qquad \ldots(i)$$

Equation of pair of tangent from P(- 3, 2)

$$SS_1 = T^2$$
$$\left(\frac{x^2}{9} + \frac{y^2}{4} - 1\right)\left(\frac{(-3)^2}{9} + \frac{(2)^2}{4} - 1\right) = \left(-\frac{3(x)}{9} + \frac{2(y)}{4} - 1\right)^2$$
$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} - 1 = \left(-\frac{x}{3} + \frac{y}{2} - 1\right)^2$$
$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} - 1 = \frac{x^2}{9} + \frac{y^2}{4} - \frac{xy}{3} - y + \frac{2x}{3}$$
$$\Rightarrow \frac{xy}{3} - \frac{2x}{3} + y - 2 = 0$$
$$\Rightarrow xy - 2x + 3y - 6 = 0$$
$$\Rightarrow (x+3)(y-2) = 0$$
$$x + 3 = 0 \text{ and } y - 2 = 0$$
$$\Rightarrow x = -3 \text{ and } y = 2$$

The tangent lines are $x = -3$ (a vertical line)

and $y = 2$ (a horizontal line)

$\therefore$ These ar perpendicular

$\therefore \quad \theta = 90^\circ$

Question9

The equation of a chord AB of an ellipse $2x^2 + y^2 = 1$ is $x - y + 1 = 0$. If O is the origin, then $\sqrt{AOB} =$

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

$\frac{\pi}{4}$

B.

$\tan^{-1} 2$

C.

$\tan^{-1}\left(\frac{1}{2}\right)$

D.

$\frac{\pi}{6}$

Answer: B

Solution:

We have, $2x^2 + y^2 = 1$

And $x - y = -1$

Now make homogeneous equation of given ellipse.

$$\begin{aligned} & 2x^2 + y^2 = (y - x)^2 \\ \Rightarrow\; & 2x^2 + y^2 = y^2 + x^2 - 2xy \\ \Rightarrow\; & x^2 + 2xy + 0 \cdot y^2 = 0 \end{aligned}$$

Angle AOB

$$\begin{aligned} & \tan\theta = \frac{2\sqrt{1-0}}{1} = 2 \\ \therefore\; & \theta = \tan^{-1} 2 \end{aligned}$$

Question10

The square of the slope of a common tangent drawn to the circle $4x^2 + 4y^2 = 25$ and the ellipse $4x^2 + 9y^2 = 36$ is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

1

B.

$\frac{9}{11}$

C.

$\frac{2}{3}$

D.

2

Answer: B

Solution:

We have a circle $x^2 + y^2 = \frac{25}{4}$ and Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Let equation of tangent to the ellipse be

$y = mx \pm \sqrt{9m^2 + 4}$

Given this tangent also touches the circle

$\because$ Condition of tendency

$\frac{\sqrt{9m^2+4}}{\sqrt{1+m^2}} = \frac{5}{2}$

On squaring both sides, we get

$$\begin{aligned} & 4\left(9m^2+4\right) = 25\left(1+m^2\right) \\ \Rightarrow \quad & 36m^2 + 16 = 25 + 25m^2 \\ \Rightarrow \quad & 11m^2 = 9 \\ \therefore \quad & m^2 = \frac{9}{11} \end{aligned}$$

Question11

If the tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-points of the intercepts made by the tangents between the coordinate axes is

Options:

A.

$\frac{x^2}{4}+\frac{y^2}{2}=1$

B.

$\frac{x^2}{2}+\frac{y^2}{4}=1$

C.

$\frac{1}{4x^2}+\frac{1}{2y^2}=1$

D.

$\frac{1}{2x^2}+\frac{1}{4y^2}=1$

Answer: D

Solution:

Given, ellipse is $x^2+2y^2=2$

or $\frac{x^2}{2}+y^2=1$

Now, equation of tangent in parametric form is

$\frac{x}{\sqrt{2}}\cos\theta+y\sin\theta=1$

Now, coordinate of X-intercept

$=(\sqrt{2}\sec\theta,0)A$

Coordinate of Y-intercept

$=(0,\operatorname{cosec}\theta)B$

Let $P(h,k)$ be the points which is the mid-points of A and B.

$$h=\frac{\sqrt{2}\sec\theta}{2}\text{ and }k=\frac{\operatorname{cosec}\theta}{2}$$
$$\sec\theta=\sqrt{2}h\text{ and }\operatorname{cosec}\theta=2k$$
$$\cos\theta=\frac{1}{\sqrt{2}h}\text{ and }\sin\theta=\frac{1}{2k}$$

Using $\sin^2 \theta + \cos^2 \theta = 1$

$\frac{1}{2h^2} + \frac{1}{4k^2} = 1$

Taking locus of $P(h, k)$, we get

$\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

Question12

Let T_1 be the tangent drawn at a point $P(\sqrt{2}, \sqrt{3})$ on the ellipse $\frac{x^2}{4} + \frac{y^2}{6} = 1$. If (α, β) is the point where, T_1 intersects another tangent T_2 to the ellipse perpendicularly, then $\alpha^2 + \beta^2$ is equal to

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. 10

B. 52

C. 26

D. $5/12$

Answer: A

Solution:

We have, equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{6} = 1$

Equation of tangent at point $(\sqrt{2}, \sqrt{3})$ is

$\frac{\sqrt{2}x}{4} + \frac{\sqrt{3}}{6} = 1$

$3\sqrt{2}x + 2\sqrt{3}y = 12$

$\Rightarrow \quad 18x^2 + 12y^2 + 12\sqrt{6}xy = 144 \qquad \ldots(i)$

Slope of tangent T_1

$= \frac{-3\sqrt{2}}{2\sqrt{3}} = -\sqrt{\frac{3}{2}}$

Slope of tangent $T_2 = \sqrt{\frac{2}{3}}$

Equation of tangent T_r is

$$y = \sqrt{\frac{2}{3}}x \pm \sqrt{4 \times \frac{2}{3} + 6}$$
$$\sqrt{3}y - \sqrt{2}x = \pm\sqrt{26}$$
$$3y^2 + 2x^2 - 2\sqrt{6}xy = 26$$

or $\quad 18y^2 + 12x^2 - 12\sqrt{6}xy = 156 \qquad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$30\left(x^2 + y^2\right) = 300$$
$$x^2 + y^2 = 10$$

As (α, β) are the intersecting point of T_1 and T_2,

$\alpha^2 + \beta^2 = 10$

Question13

The length of the latusrectum of $16x^2 + 25y^2 = 400$ is

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. $\frac{25}{2}$

B. $\frac{25}{4}$

C. $\frac{16}{2}$

D. $\frac{32}{5}$

Answer: D

Solution:

To find the length of the latus rectum of the given ellipse, start with the equation:

$16x^2 + 25y^2 = 400$

First, rewrite it in standard form. Divide through by 400:

$\frac{x^2}{400/16} + \frac{y^2}{400/25} = 1$

This simplifies to:

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

Here, $a^2 = 25$ and $b^2 = 16$. For an ellipse in this form, the length of the latus rectum L is given by:

$L = \frac{2b^2}{a}$

Calculate L using the values of a and b:

$L = \frac{2 \times 16}{\sqrt{25}} = \frac{32}{5}$

Therefore, the length of the latus rectum is $\frac{32}{5}$.

Question14

The product of perpendiculars from the two foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ on the tangent at any point on the ellipse is

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. 6

B. 7

C. 8

D. 9

Answer: D

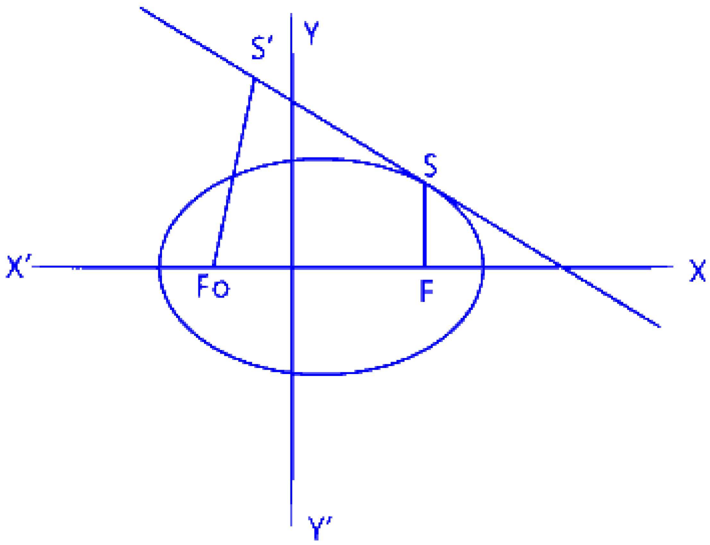
Solution:

We have, equation of ellipse

$\frac{x^2}{9} + \frac{y^2}{25} = 1 \qquad \ldots (i)$

from above equation, we obtained

$a = 3, b = 5$ i.e. $b^2 > a^2$



Here, product of perpendicular from focus is equal to length of semi-minor axis.

$\Rightarrow \quad (S'f') \times (Sf) = (a^2) = (3)^2 = 9$

The product of perpendicular from the two foci of the ellipse on the tangent at any point is 9 .

Question15

If A_1, A_2, A_3 are the areas of ellipse $x^2 + 4y^2 - 4 = 0$ its director circle and auxiliary circle respectively, then $A_2 + A_3 - A_1 =$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 11π

B. 3π

C. 7π

D. 9π

Answer: C

Solution:

Given the ellipse equation:

$x^2 + 4y^2 - 4 = 0$

This can be rearranged into the standard form of an ellipse:

$\frac{x^2}{4} + \frac{y^2}{1} = 1$

Comparing this with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we identify that $a^2 = 4$ and $b^2 = 1$, so $a = 2$ and $b = 1$.

Area of the Ellipse (A_1):

The area of an ellipse is given by the formula $A = \pi ab$.

Thus,

$A_1 = \pi \times 2 \times 1 = 2\pi$

Director Circle:

The equation of the director circle of an ellipse is $x^2 + y^2 = a^2 + b^2$.

Here,

$a^2 + b^2 = 4 + 1 = 5$

The equation of the director circle becomes:

$x^2 + y^2 = 5$

The radius r of this director circle is $\sqrt{5}$. So, the area (A_2) is:

$A_2 = \pi r^2 = \pi \times 5 = 5\pi$

Auxiliary Circle:

The auxiliary circle has the same center as the ellipse and its radius is equal to the semi-major axis of the ellipse, a. Hence, the equation is:

$x^2 + y^2 = a^2 = 4$

The radius r is $\sqrt{4} = 2$. Thus, the area (A_3) is:

$A_3 = \pi \times 4 = 4\pi$

Finally, calculate $A_2 + A_3 - A_1$:

$A_2 + A_3 - A_1 = 5\pi + 4\pi - 2\pi = 7\pi$

Therefore, the result is 7π.

Question16

If the chord of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ having $(1, 1)$ as its middle point is $x + \alpha y = \beta$, then

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $\alpha+\beta=1$

B. $\alpha+1=\beta$

C. $\alpha-1=\beta$

D. $2\alpha-1=3\beta$

Answer: B

Solution:

Given ellipse, $\frac{x^2}{4}+\frac{y^2}{9}=1$

Equation of chord with given mid-point (x_1, y_1) is

$\frac{xx_1}{4}+\frac{yy_1}{9}=\frac{x_1^2}{4}+\frac{y_1^2}{9}$

Since, (1,1) is mid-points, we get

$$\frac{x}{4}+\frac{y}{9}=\frac{1}{4}+\frac{1}{9}$$
$$\Rightarrow 9x+4y=13$$
$$\Rightarrow x+\frac{4}{9}y=\frac{13}{9}$$

But $\quad x+\alpha y=\beta \quad$ [given]

On comparing, we get

$\alpha=\frac{4}{9}, \beta=\frac{13}{9}$

So, $\quad \alpha+1=\frac{4}{9}+1$

$=\frac{4+9}{9}=\frac{13}{9}=\beta$

$\therefore \quad \alpha+1=\beta$

Question17

Let F and F^1 be the foci of the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1(b<2)$ and B is one end of the minor axis. If the area of the triangle FBF^1 is $\sqrt{3}$ sq units, then the eccentricity of the ellipse is

Options:

A. $\frac{\sqrt{3}}{2}$ or $\frac{1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{4}$ or $\frac{1}{4}$

D. $\frac{3}{4}$ or $\frac{1}{4}$

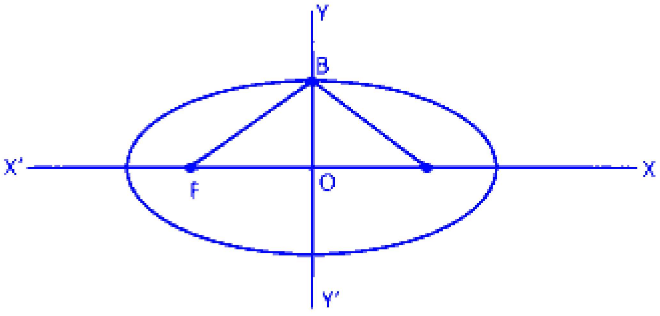
Answer: A

Solution:

Here, $a = 2$

Coordinate of F and F' are ($2e, 0$), ($-2a, 0$).

Distance between $FF' = 4e$



$$\begin{aligned} \text{Area of } \triangle FBF' &= \frac{1}{2} \times b \times 4e \\ \sqrt{3} &= 2be \\ be &= \frac{\sqrt{3}}{2} \text{ or } b = \frac{3}{2e} \\ \Rightarrow e &= \sqrt{1 - \frac{b^2}{4}} \\ \Rightarrow e^2 &= 1 - \frac{3}{4e^2 \times 4} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & 16e^4 - 16e^2 + 3 = 0 \\ \Rightarrow \quad & 16e^4 - 12e^2 - 4e^2 + 3 = 0 \\ & \left(4e^2 - 1\right)\left(4e^2 - 3\right) = 0 \\ & e^2 = \frac{1}{4} \text{ or } e^2 = \frac{3}{4} \\ & e = \frac{1}{2} \text{ or } \frac{\sqrt{3}}{2} [\because e \text{ can't be negatiec }] \end{aligned}$$

Question18

If a tangent of slope 2 to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the circle $x^2+y^2=4$, then maximum value of ab is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. 4

B. 12

C. 5

D. 7

Answer: C

Solution:

To solve this problem, we start by considering the given ellipse equation:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

The equation of a tangent to this ellipse with a slope of 2 is:

$y=2x\pm\sqrt{4a^2+b^2}$

Rearranging, the equation becomes:

$2x-y\pm\sqrt{4a^2+b^2}=0 \quad \text{(equation i)}$

This tangent also touches a circle with center $(0,0)$ and radius 2, given by the equation:

$x^2+y^2=4$

For the tangent line to touch the circle, the perpendicular distance from the center of the circle to the line should equal the radius of 2. The formula for the perpendicular distance d from a point (x_1, y_1) to a line $Ax+By+C=0$ is:

$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Applying this to equation (i), with $A=2, B=-1, C=\pm\sqrt{4a^2+b^2}$, and the center $(0,0)$, we get:

$\frac{|\pm\sqrt{4a^2+b^2}|}{\sqrt{4+1}}=2$

Solving this gives:

$\frac{\sqrt{4a^2+b^2}}{\sqrt{5}} = 2$

Squaring both sides, we obtain:

$4a^2 + b^2 = 20 \quad \text{(equation ii)}$

Now, applying the AM-GM inequality to find the maximum value of ab, we have:

$\frac{4a^2+b^2}{2} \geq \sqrt{4a^2 \cdot b^2}$

Substituting from (ii):

$\frac{20}{2} \geq \sqrt{4a^2b^2}$

Simplifying:

$10 \geq 2ab \quad \Rightarrow \quad ab \leq 5$

Therefore, the maximum value of ab is **5**.

Question19

If $4x - 3y - 5 = 0$ is a normal to the ellipse $3x^2 + 8y^2 = k$, then the equation of the tangent drawn to this ellipse at the point $(-2, m)(m > 0)$ is

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $3x + 4y - 14 = 0$

B. $3x - 4y + 10 = 0$

C. $3x - 4y + 1 = 0$

D. $4x + 3y - 3 = 0$

Answer: B

Solution:

We have, equation of ellipse is

$$3x^2 + 8y^2 = K$$
$$\Rightarrow \quad \frac{x^2}{\frac{K}{3}} + \frac{y^2}{\frac{K}{8}} = 1$$

Here, $a^2 = \frac{K}{3}$ and $b^2 = \frac{K}{8}$

Equation of normal $= 4x - 3y - 5 = 0$

Or $y = \frac{4}{3}x - \frac{5}{3}$

Here, $m = \frac{4}{3}$ and

$$\frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}} = \frac{5}{3}$$
$$\frac{\left(\frac{K}{3} - \frac{K}{8}\right) \times \frac{4}{3}}{\sqrt{\frac{K}{3} + \frac{K}{8} \times \frac{16}{9}}} = \frac{5}{3}$$
$$K^2 = 20K$$
$$K = 20 \quad [\because K \neq 0]$$

Since, $(-2, m)$ lies cilipse

$\because 3(4) + 8m^2 = 20 \Rightarrow m = \pm 1$

Equation of tangent of ellipse at $(-2, 1)$ is

$-6x + 8y = 20 \Rightarrow 3x - 4y + 10 = 0$

Question20

If the angle between the straight lines joining the foci and the ends of the minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is 90°, then it eccentricity

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $1/2$

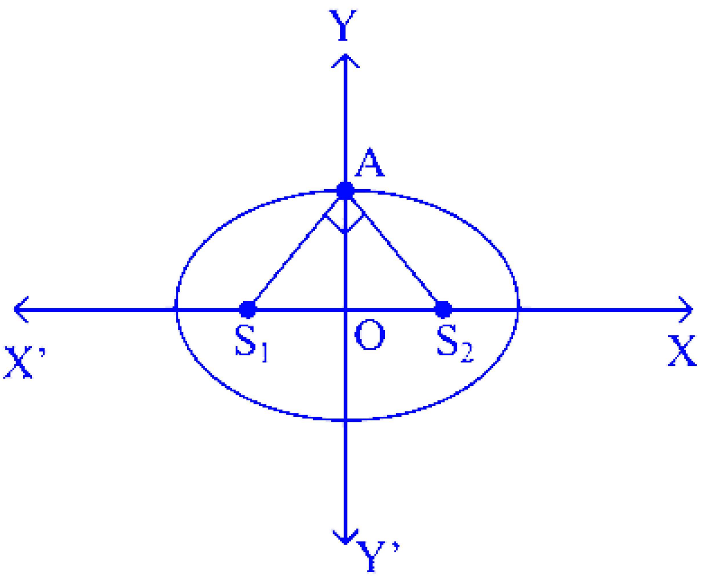
B. $1/4$

C. $1/3$

D. $1/\sqrt{2}$

Answer: D

Solution:



Apply pythagoras theorem, in $\triangle AS_1S_2$

$$(S_1A)^2 + (AS_2)^2 = (S_1S_2)^2$$
$$a^2 + a^2 = (2ae)^2$$
$$\Rightarrow \quad 2a^2 = 4a^2e^2$$
$$\therefore e = \frac{1}{\sqrt{2}}$$

Question21

The focal distances of the point $\left(\frac{4}{\sqrt{5}}, \frac{3}{\sqrt{5}}\right)$ on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ are

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $\frac{10}{3}, \frac{2}{3}$

B. $3, 1$

C. $\frac{13}{3}, \frac{5}{3}$

D. 4, 2

Answer: D

Solution:

Given, equation of ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

Thus, $a^2=4, b^2=9$

Since, $b>a$.

So, the major axis of ellipse is Y-axis

$$e^2=1-\frac{a^2}{b^2}=1-\frac{4}{9}=\frac{5}{9}$$
$$e=\frac{\sqrt{5}}{3}$$

Thus, foci of ellipse be $(0, \pm be)=(0, \pm\sqrt{5})$

So, $S=(0, \sqrt{5})$ and $s'=(0, -\sqrt{5})$

Given, point on ellipse is $p\left(\frac{4}{\sqrt{5}}, \frac{3}{\sqrt{5}}\right)$.

Thus, focal distance

$$PS=\sqrt{\left(\frac{4}{\sqrt{5}}-0\right)^2+\left(\frac{3}{\sqrt{5}}-\sqrt{5}\right)^2}=\sqrt{\frac{4^2}{5}+\frac{4}{5}}$$
$$=\sqrt{\frac{16+4}{5}}=\sqrt{4}=2$$

and focal distance $PS'=\sqrt{\left(\frac{4}{\sqrt{5}}-0\right)^2+\left(\frac{3}{\sqrt{5}}+\sqrt{5}\right)^2}$
$$=\sqrt{\frac{16+64}{5}}=\sqrt{\frac{80}{5}}$$
$$=\sqrt{16}=4$$

Thus, required focal distances are 4, 2.

Question22

A stick of length r units slides with its ends on coordinate axes. Then, the locus of the mid-point of the stick is a curve whose length is

Options:

A. $2\pi r$

B. $\pi\pi^2$

C. $\frac{1}{2}\pi r$

D.

πr

Answer: D

Solution:

Let the coordinates of ends of rod be $A(a, 0), B(0, b)$.

$AB = r \Rightarrow a^2 + b^2 = r^2 \quad \ldots \text{(i)}$

Let the mid-point of AB be (h, k)

$$h = \frac{a+0}{2} = \frac{a}{2}, k = \frac{0+b}{2} = \frac{b}{2}$$
$$\Rightarrow \quad a = 2h, b = 2k$$

From Eq. (i), $(2h)^2 + (2k)^2 = r^2$

$$\Rightarrow h^2 + k^2 = \left(\frac{r}{2}\right)^2$$
$$\begin{aligned} \text{Length} &= \text{Circumference of circle} \\ &= 2\pi \text{ (radius)} \\ &= 2\pi\left(\frac{r}{2}\right) = \pi r \end{aligned}$$

Question23

The eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ lying at a distance of 2 units from its centre is

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

Ellipse : $x^2 + 3y^2 = 6$

$\Rightarrow \quad \frac{x^2}{(\sqrt{6})^2} + \frac{y^2}{(\sqrt{2})^2} = 1$

Let the eccentric angle of the point be θ, then its coordinates are $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$.

Since, the distance of the point from the centre is 2 units.

Therefore, $(\sqrt{6}\cos\theta - 0)^2 + (\sqrt{2}\sin\theta - 0)^2 = 2^2$

$$\begin{aligned}\Rightarrow \quad & 6\cos^2\theta + 2\left(1 - \cos^2\theta\right) = 4 \\ \Rightarrow \quad & 4\cos^2\theta = 2 \\ \Rightarrow \quad & \cos\theta = \pm\frac{1}{\sqrt{2}} \\ \Rightarrow \quad & \theta = \frac{\pi}{4}, \frac{3\pi}{4}\end{aligned}$$

Question24

A point moves so that the sum of its distances from $(ae, 0)$ and $(-ae, 0)$ is $2a$, then the equation to its locus, where $b^2 = a^2\left(1 - e^2\right)$ is

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

B. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

C. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

D. $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

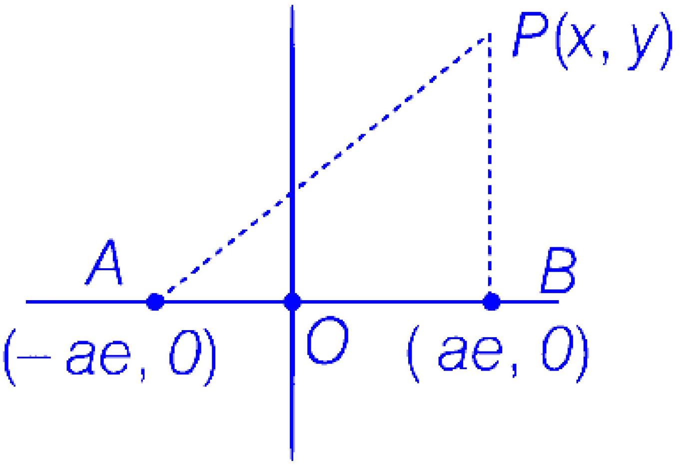
Answer: B

Solution:

Given that, $AP + BP = 2a$

Using distance formula,

$AP = \sqrt{(x+ae)^2 + y^2}$



and $BP = \sqrt{(x-ae)^2 + y^2}$

Now, $\sqrt{(x+ae)^2 + y^2} + \sqrt{(x-ae)^2 + y^2} = 2a$

Squaring both sides,

$$(x+ae)^2 + y^2 + (x-ae)^2 + y^2$$
$$+2\sqrt{(x+ae)^2 + y^2}\sqrt{(x-ae)^2 + y^2} = 4a^2$$
$$\Rightarrow \quad 2x^2 + 2y^2 + 2a^2e^2 - 4a^2$$
$$= -2\sqrt{(x+ae)^2 + y^2}\sqrt{(x-ae)^2 + y^2}$$
$$x^2 + y^2 + a^2e^2 - 2a^2 = -\sqrt{(x+ae)^2 + y^2}$$
$$\cdot\sqrt{(x-ae)^2 + y^2}$$

Squaring both sides,

$$a^4e^4 - 4a^4e^2 + 4a^4 + 2a^2x^2e^2 - 4a^2x^2$$
$$+ 2a^2y^2e^2 - 4a^2y^2 + x^4 + 2x^2y^2 + y^4$$
$$= a^4e^4 - 2a^2x^2e^2 + 2a^2y^2e^2 + x^4 + 2x^2y^2 + y^4$$
$$\Rightarrow \quad 4\left(a^4 - a^4e^2 - a^2x^2 - a^2y^2 + a^2x^2e^2\right) = 0$$
$$\Rightarrow \quad a^2\left(a^2 - a^2e^2 - x^2 - y^2 + x^2e^2\right) = 0$$
$$\Rightarrow a^2\left(1 - e^2\right) - x^2 - y^2 + x^2e^2 = 0$$

$$\Rightarrow \quad b^2 - x^2 - y^2 + x^2\left(1 - \frac{b^2}{a^2}\right) = 0 \quad \left[\because b^2 = a^2\left(1 - e^2\right)\right]$$

$$\Rightarrow \quad x^2\left[1 - \frac{b^2}{a^2} - 1\right] - y^2 + b^2 = 0$$

$$\Rightarrow x^2\left(-\frac{b^2}{a^2}\right) - y^2 = -b^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \left(\text{divide by } b^2\right)$$

Question25

If $\tan\theta_1, \tan\theta_2 = \frac{-a^2}{b^2}$, then the chord joining 2 points θ_1 and θ_2 one the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. Focus

B. Center

C. end of major axis

D. end of minor axis

Answer: B

Solution:

Let two points be $A\left(a\cos\theta_1 b\sin\theta_1\right)$ and $\left(a\cos\theta_2, b\sin\theta_2\right)$.

O : origin$(0,0)$

$$m_{OA}=\frac{b\sin\theta_1}{a\cos\theta_1}=\frac{b}{a}\tan\theta_1$$
$$m_{OB}=\frac{b}{a}\tan\theta_2$$
$$\Rightarrow \quad m_{OA}\times m_{OB}=-1\Rightarrow\frac{b^2}{a^2}\tan\theta_1\tan\theta_2=-1$$

Question26

In an ellipse, if the distance between the fociis 6 units and the length of its minor axis is8 units, then its eccentricity is

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. $\frac{1}{2}$

B. $\frac{7}{5}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{3}{5}$

Answer: D

Solution:

In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Distance between foci $=2ae=6$

$\Rightarrow \quad ae=3$
and length of minor axis $=2b=8$
$\Rightarrow \quad b=4$
$\because \quad (ae)^2=a^2-b^2$
$\Rightarrow \quad 3^2=a^2-4^2\Rightarrow a^2=25$

Now, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

Question27

If a point $P(x, y)$ moves along the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and if C is the center of the ellipse, then the sum of maximum and minimum values of CP is

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. 25

B. 9

C. 4

D. 5

Answer: B

Solution:

Given ellipse, $\frac{x^2}{25}+\frac{y^2}{16}=1$

Length of semi-major axis $=5$

Length of semi-minor axis $=4$

Centre $C=(0,0)$

If $P(x, y)$ be any point on the ellipse, then

maximum value of $CP=$ Length of the semi-major axis $=5$

minimum value of $CP=$ Length of semi-minor axis $=4$

Their sum $=5+4=9$
