

Polynomial

1. Classify the following polynomials as polynomials in one variable, two variables etc.

(i) $x^2 + x + 1$

(ii) $y^3 - 5y$

(iii) $xy + yz + zx$

(iv) $x^2 - 2xy + y^2 + 1$

2. Determine the degree of each of the following polynomials :

(i) $2x - 1$

(ii) -10

(iii) $x^3 - 9x + 3x^5$

(iv) $y^3(1 - y^4)$

3. For the polynomial

$$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6, \text{ write}$$

(i) the degree of the polynomial

(ii) the coefficient of x^3

(iii) the coefficient of x^6

(iv) the constant term

4. Write the coefficient of x^2 in each of the following :

(i) $\frac{\pi}{6}x + x^2 - 1$

(ii) $3x - 5$

(iii) $(x - 1)(3x - 4)$

(iv) $(2x - 5)(2x^2 - 3x + 1)$

5. Classify the following as a constant, linear, quadratic and cubic polynomials :

(i) $2 - x^2 + x^3$

(ii) $3x^3$

(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) $2 + x$

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

6. Give an example of a polynomial, which is :

(i) monomial of degree 1

(ii) binomial of degree 20

(iii) trinomial of degree 2

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when $x = 3$ and also when $x = -3$.

8. If $p(x) = x^2 - 4x + 3$, evaluate : $p(2) - p(-1) + p\left(\frac{1}{2}\right)$

9. Find $p(0), p(1), p(-2)$ for the following polynomials :

(i) $p(x) = 10x - 4x^2 - 3$

(ii) $p(y) = (y + 2)(y - 2)$

10. Verify whether the following are True or False :

(i) -3 is a zero of $x - 3$

(ii) $-\frac{1}{3}$ is a zero of $3x + 1$

(iii) $\frac{-4}{5}$ is a zero of $4 - 5y$

(iv) 0 and 2 are the zeroes of $t^2 - 2t$

(v) -3 is a zero of $y^2 + y - 6$

11. Find the zeroes of the polynomial in each of the following :

(i) $p(x) = x - 4$

(ii) $g(x) = 3 - 6x$

(iii) $q(x) = 2x - 7$

(iv) $h(y) = 2y$

12. Find the zeroes of the polynomial :

$$p(x) = (x - 2)^2 - (x + 2)^2$$

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial : $x^4 + 1$; $x - 1$

14. By Remainder Theorem find the remainder, when $p(x)$ is divided by $g(x)$, where

(i) $p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$

(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1$

(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - \frac{3}{2}x$

15. Check whether $p(x)$ is a multiple of $g(x)$ or not :

(i) $p(x) = x^3 - 5x^2 + 4x - 3$, $g(x) = x - 2$

(ii) $p(x) = 2x^3 - 11x^2 - 4x + 5$, $g(x) = 2x + 1$

16. Show that :

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

17. Determine which of the following polynomials has $x - 2$ a factor :

(i) $3x^2 + 6x - 24$

(ii) $4x^2 + x - 2$

18. Show that $p - 1$ is a factor of $p^{10} - 1$ and also of $p^{11} - 1$.

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by $x + 2$?

20. If $x + 2a$ is a factor of $x^5 - 4a^2x^3 + 2x + 2a + 3$, find a .

21. Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$

22. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a .

23. Factorise :

(i) $x^2 + 9x + 18$

(ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$

(iv) $84 - 2r - 2r^2$

24. Factorise :

(i) $2x^3 - 3x^2 - 17x + 30$

(ii) $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x - 4$

(iv) $3x^3 - x^2 - 3x + 1$

25. Using suitable identity, evaluate the following:

(i) 103^3

(ii) 101×102

(iii) 999^2

26. Factorise the following:

(i) $4x^2 + 20x + 25$

(ii) $9y^2 - 66yz + 121z^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

27. Factorise the following :

(i) $9x^2 - 12x + 3$

(ii) $9x^2 - 12x + 4$

28. Expand the following :

(i) $(4a - b + 2c)^2$

(ii) $(3a - 5b - c)^2$

(iii) $(-x + 2y - 3z)^2$

29. Factorise the following :

(i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$

(ii) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$

(iii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

30. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

31. Expand the following :

(i) $(3a - 2b)^3$

(ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

(iii) $\left(4 - \frac{1}{3x}\right)^3$

32. Factorise the following :

(i) $1 - 64a^3 - 12a + 48a^2$

(ii) $8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$

33. Find the following products :

(i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$ (ii) $(x^2 - 1)(x^4 + x^2 + 1)$

34. Factorise :

(i) $1 + 64x^3$ (ii) $a^3 - 2\sqrt{2}b^3$

35. Find the following product :

$(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

36. Factorise :

(i) $a^3 - 8b^3 - 64c^3 - 24abc$ (ii) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$.

37. Without actually calculating the cubes, find the value of :

(i) $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ (ii) $(0.2)^3 - (0.3)^3 + (0.1)^3$

38. Without finding the cubes, factorise

$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

39. Find the value of

(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$
(ii) $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

40. Give possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a - 3$.

41) If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a .

42) The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a . Also find the remainder when $p(x)$ is divided by $x + 2$.

43) If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

44) Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.
[Hint: Factorise $x^2 - 3x + 2$]

45) Simplify $(2x - 5y)^3 - (2x + 5y)^3$.

46) Multiply $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ by $(-z + x - 2y)$.

47) If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

48) If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

49) Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.

50) Which of the following expressions are polynomials? Justify your answer:

(i) 8

(ii) $\sqrt{3}x^2 - 2x$

(iii) $1 - \sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$

(v) $\frac{(x-2)(x-4)}{x}$

(vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$

(viii) $\frac{1}{2x}$

51) Write whether the following statements are **True** or **False**. Justify your answer.

(i) A binomial can have at most two terms

(ii) Every polynomial is a binomial

(iii) A binomial may have degree 5

(iv) Zero of a polynomial is always 0

(v) A polynomial cannot have more than one zero

(vi) The degree of the sum of two polynomials each of degree 5 is always 5.