

Circles

Selected NCERT Questions

1. From a point Q , the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

Sol. Let QT be the tangent and OT be the radius of circle. Therefore

$$OT \perp QT \quad \text{i.e., } \angle OTQ = 90^\circ$$

$$\text{and } OQ = 25 \text{ cm and } QT = 24 \text{ cm}$$

Now, by Pythagoras Theorem, we have

$$OQ^2 = QT^2 + OT^2 \quad \Rightarrow \quad 25^2 = 24^2 + OT^2$$

$$\Rightarrow \quad OT^2 = 25^2 - 24^2 = 625 - 576$$

$$OT^2 = 49 \quad \therefore \quad OT = 7 \text{ cm}$$

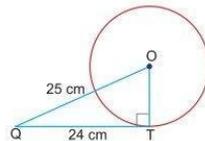


Fig. 8.7

2. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

[CBSE (F) 2014, CBSE Delhi 2017, CBSE 2019(30/5/2)]

Sol. Let AB be the diameter of the given circle with centre O , and two tangents PQ and LM are drawn at the end of diameter AB respectively.

Now, since the tangent at a point to a circle is perpendicular to the radius through the point of contact.

$$\text{Therefore, } OA \perp PQ \text{ and } OB \perp LM$$

$$\text{i.e., } AB \perp PQ \text{ and also } AB \perp LM$$

$$\Rightarrow \quad \angle BAQ = \angle ABL \quad (\text{Each } 90^\circ)$$

$$\therefore \quad PQ \parallel LM \quad (\because \angle BAQ \text{ and } \angle ABL \text{ are alternate angles})$$

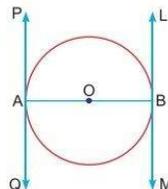


Fig. 8.8

3. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Sol. Let the tangents to a circle with centre O be ABC and XYZ .

Construction : Join OB and OY .

Draw $OP \parallel AC$

$$\therefore \quad AB \parallel PO$$

$$\therefore \quad \angle ABO + \angle POB = 180^\circ \quad (\text{Adjacent interior angles})$$

$$\angle ABO = 90^\circ \quad (\text{A tangent to a circle is perpendicular to the radius through the point of contact})$$

$$90^\circ + \angle POB = 180^\circ \quad \Rightarrow \quad \angle POB = 90^\circ$$

$$\text{Similarly, } \angle POY = 90^\circ$$

$$\angle POB + \angle POY = 90^\circ + 90^\circ = 180^\circ$$

Hence, BOY is a straight line passing through the centre of the circle.

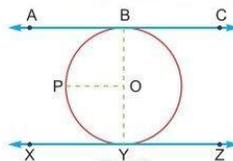


Fig. 8.9

4. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. Let O be the common centre of two concentric circles and let AB be a chord of larger circle touching the smaller circle at P . Join OP .

Since OP is the radius of the smaller circle and AB is tangent to this circle at P ,

$$\therefore \quad OP \perp AB$$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord.

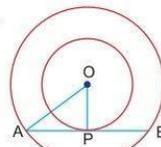


Fig. 8.10

Therefore, $AP = BP$

In right $\triangle APO$ we have

$$OA^2 = AP^2 + OP^2$$

$$5^2 = AP^2 + 3^2 \quad \Rightarrow \quad 25 - 9 = AP^2$$

$$AP^2 = 16 \quad \Rightarrow \quad AP = 4$$

$$\text{Now, } AB = 2 \cdot AP = 2 \times 4 = 8 \quad (\because AP = PB)$$

Hence, the length of the chord of the larger circle which touches the smaller circle is 8 cm.

5. A quadrilateral $ABCD$ is drawn to circumscribe a circle (see Fig. 8.11). Prove that

$$AB + CD = AD + BC$$

[CBSE (AI) 2016]

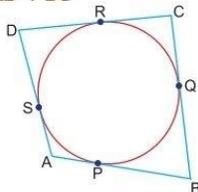


Fig. 8.11

Sol.

Given a quadrilateral circumscribing a circle with centre O , such that it touches side AB, BC, CD, AD at P, Q, R, S

To prove = $AB + CD = BC + DA$

Proof = Length of tangent drawn from external point are equal

$AP = AS$	\rightarrow (at A)	— (1)
$BP = BQ$	\rightarrow (at B)	— (2)
$DR = DS$	\rightarrow (at C)	— (3)
$CR = CQ$	\rightarrow (at D)	— (4)

Adding eq (1), (2), (3), (4)

$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$\underline{AB + CD = AD + BC}$$

Hence proved. [Topper's Answer 2016]

6. In Fig. 8.12, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B . Prove that $\angle AOB = 90^\circ$.

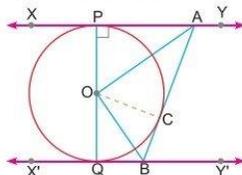
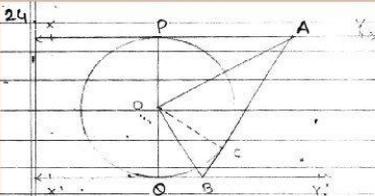


Fig. 8.12

[CBSE (AI) 2017, 2019(30/2/1)]

Sol.



Given: $XY \parallel X'Y'$ - tangents.
 PQ is diameter, OC is radius.
 Tangent ACB touches XY at A and $X'Y'$ at B .

To prove: $\angle AOB = 90^\circ$.

proof: $XY \parallel X'Y'$ and AB is transversal.

$$\therefore \angle XAB + \angle ABX' = 180^\circ \quad \text{--- co-interior angles}$$

$$\text{or } \angle PAB + \angle Q'AB = 180^\circ \quad \text{--- (1)}$$

It is known that tangents from a same point are equally inclined to the line joining centre to that point.

$$\Rightarrow \angle PAO = \angle CAO \quad \text{and} \quad \angle QBO = \angle CBO$$

In (1),

$$\checkmark 2\angle CAO + 2\angle CBO = 180^\circ$$

$$\text{or } 2\angle BAO + 2\angle ABO = 180^\circ$$

$$\angle BAO + \angle ABO = 90^\circ \quad \text{--- (2)}$$

In ΔAOB ,

$$\angle BAO + \angle ABO + \angle AOB = 180^\circ \quad \text{--- angle sum.}$$

$$\text{From (2), } 90^\circ + \angle AOB = 180^\circ$$

$$\therefore \angle AOB = 90^\circ$$

Hence, proved.

[Topper's Answer 2017]

7. Prove that the parallelogram circumscribing a circle is a rhombus.

[CBSE Delhi 2014; CBSE 2019 (30/5/1)]

Sol. Let $ABCD$ be a parallelogram such that its sides touch a circle with centre O .

We know that the tangents to a circle from an exterior point are equal in length.

Therefore, we have

$$AP = AS \quad (\text{Tangents from } A) \quad \dots (i)$$

$$BP = BQ \quad (\text{Tangents from } B) \quad \dots (ii)$$

$$CR = CQ \quad (\text{Tangents from } C) \quad \dots (iii)$$

$$\text{And } DR = DS \quad (\text{Tangents from } D) \quad \dots (iv)$$

Adding (i), (ii), (iii) and (iv), we have

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$

$$AB + AB = BC + BC \quad (\because ABCD \text{ is a parallelogram } \therefore AB = CD, BC = DA)$$

$$2AB = 2BC \quad \Rightarrow \quad AB = BC$$

Thus, $AB = BC = CD = AD$

Hence, $ABCD$ is a rhombus.

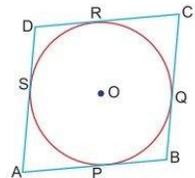


Fig. 8.13

8. A triangle ABC [Fig. 8.14] is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC . [Competency Based Question]

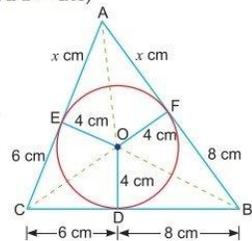


Fig. 8.14

Sol. Let $\triangle ABC$ be drawn to circumscribe a circle with centre O and radius 4 cm and circle touches the sides BC , CA and AB at D , E and F respectively.

We have given that $CD = 6$ cm and $BD = 8$ cm

$$\therefore BF = BD = 8 \text{ cm and } CE = CD = 6 \text{ cm}$$

(Length of two tangents drawn from an external point of circle are equal)

Now, let $AF = AE = x$ cm

Then, $AB = c = (x + 8)$ cm, $BC = a = 14$ cm, $CA = b = (x + 6)$ cm

$$\therefore 2s = (x + 8) + 14 + (x + 6)$$

$$\Rightarrow 2s = 2x + 28 \quad \text{or} \quad s = x + 14$$

$$s - a = (x + 14) - 14 = x$$

$$s - b = (x + 14) - (x + 6) = 8$$

$$s - c = (x + 14) - (x + 8) = 6$$

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{(x+14)(x)(8)(6)} = \sqrt{48x(x+14)} \end{aligned}$$

Also, $\text{area}(\triangle ABC) = \text{area}(\triangle OBC) + \text{area}(\triangle OCA) + \text{area}(\triangle OAB)$

$$\begin{aligned} &= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times CA \times OE + \frac{1}{2} \times AB \times OF \\ &= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (x + 6) \times 4 + \frac{1}{2} \times (x + 8) \times 4 \\ &= 28 + 2x + 12 + 2x + 16 = 4x + 56 \end{aligned}$$

$$\therefore \sqrt{48x(x+14)} = 4x + 56 \Rightarrow \sqrt{48x(x+14)} = 4(x + 14)$$

Squaring both sides, we have

$$48x(x + 14) = 16(x + 14)^2$$

$$\Rightarrow 48x(x + 14) - 16(x + 14)^2 = 0$$

$$\Rightarrow 16(x + 14)[3x - (x + 14)] = 0$$

$$\Rightarrow 16(x + 14)(2x - 14) = 0$$

$$\text{either } 16(x + 14) = 0 \quad \text{or} \quad 2x - 14 = 0$$

$$\Rightarrow x = -14 \quad \text{or} \quad 2x = 14$$

$$\Rightarrow x = -14 \quad \text{or} \quad x = 7$$

But x cannot be negative so $x \neq -14$

$$\therefore x = 7 \text{ cm}$$

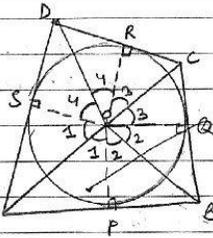
Hence, the sides $AB = x + 8 = 7 + 8 = 15$ cm and

$$AC = x + 6 = 7 + 6 = 13 \text{ cm.}$$

9. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. [CBSE 2019 (30/3/1)]

Sol.

14.



Construction: Constructed a quadrilateral ABCD, circumscribing a circle (centre O). Circle touches AB, BC, CD, DA at P, Q, R, S respectively.

To prove: $\angle AOB + \angle COD = 180^\circ$
Or $\angle AOD + \angle BOC = 180^\circ$

We know, that tangents from same exterior point subtend equal angle at the centre of circle with radius.

$$\therefore \angle AOP = \angle AOS = \angle 1 \text{ (say)}$$

$$\text{Similarly, } \angle BOR = \angle BOQ = \angle 2$$

$$\angle COQ = \angle COR = \angle 3$$

$$\angle DOR = \angle DOS = \angle 4.$$

$$\therefore \angle AOP + \angle BOR + \angle COQ + \angle COR + \angle DOR + \angle DOS + \angle AOS = 360^\circ$$

[Complete angle around a point]

$$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 3 + 2\angle 4 = 360^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^\circ$$

$$\Rightarrow \angle AOB + \angle COD = 180^\circ$$

$$\text{Or } (\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 180^\circ$$

$$\Rightarrow \angle AOD + \angle BOC = 180^\circ$$

Hence, proved!

[Topper's Answer 2019]

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. A circle with centre O is shown below.

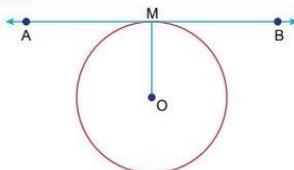


Fig. 8.15

Which of the following statements is true?

[Competency Based Question]

- There can be only one line passing through point M such that it is parallel to OM .
 - There can be only one line passing through point M such that it is perpendicular to OM .
 - There can be any number of lines passing through point M such that they are parallel to OM .
 - There can be any number of lines passing through point M such that they are perpendicular to OM .
2. In Fig. 8.16, RJ and RL are tangents of the circle. What is the measure of $\angle JRL$?

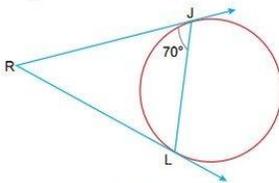


Fig. 8.16

- $180^\circ - 70^\circ$
 - $180^\circ - 140^\circ$
 - $90^\circ - 70^\circ$
 - $90^\circ - 140^\circ$
3. In Fig. 8.17, TP and TQ are tangents drawn to the circle with centre at O , if $\angle POQ = 115^\circ$ then $\angle PTQ$ is

[CBSE 2020(30/4/1)]

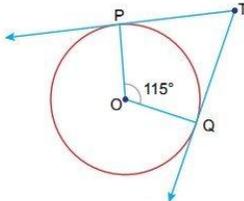


Fig. 8.17

- 115°
 - 57.5°
 - 55°
 - 65°
4. In Fig. 8.18, the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm. Then radius of the circle is

[NCERT Exemplar]

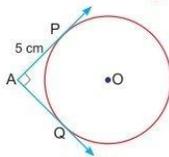


Fig. 8.18

- 10 cm
- 7.5 cm
- 5 cm
- 2.5 cm

5. In Fig. 8.19, the perimeter of $\triangle ABC$ is

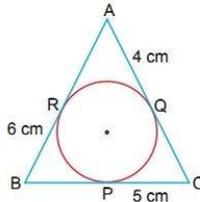


Fig. 8.19

- (a) 30 cm (b) 60 cm (c) 45 cm (d) 15 cm
6. In Fig. 8.20, PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60^\circ$. Then $\angle PRQ$ is equal to [NCERT Exemplar]

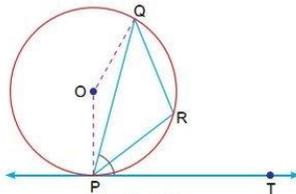


Fig. 8.20

- (a) 135° (b) 150° (c) 120° (d) 110°
7. In Fig. 8.21, PQ is tangent to the circle with centre at O , at the point B . If $\angle AOB = 100^\circ$, then $\angle ABP$ is equal to [CBSE 2020(30/5/1)]

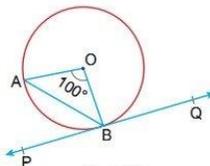


Fig. 8.21

- (a) 50° (b) 40° (c) 60° (d) 80°
8. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to [NCERT Exemplar]
- (a) $\frac{3}{2}\sqrt{3}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm
9. Two circles touch each other externally at C and AB is common tangent of circles, then $\angle ACB$ is

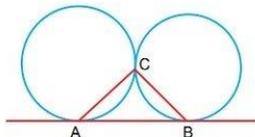


Fig. 8.22

- (a) 70° (b) 60° (c) 100° (d) 90°

10. In Fig. 8.23, if PQR is the tangent to a circle at Q whose centre is O , AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal [NCERT Exemplar]

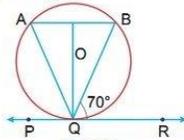


Fig. 8.23

- (a) 20° (b) 40° (c) 35° (d) 45°
11. In Fig. 8.24, from an external point P , two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O . If $\angle QPR = 90^\circ$, then length of PQ is [CBSE 2020(30/5/1)]

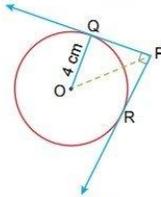


Fig. 8.24

- (a) 3 cm (b) 4 cm (c) 2 cm (d) $2\sqrt{2}$ cm
12. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is [NCERT Exemplar]
- (a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm
13. In Fig. 8.25, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to [NCERT Exemplar]

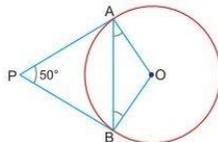


Fig. 8.25

- (a) 25° (b) 30° (c) 40° (d) 50°
14. From an external point Q , the length of the tangent to a circle is 5 cm and the distance of Q from the centre is 8 cm. The radius of the circle is [CBSE 2020(30/4/1)]
- (a) 39 cm (b) 3 cm (c) $\sqrt{39}$ cm (d) 7 cm
15. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is [Competency Based Question]

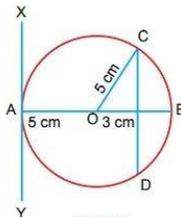


Fig. 8.26

- (a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

Answers

1. (b) 2. (b) 3. (d) 4. (c) 5. (a) 6. (c) 7. (a)
 8. (d) 9. (d) 10. (b) 11. (b) 12. (b) 13. (a) 14. (c)
 15. (d)

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. The length of the tangent to a circle from a point P , which is 17 cm away from the centre, is 15 cm. What is the radius of the circle?

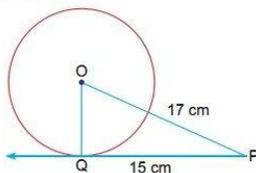


Fig. 8.27

- Sol.** $\because OQ \perp PQ$ (\because Radius is perpendicular to tangent at point of contact)
 $\therefore PQ^2 + OQ^2 = OP^2$
 $\Rightarrow OQ^2 + (15)^2 = (17)^2$
 or $OQ = \sqrt{289 - 225}$
 $= \sqrt{64} = 8 \text{ cm}$

2. From an external point P , tangents PA and PB are drawn to a circle with centre O . If $\angle PAB = 50^\circ$, then find $\angle AOB$.

[CBSE Delhi 2016]

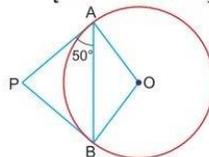


Fig. 8.28

- Sol.** $\because PA = PB$
 $\Rightarrow \angle BAP = \angle ABP = 50^\circ$ (Fig. 8.28)
 $\therefore \angle APB = 180^\circ - 50^\circ - 50^\circ = 80^\circ$
 and $\angle AOB = 180^\circ - 80^\circ = 100^\circ$

3. In Fig. 8.29, $\triangle ABC$ is circumscribing a circle. Find the length of BC .

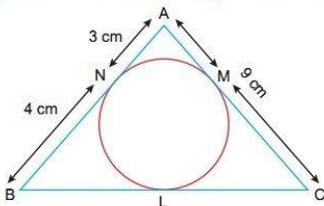


Fig. 8.29

- Sol.** $AN = AM = 3 \text{ cm}$ (Tangents drawn from an external point)
 $BN = BL = 4 \text{ cm}$ (Tangents drawn from an external point)
 $CL = CM = AC - AM = 9 - 3 = 6 \text{ cm}$
 $\Rightarrow BC = BL + CL = 4 + 6 = 10 \text{ cm.}$

4. In Fig. 8.30, AB and CD are common tangents to circles which touch each other at D . If $AB = 8$ cm, then find the length of CD . [CBSE 2020 (30/4/1)]

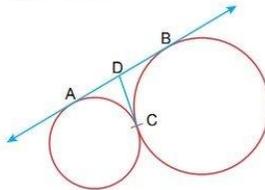


Fig. 8.30

Sol. As $AD = CD = BD$ (\because Tangents from external point are equal)

$$\therefore CD = 4 \text{ cm}$$

$\frac{1}{2}$

$\frac{1}{2}$

[CBSE Marking Scheme 2020 (30/4/1)]

5. In Fig. 8.31, there are two concentric circles with centre O . PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm, find the length of PS . [CBSE Delhi 2017(C)]

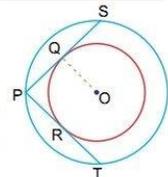


Fig. 8.31

Sol. $PQ = PR = 5$ cm (\because Tangents drawn from external point are equal)

$$\therefore PS = 2PQ = 10 \text{ cm} \quad (\because \text{Perpendicular drawn from centre to the chord bisects the chord})$$

6. Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE 2019 (30/3/1)]

Sol.

Q. Given, 2 concentric circles

$OP = OA = a$
 $OM = b$
 To find - PQ
 $PM = \sqrt{OP^2 - OM^2} \Rightarrow PM = \sqrt{a^2 - b^2}$
 $PQ = 2PM \Rightarrow PQ = 2\sqrt{a^2 - b^2}$ units [Topper's Answer 2019]

7. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O , is 60° , then find the length of OP . [CBSE (AI) 2017]

Sol.

Q. Tangents are equally inclined to line joining the external point P to centre O .
 $\therefore \angle APO = \angle BPO = \frac{60^\circ}{2} = 30^\circ$
 also radius \perp tangent at point of contact.
 \therefore in right ΔOAP , $\angle APO = 30^\circ$.
 Now $\sin 30^\circ = \frac{AO}{OP}$
 $\frac{1}{2} = \frac{a}{OP}$ \therefore radius = a
 $\therefore OP = 2a$ [Topper's Answer 2017]

Short Answer Questions-I

Each of the following questions are of 2 marks.

1. If from an external point P of a circle with centre O , two tangents PQ and PR are drawn such that $\angle QPR = 120^\circ$, prove that $2PQ = PO$.
[CBSE Delhi 2014, (F) 2016]

Sol. Given, $\angle QPR = 120^\circ$

\therefore Radius is perpendicular to the tangent at the point of contact.

$\therefore \angle OQP = 90^\circ$ and

$\angle QPO = 60^\circ$ (Tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point.)

$$\text{In } \Delta QPO, \cos 60^\circ = \frac{PQ}{PO} \Rightarrow \frac{1}{2} = \frac{PQ}{PO} \Rightarrow 2PQ = PO$$

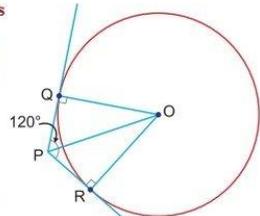


Fig. 8.32

2. In Fig. 8.33, AB is the diameter of a circle with centre O and AT is a tangent if $\angle AOQ = 58^\circ$, find $\angle ATQ$.

Sol. We have,

$$\angle AOQ = 58^\circ$$

$$\therefore \angle ABQ = \frac{1}{2} \angle AOQ = \frac{1}{2} \times 58^\circ = 29^\circ$$

(Angle on the circumference of the circle by the same arc)

$$\angle BAT = 90^\circ \quad (\because OA \perp AT)$$

$$\therefore \angle ATQ = 90^\circ - 29^\circ = 61^\circ$$

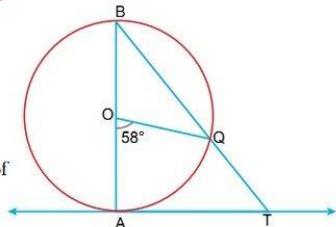


Fig. 8.33

3. In Fig. 8.34, XP and XQ are two tangents to the circle with centre O , drawn from an external point X . ARB is another tangent, touching the circle at R . Prove that $XA + AR = XB + BR$.
[CBSE (F) 2014]

Sol. In the given figure,

$$AP = AR$$

$$BR = BQ$$

$$XP = XQ$$

(Tangents to a circle from an external point are equal)

$$XA + AP = XB + BQ$$

$$XA + AR = XB + BR \quad [AP = AR, BQ = BR]$$

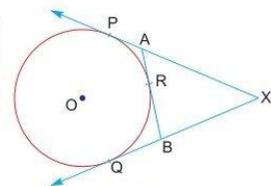


Fig. 8.34

4. In Fig. 8.35, are two concentric circles of radii 6 cm and 4 cm with centre O . If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8 cm, find the length of BP .
[CBSE (F) 2016]

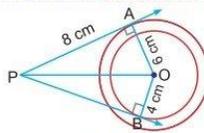


Fig. 8.35

Sol. $OA = 6$ cm, $OB = 4$ cm, $AP = 8$ cm

$$OP^2 = OA^2 + AP^2 = 36 + 64 = 100$$

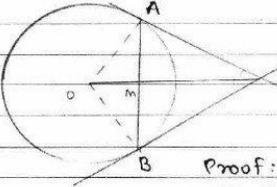
$$\Rightarrow OP = 10 \text{ cm}$$

$$BP^2 = OP^2 - OB^2 = 100 - 16 = 84$$

$$\Rightarrow BP = 2\sqrt{21} \text{ cm}$$

5. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. [CBSE (AI) 2017]

Sol.

6. 

Given: chord AB.
tangents AP and BP at A & B
P To prove: $AP = BP$ $\angle PAM = \angle PBM$
Construction: Join centre O to P
let OP meet AB at M.

Proof:

In $\triangle AMP$ and $\triangle BMP$
 $AP = BP$ - tangents from same point to a circle are equal.
 $MP = MP$ - common side
 $\angle APM = \angle BPM$ - tangents are equally inclined to line joining the points to circle's centre. *evidence*

by SAS criterion,
 $\triangle AMP \cong \triangle BMP$.
 by cpct, $\angle PAM = \angle PBM$
 Hence, tangents at endpoints of a chord make equal angles with it [Topper's Answer 2017]

6. PB is a tangent to the circle with centre O to B. AB is a chord of length 24 cm at a distance of 5 cm from the centre, if the tangent is of length 20 cm, find the length of PO.

Sol. Join OB,

Now, In right angled $\triangle OMB$, we have

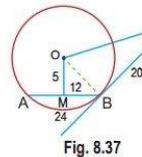
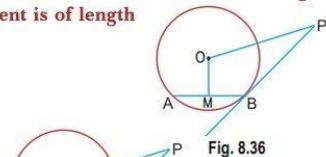
$$OB^2 = 5^2 + 12^2 = 169 \Rightarrow OB = 13 \text{ cm}$$

\therefore In right $\triangle OBP$, we have

$$OP^2 = OB^2 + BP^2 = 13^2 + 20^2 = 169 + 400$$

$$\Rightarrow OP^2 = 569$$

$$\therefore OP = \sqrt{569} = 23.66 \text{ cm}$$



7. In Fig. 8.38, a circle is inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF. [CBSE Delhi 2016]

Sol. Let

$$AD = AF = x$$

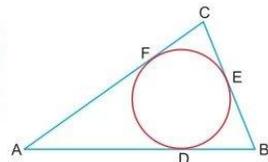
$$\therefore DB = BE = 12 - x$$

$$\text{and } CF = CE = 10 - x$$

$$BC = BE + EC$$

$$\Rightarrow 8 = 12 - x + 10 - x \Rightarrow x = 7$$

$$\therefore AD = 7 \text{ cm, } BE = 12 - 7 = 5 \text{ cm, } CF = 10 - 7 = 3 \text{ cm}$$



8. In Fig. 8.39 from an external point P , two tangents PT and PS are drawn to a circle with centre O and radius r . If $OP = 2r$, show that $\angle OTS = \angle OST = 30^\circ$.
[Competency Based Question]

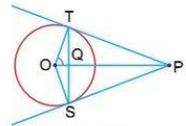


Fig. 8.39

Sol.

7) Given - PT & PS are two tangents drawn from P to circle $C(O, r)$ & $OP = 2r$
 To prove = $\angle OTS = \angle OST = 30^\circ$
 Proof -
 In $\triangle OPT$,
 Let $\angle TOP = \theta$
 $\cos \theta = \frac{OT}{OP} = \frac{r}{2r}$ $\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$
 $\cos \theta = \frac{1}{2}$
 also $\cos 60^\circ = \frac{1}{2}$, then
 $\theta = 60^\circ$
 similarly, $\angle SOP = 60^\circ$, and $\angle SOT = 120^\circ$
 In $\triangle OST$,
 applying (By the) angle sum property of \triangle -
 $\angle OTS + \angle OST + \angle TOS = 180^\circ$
 $\angle OTS + \angle OST = 180^\circ - \angle TOS$
 $\angle OTS + \angle OST = 60^\circ$
 Since $OT = OS$ (radius of circle) $\Rightarrow \angle OTS = \angle OST$
 $\Rightarrow 2\angle OST = 60^\circ \Rightarrow \angle OST = 30^\circ$
 $\Rightarrow \angle OTS = \angle OST = 30^\circ$
 Hence Proved
 [Topper's Answer 2016]

9. In Fig. 8.40, PA and PB are tangents to the circle from an external point P . CD is another tangent touching the circle at Q . If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.
[CBSE Delhi 2017]

Sol. $PA = PC + CA = PC + CQ$ [$\because CA = CQ$ (tangents drawn from external point are equal)]

$$\Rightarrow 12 = PC + 3 \Rightarrow PC = 9 \text{ cm}$$

$$\therefore PA = PB \Rightarrow PA - AC = PB - BD \Rightarrow PC = PD$$

$$\therefore PD = 9 \text{ cm}$$

$$\text{Hence, } PC + PD = 18 \text{ cm}$$

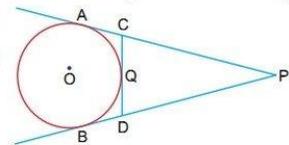


Fig. 8.40

Short Answer Questions-II

Each of the following questions are of 3 marks.

1. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R , respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.

[CBSE 2020 (30/2/1)]

Sol. Given: A triangle ABC and a circle touching the side BC at P and AB, AC produce to at Q and R respectively.

To prove: $AQ = \frac{1}{2}(BC + CA + AB)$

Proof: We know that lengths of tangents drawn from an external point to a circle are equal.

$$\Rightarrow AQ = AR, BQ = BP, CP = CR$$

$$\begin{aligned} \text{Now, perimeter of } \triangle ABC &= AB + BC + CA \\ &= (AQ - BQ) + (BP + PC) + (AR - CR) \\ &= AQ - BQ + BQ + CR + AQ - CR \\ &\quad (\because AQ = AR, BP = BQ, PC = CR) \\ &= 2AQ \end{aligned}$$

$$\therefore AB + BC + CA = 2AQ$$

$$\therefore AQ = \frac{1}{2}(AB + BC + CA)$$

$$\Rightarrow AQ = \frac{1}{2}(BC + CA + AB) \quad \text{Proved}$$

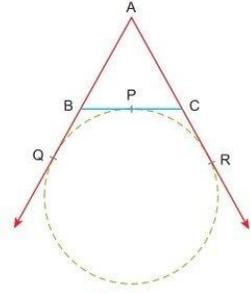


Fig. 8.41

2. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

[CBSE 2020(30/3/1)]

Sol. Given: A circle $C(O, r)$ and a tangent AB at a point P .

To Prove: $OP \perp AB$.

Construction: Take any point Q , other than P , on the tangent AB . Join OQ . Suppose OQ meets the circle at R .

Proof: We know that among all line segments joining the point O to a point on AB , the shortest one is perpendicular to AB . So, to prove that $OP \perp AB$ it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB .

$$\text{Clearly, } OP = OR \quad (\text{Radii of the same circle})$$

$$\text{Now, } OQ = OR + RQ$$

$$\Rightarrow OQ > OR$$

$$\Rightarrow OQ > OP \quad (\because OP = OR)$$

Thus, OP is shorter than any other segment joining O to any point on AB .

Hence, $OP \perp AB$.

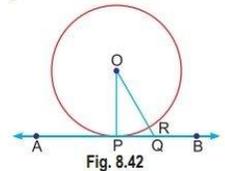


Fig. 8.42

3. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by $r = \frac{a + b - c}{2}$. [NCERT Exemplar]

Sol. Let the circle touches the sides BC, CA and AB of the right triangle ABC at D, E and F respectively, where $BC = a, CA = b$, and $AB = c$.

(See fig. 8.43)

Then $AE = AF$

and $BD = BF$

Also $CE = CD = r$

i.e., $AF = AE = b - r, a - r = BF = BD$

or $AB = c = AF + BF = b - r + a - r$

This gives, $r = \frac{a+b-c}{2}$

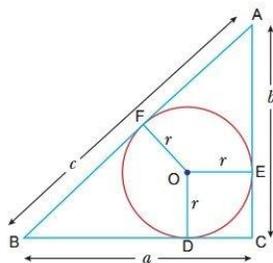


Fig. 8.43

Long Answer Questions

Each of the following questions are of 5 marks.

1. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

[CBSE Delhi 2014, (F) 2014, Delhi 2016, (AI) 2016, (F) 2016, CBSE Delhi 2017, (AI) 2017, (F) 2017, Delhi 2017 (C) 2018]

Sol.

19) Given: Circle (O, r) . AP and PB are tangents drawn to the circle.

To prove: $PA = PB$.

Construction: Join OA, OB and OP.

Proof: $OA = OB$ [radius]. (side).
 $\angle OAP = \angle OBP = 90^\circ$ (right angle).
 $[\because \text{radius is perpendicular to tangent at point of contact}]$.
 $OP = OP$ (hypotenuse).
 So in $\triangle OAP$ and $\triangle OBP$,
 by R.H.S congruency,
 $\rightarrow \triangle OAP \cong \triangle OBP$.
 by CPCT,
 $\Rightarrow AP = BP$.
 hence proved.

[Topper's Answer 2018]

2. In Fig. 8.44, two equal circles, with centres O and O' , touch each other at X . OO' produced meets the circle with centre O' at A . AC is tangent to the circle with centre O , at the point C . $O'D$ is perpendicular to AC . Find the value of $\frac{DO'}{CO}$.
- [CBSE (AI) 2016]

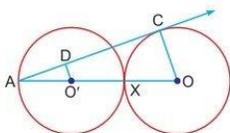
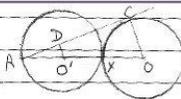


Fig. 8.44

Sol.

22) We have -

Two equal circles, O & O' , touching each other at X and $O'D \perp AC$.



To find = $\frac{DO'}{CO}$

Let radius of each circle = r

In $\triangle ADO'$ & $\triangle ACO$ -

$\angle A = \angle A$ (common)

$\angle ADO' = \angle ACO$ (each 90°)

$\Rightarrow \triangle ADO' \sim \triangle ACO$ by AA similarity

$$\Rightarrow \frac{AO'}{AO} = \frac{O'D}{OC} = \frac{AD}{AC}$$

($AO' = r$ & $AO = 3r$)

$$\Rightarrow \frac{r}{3r} = \frac{O'D}{OC} \Rightarrow \frac{1}{3} = \frac{DO'}{CO}$$

$$\text{Ans: } \frac{DO'}{CO} = \frac{1}{3}$$

[Topper's Answer 2016]

3. In Fig. 8.45, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T . Find the length of TP . [CBSE (AI) 2014]

Sol. Given: $PQ = 16$ cm and $PO = 10$ cm

To find: TP

$$PR = RQ = \frac{16}{2} = 8 \text{ cm}$$

(Perpendicular from the centre bisects the chord)

In $\triangle OPR$,

$$OR = \sqrt{OP^2 - PR^2} = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} \\ = \sqrt{36} = 6 \text{ cm}$$

Let $\angle POR$ be θ .

$$\text{In } \triangle POR, \quad \tan \theta = \frac{PR}{RO} = \frac{8}{6} = \frac{4}{3}$$

We know, $OP \perp TP$ (Point of contact of a tangent is perpendicular to the line from the centre)

$$\text{In } \triangle OTP, \quad \tan \theta = \frac{TP}{OP} \Rightarrow \frac{4}{3} = \frac{TP}{10}$$

$$TP = \frac{10 \times 4}{3} = \frac{40}{3} = 13.33 \text{ cm.}$$

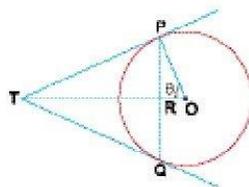


Fig. 8.45

4. A Vidyalaya has a circular playground of diameter 13 metres. There are two gates on the boundary, diametrically opposite to each other as shown in Fig. 8.46.

A physical teacher told the students of class X to make a hole on the boundary so that a pole has to be erected in the hole. But the hole should be made in such a way that the difference of its distance from both gates should be 7 metres.

Find the distances of the erected pole from the two gates.

[Competency Based Question]

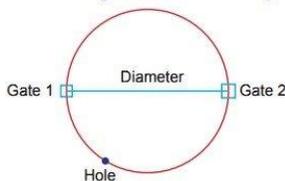


Fig. 8.46

Sol. Let A and B be the position of gate 1 and gate 2, also C be the hole.

Let x m and y m be the distances of the hole (in which pole is erected) from gate 1 and gate 2 respectively. (Fig. 8.47)

$$\therefore y - x = 7 \quad \dots(i)$$

Now,

$$\angle ACB = 90^\circ \quad (\text{Angle in the semicircle})$$

$$\therefore AB^2 = AC^2 + BC^2 \quad (\text{Pythagoras theorem})$$

$$(13)^2 = x^2 + y^2$$

$$\Rightarrow 169 = x^2 + (7 + x)^2 \quad [\text{From (i)}]$$

$$\Rightarrow x^2 + 49 + x^2 + 14x = 169 \quad \Rightarrow 2x^2 + 14x - 120 = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0 \quad \Rightarrow x^2 + 12x - 5x - 60 = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x - 5 = 0 \quad (\text{But } x + 12 \neq 0 \text{ since } x \neq -12)$$

$$\therefore x = 5$$

$$\text{and } y = 7 + x = 7 + 5 = 12 \text{ metre}$$

\therefore Distances are 5 metres from gate 1 and 12 metres from gate 2.

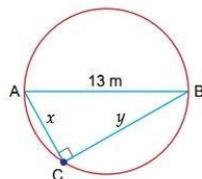


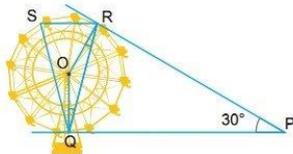
Fig. 8.47

Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (i) to (v).

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below. [CBSE Question Bank]

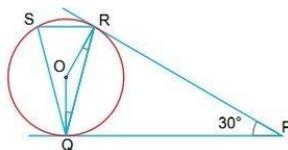


(i) In the given figure $\angle ROQ$ is equal to

(a) 60°

(b) 100°

Fig. 8.48



[Competency Based Question]

(c) 150°

(d) 90°

(ii) The measurement of $\angle RQP$ is

- (a) 75° (b) 60° (c) 30° (d) 90°

(iii) The measurement of $\angle RSQ$ is

- (a) 60° (b) 75° (c) 100° (d) 30°

(iv) The measurement of $\angle ORP$ is

- (a) 90° (b) 70° (c) 100° (d) 60°

(v) Reflex angle of $\angle ROQ$ is

- (a) 180° (b) 150° (c) 210° (d) 360°

Sol. (i) In quadrilateral $PROQ$, we have

$$\begin{aligned}\angle RPQ + \angle PQO + \angle QOR + \angle ORP &= 360^\circ \\ \Rightarrow 30^\circ + 90^\circ + \angle ROQ + 90^\circ &= 360^\circ \quad (\because OR \perp PR, OQ \perp PQ) \\ \Rightarrow \angle ROQ &= 360^\circ - 210^\circ = 150^\circ \quad \Rightarrow \angle ROQ = 150^\circ\end{aligned}$$

\therefore Option (c) is correct.

(ii) In $\triangle OQR$, we have

$$\begin{aligned}\angle OQR &= \angle ORQ && (\because OQ = OR, \text{ radii of a circle}) \\ \angle QOR + \angle OQR + \angle ORQ &= 180^\circ \\ 150^\circ + \angle OQR + \angle OQR &= 180^\circ \quad \Rightarrow \quad 2\angle OQR = 30^\circ \\ & && \Rightarrow \quad \angle OQR = 15^\circ \\ \angle RQP &= 90^\circ - 15^\circ = 75^\circ \quad (\because OQ \perp PQ)\end{aligned}$$

\therefore Option (a) is correct.

(iii) We have,

$$\begin{aligned}\angle RSQ &= \frac{1}{2} \angle QOR \\ \Rightarrow \angle RSQ &= \frac{1}{2} \times 150^\circ = 75^\circ\end{aligned}$$

{ Angle at the centre is double
that of circumference made by
same segment.

\therefore Option (b) is correct.

(iv) Since, OR is radius and PR is tangent at R

$$\therefore OR \perp PR \quad \Rightarrow \quad \angle ORP = 90^\circ$$

\therefore Option (a) is correct.

(v) Reflex of $\angle ROQ = 360^\circ - \angle ROQ = 360^\circ - 150^\circ = 210^\circ$

\therefore Option (c) is correct.

2. Read the following and answer any four questions from (i) to (v).

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a $\triangle ABC$, such that it touches the sides AB , BC and CA at points D , E and F respectively. The lengths of sides AB , BC and CA are 12 cm, 8 cm and 10 cm respectively. [CBSE Question Bank]

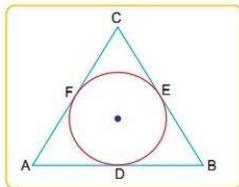
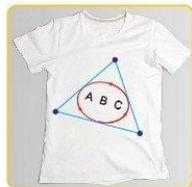


Fig. 8.49

- (i) The length of AD is
 (a) 7 cm (b) 8 cm (c) 5 cm (d) 9 cm
- (ii) The length of BE is
 (a) 8 cm (b) 5 cm (c) 2 cm (d) 9 cm
- (iii) The length of CF is
 (a) 9 cm (b) 5 cm (c) 2 cm (d) 3 cm
- (iv) If radius of the circle is 4 cm, then the area of $\triangle OAB$ is
 (a) 20 cm^2 (b) 36 cm^2 (c) 24 cm^2 (d) 48 cm^2
- (v) The area of $\triangle ABC$ is
 (a) 50 cm^2 (b) 60 cm^2 (c) 100 cm^2 (d) 90 cm^2

Sol.

(i) Let $AD = x \text{ cm}$

$$\therefore DB = (12 - x) \text{ cm}$$

$$DB = BE = (12 - x) \text{ cm} \quad (\text{Tangents from external point are equal})$$

$$CE = 8 - (12 - x) = (x - 4) \text{ cm}$$

Also, $CE = CF = (x - 4) \text{ cm}$

and $AF = 10 - (x - 4) = (14 - x) \text{ cm}$

$$AF = AD \Rightarrow 14 - x = x \Rightarrow 14 = 2x \Rightarrow x = 7 \text{ cm}$$

We have, $AD = x \text{ cm} = 7 \text{ cm}$

\therefore Option (a) is correct.

(ii) Length of $BE = (12 - x) \text{ cm} = 12 - 7 = 5 \text{ cm}$

\therefore Option (b) is correct.

(iii) Length of $CF = (x - 4) \text{ cm} = 7 - 4 = 3 \text{ cm}$

\therefore Option (d) is correct.

(iv) We have, Radius of the circle is 4 cm.

$$\therefore OD = 4 \text{ cm}$$

$$\therefore \text{Area of } \triangle OAB = \frac{1}{2} \times AB \times OD = \frac{1}{2} \times 12 \times 4 = 24 \text{ cm}^2$$

\therefore Option (c) is correct.

(v) We have,

$$\text{Area of } \triangle ABC = ar(\triangle OAB) + ar(\triangle OBC) + ar(\triangle OCA)$$

$$= 24 + \frac{1}{2} \times 8 \times 4 + \frac{1}{2} \times 10 \times 4 = 24 + 16 + 20 = 60 \text{ cm}^2$$

\therefore Option (b) is correct.

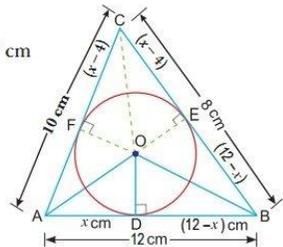


Fig. 8.50

PROFICIENCY EXERCISE

Objective Type Questions:

[1 mark each]

1. Choose and write the correct option in each of the following questions.

- (i) A circle passes through point A . How many tangents and secants to the circle are possible that pass through A ?
- (a) Tangent: 1; Secant: 1 (b) Tangent: Infinite; Secant: 1
 (c) Tangent: 1; Secant: Infinite (d) Tangent: Infinite; Secant: Infinite

(ii) In Fig. 8.51, $\angle RPS = 25^\circ$, $\angle ROS$ is

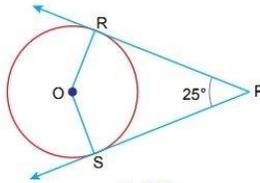


Fig. 8.51

- (a) 135° (b) 145° (c) 165° (d) 155°

(iii) A circle is shown below.

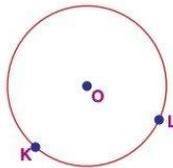


Fig. 8.52

- I. There is only one line passing through point K which makes an angle of 90° with OK.
- II. The shortest distance of a tangent passing through point L from the centre O is equal to the radius of the circle, OL.
- III. One tangent can pass through two points K and L of a circle.

Which statement(s) is/are correct?

[Competency Based Question]

- (a) I and II (b) III (c) II and III (d) I, II and III

(iv) A figure is shown below.

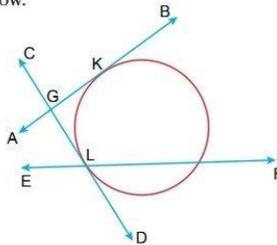


Fig. 8.53

Which of the following is true?

[Competency Based Question]

- (a) Lines AB and CD are the tangents and line EF is a secant to the circle.
 - (b) Lines AB and CD are the secants and line EF is a tangent to the circle.
 - (c) Line AB is a tangent and lines EF and CD are the secants to the circle.
 - (d) Line AB is a secant and lines EF and CD are the tangents to the circle.
- (v) If angle between two radii of a circle is 130° , the angle between the tangents at the ends of the radii is
- [NCERT Exemplar]
- (a) 90° (b) 50° (c) 70° (d) 40°

- (vi) In Fig. 8.54, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A , then $\angle BAT$ is equal to [NCERT Exemplar]

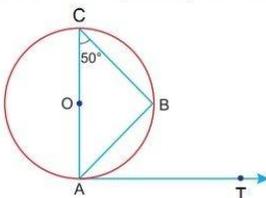


Fig. 8.54

- (a) 65° (b) 60° (c) 50° (d) 40°
- (vii) The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm is
- (a) 10 cm (b) 5 cm (c) $\sqrt{7}$ cm (d) $2\sqrt{7}$ cm

■ **Very Short Answer Questions:**

[1 mark each]

2. In Fig. 8.55, PQ is a tangent of length 6 cm to the circle with centre O and $\angle OQP = 60^\circ$. Find OQ .

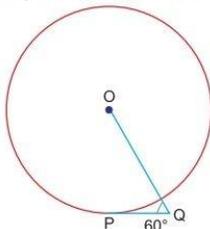


Fig. 8.55

3. PQ is a tangent drawn from an external point P to a circle with centre O , QOR is the diameter of the circle. If $\angle POR = 120^\circ$, what is the measure of $\angle OPQ$? [CBSE (F) 2017]

4. In Fig. 8.56, RQ is a chord of the circle and POQ is its diameter such that $\angle RPQ = 30^\circ$. If QT is the tangent to the circle at the point Q , then find $\angle RQT$.

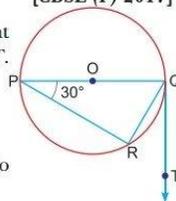


Fig. 8.56

5. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 110° , find $\angle POA$.

6. In Fig. 8.57, if $\angle AOB = 125^\circ$, then find $\angle COD$. [NCERT Exemplar]

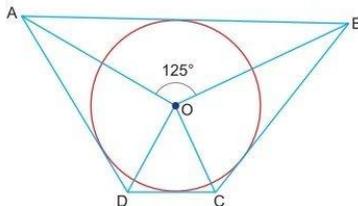


Fig. 8.57

7. In Fig 8.58, $\triangle ABC$ is circumscribing a circle. Find the length of AB .

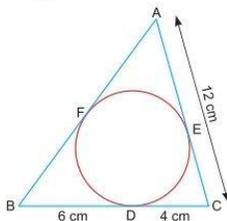


Fig. 8.58

8. At one end A of a diameter AB of a circle of radius 13 cm, tangent XAY is drawn to the circle. A chord CD is parallel to XY and is at a distance of 18 cm from A . What will be the length of CD ?

■ Short Answer Questions-I:

[2 marks each]

9. In Fig. 8.59, ABC is a triangle in which $\angle B = 90^\circ$, $BC = 48$ cm and $AB = 14$ cm. A circle is inscribed in the triangle, whose centre is O . Find radius r of in-circle. [CBSE 2018 (30/1)]

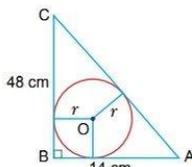


Fig. 8.59

10. In Fig. 8.60, BOA is a diameter of a circle with centre O and the tangent at a point P meets BA extended at T . If $\angle ABP = 40^\circ$, then find the value of $\angle PTA$.

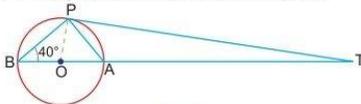


Fig. 8.60

11. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O , is 60° , then find the length of OP . [CBSE (AI) 2017]
12. In Fig. 8.61, BA and BC are tangents to the circle drawn from an external point B . PQ is a third tangent touching the circle at R . If $BC = 12$ cm and $PR = 3$ cm, what is the perimeter of $\triangle BPQ$?

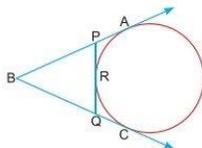


Fig. 8.61

13. In Fig. 8.62, there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external point P , tangents PA and PB are drawn to these circles. If $AP = 12$ cm, find the length of BP .

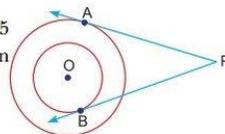
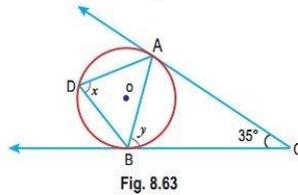


Fig. 8.62

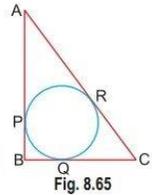
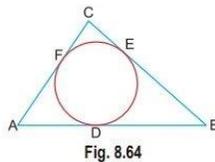
14. In the given figure, find the value of x and y .



■ Short Answer Questions–II:

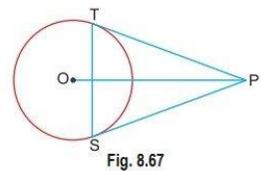
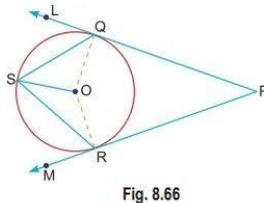
[3 marks each]

15. Two concentric circles are of radii 8 cm and 5 cm. Find the length of the chord of the larger circle which touches the smaller circle.
16. A circle is inscribed in a $\triangle ABC$ having sides $AB = 14$ cm, $BC = 10$ cm and $AC = 12$ cm in Fig. 8.64. Find AD , BE and CF .

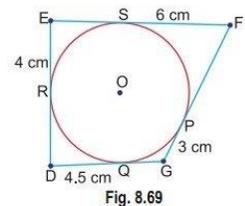
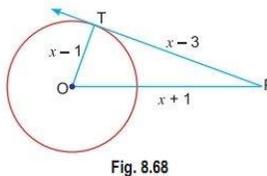


17. In Fig. 8.65, ABC is a right triangle, right angled at B such that $BC = 6$ cm and $AB = 8$ cm. Find the radius of its incircle.
18. Two tangents PQ and PR are drawn from an external point to a circle with centre O . Prove that $QORP$ is a cyclic quadrilateral.
19. In Fig. 8.66, PQL and PRM are tangents to the circle with centre O at the points Q and R , respectively and S is a point on the circle such that $\angle SQL = 50^\circ$ and $\angle SRM = 60^\circ$. Find $\angle QSR$.

[Competency Based Question]



20. In Fig. 8.67, from a point P two tangents PT and PS are drawn to a circle with centre O and radius r . If $OP = 2r$, then show that $\triangle TPS$ is an equilateral triangle.
21. Find the actual lengths of sides of $\triangle OTP$. (Fig. 8.68.)



22. Find the perimeter of $DEFG$. (Fig. 8.69).

23. If $d_1, d_2, (d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.
24. Let s denote the semi-perimeter of a triangle ABC in which $BC = a, CA = b, AB = c$. If a circle touches the sides BC, CA, AB , at D, E, F , respectively, prove that $BD = s - b$.

■ Long Answer Questions:

[5 marks each]

25. If a hexagon $ABCDEF$ circumscribe a circle, prove that $AB + CD + EF = BC + DE + FA$.
[NCERT Exemplar]
26. A is a point at a distance 10 cm from the centre O of a circle of radius 6 cm. AP and AQ are the tangents to the circle at P and Q . If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C , find the perimeter of the ΔABC .
27. In Fig. 8.70, O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E . If AB is a tangent to the circle at E , find the length of AB , where TP and TQ are two tangents to the circle.
[CBSE Delhi 2016]

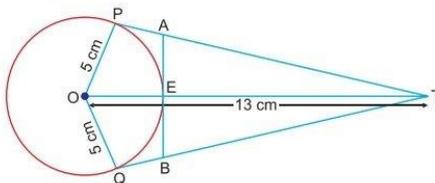


Fig. 8.70

28. If an isosceles triangle ABC , in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.
[NCERT Exemplar]
29. In Fig. 8.71, the tangent at a point C of a circle and a diameter AB when extended intersect at P . If $\angle PCA = 130^\circ$, find $\angle CBA$.

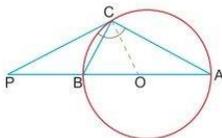


Fig. 8.71

Answers

- | | | | | | | |
|---|----------------------------|---------------------|---------------------------------|-----------------------|-----------|-----------|
| 1. (i) (c) | (ii) (d) | (iii) (a) | (iv) (a) | (v) (b) | (vi) (c) | (vii) (d) |
| 2. 12 cm | 3. $\angle OPQ = 30^\circ$ | 4. 30° | 5. 35° | 6. 55° | | |
| 7. 14 cm | 8. 24 cm | 9. $r = 6$ cm | 10. $\angle PTA = 10^\circ$ | 11. $2a$ | 12. 24 cm | |
| 13. $4\sqrt{10}$ cm | 14. $x = y = 72.5^\circ$ | 15. $2\sqrt{39}$ cm | | | | |
| 16. $AD = 8$ cm, $BE = 6$ cm, $CF = 4$ cm | 17. 2 cm | 18. 70° | 19. 70° | 20. 6 cm, 8 cm, 10 cm | | |
| 21. 35 cm | 22. 16 cm | 23. $AB = 6.6$ cm | 24. $8\sqrt{2}$ cm ² | 25. 50° | | |

Self-Assessment

Time allowed: 1 hour

Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions.

(3 × 1 = 3)

- (i) In Fig. 8.72, PQ and PR are tangents drawn from P to a circle with centre O . If $\angle OPQ = 35^\circ$, then

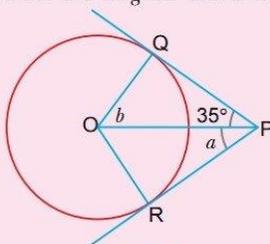


Fig. 8.72

- (a) $a = 30^\circ, b = 60^\circ$ (b) $a = 35^\circ, b = 55^\circ$ (c) $a = 40^\circ, b = 50^\circ$ (d) $a = 45^\circ, b = 45^\circ$
- (ii) If tangents AB and AC from a point A to a circle with centre O are inclined to each other at angle of 70° , then $\angle AOB$ is equal to
- (a) 110° (b) 55° (c) 70° (d) 60°
- (iii) In Fig. 8.73, if AC is a tangent to the circle at A and O is the center of the circle, then $\angle AOB$ is equal to

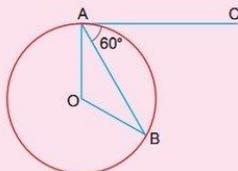


Fig. 8.73

- (a) 100° (b) 110° (c) 120° (d) 90°
2. Solve the following questions. (2 × 1 = 2)
- (i) Radii of two concentric circles are 5 cm and x cm, if x is radius of smaller circle and length of each chord of one circle which is tangent to other, is 6 cm, find x .
- (ii) In a circle of radius 3 cm, a point lies 5 cm away from its centre. What is the length of the tangent to the circle through this point?

SECTION B

■ Solve the following questions.

(4 × 2 = 8)

3. In Fig. 8.74, PA and PB are tangents to the circle drawn from an external point P . CD is the third tangent touching the circle at Q . If $PA = 15$ cm, find the perimeter of $\triangle PCD$.

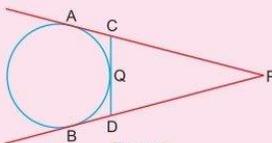


Fig. 8.74

4. In Fig. 8.75, common tangents AB and CD to two circles with centres O_1 and O_2 intersect at E .
Prove that $AB = CD$.
[CBSE (AI) 2014]

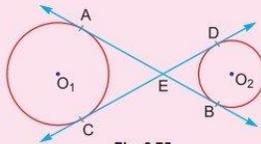


Fig. 8.75

5. In Fig. 8.76, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 68^\circ$ find $\angle ATQ$.

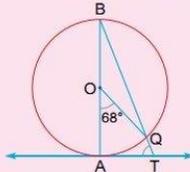


Fig. 8.76

6. Two tangents TP and TQ are drawn to a circle with centre O from an external point T (Fig. 8.77).
Prove that $\angle PTQ = 2 \angle OPQ$.
[CBSE Delhi 2017, Delhi 2017 (C)]

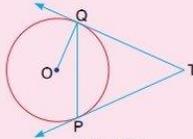


Fig. 8.77

■ Solve the following questions.

(4 × 3 = 12)

7. In Fig. 8.78, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135^\circ$. Find the length of chord AB .
[CBSE (F) 2017]

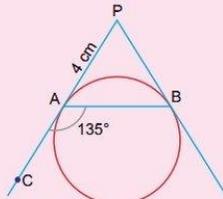


Fig. 8.78

8. In Fig. 8.79, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90^\circ$. If $AD = 23$ cm, $AB = 29$ cm and $DS = 5$ cm, find the radius (r) of the circle.

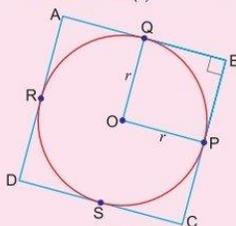


Fig. 8.79

9. In Fig. 8.80, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ . Find $\angle RQS$. [NCERT Exemplar]

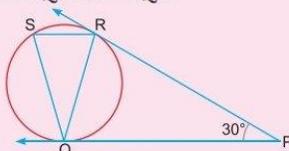


Fig. 8.80

10. If an isosceles triangle ABC , in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle. [NCERT Exemplar]

■ Solve the following questions.

(3 × 5 = 15)

11. In Fig. 8.81, O is the centre of the circle and TP is the tangent to the circle from an external point T . If $\angle PBT = 30^\circ$, prove that $BA : AT = 2 : 1$. [CBSE (F) 2015]

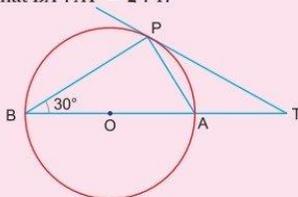


Fig. 8.81

12. In Fig. 8.82, AB is a chord of a circle, with centre O , such that $AB = 16$ cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P . Find the length of PA . [CBSE (F) 2015]

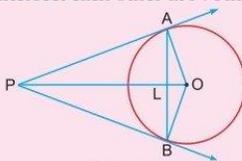


Fig. 8.82

13. In Fig. 8.83, the common tangent, AB and CD of two circles with centres O and O' intersect at E . Prove that the points O, E, O' are collinear. [NCERT Exemplar]

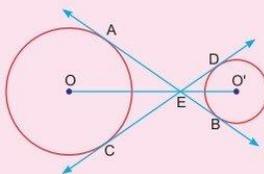


Fig. 8.83

Answers

1. (i) (b) (ii) (b) (iii) (c)
 2. (i) 4 cm (ii) 4 cm 3. 30 cm 5. 56° 7. $4\sqrt{2}$ cm 8. 11 cm
 9. 30° 10. $8\sqrt{2}$ cm² 12. $\frac{40}{3}$ cm