Probability

1. If the letters of the word **ALGORITHM** are arranged at random in a row what is the probability the letters GOR must remain together as a unit?

2. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks?

   [**Hint:** First find the probability that the couple has adjacent desks, and then subtract it from 1.]

3. Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.

4. An experiment consists of rolling a die until a 2 appears.

   (i) How many elements of the sample space correspond to the event that the 2 appears on the kth roll of the die?

   (ii) How many elements of the sample space correspond to the event that the 2 appears not later than the kth roll of the die?

   [Hint:(a) First $(k-1)$ rolls have 5 outcomes each and k^{th} rolls should result in 1 outcomes. (b)$1 + 5 + 5^2 + \dots + 5^{k-1}$]

5. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

6. In a large metropolitan area, the probabilities are .87, .36, .30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?

7. If A and B are mutually exclusive events, $P(A) = 0.35$ and $P(B) = 0.45$, find

   (a) P (A')   (b)   P (B')   (c)   P(A∪B) (d)   P(A∩B)
   (e)   P(A∩B') (f)   P(A'∩B')

8. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26, .08. Find the probabilities that a particular surgery will be rated

   (a) complex or very complex;

   (b) neither very complex nor very simple;

   (c) routine or complex

   (d) routine or simple

9. Four candidates A, B, C, D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B, and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what are the probabilities that

(a) C will be selected?

(b) A will not be selected?

10. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes

S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(a) Determine P (John promoted)

        P (Rita promoted)

        P (Aslam promoted)

        P (Gurpreet promoted)

(b) If A = {John promoted or Gurpreet promoted}, find P (A).

11. The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $P(A \cap B) = .07$). Determine

(a)   P (A)

(b) $P(B \cap \bar{C})$

(c) $P(A \cup B)$

(d) $P(A \cap \bar{B})$

(e) $P(B \cap C)$

(f)   Probability of exactly one of the three occurs.
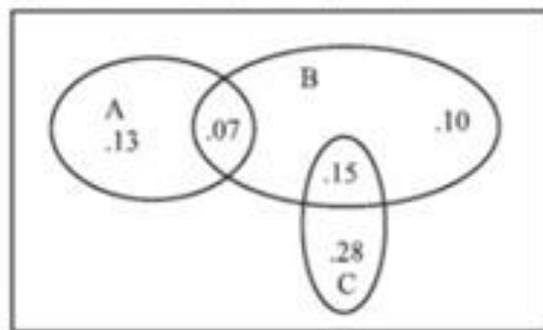

Fig. 16.2

12. One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2).

Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) Write the sample space showing all possible outcomes

(b) What is the probability that two black balls are chosen?

(c) What is the probability that two balls of opposite colour are chosen?

13. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the Probability that

(a) All the three balls are white

(b) All the three balls are red

(c) One ball is red and two balls are white

14. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that

(a) Four S's come consecutively in the word

(b) Two I's and two N's come together

(c) All A's are not coming together

(d) No two A's are coming together.

15. A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.

16. A sample space consists of 9 elementary outcomes $e_1, e_2, ..., e_9$ whose probabilities are

$P(e_1) = P(e_2) = .08, P(e_3) = P(e_4) = P(e_5) = .1$

$P(e_6) = P(e_7) = .2, P(e_8) = P(e_9) = .07$

Suppose $A = \{e_1, e_5, e_8\}$, $B = \{e_2, e_5, e_8, e_9\}$

(a) Calculate P (A), P (B), and P (A ∩ B)

(b) Using the addition law of probability, calculate P(A ∪ B)

(c) List the composition of the event A ∪ B, and calculate P (A ∪ B) by adding the probabilities of the elementary outcomes.

(d) Calculate P ($\bar{B}$) from P (B), also calculate P ($\bar{B}$) directly from the elementary outcomes of $\bar{B}$

17. Determine the probability p, for each of the following events.

(a) An odd number appears in a single toss of a fair die.

(b) At least one head appears in two tosses of a fair coin.

(c) A king, 9 of hearts, or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.

(d) The sum of 6 appears in a single toss of a pair of fair dice.

Objective Type Questions

18. In a non-leap year, the probability of having 53 tuesdays or 53 wednesdays is

(A) $\dfrac{1}{7}$　　(B) $\dfrac{2}{7}$　　(C) $\dfrac{3}{7}$　　(D) none of these

19. Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive

(A) $\dfrac{186}{190}$　　(B) $\dfrac{187}{190}$　　(C) $\dfrac{188}{190}$　　(D) $\dfrac{18}{^{20}C_3}$

20. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours

(A) $\dfrac{29}{52}$　　(B) $\dfrac{1}{2}$　　(C) $\dfrac{26}{51}$　　(D) $\dfrac{27}{51}$

21. Seven persons are to be seated in a row. The probability that two particular persons sit next to each other is

(A) $\dfrac{1}{3}$　　(B) $\dfrac{1}{6}$　　(C) $\dfrac{2}{7}$　　(D) $\dfrac{1}{2}$

22. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

(A) $\dfrac{1}{5}$　　(B) $\dfrac{4}{5}$　　(C) $\dfrac{1}{30}$　　(D) $\dfrac{5}{9}$

23. If A and B are mutually exclusive events, then

(A) $P(A) \le P(\bar{B})$　　　　(B) $P(A) \ge P(\bar{B})$

(C) $P(A) < P(\bar{B})$　　　　(D) none of these

24. If $P(A \cup B) = P(A \cap B)$ for any two events A and B, then
    (A) $P(A) = P(B)$
    (B) $P(A) > P(B)$
    (C) $P(A) < P(B)$
    (D) none of these

25. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is
    (A) $\frac{1}{432}$
    (B) $\frac{12}{431}$
    (C) $\frac{1}{132}$
    (D) none of these

26. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is
    (A) $\frac{1}{3}$
    (B) $\frac{4}{11}$
    (C) $\frac{2}{11}$
    (D) $\frac{3}{11}$

27. If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is
    (A) $> .5$
    (B)
    .5 (C)
    $\le .5$ (D) 0

28. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is
    (A) 0.4
    (B) 0.8
    (C) 1.2
    (D) 1.6

29. If M and N are any two events, the probability that at least one of them occurs is
    (A) $P(M) + P(N) - 2P(M \cap N)$
    (B) $P(M) + P(N) - P(M \cap N)$
    (C) $P(M) + P(N) + P(M \cap N)$
    (D) $P(M) + P(N) + 2P(M \cap N)$

True or False Type Questions

30. The probability that a person visiting a zoo will see the giraffee is 0.72, the probability that he will see the bears is 0.84 and the probability that he will see both is 0.52.

31. The probability that a student will pass his examination is 0.73, the probability of the student getting a compartment is 0.13, and the probability that the student will either pass or get compartment is 0.96.

32. The probabilities that a typist will make 0, 1, 2, 3, 4, 5 or more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.

33. If A and B are two candidates seeking admission in an engineering College. The probability that A is selected is .5 and the probability that both A and B are selected is at most .3. Is it possible that the probability of B getting selected is 0.7?

34. The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

35. The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4.

36. The sum of probabilities of two students getting distinction in their final examinations is 1.2.