Probability

1. If P(A|B) = P(A'|B), then which of the following statements is true ? (2024)

(A) $P(A) = P(A')$

(B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = 1/2\,P(B)$

(D) $P(A \cap B) = 2\,P(B)$

Ans. (C) $P(A \cap B) = 1/2\,P(B)$

2.

E and F are two independent events such that $P(\bar{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

(2024)

Ans.

$$P(\bar{E}) = 0.6 \Rightarrow P(E) = 0.4$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow 0.6 = 0.4 + P(F) - 0.4\,P(F) \Rightarrow P(F) = \frac{1}{3}$$

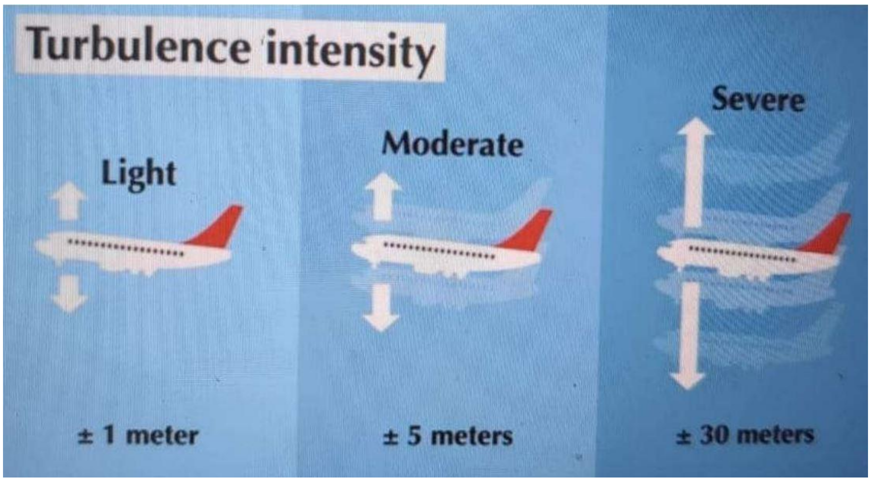
$$P(\bar{E} \cup \bar{F}) = 1 - P(E \cap F)$$

$$= 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$$

3. Case Study Based Question : (2024)

According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

(i) Find the probability that an airplane reached its destination late.

Ans. Let A denote the event of airplane reaching its destination late

E_1= severe turbulence

E_2 = moderate turbulence

E_3 = light turbulence

$P(A) = P(E_1)\, \boldsymbol{P}(\boldsymbol{A}|E_1) + \boldsymbol{P}(E_2)\boldsymbol{P}(\boldsymbol{A}|E_2) + \boldsymbol{P}(E_3)\boldsymbol{P}(\boldsymbol{A}|E_3)$

$$= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$$

$$= \frac{1}{3}\left(\frac{109}{100}\right) = \frac{109}{300}$$

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

Ans. $P(E_2|A) = P(E_2)P(A|E_2)/P(A)$

$$= \frac{\frac{1}{3} \times \frac{37}{100}}{\frac{109}{300}}$$

$$= \frac{37}{109}$$

Previous Years' CBSE Board Questions

13.1 Introduction

MCQ

1. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is
   (a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$ *(2023)*

2. A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is
   (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 *(2020)*

VSA (1 mark)

3. A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6. *(2021 C)*

4. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black. *(2020)*

5. From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is ________. *(2020C)*

6. A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is ________. *(2020)* Ap

SA I (2 marks)

7. A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour. *(Term II, 2021-22)*

8. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7. *(Term II, 2021-22 C)*

9. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.6$, then find $P(B' \cap A)$. *(2020)* An

10. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected. *(AI 2019)*

LA I (4 marks)

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black. *(Delhi 2015)* Cr

13.2 Conditional Probability

MCQ

12. For two events A and B, if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is
    (a) 0.24 (b) 0.3 (c) 0.48 (d) 0.96 *(2023)*

13. If for any two events A and B,
    $P(A)=\frac{4}{5}$ and $P(A \cap B)=\frac{7}{10}$, then $P(B/A)$ is
    (a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$ *(2023)*

14. **In the following questions, a statements are Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:**

    Assertion (A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

    Reason (R) : Let E and F be two events with a random experiment, then $P(F/E)=\frac{P(E \cap F)}{P(E)}$.

    (a) Both (A) and (R) are true and (R) is the correct explanation of (A)
    (b) Both (A) and (R) are true, but (R) is not the correct explanation of the (A)
    (c) (A) is true, and (R) is False.
    (d) (A) is false, but (R) is true. *(2023)*

15. A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is
    (a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ *(2020)* Ap

SA I (2 marks)

16. Find $[P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$. *(2020)*

17. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number. *(AI 2019)* An

18. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find $P(B/A)$. *(2019)*

19. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. *(2018)*

LA I (4 marks)

20. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls? Given that
(i) the youngest is a girl.
(ii) atleast one is a girl. *(Delhi 2014)* Ap

21. A couple has 2 children. Find the probability that both are boys, if it is known that
(i) one of them is a boy,
(ii) the older child is a boy. *(Delhi 2014C)*

LA II (5/6 marks)

22. Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'. *(Delhi 2014C)*

13.3 Multiplication Theorem on Probability

LA I (4 marks)

23. A bag contains 3 red and 7 black balls. Two balls are selected at random one-by-one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red? *(Delhi 2014C)* Ev

13.4 Independent Events

MCQ

24. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{3}{4}$ (d) 1 *(2020)* R

VSA (1 mark)

25. A problem is given to three students whose probabilities of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it. *(2020)* U

SA I (2 marks)

26. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit. *(Term II, 2021-22)*

27. Events A and B are such that
$$P(A)=\frac{1}{2},\ P(B)=\frac{7}{12} \text{ and } P(\bar{A}\cup\bar{B})=\frac{1}{4}$$
Find whether the events A and B are independent or not. *(Term II, 2021-22)* Ap

28. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three? *(2020)*

29. Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, find $P(A' \cap B')$. *(2020)*

30. The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time. *(2019)*

31. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not. *(Delhi 2019)* An

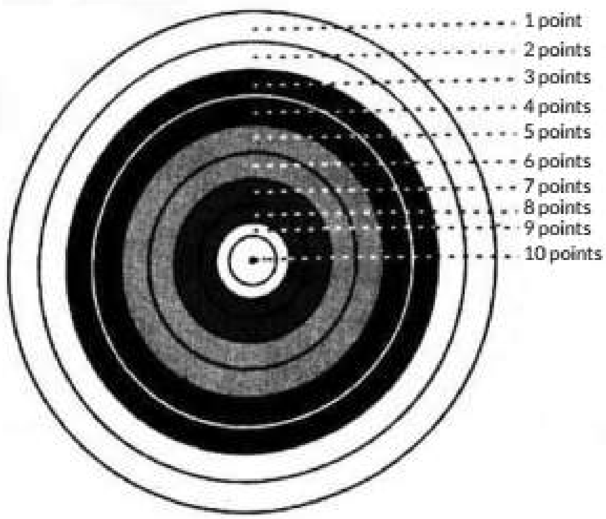
OR

A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events. *(AI 2017)*

32. Prove that if E and F are independent events, then the events E' and F' are also independent. *(Delhi 2017)*

LA I (4 marks)

33. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.
Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9.



Based on the above information, answer the following questions :

If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points.

(b) both of them earn 10 points.

(Term II, 2021-22, 2020)

34. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins. *(Delhi 2016)*

35. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) exactly one of them solved the problem.

(Delhi 2015C) Ap

LA II (5/6 marks)

36. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B). *(Delhi 2015)*

13.5 Bayes' Theorem

SA I (2 marks)

37. There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour. *(Term II, 2021-22)*

38. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin. *(2020)* Ev

LA I (4 marks)

39. **Case study :** A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job?

(ii) What is the probability that construction will be completed on time?

(iii) What is the probability that many workers are not present given that the construction work is completed on time?

OR

What is the probability that all workers were present given that the construction job was completed on time? *(2023)*

40. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II. *(Term II, 2021-22)*

41. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black. *(2020)* Ev

42. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag. *(2020 C)*

43. A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin? *(2020)*

44. In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y. *(2020)* Ev

45. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die? *(2018)* An

46. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society? *(Delhi 2017)*

47. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? Is regularity required only in school? Justify your answer. *(AI 2017)*

48. Three persons *A*, *B* and *C* apply for a job of Manager in a Private Company. Chances of their selection (*A*, *B* and *C*) are in the ratio 1 : 2 : 4. The probabilities that *A*, *B* and *C* can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of *C*. *(Delhi 2016)* An

49. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white? *(AI 2016)*

50. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective. *(Foreign 2015)* An

LA II (5/6 marks)

51. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly? *(2023)*

52. A manufacturer has three machine operators *A*, *B* and *C*. The first operator *A* produces 1% of defective items, whereas the other two operators *B* and *C* produces 5% and 7% defective items respectively. *A* is on the job for 50% of the time. *B* on the job 30% of the time and *C* on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by *A*? *(Delhi 2019)* Ap

53. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist? *(AI 2019)* Ev

54. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and '*n*' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of '*n*'. *(2019)*

55. Bag *A* contains 3 red and 5 black balls, while bag *B* contains 4 red and 4 black balls. Two balls are transferred at random from bag *A* to bag *B* and then a ball is drawn from bag *B* at random. If the ball drawn from bag *B* is found to be red find the probability that two red balls were transferred from *A* to *B*. *(Foreign 2016)*

56. In a factory which manufactures bolts, machines *A*, *B* and *C* manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine *B*. *(AI 2015)*

57. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random without replacement from the bag and are found to be both red. Find the probability that the balls are drawn from the first bag. *(Delhi 2015C)* An

58. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly? *(AI 2015C)*

59. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade. *(Delhi 2014)* An

60. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two-headed coin? (AI 2014)

61. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident for them are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver or a car driver? (Foreign 2014) Ev

62. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is '1'. Find the probability that it is actually 1. (Delhi 2014C) Ap

63. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white? (AI 2014C)

Random Variables and its Probability Distributions

SA I (2 marks)

64. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X. (Term II, 2021-22)

65. A coin is tossed twice. The following table shows the probability distribution of number of tails :

X	0	1	2
$P(X)$	K	$6K$	$9K$

(a) Find the value of K.
(b) Is the coin tossed biased or unbiased? Justify your answer. (Term II, 2021-22)

66. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X. (Term II, 2021-22)

67. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number,

$$P(X=x)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'. (Delhi 2019) An

LA I (4 marks)

68. The probability distribution of a random variable X, where k is a constant is given below :

$$P(X=x)=\begin{cases} 0.1, & \text{if } x=0 \\ kx^2, & \text{if } x=1 \\ kx, & \text{if } x=2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine
(a) the value of k
(b) $P(x \le 2)$
(c) Mean of the variable X (2020) An

69. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples. (2020)

70. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p. (Delhi 2017) An

71. In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses. (AI 2016)

72. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X=x)=\begin{cases} kx & ,\text{if } x=0 \text{ or } 1 \\ 2kx & ,\text{if } x=2 \\ k(5-x) & ,\text{if } x=3 \text{ or } 4 \\ 0 & ,\text{if } x>4 \end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in
(i) exactly one college
(ii) atmost 2 colleges
(iii) atleast 2 colleges. (Foreign 2016) An

73. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution. (AI 2015) Ev

74. From a lot of 15 bulbs which include 5 defectives, a sample of 2 bulbs is drawn at random (without replacement). Find the probability distribution of the number of defective bulbs. (Delhi 2015C)

75. Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution. (Foreign 2014) Ev

76. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X. (AI 2014C)

LA II (5/6 marks)

77. A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize. *(2023)*

78. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution. *(AI 2014)*

79. In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses. *(AI 2014C)* An

CBSE Sample Questions

13.1 Introduction

SA II (3 marks)

1. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made? *(2022-23)*

13.2 Conditional Probability

SA I (2 marks)

2. Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P(\bar{E}|\bar{F})$. *(2020-21)* Ap

13.4 Independent Events

VSA (1 mark)

3. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved? *(2020-21)*

13.5 Bayes' Theorem

MCQ

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

4. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms, Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following:

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is
(a) 0.0210 (b) 0.04 (c) 0.47 (d) 0.06

(ii) The probability that Sonia processed the form and committed an error is
(a) 0.005 (b) 0.006 (c) 0.008 (d) 0.68

(iii) The total probability of committing an error in processing the form is
(a) 0 (b) 0.047 (c) 0.234 (d) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay is
(a) 1 (b) $\frac{30}{47}$ (c) $\frac{20}{47}$ (d) $\frac{17}{47}$

(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i|A)$ is
(a) 0 (b) 0.03 (c) 0.06 (d) 1

(2020-21)

LA I (4 marks)

5. There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B? *(2022-23)*

6. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone? *(Term II, 2021-22)*

Random Variables and its Probability Distributions

SA I (2 marks)

7. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement. *(Term II, 2021-22)* Ev

8. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome? *(2020-21)*

SA II (3 marks)

9. Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size. *(2022-23)*

Detailed SOLUTIONS

1. (c) : Since each coin turns up on either a head or tail.

$\therefore$ Total possible outcomes $= 2^5 = 32$

Let A be the event that all tails comes up.

$\therefore$ $n(A) = 1$ {i.e., (T, T, T, T, T)}

So, required probability $= 1 - P(A) = 1 - \frac{1}{32} = \frac{31}{32}$

2. (d) : Here, $A = \{4, 5, 6\}$, $B = \{1, 2, 3, 4\}$

$A \cap B = \{4\}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$

3. We have the sample space associated with the given random experiment as follows :

$\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, H), (T, T)\}$

So, the total number of elementary events $= 8 = n(S)$

There is only one way in which head and number 6 occurring i.e., $(H, 6)$

$\therefore$ $n(E) = 1$

So, the required probability $= \frac{n(E)}{n(S)} = \frac{1}{8}$

4. Required probability $= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$

$= 2 \times \frac{26}{52} \times \frac{26}{51} = \frac{26}{51}$

5. We have, $n(s) = 52$

Probability that first drawn card is red i.e., $P(R_1) = \frac{26}{52}$

Probability that second drawn card is black i.e., $P(B) = \frac{26}{51}$

Probability that third drawn card is red i.e., $P(R_2) = \frac{25}{50}$

So, required probability $= P(R_1) \times P(B) \times P(R_2)$

$= \frac{26}{52} \times \frac{26}{51} \times \frac{25}{50} = \frac{13}{102}$

6. Required probability $= \frac{{}^4C_1 \times {}^3C_1 \times {}^2C_1}{{}^9C_3}$

$= \frac{4 \times 3 \times 2}{\frac{9 \times 7 \times 8}{3 \times 2}} = \frac{2}{7}$

7. B_1 contains 1 white ball and 3 red balls.
B_2 contains 2 white balls and 3 red balls.
P(two ball drawn of same colour)
= P(white ball of B_1 and white ball of B_2) or P(red ball of B_1 and red ball of B_2)

$$=\frac{1}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}=\frac{2}{20}+\frac{9}{20}=\frac{11}{20}$$

8. Since the bag contains cards numbered 1 to 25.
So, the numbers which are multiple of 7 are {7, 14, 21}.

Required probability $=\frac{3}{25}\times\frac{2}{24}=\frac{1}{100}$

9. We have, $P(A) = 0.4$, $P(B) = 0.3$ and $P(A\cup B) = 0.6$
9, $P(A\cap B) = P(A) + P(B) - P(A\cup B) = 0.4 + 0.3 - 0.6 = 0.1$
Now, $P(B'\cap A) = P(A-B) = P(A) - P(A\cap B) = 0.4 - 0.1 = 0.3$

10. Total number of students = 8
The number of ways to select 4 students out of 8 students

$$= {}^8C_4 = \frac{8!}{4!4!} = 70$$

The number of ways to select 2 boys and 2 girls

$$= {}^3C_2\times{}^5C_2 = \frac{3!}{2!1!}\times\frac{5!}{2!3!} = 3\times 10 = 30$$

$\therefore$ Required probability $=\frac{30}{70}=\frac{3}{7}$.

11. Probability of choosing bag A = $P(A)$ = $\frac{2}{6}=\frac{1}{3}$

Probability of choosing bag B = $P(B)$ = $\frac{4}{6}=\frac{2}{3}$

Let E_1 and E_2 be the events of drawing a red and a black ball from bag A and B respectively.

$\therefore$ $P(E_1) = \frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2}$ and $P(E_2) = \frac{{}^7C_1\times{}^3C_1}{{}^{10}C_2}$

$\therefore$ Required probability = $P(A)\times P(E_1) + P(B)\times P(E_2)$

$$=\frac{1}{3}\times\frac{{}^6C_1\times{}^4C_1}{{}^{10}C_2}+\frac{2}{3}\times\frac{{}^7C_1\times{}^3C_1}{{}^{10}C_2}=\frac{8}{45}+\frac{14}{45}=\frac{22}{45}$$

12. (d): We have, $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$

We know that $P(B/A)=\frac{P(A\cap B)}{P(A)}$

$\Rightarrow$ $0.6=\frac{P(A\cap B)}{0.4}$

$\Rightarrow$ $P(A\cap B) = 0.24$

Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.4 + 0.8 - 0.24 = 0.96$

Hence, $P(A\cup B) = 0.96$

13. (c): We know that, $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$

14. (a): Sample space = {HH, HT, TH, TT}
Let A be the event of coming up two heads

$\therefore$ $A = \{HH\} \Rightarrow P(A)=\frac{1}{4}$

and B be the event of coming up atleast one head

$\therefore$ $B = \{HH, HT, TH\} \Rightarrow P(B) = \frac{3}{4}$

Also, $A\cap B = \{HH\} \Rightarrow P(A\cap B) = \frac{1}{4}$

So, required probability $=P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$

So, assertion is true.

Also, reason is true and it is the correct explanation of assertion.

15. (c) : Let A be the event that the card is a spade and B be the event that the picked card is a queen.

We have a total of 13 spades and 4 queen cards.

Also only one queen is from spade.

$\therefore$ $P(A|B) = \frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{52}}{\frac{4}{52}}=\frac{1}{4}$

Key Points

- A standard 52-card deck comprises 13 cards in each of the four suits : clubs, diamonds, hearts and spades.

16. We have, $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{10}$

Now,

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1+4-6}{10}=\frac{1}{10}$$

$\therefore$ $[P(B/A)+P(A/B)]=\frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)}$

$$=\frac{\frac{1}{10}}{3/10}+\frac{\frac{1}{10}}{2/5}=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

17. The sample space, S is given by
$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Let A be the event that number on the drawn card is odd, and B be the event that number on the drawn card is greater than 5.

$\therefore$ $A = \{1, 3, 5, 7, 9, 11\}$
$B = \{6, 7, 8, 9, 10, 11, 12\}$

and, $A\cap B=\{7,9,11\}$

Now, $P(A)=\frac{n(A)}{n(S)}=\frac{6}{12}$,

$P(B)=\frac{n(B)}{n(S)}=\frac{7}{12}$

$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{3}{12}$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{3/12}{7/12}=\frac{3}{7}$

Hence, required probability is $\frac{3}{7}$.

18. Let M, F and S denote mother, father and son respectively.
Sample space $S = \{MFS, MSF, FMS, FSM, SMF, SFM\}$
Given, A = Son on one end *i.e.*, $\{MFS, FMS, SMF, SFM\}$
and B = Father in the middle *i.e.*, $B = \{MFS, SFM\}$
$A \cap B = \{MFS, SFM\}$

$$P(A)=\frac{4}{6}=\frac{2}{3}, P(B)=\frac{2}{6}=\frac{1}{3} \text{ and } P(A\cap B)=\frac{2}{6}=\frac{1}{3}$$

$$\therefore \quad P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{1/3}{2/3}=\frac{1}{2}$$

19. E: Sum 8' and F: 'red die resulted in a number less than 4'
i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$
i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$
Hence, $E\cap F = \{(5, 3), (6, 2)\}$, $P(E) = 5/36$,
$P(F) = 18/36$, $P(E \cap F) = 2/36$
$\therefore$ Required probability = $P(E/F)$

$$=\frac{P(E\cap F)}{P(F)}=\frac{2/36}{18/36}=\frac{2}{18}=\frac{1}{9}.$$

20. Let G_i (i = 1, 2) and B_i (i = 1, 2) denote the i^{th} child is a girl or a boy respectively.
Then sample space is,
$S = \{G_1G_2, G_1B_2, B_1G_2, B_1B_2\}$
Let A be the event that both children are girls, B be the event that the youngest child is a girl and C be the event that at least one of the children is a girl.
Then $A = \{G_1G_2\}$, $B = \{G_1G_2, B_1G_2\}$
and $C = \{B_1G_2, G_1G_2, G_1B_2\}$
$\Rightarrow$ $A \cap B = \{G_1G_2\}$ and $A\cap C = \{G_1G_2\}$

(i) Required probability = $P(A/B) = \dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4}{2/4}=\dfrac{1}{2}$

(ii) Required probability = $P(A/C) = \dfrac{P(A\cap C)}{P(C)}=\dfrac{1/4}{3/4}=\dfrac{1}{3}$

21. Let B_i(i = 1, 2) and G_i(i = 1, 2) denote the i^{th} child is a boy or a girl respectively.
Then sample space is, $S = \{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$
Let A be the event that both are boys, B be the event that one of them is a boy and C be the event that the older child is a boy.
$A = \{B_1B_2\}$, $B = \{G_1B_2, B_1G_2, B_1B_2\}$
$C = \{B_1B_2, B_1G_2\} \Rightarrow A\cap B = \{B_1B_2\}$ and $A\cap C = \{B_1B_2\}$

(i) Required probability = $P(A/B) = \dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4}{3/4}=\dfrac{1}{3}$

(ii) Required probability = $P(A/C) = \dfrac{P(A\cap C)}{P(C)}=\dfrac{1/4}{2/4}=\dfrac{1}{2}$

22. The sample space S of the given random experiment is
$S = \{(H, H), (H, T), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$
Let A be the event that the die shows a number greater than 4 and B be the event that there is at least one tail.
$\therefore$ $A = \{(T, 5), (T, 6)\}$
and $B = \{(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6), (H, T)\}$
$A\cap B = \{(T, 5), (T, 6)\}$

$$\therefore \quad P(B) = P\{(T, 1)\} + P\{(T, 2)\} + P\{(T, 3)\} + P\{(T, 4)\} + P\{(T, 5)\} + P\{(T, 6)\} + P\{(H, T)\}$$

$$=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{4}=\frac{3}{4}$$

$$P(A\cap B)=P\{(T,5)\}+P\{(T,6)\}=\frac{1}{12}+\frac{1}{12}=\frac{1}{6}$$

$\therefore$ Required probability $=P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/6}{3/4}=\dfrac{2}{9}$

23. Let A be the event of drawing a red ball in first draw and B be the event of drawing a red ball in second draw.

$$\therefore \quad P(A)=\frac{^3C_1}{^{10}C_1}=\frac{3}{10}$$

Now, $P(B/A)$ = Probability of drawing a red ball in the second draw, when a red ball already has been drawn in the first draw $=\dfrac{^2C_1}{^9C_1}=\dfrac{2}{9}$

$\therefore$ The required probability = $P(A \cap B)$

$$= P(A)\cdot P(B/A) = \frac{3}{10}\times\frac{2}{9}=\frac{1}{15}$$

Answer Tips

- Conditional probability is calculated by multiplied the probability of the preceding event by the renewed probability of the succeeding event.

24. (c) : Given, A and B are independent events.
Also, $P(A) = \dfrac{1}{3}$ and $P(B) = \dfrac{1}{4}$

Now, $P(B'|A) = \dfrac{P(B'\cap A)}{P(A)}$

$= \dfrac{P(B')P(A)}{P(A)}$ [$\because$ A, B are independent events]

$= P(B') = 1 - P(B) = 1-\dfrac{1}{4}=\dfrac{3}{4}$

25. Let A, B, C be respectively the events of solving problem by three students and $P(A)$, $P(B)$, $P(C)$ be their probability of solving the problem respectively.

$$\therefore \quad P(A) = \frac{1}{3}, P(B) = \frac{1}{4} \text{ and } P(C) = \frac{1}{6}$$

Required probability = $1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$
$= 1 - P(\bar{A})\, P(\bar{B})\, P(\bar{C})$
($\because$ A, B, C are independent $\therefore$ $\bar{A}, \bar{B}, \bar{C}$ are also independent)
$= 1 - [1 - P(A)]\, [1 - P(B)]\, [1 - P(C)]$

$$=1-\frac{2}{3}\times\frac{3}{4}\times\frac{5}{6}=1-\frac{5}{12}=\frac{7}{12}$$

26.

$\boxed{\text{Question : 4}}$

(option-2)

P(A hits) = $\frac{1}{3}$ (P(A))

P(B hits) = $\frac{2}{5}$ (P(B))

P(A doesn't hit) = $1-\frac{1}{3}$, [P(A) + P(A') = 1]

P(A') = $\frac{2}{3}$

P(B doesn't hit) = $1-\frac{2}{5}$

P(B') = $\frac{3}{5}$

As these are independent events = P(not hitting)
= P(A')·P(B')
[P(A∩B) = P(A).P(B)]

~~Required~~

P(target is hit) = 1 - P(target not hit)

= 1 - P(A') × P(B')

= $1-\frac{2}{3}\times\frac{3}{5}$

= $1-\frac{2}{5}$

= $\frac{3}{5}$

Answer: Probability of target being hit is $\frac{3}{5}$

[Topper's Answer, 2022]

27. Given : $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}, \; P(\bar{A}\cup\bar{B})=\frac{1}{4}$

To find whether A and B are independent or not.

Two events are independent if $P(A\cap B)=P(A)\cdot P(B)$

$$P(\bar{A}\cup\bar{B})=1-P(A\cap B)$$

$\Rightarrow \; \frac{1}{4}=1-P(A\cap B) \Rightarrow P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$

and $P(A)\cdot P(B)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}$

Since $P(A\cap B)\neq P(A)\cdot P(B)$

$\therefore$ A and B are not independent.

28. Let G be the event of a green signal.

Required probability = $P(GGG') + P(G'GG)$

$$=\left(\frac{3}{10}\right)^2\frac{7}{10}+\frac{7}{10}\left(\frac{3}{10}\right)^2=\frac{9}{100}\cdot\frac{7}{10}+\frac{7}{10}\cdot\frac{9}{100}$$

$$=\frac{63}{1000}+\frac{63}{1000}=\frac{126}{1000}=\frac{63}{500}$$

29. Given, A and B are independent events. So, A′ and B′ are also independent events.

Now, $P(A'\cap B') = P(A')\times P(B')$

$= [1 - P(A)][1 - P(B)] = [1 - 0.3][1 - 0.6]$

[Given, P(A) = 0.3 and P(B) = 0.6]

$= 0.7 \times 0.4 = 0.28$

30. Let A denotes the student A coming school on time and B denotes the student B coming school on time.

$\therefore \; P(A)=\frac{2}{7}$ and $P(B)=\frac{4}{7}$

So, we have, $P(\bar{A})=1-P(A)=1-\frac{2}{7}=\frac{5}{7}$

and $P(\bar{B})=1-P(B)=1-\frac{4}{7}=\frac{3}{7}$

$\therefore$ Probability of only one of them coming to school on time $=P(\bar{A}\cap B)+P(A\cap\bar{B})$

$=P(\bar{A})\times P(B)+P(A)\times P(\bar{B})$

$=\frac{5}{7}\times\frac{4}{7}+\frac{2}{7}\times\frac{3}{7}=\frac{20}{49}+\frac{6}{49}=\frac{26}{49}$

31. We have, $S = \{1, 2, 3, 4, 5, 6\}$ and A be the event that number is even $= \{2, 4, 6\}$

$\Rightarrow P(A)=\frac{3}{6}=\frac{1}{2}$

B be the event that number is red $= \{1, 2, 3\}$

$\Rightarrow P(B)=\frac{3}{6}=\frac{1}{2}$

and $A \cap B = \{2\}$

$\Rightarrow P(A\cap B)=\frac{1}{6}$ ...(i)

Also, $P(A)\cdot P(B)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ ...(ii)

From (i) and (ii),

$P(A) \cdot P(B) \neq P(A \cap B)$

So, A and B are not independent.

32. Since, E and F are independent events.

$\therefore \quad P(E \cap F) = P(E)\, P(F)$ ...(i)

Now, $P(E' \cap F') = 1 - P(E \cup F)$ $[\because P(E' \cap F') = P((E \cup F)')]$

$= 1 - [P(E) + P(F) - P(E \cap F)]$

$= 1 - P(E) - P(F) + P(E)\, P(F)$ [Using (i)]

$= (1 - P(E))\,(1 - P(F)) = P(E')\, P(F')$

Hence, E' and F' are also independent events.

33. (a) We have, $P(A) = 0.8$, $P(B) = 0.9$

P(exactly one of them earn 10 points) $= P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= P(A) + P(B) - 2P(A)\, P(B)$ $(\because A \,\&\, B$ are independent)

$= 0.8 + 0.9 - 0.8 \times 0.9 \times 2$

$= 0.26$

(b) P(both of them earn 10 points) $= P(A \cap B)$

$= P(A)\, P(B) = 0.8 \times 0.9 = 0.72$

34. Total outcomes = 36

Favourable outcomes for A to win

$= \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

$\therefore$ Probability of A to win, $P(A) = \frac{6}{36}=\frac{1}{6}$

Probability of A to lose, $P(\bar{A})=1-\frac{1}{6}=\frac{5}{6}$

Favourable outcomes for B to win $= \{(4, 6), (6, 4), (5, 5)\}$

$\therefore$ Probability of B to win, $P(B) = \frac{3}{36}=\frac{1}{12}$

Probability of B to lose, $P(\bar{B})=1-\frac{1}{12}=\frac{11}{12}$

$\therefore$ Required probability

$=P(\bar{A})P(B)+P(\bar{A})P(\bar{B})P(\bar{A})P(B)$

$+P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(\bar{A})P(B)+....$

$=\frac{5}{6}\times\frac{1}{12}+\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{1}{12}+\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{11}{12}\times\frac{5}{6}\times\frac{1}{12}+....$

$=\frac{5/72}{1-\frac{5}{6}\times\frac{11}{12}}=\frac{5}{17}$

35. Let X and Y denote the respective events of solving the given specific problem by A and B,

then $P(X) = \frac{1}{2}$ and $P(Y)=\frac{1}{3}$

(i) P (problem is solved)

$=P(X\cup Y)=1-P(\bar{X})P(\bar{Y}) =1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)=1-\frac{1}{2}\times\frac{2}{3}=\frac{2}{3}.$

(ii) P (Exactly one of A and B solves the problem)

$P(X)\cdot P(\bar{Y})+P(\bar{X})\cdot P(Y)$

$=\frac{1}{2}\left(1-\frac{1}{3}\right)+\left(1-\frac{1}{2}\right)\frac{1}{3}=\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{2}\left(\frac{2}{3}+\frac{1}{3}\right)=\frac{1}{2}$

Concept Applied

- $P(\text{not } A) = 1 - P(A)$

36. It is given that A and B are independent events and $P(\bar{A}\cap B)=\frac{2}{15}$

$\Rightarrow \quad P(\bar{A})P(B)=\frac{2}{15}$ ...(i)

Also, $P(A\cap\bar{B})=\frac{1}{6} \Rightarrow P(A)P(\bar{B})=\frac{1}{6}$ ...(ii)

Let $p = P(A) \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - p$

and $q = P(B) \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - q$

Now, from (i) and (ii), we get

$(1 - p)q = \frac{2}{15}$ ...(iii)

and $p(1 - q) = \frac{1}{6}$ ...(iv)

Subtracting (iii) from (iv), we get

$p - q = \frac{1}{6}-\frac{2}{15}=\frac{1}{30}\Rightarrow p=q+\frac{1}{30}$

Putting this value of p in (iii), we get

$\left(1-q-\frac{1}{30}\right)q=\frac{2}{15} \Rightarrow \frac{29}{30}q-q^2=\frac{2}{15}$

$\Rightarrow \quad 30q^2 - 29q + 4 = 0 \Rightarrow 30q^2 - 24q - 5q + 4 = 0$

$\Rightarrow \quad 6q(5q - 4) - 1(5q - 4) = 0 \Rightarrow (5q - 4)(6q - 1) = 0$

$\Rightarrow \quad q=\frac{4}{5}$ or $\frac{1}{6}$

For $q=\frac{4}{5}$, from (iv), we have

$p\left(1-\frac{4}{5}\right)=\frac{1}{6}\Rightarrow p\left(\frac{1}{5}\right)=\frac{1}{6}\Rightarrow p=\frac{5}{6}$

For $q=\frac{1}{6}$, from (iv), we have

$p\left(1-\frac{1}{6}\right)=\frac{1}{6}\Rightarrow p\left(\frac{5}{6}\right)=\frac{1}{6}\Rightarrow p=\frac{1}{5}$

$\therefore P(A)=\frac{5}{6}, P(B)=\frac{4}{5}$ or $P(A)=\frac{1}{5}, P(B)=\frac{1}{6}$

Commonly Made Mistake

- Remember the difference between exclusive and exhaustive events.

37. Let E_1 be the event that bag I is chosen, E_2 be the event that bag II is chosen and A be the event that red ball is drawn.

Clearly, E_1 and E_2 are mutually exclusive and exhaustive events.

Since, one of the bag is chosen at random

$\therefore \quad P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$ $P(A|E_1)=\frac{1}{4}$ and $P(A|E_2)=\frac{3}{8}$

By using law of total probability, we get
$P(A) = P(E_1)\, P(A \mid E_1) + P(E_2)\, P(A \mid E_2)]$

$$=\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{8}=\frac{1}{8}+\frac{3}{16}=\frac{5}{16}$$

38. Let E_1, E_2 and A denote the events defined as follow :

E_1 = selecting a purse 1

E_2 = selecting a purse 2

A = drawing a silver coin

Since one of two purses is selected randomly

$\therefore\ P(E_1)=\frac{1}{2}$ and $P(E_2)=\frac{1}{2}$

Now, $P(A/E_1)=\frac{3}{9}=\frac{1}{3}$ and $P(A/E_2)=\frac{4}{7}$

Using the total law of probability, we have,

Required probability, $P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot (A/E_2)$

$$\Rightarrow\ P(A)=\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\times\frac{4}{7}\ =\frac{1}{6}+\frac{2}{7}=\frac{19}{42}$$

39. Given, E_1 : represent the event when many workers were not present for the job.

$P(E_1) = 0.65$

E_2 : represent the event when all workers were present.

$P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

E = represent completing the construction work on time.

(i) Required probability $=P(E_2) = 0.35$

(ii) Given, $P(E/E_1)=0.35$ and $P(E/E_2)=0.80$

$P(E)=P(E_1)P(E/E_1)+P(E_2)P(E/E_2)$

($\because$ Law of total probability)

$= 0.65 \times 0.35 + 0.35 \times 0.80 = 0.2275 + 0.28 = 0.5075$

(iii) (a) We have to find $P(E_1|E)$

By using Bayes' theorem,

$$P(E_1|E)=\frac{P(E_1)\cdot P(E|E_1)}{P(E_1)\cdot P(E|E_1)+P(E_2)\cdot P(E|E_2)}$$

$$=\frac{P(E_1)\cdot P(E|E_1)}{P(E)}=\frac{0.65\times0.35}{0.5075}\approx 0.448$$

OR

(b) We have to find $P(E_2|E)$

By using Bayes' theorem

$$P(E_2|E)=\frac{P(E_2)\cdot P(E|E_2)}{P(E_1)\cdot P(E|E_1)+P(E_2)\cdot P(E|E_2)}=\frac{P(E_2)\cdot P\left(\frac{E}{E_2}\right)}{P(E)}$$

$$=\frac{0.35\times0.80}{0.5075}=\frac{0.28}{0.5075}\approx 0.551$$

40. Let E_1, E_2 and A denote the events defined as follows :

E_1 = Selecting box I

E_2 = Selecting box II

A = Getting a red ball

$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}$

$P(A|E_1) = \frac{3}{9}=\frac{1}{3}, P(A|E_2)=\frac{5}{10}=\frac{1}{2}$

Using Bayes' Theorem

$$P(E_2|A)=\frac{P(E_2)\cdot P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{2+3}{12}}=\frac{1}{4}\times\frac{12}{5}=\frac{3}{5}$$

$\therefore$ The probability that a red ball comes out from box II is $\frac{3}{5}$.

41. Let E_1 be the event that ball transferred from bag I to bag II is red, E_2 be the event that ball transferred from bag I to bag II is black and B be the event that ball drawn from bag II is black.

So, $P(E_1)=\frac{3}{8}$, $P(E_2)=\frac{5}{8}$

$P(B|E_1)=\frac{3}{8}$, $P(B|E_2)=\frac{4}{8}$

So, required probability $= P(E_2 \mid B)$

$$=\frac{P(E_2)\times P(B|E_2)}{P(E_1)\times P(B|E_1)+P(E_2)\times P(B|E_2)}=\frac{\frac{5}{8}\times\frac{4}{8}}{\frac{3}{8}\times\frac{3}{8}+\frac{5}{8}\times\frac{4}{8}}=\frac{20}{9+20}=\frac{20}{29}$$

42. Let E_1, E_2 and A denote the events defined as follows :

E_1 = First bag is chosen

E_2 = Second bag is chosen

and A = two balls drawn at random are red

Since, one of the bag is chosen at random

$\therefore\ P(E_1)=P(E_2)=\frac{1}{2}$

If E_1 has already occurred, *i.e.*, first bag is chosen.

Therefore, the probability of drawing two red balls in this case $=P(A|E_1)=\frac{^5C_2}{^9C_2}=\frac{10}{36}$

Similarly, $P(A|E_2)=\frac{^3C_2}{^9C_2}=\frac{3}{36}$

By Bayes' theorem,

Required probability, $P(E_2|A)=\dfrac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$

$$=\frac{\frac{1}{2}\times\frac{3}{36}}{\frac{1}{2}\times\frac{10}{36}+\frac{1}{2}\times\frac{3}{36}}=\frac{\frac{3}{72}}{\frac{10}{72}+\frac{3}{72}}=\frac{\frac{3}{72}}{\frac{13}{72}}=\frac{3}{13}$$

43. Let E_1 be the event of choosing a biased coin and E_2 be the event of choosing an unbiased coin.

$\Rightarrow\quad P(E_1) = P(E_2) = \frac{1}{2}$

Given, probability of biased coin has the chance of showing heads is 60%

$\therefore$ Probability of biased coin has the chance of showing tail is 40%

Let A be the event of showing tail.

$\therefore \quad P(A|E_1)=\frac{40}{100}=\frac{2}{5} \; P(A|E_2)=\frac{1}{2}$

Using Bayes' theorem, we get

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)(A|E_1)+P(E_2)(A|E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1}{5}+\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{9}{20}}=\frac{5}{9}$$

44. Let E_1 be the event of getting ghee from shop X, E_2 be the event of getting ghee from shop Y and A be the event of getting ghee of type B.

$\therefore \quad P(E_1)=\frac{1}{2},\; P(E_2)=\frac{1}{2},\; P(A|E_1)=\frac{40}{70}=\frac{4}{7},$

$P(A|E_2)=\frac{60}{110}=\frac{6}{11}$

Using Bayes' Theorem, we have

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$\Rightarrow \quad P(E_2|A)=\frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}}=\frac{\frac{6}{11}}{\frac{4}{7}+\frac{6}{11}}=\frac{42}{44+42}=\frac{42}{86}=\frac{21}{43}$$

45. Let E_1 be the event that the outcome on the die is 1 or 2, E_2 be the event that the outcome on the die is 3, 4, 5, 6.

$\therefore \; P(E_1)=\frac{2}{6}=\frac{1}{3}$ and $P(E_2)=\frac{4}{6}=\frac{2}{3}$

Let A be the event of getting exactly one tail.

Now, $P(A|E_1)$ be the probability of getting exactly one tail by tossing the coin three times if she gets 1 or 2 $=\frac{3}{8}$ and $P(A|E_2)$ be the probability of getting exactly one tail in a single throw of coin if she gets 3, 4, 5, 6 $=\frac{1}{2}$

The probability that the girl threw 3, 4, 5, 6 with the die, if she obtained exactly one tail is given by $P(E_2|A)$.

$$\therefore \quad P(E_2|A)=\frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{3}{8}+\frac{2}{3}\cdot\frac{1}{2}}=\frac{8}{11}$$

Concept Applied

- If $E_1, E_2, E_3, \ldots, E_n$ are mutually exclusive and exhaustive events associated with a random experiment and A is any event associated with the experiment, then

$$P(E_i|A)=\frac{P(E_i)P(A|E_i)}{\sum_i P(E_i)P(A|E_i)}, \quad \text{where } i=1, 2, 3, \ldots, n$$

46. Let E_1 be the event that '6' occurs, E_2 be the event that '6' does not occur and A be the event that the man reports that it is '6'.

$\therefore \quad P(E_1)=\frac{1}{6},\; P(E_2)=\frac{5}{6}$

Now, $P(A|E_1)$ be the probability that the man reports that there is '6' on the die and '6' actually occurs

= Probability that the man speaks the truth $=\frac{4}{5}$

And $P(A|E_2)$ be the probability that the man reports that there is '6' when actually '6' does not occurs

= Probability that man does not speaks the truth

$=1-\frac{4}{5}=\frac{1}{5}$

$\therefore \quad$ Required probability $=P(E_1|A)$

$$=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}=\frac{4}{4+5}=\frac{4}{9}$$

Yes, we are agree that the value of truthfulness leads to more respect in the society.

47. Let E_1 be event of students which have 100% attendance, E_2 be the event of students which are irregular and A be the event of students which have an A grade.

Then, $P(E_1)=0.3$, $P(E_2)=0.7$, $P(A|E_1)=0.7$ and $P(A|E_2)=0.1$

So, P(Probability that student has 100% attendance given that he has A grade)

$$=P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

[Using Bayes' theorem]

$$=\frac{0.3\times0.7}{0.3\times0.7+0.7\times0.1}$$

$$=\frac{0.3\times0.7}{0.7(0.3+0.1)}=\frac{0.3}{0.4}=\frac{3}{4}=0.75$$

As per answer, the probability of regular students having grade A is more than 50%. So, the regularity is required. No, regularity is required everywhere as it maintains our respect in society.

48. Let I be the event that changes take place to improve profits.

Probability of selection of A, $P(A)=\frac{1}{7}$

Probability of selection of B, $P(B)=\frac{2}{7}$

Probability of selection of C, $P(C)=\frac{4}{7}$

Probability that A does not introduce changes, $P(\bar{I}|A)=1-0.8=0.2$

Probability that B does not introduce changes, $P(\bar{I}|B)=1-0.5=0.5$

Probability that C does not introduce changes, $P(\bar{I}|C)=1-0.3=0.7$

So, required probability $=P(C|\bar{I})$

$$=\frac{P(C)P(\bar{I}|C)}{P(A)P(\bar{I}|A)+P(B)P(\bar{I}|B)+P(C)P(\bar{I}|C)}$$

$$=\frac{\frac{4}{7}\times 0.7}{\frac{1}{7}\times 0.2+\frac{2}{7}\times 0.5+\frac{4}{7}\times 0.7}=0.7$$

49. Consider the following events.

E: Two balls drawn are white

A: There are 2 white balls in the bag

B: There are 3 white balls in the bag

C: There are 4 white balls in the bag

$P(A) = P(B) = P(C) = \frac{1}{3}$

$P(E|A)=\frac{^2C_2}{^4C_2}=\frac{1}{6}$, $P(E|B)=\frac{^3C_2}{^4C_2}=\frac{3}{6}=\frac{1}{2}$

$P(E|C)=\frac{^4C_2}{^4C_2}=1$

$$\therefore \quad P(C|E)=\frac{P(C)\cdot P(E|C)}{P(A)\cdot P(E|A)+P(B)\cdot P(E|B)+ \quad P(C)\cdot P(E|C)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1}=\frac{3}{5}$$

50. Let A be the event that the bulb is defective.

$\therefore \quad P(E_1)=\frac{50}{100}$, $P(E_2)=\frac{25}{100}$, $P(E_3)=\frac{25}{100}$

$P(A|E_1)=\frac{4}{100}$, $P(A|E_2)=\frac{4}{100}$, $P(A|E_3)=\frac{5}{100}$

$\therefore$ Required probability, $P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$

$$=\frac{50}{100}\times\frac{4}{100}+\frac{25}{100}\times\frac{4}{100}+\frac{25}{100}\times\frac{5}{100}$$

$$=\frac{200+100+125}{10000}=\frac{425}{10000}=\frac{17}{400}$$

51. Let E_1, E_2 and A be the events defined as follows :

E_1: The student knows the answer

E_2: The student guesses the answer

A: The student answers correctly

We have, $P(E_1)=\frac{3}{5}, P(E_2)=\frac{2}{5}$

Also, $P(A|E_2)=\frac{1}{3}$ and $P(A|E_1) = 1$

$\therefore$ Required probability

$$=P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{3}{5}\cdot 1}{\frac{3}{5}\cdot 1+\frac{2}{5}\cdot\frac{1}{3}}=\frac{3\times 3}{3\times 3+2}=\frac{9}{11}.$$

52. Let E_1, E_2, E_3 and E be the events defined as follows:

E_1: The item is manufactured by operator A

E_2: The item is manufactured by operator B

E_3: The item is manufactured by operator C

E: The item is defective.

$\therefore$ $P(E_1)=\frac{50}{100}=\frac{5}{10}$, $P(E_2)=\frac{30}{100}=\frac{3}{10}$, $P(E_3)=\frac{20}{100}=\frac{2}{10}$

$P(E|E_1)=\frac{1}{100}$; $P(E|E_2)=\frac{5}{100}$; $P(E|E_3)=\frac{7}{100}$

Now, we have, to find $P(E_1/E)$ (*i.e.*, item is defective and it is produced by operator A)

$$\therefore \quad P(E_1/E)=\frac{P(E_1)P(E|E_1)}{P(E_1)P(E|E_1)+P(E_2)P(E|E_2)+P(E_3)P(E|E_3)}$$

$$=\frac{\frac{5}{10}\times\frac{1}{100}}{\frac{5}{10}\times\frac{1}{100}+\frac{3}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{7}{100}}=\frac{5}{5+15+14}=\frac{5}{34}$$

53. Total number of persons insured

$= 3000 + 6000 + 9000 = 18000$

Let E_1, E_2 and E_3 be the event that the person is a cyclist, a scooter driver and a car driver respectively.

$\therefore$ $P(E_1)=\frac{3000}{18000}=\frac{1}{6}$, $P(E_2)=\frac{6000}{18000}=\frac{1}{3}$

and $P(E_3)=\frac{9000}{18000}=\frac{1}{2}$

Let E be the event that insured person meets with an accident.

$\therefore \quad P(E|E_1) = 0.3, P(E|E_2) = 0.05, P(E|E_3) = 0.02$

By Bayes' theorem,

$\therefore$ Required probability

$$=\frac{P(E|E_1)\cdot P(E_1)}{P(E|E_1)\cdot P(E_1)+P(E|E_2)\cdot P(E_2)+P(E|E_3)P(E_3)}$$

$$=\frac{0.3\times\frac{1}{6}}{0.3\times\frac{1}{6}+0.05\times\frac{1}{3}+0.02\times\frac{1}{2}}=\frac{\frac{0.3}{6}}{\frac{0.3+0.1+0.06}{6}}=\frac{0.3}{0.46}=\frac{15}{23}.$$

54. Let us consider the following events

E_1 = bag I is selected

E_2 = bas II is selected

A = getting a red ball

Here $P(E_1)=P(E_2)=\frac{1}{2}$ $P(A|E_1)=\frac{3}{9}=\frac{1}{3}$ and $P(A|E_2)=\frac{5}{5+n}$

By Baye's theorem, we have

$$P(E_2|A)=\frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$\Rightarrow \frac{3}{5}=\frac{\frac{1}{2}\times\frac{5}{5+n}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{5}{5+n}} \Rightarrow \frac{3}{5}=\frac{\frac{5}{5+n}}{\frac{1}{3}+\frac{5}{5+n}}$$

$$\Rightarrow \frac{3}{5}=\frac{\frac{5}{5+n}}{(5+n+15)/[3(5+n)]}=\frac{5}{5+n}\times\frac{3(n+5)}{n+20}$$

$$\Rightarrow \frac{3}{5}=\frac{15}{n+20}\Rightarrow 3n+60=75 \Rightarrow 3n=15 \Rightarrow n=5$$

Hence, the value of n is 5.

55. Let E_1, E_2, E_3 and C be the events as defined below :

E_1: Two red balls are transferred from bag A to bag B.

E_2: One red ball and one black ball is transferred from bag A to bag B.

E_3: Two black balls are transferred from bag A to bag B.

C: Ball drawn from bag B is red.

So, $P(E_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28}$, $P(E_2) = \frac{^3C_1 \times {}^5C_1}{^8C_2} = \frac{15}{28}$

$P(E_3) = \frac{^5C_2}{^8C_2} = \frac{10}{28}$

Also, $P(C|E_1) = \frac{6}{10}$, $P(C|E_2) = \frac{5}{10}$, $P(C|E_3) = \frac{4}{10}$

$\therefore$ Required probability, $P(E_1|C)$

$$= \frac{P(E_1)P(C|E_1)}{P(E_1)P(C|E_1)+P(E_2)P(C|E_2)+P(E_3)P(C|E_3)}$$

$$= \frac{\frac{3}{28}\times\frac{6}{10}}{\frac{3}{28}\times\frac{6}{10}+\frac{15}{28}\times\frac{5}{10}+\frac{10}{28}\times\frac{4}{10}} = \frac{18}{18+75+40} = \frac{18}{133}$$

56. Let E_1, E_2, E_3 and E be the events as follows:

E_1: The bolt is manufactured by the machine A

E_2: The bolt is manufactured by the machine B

E_3: The bolt is manufactured by the machine C

E: The bolt is defective.

$\therefore$ $P(E_1) = \frac{30}{100} = \frac{3}{10}$; $P(E_2) = \frac{50}{100} = \frac{5}{10}$;

$P(E_3) = \frac{20}{100} = \frac{2}{10}$

$P(E|E_1) = \frac{3}{100}$; $P(E|E_2) = \frac{4}{100}$; $P(E|E_3) = \frac{1}{100}$

Now, $P(E_2|E) = \frac{P(E_2)\cdot P(E|E_2)}{\sum_{i=1}^{3} P(E_i)\cdot P(E|E_i)}$

$$= \frac{\frac{5}{10}\cdot\frac{4}{100}}{\frac{3}{10}\cdot\frac{3}{100}+\frac{5}{10}\cdot\frac{4}{100}+\frac{2}{10}\cdot\frac{1}{100}} = \frac{20}{9+20+2} = \frac{20}{31}$$

$\therefore$ Required probability = The probability that bolt is defective and not manufactured by machine B.

$$= 1 - P(E_2|E) = 1 - \frac{20}{31} = \frac{11}{31}.$$

57. Let E_1 and E_2 denote the events of selection of first bag and second bag respectively. Let A be the event that 2 balls drawn are both red.

$\therefore$ $P(E_1) = \frac{1}{2} = P(E_2)$

Now, $P(A|E_1) = \frac{^4C_2}{^8C_2} = \frac{4\cdot 3}{8\cdot 7} = \frac{3}{14}$ $P(A|E_2) = \frac{^2C_2}{^8C_2} = \frac{1\times 2}{8\cdot 7} = \frac{1}{28}$

The required probability = $P(E_1|A)$

$$= \frac{P(E_1)\cdot P(A|E_1)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)} = \frac{\frac{1}{2}\cdot\frac{3}{14}}{\frac{1}{2}\cdot\frac{3}{14}+\frac{1}{2}\cdot\frac{1}{28}} = \frac{3\times 2}{3\times 2+1} = \frac{6}{7}.$$

58. Let E_1, E_2 and A be the events defined as follows:

E_1: The student knows the answer

E_2: The student guesses the answer

A: The student answers correctly

We have, $P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$

Also, $P(A|E_2) = \frac{1}{3}$ and $P(A|E_1) = 1$

$\therefore$ Required probability

$$= P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{3}{5}\cdot 1}{\frac{3}{5}\cdot 1+\frac{2}{5}\cdot\frac{1}{3}} = \frac{3\times 3}{3\times 3+2} = \frac{9}{11}.$$

59. Let E_1, E_2, E_3, E_4 and A be the events defined as below:

E_1: Missing card is a card of heart.

E_2: Missing card is a card of spade.

E_3: Missing card is a card of club.

E_4: Missing card is a card of diamond.

A: Drawing three spade cards from the remaining cards.

Now, $P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{13}{52} = \frac{1}{4}$

$P(A|E_2) = \frac{^{12}C_3}{^{51}C_3}$

$P(A|E_1) = P(A|E_3) = P(A|E_4) = \frac{^{13}C_3}{^{51}C_3}$

$\therefore$ Required probability = $P(E_2|A)$

$$= \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{4}\times\frac{^{12}C_3}{^{51}C_3}}{\frac{1}{4}\times\frac{^{13}C_3}{^{51}C_3}+\frac{1}{4}\times\frac{^{12}C_3}{^{51}C_3}+\frac{1}{4}\times\frac{^{13}C_3}{^{51}C_3}+\frac{1}{4}\times\frac{^{13}C_3}{^{51}C_3}}$$

$$= \frac{220}{286+220+286+286} = \frac{220}{1078} = \frac{10}{49}$$

60. Let A be the two-headed coin, B be the biased coin showing up heads 75% of the times and C be the biased coin showing up tails 40% (*i.e.*, showing up heads 60%) of the times.

Let E_1, E_2 and E_3 be the events of choosing coins of the type A, B and C respectively. Let S be the event of getting a head. Then

$P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{3}$, $P(E_3) = \frac{1}{3}$

$P(S|E_1) = 1$, $P(S|E_2) = \frac{75}{100} = \frac{3}{4}$,

$P(S|E_3) = \frac{60}{100} = \frac{3}{5}$

$\therefore$ Required probability $= P(E_1|S) = \frac{P(E_1)\cdot P(S|E_1)}{\sum_{i=1}^{3} P(E_i)\cdot P(S|E_i)}$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{3}{5}} = \frac{20}{20+15+12} = \frac{20}{47}$$

61. Let the events are defined as

E_1: Person is a scooter driver

E_2: Person is a car driver

E_3 : Person is a truck driver

A : Person meets with an accident.

Then, $P(E_1)=\frac{2000}{12000}=\frac{1}{6}, P(E_2)=\frac{4000}{12000}=\frac{2}{6},$

$P(E_3)=\frac{6000}{12000}=\frac{3}{6}.$

Also, $P(A|E_1)=0.01=\frac{1}{100}, P(A|E_2)=0.03=\frac{3}{100},$

$P(A|E_3)=0.15=\frac{15}{100}.$

$\therefore$ Required probability = 1 – P(the person who meets with accident is a truck driver)

i.e., Required probability = $1 - P(E_3|A)$

$$=1-\frac{P(A|E_3)P(E_3)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)+P(A|E_3)P(E_3)}$$

$$=1-\frac{\frac{15}{100}\times\frac{3}{6}}{\frac{1}{100}\times\frac{1}{6}+\frac{3}{100}\times\frac{2}{6}+\frac{15}{100}\times\frac{3}{6}}=1-\frac{45}{1+6+45}$$

$$=1-\frac{45}{52}=\frac{7}{52}.$$

62. Let E_1 be the event that '1' occurs, E_2 be the event that '1' does not occur and A be the event that the man reports that it is '1'.

$\therefore \quad P(E_1)=\frac{1}{6},\ P(E_2)=\frac{5}{6}$

Now, $P\left(\frac{A}{E_1}\right)$ be the probability that the man reports that there is '1' on the die given that '1' actually occurs.

So, $P\left(\frac{A}{E_1}\right)$ = Probability that the man speaks the truth

$=\frac{3}{5}$

And $P\left(\frac{A}{E_2}\right)$ be the probability that the man reports that there is '1' when actually '1' does not occur.

So, $P\left(\frac{A}{E_2}\right)$ = Probability that man does not speak the truth

$=1-\frac{3}{5}=\frac{2}{5}.$

$\therefore$ Required probability = $P\left(\frac{E_1}{A}\right)$

$$=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}=\frac{\frac{1}{6}\times\frac{3}{5}}{\frac{1}{6}\times\frac{3}{5}+\frac{5}{6}\times\frac{2}{5}}=\frac{3}{13}.$$

63. Consider the following events.

E : Two balls drawn are white

A : There are 2 white balls in the urn

B : There are 3 white balls in the urn

C : There are 4 white balls in the urn

$P(A)=P(B)=P(C)=\frac{1}{3}$

$P(E|A)=\frac{^2C_2}{^4C_2}=\frac{1}{6},\ P(E|B)=\frac{^3C_2}{^4C_2}=\frac{3}{6}=\frac{1}{2}$

$P(E|C)=\frac{^4C_2}{^4C_2}=1$

$$\therefore \quad P(C|E)=\frac{P(C)\cdot P(E|C)}{P(A)\cdot P(E|A)+P(B)\cdot P(E|B)+P(C)\cdot P(E|C)}$$

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times 1}=\frac{3}{5}$$

64. $2P(X=x_1)=3P(X=x_2)=P(X=x_3)=5P(X=x_4)$

Let $P(X=x_3)=k$.

So $P(X=x_1)=\frac{k}{2}$; $P(X=x_2)=\frac{k}{3}$; $P(X=x_4)=\frac{k}{5}$

We know that sum of all probabilities in probability distribution is 1.

So, $P(X=x_1)+P(X=x_2)+P(X=x_3)+P(X=x_4)=1$

$\Rightarrow \quad \frac{k}{2}+\frac{k}{3}+k+\frac{k}{5}=1$

$\Rightarrow \quad \frac{15k+10k+30k+6k}{30}=1 \Rightarrow 61k=30$

$\Rightarrow \quad k=\frac{30}{61}$

So, probability distribution of X :

$P(X=x_1)=\frac{30}{61\times 2}=\frac{15}{61}$; $P(X=x_2)=\frac{30}{61\times 3}=\frac{10}{61}$

$P(X=x_3)=\frac{30}{61}$; $P(X=x_4)=\frac{30}{61\times 5}=\frac{6}{61}$

65. (a) : We know $\Sigma p(x_i)=1$

$\Rightarrow \quad K+6K+9K=1 \Rightarrow 16K=1 \Rightarrow K=\frac{1}{16}$

(b) $\quad$ P(getting 2 heads) $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ (if the coin was unbiased)

But from given p.d. table, P(getting 2 heads) $=\frac{1}{16}\neq\frac{1}{4}$

$\therefore \quad$ Coin tossed is biased

66. Given, the number of red balls in a bag is = 2

The number of blue balls in a bag is = 3

So, total number of balls in a bag is = 2 + 3 = 5

Since, two balls are drawn at random without replacement and X denotes the number of red balls. So X can be 0, 1 and 2.

Case I : When no red ball is drawn, $X = 0$

$P(X=0)=P(BB)=P(B)\cdot P(B)$

$=\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}=\frac{3}{10}$

Case II : When one red ball is drawn, $X = 1$

$P(X=1)=P(RB)+P(BR)=P(R)P(B)+P(B)P(R)$

$=\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}+\frac{6}{20}=\frac{12}{20}=\frac{3}{5}$

Case III : When two red ball are drawn, $X = 2$

$P(X = 2) = P(RR) = P(R) \cdot P(R)$

$= \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

Hence, the required probability distribution is given by

X	0	1	2
$P(X)$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{10}$

67. We have, $P(X=x)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$

Since, $\Sigma P(x_i) = 1 \Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

68. The probability distribution of x is

$X = x$	0	1	2	3
$P(X = x)$	0.1	k	$2k$	$3k$

(a) $\because \sum P(X) = 1 \Rightarrow 0.1 + k + 2k + 3k = 1$

$\Rightarrow 6k = 1 - 0.1 \Rightarrow 6k = 0.9 \Rightarrow k = \frac{0.9}{6} = 0.15$

(b) $P(x \le 2) = P(0) + P(1) + P(2) = 0.1 + 0.15 + 0.3 = 0.55$

(c) Mean, $\bar{X} = \sum X \cdot P(X)$

$= 0.15 \times 1 + 2 \times 0.3 + 3 \times 0.45 = 2.1$

69. We have, number of rotten apples = 3 and number of good apples = 7

$\therefore$ Total number of apples = 10

Let X be number of rotten apples.

So, X can take values 0, 1, 2, 3

Let E be the event of getting a rotten apple.

$P(E) = \frac{3}{10}, P(\bar{E}) = \frac{7}{10}$

Now, $P(X = 0) = {}^3C_0 \cdot \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10} = \frac{343}{1000}$

$P(X = 1) = {}^3C_1 \cdot \frac{3}{10} \cdot \frac{7}{10} \cdot \frac{7}{10} = \frac{441}{1000}$

$P(X = 2) = {}^3C_2 \cdot \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{7}{10} = \frac{189}{1000}$

$P(X = 3) = {}^3C_3 \cdot \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10} = \frac{27}{1000}$

So, probability distribution table is given by

X	0	1	2	3
$P(X)$	$\frac{343}{1000}$	$\frac{441}{1000}$	$\frac{189}{1000}$	$\frac{27}{1000}$

Now, mean $(\bar{X}) = \sum X \cdot P(X)$

$= 0 \times \frac{343}{1000} + 1 \times \frac{441}{1000} + 2 \times \frac{189}{1000} + 3 \times \frac{27}{1000}$

$= \frac{441}{1000} + \frac{378}{1000} + \frac{81}{1000} = \frac{900}{1000} = \frac{9}{10}$

70. We have, $P(X = 0) = P(X = 1) = p$

Let $P(X = 2) = P(X = 3) = k$

Since, X is a random variable taking values 0, 1, 2, 3

$\therefore P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$

$\Rightarrow p + p + k + k = 1 \Rightarrow 2p + 2k = 1 \Rightarrow p + k = \frac{1}{2}$ ...(i)

Now, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

$\Rightarrow p(0) + p(1) + k(4) + k(9) = 2[p(0) + p(1) + k(2) + k(3)]$

$\Rightarrow p + 13k = 2p + 10k$

$\Rightarrow p - 3k = 0$ ...(ii)

Subtracting (ii) from (i), we get $4k = \frac{1}{2} \Rightarrow k = \frac{1}{8}$

$\therefore$ From (i), we get $p = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

71. Let X be the amount he wins/loses.

Then, X can take values $-3, 3, 4, 5$.

$P(X = 5) = P$(Getting a number greater than 4 in the first throw) $= \frac{2}{6} = \frac{1}{3}$

$P(X = 4) = P$(Getting a number less than or equal to 4 in the first throw and a number greater than 4 in the second throw) $= \frac{4}{6} \times \frac{2}{6} = \frac{2}{9}$

$P(X = 3) = P$(Getting a number less than or equal to 4 in the first two throws and a number greater than 4 in the third throw) $= \frac{4}{6} \times \frac{4}{6} \times \frac{2}{6} = \frac{4}{27}$

$P(X = -3) = P$(Getting a number less than or equal to 4 in all three throws) $= \frac{4}{6} \times \frac{4}{6} \times \frac{4}{6} = \frac{8}{27}$

$\therefore$ The probability distribution is

X	5	4	3	-3
$P(X)$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{4}{27}$	$\frac{8}{27}$

$\therefore E(X) = \sum XP(X) = 5\left(\frac{1}{3}\right) + 4\left(\frac{2}{9}\right) + 3\left(\frac{4}{27}\right) - 3\left(\frac{8}{27}\right)$

$= \frac{57}{27} = \frac{19}{9}$

Hence, expected value of the amount he wins/loses is $\frac{19}{9}$.

72. The probability distribution of X is

X	0	1	2	3	4
$P(X)$	0	k	$4k$	$2k$	k

The given distribution is a probability distribution.

$\therefore \sum_{i=0}^{4} p_i = 1$

$\Rightarrow 0 + k + 4k + 2k + k = 1 \Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8} = 0.125$

(i) P (getting admission in exactly one college)

$= P(X = 1) = k = 0.125$

(ii) P (getting admission in atmost 2 colleges)
$= P(X \le 2) = 0 + k + 4k = 5k = 0.625$

(iii) P (getting admission in atleast 2 colleges)
$= P(X \ge 2) = 4k + 2k + k = 7k = 0.875$

73. Let X denote the number of spade cards in a sample of 3 cards drawn from a well-shuffled pack of 52 cards.
Since there are 13 spade cards in the pack, so in a sample of 3 cards drawn, either there is no spade card or one spade card or two spade cards or 3 spade cards. Thus $X = 0, 1, 2$ and 3.
Now, $P(X = 0)$ = Probability of getting no spade card

$= \frac{39}{52} \cdot \frac{39}{52} \cdot \frac{39}{52} = \frac{27}{64}$

$P(X = 1)$ = Probability of getting one spade card

$= \frac{13}{52} \cdot \frac{39}{52} \cdot \frac{39}{52} + \frac{39}{52} \cdot \frac{13}{52} \cdot \frac{39}{52} + \frac{39}{52} \cdot \frac{39}{52} \cdot \frac{13}{52} = \frac{27}{64}$

$P(X = 2)$ = Probability of getting 2 spade cards

$= \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{39}{52} + \frac{13}{52} \cdot \frac{39}{52} \cdot \frac{13}{52} + \frac{39}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{9}{64}$

$P(X = 3)$ = Probability of getting 3 spade cards

$= \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{64}$

Hence, the probability distribution of X is

X	0	1	2	3
$P(X)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Now, mean of this distribution is given by

$\bar{X} = 0 \times \frac{27}{64} + 1 \times \frac{27}{64} + 2 \times \frac{9}{64} + 3 \times \frac{1}{64} = \frac{48}{64} = \frac{3}{4}$

74. Let X denote the number of defective bulbs in a sample of 2 bulbs which are to be drawn.
Here, number of defective bulbs = 5
Number of non-defective bulbs = 15 − 5 = 10
$\therefore$ X can take values 0, 1, 2.
Now, $P(X = 0)$ = Probability of getting no defective bulb
= Probability of getting 2 non-defective bulbs.

$= \frac{{}^{10}C_2}{{}^{15}C_2} = \frac{10 \times 9}{15 \times 14} = \frac{3}{7} = \frac{9}{21}$

$P(X = 1)$ = Probability of getting 1 defective bulb

$= \frac{{}^{5}C_1 \times {}^{10}C_1}{{}^{15}C_2} = \frac{5 \times 10 \times 2}{15 \times 14} = \frac{10}{21}$

$P(X = 2)$ = Probability of getting 2 defective bulbs

$= \frac{{}^{5}C_2}{{}^{15}C_2} = \frac{5 \times 4}{15 \times 14} = \frac{2}{21}$

Thus the probability distribution of X is given by

X	0	1	2
$P(X)$	$\frac{9}{21}$	$\frac{10}{21}$	$\frac{2}{21}$

Concept Applied

- A combination determines the number of possible selection in a collection of items where the order of the selection does not matter.

75. Let X denote the number of red cards. So X can take values 0, 1, 2, 3.
Total number of cards = 52
Number of red cards = 26.

Now, $P(X = 0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26 \times 25 \times 24}{52 \times 51 \times 50} = \frac{4}{34}$

$P(X = 1) = \frac{{}^{26}C_1 \times {}^{26}C_2}{{}^{52}C_3} = \frac{26 \times 26 \times 25 \times 6}{2 \times 52 \times 51 \times 50} = \frac{13}{34}$

$P(X = 2) = \frac{{}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3} = \frac{26 \times 25 \times 26 \times 6}{2 \times 52 \times 51 \times 50} = \frac{13}{34}$

$P(X = 3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{26 \times 25 \times 24}{52 \times 51 \times 50} = \frac{4}{34}$

$\therefore$ Probability distribution of X is given by

X	0	1	2	3
$P(X)$	$\frac{4}{34}$	$\frac{13}{34}$	$\frac{13}{34}$	$\frac{4}{34}$

Mean $(\bar{X}) = \sum XP(X)$

$= 0\left(\frac{4}{34}\right) + 1\left(\frac{13}{34}\right) + 2\left(\frac{13}{34}\right) + 3\left(\frac{4}{34}\right) = \frac{3}{2}$

76. Here the ages of the given 15 students are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years.
$\therefore$ The required probability distribution of X is given by

X	14	15	16	17	18	19	20	21
$P(X)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean, $\bar{X} = \sum XP(X)$

$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$

$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21) = \frac{263}{15}$

77. The probability of drawing a ticket out of $10 = \frac{1}{10}$
The probability of drawing a ticket with prize of ₹ 8 is $2 \times \frac{1}{10}$.
The probability of drawing a ticket with prize of ₹ 4 is $5 \times \frac{1}{10}$.
The probability of drawing a ticket with prize of ₹ 2 is $3 \times \frac{1}{10}$.

We can show this on a table as :

Number of tickets	2	5	3
X	8	4	2
$P(X)$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{3}{10}$

$\because$ Mean = $\sum X_i P(X_i)$

Hence the mean prize

$8\times\frac{2}{5}+4\times\frac{5}{10}+2\times\frac{3}{10}=\frac{8}{5}+2+\frac{3}{5}=₹\frac{21}{5}₹\,4.20$

78. The first six positive integers are 1, 2, 3, 4, 5 and 6. Let X be the larger number of two numbers selected the possible outcomes are :

Sample space S is given by

S = {(1, 2) (1, 3), (1, 4) (1, 5) (1, 6), (2, 1), (2, 3) .... (2, 6), (3, 1), (3, 2), (3, 4) .... (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2) .... (5, 4), (5, 6), (6, 1), (6, 2) .... (6, 5)}

$\therefore$ X can take values 2, 3, 4, 5, or 6.

Total number of ways = ${}^6C_2$ = 15

The probability distribution of a random variable X is given by

X	2	3	4	5	6
$P(X)$	1/15	2/15	3/15	4/15	5/15

$\therefore$ Mean = $\sum XP(X)$

$= 2\times\frac{1}{15}+3\times\frac{2}{15}+4\times\frac{3}{15}+5\times\frac{4}{15}+6\times\frac{5}{15}$

$= \frac{2}{15}+\frac{6}{15}+\frac{12}{15}+\frac{20}{15}+\frac{30}{15}=\frac{70}{15}=\frac{14}{3}.$

79. When a die is thrown, probability of getting a six = $\frac{1}{6}$

$\therefore$ Probability of not getting a six = $1-\frac{1}{6}=\frac{5}{6}.$

If he gets a six in first throw, then, probability of getting a six = $\frac{1}{6}$.

If he does not get a six in first throw, but he gets a six in the second throw, then

Probability = $\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}.$

Probability that he does not get a six in first two throws and he gets a six in third throw $=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$

Probability that he does not get a six in any of the three throws $=\left(\frac{5}{6}\right)^3=\frac{125}{216}.$

In first throw he gets a six, will receive ₹ 5.

If he gets a six in second throw, he will receive ₹ (5 – 1) = 4

If he gets a six in third throw, he will receive ₹ (–1 – 1 + 5) = ₹ 3

If he does not get a six in all three throws, he will receive ₹ (–1 – 1 – 1) = ₹ –3.

Let X be the amount he wins/losses.

Then, X can take values –3, 3, 4, 5

$\therefore$ The probability distribution is

X	5	4	3	–3
$P(X)$	1/6	5/36	25/216	25/216

$\therefore$ Expected value $=\frac{1}{6}\times5+\left(\frac{5}{36}\right)\times4+\left(\frac{25}{216}\right)\times3+\left(\frac{125}{216}\right)\times(-3)$

$=\frac{5}{6}+\frac{20}{36}+\frac{75}{216}-\frac{375}{216}=0$

He neither loses or wins.

CBSE Sample Questions

1. P(not obtaining an odd person in a single round) = P(All three of them throw tails or All three of them throw heads)

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ (1½)

P(obtaining an odd person in a single round)

= 1 – P(not obtaining an odd person in a single round) = 3/4

Required probability = P(In first round there is no odd person' and 'In second round there is no odd person' and 'In third round there is an odd person)

$=\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{64}$ (1½)

2. $P(\bar{E}|\bar{F})=\frac{P(\bar{E}\cap\bar{F})}{P(\bar{F})}=\frac{P(\overline{E\cup F})}{P(\bar{F})}=\frac{1-P(E\cup F)}{1-P(F)}$ ... (i)

Now, $P(E\cup F) = P(E) + P(F) - P(E\cap F) = 0.8 + 0.7 - 0.6 = 0.9$ (1)

Substituting value of $P(E\cup F)$ in (i), we get (1/2)

$P(\bar{E}|\bar{F})=\frac{1-0.9}{1-0.7}=\frac{0.1}{0.3}=\frac{1}{3}$ (1/2)

3. Given, $P(A)=\frac{1}{3}$ then $P(A)=1-\frac{1}{3}=213$ and

$P(B)=\frac{1}{4}$ then $P(B)=1-\frac{1}{4}=\frac{3}{4}$

Required probability

= 1 – P(problem is not solved)

$=1-P(\bar{A})\cdot P(\bar{B})=1-\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$ (1)

4. Let A be the event of commiting an error and E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form.

(i) (b) : Required probability = $P(A|E_2)$

$=\frac{P(A\cap E_2)}{P(E_2)}=\frac{\left(0.04\times\frac{20}{100}\right)}{\left(\frac{20}{100}\right)}=0.04$ (1)

(ii) (c) : Required probability = $P(A\cap E_2)$

$=0.04\times\frac{20}{100}=0.008$ (1)

(iii) (b) : Total probability is given by

$$P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)$$

$$= \frac{50}{100}\times 0.06 + \frac{20}{100}\times 0.04 + \frac{30}{100}\times 0.03 = 0.047 \quad (1)$$

(iv) (d) : Using Bayes' theorem, we have

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$= \frac{0.5\times 0.06}{0.5\times 0.06 + 0.2\times 0.04 + 0.3\times 0.03} = \frac{30}{47}$$

$\therefore$ Required probability = $P(\bar{E}_1|A)$

$$= 1 - P(E_1|A) = 1 - \frac{30}{47} = \frac{17}{47} \quad (1)$$

(v) (d) : $\sum_{i=1}^{3} P(E_i|A) = P(E_1|A) + P(E_2|A) + P(E_3|A)$

$= 1$

[$\because$ Sum of posterior probabilities is 1] (1)

5. (i) Let P be the event that the shell fired from A hits the plane. Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible the trial, with the guns operating independently:

$$E_1 = PQ, E_2 = \bar{P}\bar{Q}, E_3 = \bar{P}Q, E_4 = P\bar{Q}$$

Let E = The shell fired from exactly one of them hits the plane. (1/2)

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$,

$P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$ (1/2)

$$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1 \quad (1/2)$$

$$P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right) + P(E_4)\cdot P\left(\frac{E}{E_4}\right) \quad (1/2)$$

$= 0.14 + 0.24 = 0.38$

(ii) By Bayes' Theorem, $P\left(\frac{E_3}{E}\right)$ (1/2)

$$= \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right) + P(E_4)\cdot P\left(\frac{E}{E_4}\right)} \quad (1)$$

$$= \frac{0.14}{0.38} = \frac{7}{19} \quad (1/2)$$

6. Let E_1 = The policyholder is accident prone.

E_2 = The policyholder is not accident prone.

E = The new policyholder has an accident withing a year of purchasing a policy.

(i) $P(E) = P(E_1) \times P(E|E_1) + P(E_2) \times P(E|E_2)$

$$= \frac{20}{100}\times\frac{6}{10} + \frac{80}{100}\times\frac{2}{10} = \frac{7}{25} \quad (2)$$

(ii) Using Bayes' Theorem, we have $P(E_1|E)$

$$= \frac{P(E_1)\times P(E|E_1)}{P(E)} = \frac{\frac{20}{100}\times\frac{6}{10}}{\frac{7}{25}} = \frac{3}{7} \quad (2)$$

7. Let X be the random variable defined as the number of red balls.

Then, $X = 0, 1$ (1/2)

$$P(X=0) = \frac{3}{4}\times\frac{2}{3} = \frac{6}{12} = \frac{1}{2} \quad (1/2)$$

$$P(X=1) = \frac{1}{4}\times\frac{3}{3} + \frac{3}{4}\times\frac{1}{3} = \frac{6}{12} = \frac{1}{2} \quad (1/2)$$

Probability Distribution Table :

X	0	1
$P(X)$	$\frac{1}{2}$	$\frac{1}{2}$

(1/2)

8. Let X denotes the number of milk chocolates drawn. Then probability distribution table is

X	$P(X)$
0	$\frac{4}{6}\times\frac{3}{5} = \frac{12}{30}$
1	$\left(\frac{2}{6}\times\frac{4}{5}\right) + \left(\frac{4}{6}\times\frac{2}{5}\right) = \frac{16}{30}$
2	$\frac{2}{6}\times\frac{1}{5} = \frac{2}{30}$

(1½)

Most likely outcome is getting one chocolate of each type. (1/2)

9. Suppose X denotes the Random Variable defined by the number of defective items.

$$P(X=0) = \frac{4}{6}\times\frac{3}{5} = \frac{2}{5} \quad (1/2)$$

$$P(X=1) = \left(\frac{2}{6}\times\frac{4}{5}\right) + \left(\frac{4}{6}\times\frac{2}{5}\right) = \frac{8}{15} \quad (1/2)$$

$$P(X=2) = \frac{2}{6}\times\frac{1}{5} = \frac{1}{15} \quad (1/2)$$

x_i	0	1	2
p_i	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$
$p_i x_i$	0	$\frac{8}{15}$	$\frac{2}{15}$

$$\therefore \text{Mean} = \sum p_i x_i = \frac{10}{15} = \frac{2}{3} \quad (1½)$$